Supply Disruption with a Risk-Averse Buyer

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We consider a supply chain with two unreliable suppliers competing to supply one risk-averse buyer. We model the interaction among the supply chain participants as a Nash game among the suppliers, and Stackelberg games between the suppliers and the buyer. By introducing risk aversion, the buyer implements a spectrum of distinctive diversification and order inflation strategies, instead of the extreme ordering strategies of single sourcing, duplicate sourcing and various boundary ordering strategies in the risk-neutral case. In the case of exogenous wholesale price, we fully characterize the buyer’s optimal order quantities. We find that the more reliable supplier can possibly increase his market share by increasing his wholesale price. In the case of endogenous wholesale prices, we find that cases of non-existence, uniqueness and multiplicity of equilibria can all possibly occur depending on the level of risk aversion, and we characterize the equilibrium in some cases. We also find that as the buyer becomes more risk averse he is less sensitive to the wholesale prices in terms of order quantities, giving more power to the suppliers to exploit the buyer’s risk.

Key words: supply disruptions; supply diversification; competition; equilibrium pricing; risk aversion

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1. Introduction

Much has been written about multi-supplier sourcing and supplier diversification. A benefit of implementing multi-supplier sourcing is that it encourages competition among suppliers, which results in lower procurement costs for the buyer. It also enables the buyer to mitigate his risk by diversifying his supply when facing supply disruptions. Supply chain managers have been increasingly concerned about supply disruptions due to natural disasters such as earthquakes and hurricanes, man-made breakdowns such as strikes and terrorist attacks or financial defaults. Some recent examples include (Lohr (2011):
In February 1997, Toyota was forced to shut down its assembly plants because of a fire at Aisin, which provided 90% of all brake components and practically all brake valves for Toyota (Nishiguchi and Beaudet 1998). Thereafter, Toyota sought multiple parallel suppliers for each part (Treece 1997).

In April 2010, a volcano in Iceland erupted shutting down production plants all over the world that rely on key parts coming from Europe. Nissan stopped production of three auto models in Japan, and in Germany BMW also cut production.¹

In March 2011, a major earthquake and tsunami hit Japan (Fisher 2011), shutting down plants that supply much of the world’s silicon wafers, auto parts, flash memory, and other components.

Our focus in this paper is on such catastrophic supply chain disruptions, which persist long enough for complete inventory orders to be missed. There is empirical evidence (see Schweitzer and Cachon (2000)) that inventory decision makers exhibit risk-averse behaviour. Nevertheless, a recent McKinsey global survey revealed that two-thirds of executives interviewed are facing growing risks from disruptions to their supply chains - yet many are unprepared to manage those risks (McKinsey 2006).

When there is risk of supply disruptions, procurement managers may diversify their supplier base, and also inflate their order quantities to ensure their needs are covered. Traditionally, in the situation of Bernoulli all-or-nothing supply disruption, the uncertainties of demand and supply are combined to drive the diversification and order inflation behavior (e.g., Anupindi and Akella 1993). When supplies are certain while the demand is uncertain, only the supplier with the lowest wholesale price will be included in the buyer’s supplier base. Oftentimes, the procurement managers treat their demand as known by tracking the MRP (material requirement planning) system, which means they know their needs exactly. The situation of deterministic demand and possible supply disruptions is investigated in Babich et al. (2007). In a model with two suppliers they show that a risk-neutral profit-maximizing buyer’s optimal ordering strategies are single sourcing (order full

demand from one supplier), duplicate sourcing (order full demand from each of the two suppliers) or boundary ordering strategies (the buyer is indifferent between two suppliers and splits the order arbitrarily between them). While these strategies exhibit diversification and order inflation behavior, their model does not explain what drives non-boundary behavior, in which buyers may prefer order quantities in the continuum between these extremes.

There are many potential reasons why firms diversify their supplier base and inflate their orders. For example, procurement managers fear the dependency on a single supplier associated with several kinds of risk like supply disruptions due to natural disasters, man-made breakdowns or financial defaults, and increasing prices in global sourcing due to exchange rate volatility; they want to lower the procurement costs by enhancing supplier competition in procurement auctions and competitive bidding; they inflate their orders in anticipation of unreliable supply and potential delay; they want to build up inventory in anticipation of possible future capacity shortage; the bullwhip effect of the exaggerated order swings caused by the information distortion; order rationing in times of shortages leads them to inflate their orders in order to gain a better share of the items in short supply (shortage game).

In this paper we analyze a behavioral driver for these phenomena: the procurement managers behave in a risk-averse way. For example, a procurement manager facing a certain procurement or revenue target would aim to reduce the variance of the procurement quantity to make himself strictly better off. We show that even when demand is deterministic, a risk-averse manager implements a spectrum of distinctive diversification and order inflation strategies under the possibility of supply disruptions. The contributions of our paper are:

- To provide a behavioral explanation, namely risk aversion, for a continuum of diversification and order inflation phenomena.

- To characterize how these behaviors change as functions of risk aversion level and probability of supply disruptions, as well as how endogenously determined prices change with risk aversion.
We consider a decentralized supply chain with two risk-neutral unreliable suppliers competing to supply one risk-averse buyer. The products produced by the two suppliers are perfectly substitutable. The buyer uses these intermediate products to produce final products and sell to the downstream customers. In this supply chain, competition exists between two suppliers who compete for the business with the buyer. Diversification and order inflation exist as the buyer uses both suppliers and orders more than what he needs in order to hedge against the supply disruption risk. The supply chain participants play a noncooperative game with a Nash game among the suppliers and Stackelberg games between the suppliers and the buyer. The suppliers start out setting their wholesale prices and the buyer follows by determining how much he needs to order from each supplier. We characterize the equilibrium behavior of the supply chain participants and analyze the effects of supply disruptions and risk aversion on suppliers’ pricing strategies and buyer’s ordering strategy.

Throughout the paper, we make three basic modeling assumptions to give a parsimonious model, rather than the most general and complete, that isolates the impact of catastrophic risks on order inflation and supplier diversification behaviours. First, we employ a Bernoulli model of supply disruption, in which with some probability a supplier will not deliver any quantity, and otherwise delivers everything ordered. More detailed models are possible, however our motivation comes from catastrophic events such as described above. A model that also allows for random fractional supply, due to minor disruptions or technological yield issues, would be more general in industrial settings to which they apply, but would also blur the impact of catastrophic risks, relative to fractional yield risks, on ordering behaviour.

Secondly, distinguished from the random yield and supply disruption literature, this paper uncouples the convoluted uncertainties of demand and supply which are typically combined to impact the diversification behavior of the buyer. Doing so sheds light on the factors that drive this behavior when the buyer’s demand is deterministic under the possibility of supply disruptions. Oftentimes, in practice, procurement managers order against production plans handed to them with pre-determined parts quantities. A more general model would allow for demand uncertainty
as well, but then optimal order quantities would be affected by more than just catastrophic risks whose effect we wish to isolate.

Finally, we model risk aversion explicitly by imposing a concave utility function, which lets us analyze the dynamics of the supply chain participants’ behaviour parametrically on the risk aversion level. In particular, we employ the negative exponential utility function (Mas-Collel et al. 1995), which exhibits constant absolute risk aversion. This function has been, and continues to be, used widely throughout the literature and practice in economics, finance, decision analysis, and supply chains (see (Chen et al. 2007), (Choi and Ruszczyński 2011), and (Giri 2011)). Kirkwood (2004) demonstrates that it can provide a reasonable approximation to risk aversion exhibited in more general utility functions. Because it also simplifies analysis, it provides a useful model for investigating risk aversion in our setting. Nevertheless, other alternatives exist, for example see Choi et al. (2011).

When the buyer faces exogenously fixed wholesale prices, we fully characterize the conditions for the buyer’s optimal order quantities to be in different order regions for both a risk-neutral model (Babich et al. 2007) and a risk-averse model. We find that a risk-averse buyer implements a spectrum of distinctive diversification and order inflation strategy. This is in contrast with a risk-neutral buyer. Contradicting the results of the classical risk-averse newsvendor case with deterministic supply and stochastic demand, namely order deflation compared with a risk-neutral newsvendor driven by a wealth effect, we find that an increase in risk aversion can lead to higher total order quantity. This can be explained by the fact that a risk-averse buyer who is concerned about the tradeoff between the expected profit and profit variance might potentially reduce profit variability by diversifying his portfolio and ordering more from the reliable supplier and in total. Reversing the traditional intuition, our results show that a more reliable supplier can possibly increase his market share by increasing his wholesale price. The intuition is that by increasing his wholesale price the more reliable supplier makes the buyer’s procurement more expensive. The buyer would rather reduce the order quantity to the less reliable supplier than to him, resulting in an increase in the more reliable supplier’s market share, if the reliability benefits outweigh the cost benefits. The
economic implication is that a more expensive but more reliable supplier does not need to compete as fiercely on price with a cheaper but less reliable supplier because he has comparative advantage in supply reliability. This implies that investing in reliability rather than cost reduction may yield higher returns to a supplier. This is in contrast with the result of Dada et al. (2007). They show that although reliability affects how much is ordered from a selected supplier, cost generally takes precedence over reliability when it comes to selecting suppliers in the first place.

In the case when the buyer faces endogenous wholesale prices we analyze both suppliers’ and the buyer’s behavior in equilibrium. We show that there is a unique equilibrium solution to the game between suppliers and the risk-neutral buyer. For the setting with a risk-averse buyer, we characterize the pure-strategy equilibrium for some cases. We also prove that a pure-strategy equilibrium can only exist in certain regions, which reduces search effort.

We undertake a comprehensive numerical study to further investigate how risk aversion influences the equilibrium wholesale prices and order quantities. Cases of non-existence, uniqueness and multiplicity of equilibria can all possibly occur depending on the level of risk aversion. When the buyer has low risk aversion, he is very sensitive to the wholesale prices in terms of order quantities. Therefore the equilibrium wholesale prices are relatively low, induced by the fierce price competition between two suppliers. As the buyer becomes more risk averse he is less sensitive to the wholesale prices in terms of order quantities and the benefits of diversification outweigh the benefits of competition. Consequently, with increasing risk aversion, the equilibrium wholesale prices are increasing, while the buyer’s total order quantity and expected profit are decreasing. The suppliers exploit the buyer’s risk by setting higher wholesale prices without the fear of losing too much business, since a highly risk-averse buyer is less sensitive to the wholesale prices and has strong incentives for diversification. The more risk averse the buyer is, the more power the suppliers have to exploit the buyer’s risk.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. Section 3 introduces the supply chain models: Model I (benchmark risk-neutral model) and Model II (risk-averse model). Section 4 analyzes the buyer’s behavior when the wholesale prices are
exogenously fixed, and Section 5 analyzes the equilibrium results when the suppliers’ wholesale
prices are endogenously determined. The numerical results on the equilibrium analysis are reported.
Finally, Section 6 provides some concluding remarks.

2. Literature Review

Typically the literature on supply uncertainty assumes that supply is either subject to yield uncer-
tainty or supply disruptions. Yield uncertainty occurs when the quantity of supply produced or
delivered is a random variable. Supply disruptions are binary events and occur when supply is
subject to partial or complete failure. Compared with the effects of yield uncertainty, the effects
of supply disruptions are less frequent but more severe when they occur. The yield uncertainty
can be modeled in various ways based on certain classes of real systems. We refer the interested
More recent work includes Giri (2011), who considers ordering from a second supplier as a recourse
action after a first supplier’s yield is revealed. Supply disruptions can be modeled as events which
occur randomly and may have random time length. See Kleindorfer and Saad (2005) for a review
of the conceptual and empirical work in the supply disruption management.

The Bernoulli yield model is often used to model supply failures in the literature on random
yield management (e.g., Anupindi and Akella 1993, Tomlin and Wang 2005, Swaminathan and
Shanthikumar 1999) and supply disruption management (e.g., Parlar et al. 1995, Babich et al.
2007), in which an order placed may either be realized in full or completely fail. Bernoulli supply
processes arise due to supply disruptions, batch failures or acceptance sampling, as well as supplier
delays resulting in untimely deliveries. In this paper we also use Bernoulli yield to model the
suppliers’ delivery rates.

The majority of the literature on supply uncertainty is focused on single-supplier models. With
only one supplier these papers mainly focus on developing the optimal inventory policy. Here we
restrict our attention to multiple-supplier models. Minner (2005) reviews inventory models with
multiple supply options and discusses their contribution to supply chain management. Multisourc-
ing provides a risk-mitigation tool due to supplier diversification. Dual sourcing, as a special case
of multisourcing, has been investigated in various papers (e.g., Anupindi and Akella 1993, Swaminathan and Shanthikumar 1999, Tomlin 2006, Babich et al. 2007). Models with an arbitrary set of potential suppliers are studied by Ilan and Yadin (1985), Agrawal and Nahmias (1997), Dada et al. (2007) and Federgruen and Yang (2008, 2009a). The central problem is generally to select which of the given set of suppliers to retain, and how to allocate the aggregate supply quantity among them.

A continuous and differentiable demand distribution is often combined with supply uncertainty to derive the operational strategies. Anupindi and Akella (1993) study one- and multi-period procurement problems with two unreliable suppliers assuming Bernoulli random yield. They address the operational issue of allocating quantities and its implications for the inventory policy of the buyer when the buyer faces stochastic and continuously distributed demand and unreliable supply. They establish that the optimal supplier set is consecutive in the effective cost rates: if the on-hand inventory is lower than a threshold, both suppliers are used; if the on-hand inventory is between this threshold and the other higher threshold, only the cheaper supplier is used. No orders are placed for higher inventory. Dada et al. (2007) also establish the consecutiveness property of the optimal supplier set for multiple suppliers in a single period with zero on-hand inventory using a general reliability construct that embraces stochastic proportional yield as well as random capacity. Assuming the buyer pays each supplier only for the units actually delivered, they show that although reliability affects how much is ordered from a selected supplier, cost generally takes precedence over reliability when it comes to selecting suppliers in the first place. In our paper we show that investing in reliability rather than cost reduction may yield higher returns to a supplier when the buyer is risk averse.

Swaminathan and Shanthikumar (1999) show, both in a one- and multi-period model, that the consecutiveness property of the optimal set of the suppliers based on the purchasing cost may break down when demand is discrete. They point out that “The structure of the optimal policy derived in Anupindi and Akella, although technically correct, is misleading because it emphasizes that it is never optimal to place an order with the more expensive supplier alone”. In reality the buyer
might choose a more expensive but more reliable supplier and eliminate a cheaper but less reliable supplier. When the buyer pays for all units ordered from the suppliers instead of only the units actually delivered which is assumed in this paper, the consecutiveness property of the optimal set of the suppliers based on the purchasing cost may also breakdown even with continuous demand. Assuming the buyer pays for all units ordered, and normally distributed demand, Federgruen and Yang (2009a) establish the consecutiveness property where the indexing is based on reliability-adjusted costs.

In addition to providing a risk-mitigation tool due to supplier diversification, multisourcing provides another dimension of benefits due to supplier competition. Very few papers consider strategic interaction among suppliers. All the multisourcing papers cited above assume the wholesale prices set by the suppliers are exogenous except Babich et al. (2007). They analyze price competition of two all-or-nothing suppliers with correlated default risk in a Stackelberg game. Reversing the traditional intuition, they show that the retailers may prefer positively correlated defaults because the price competition benefits, induced by high correlations, more than offset the diversification benefits. By considering the strategic suppliers competing on price, in their model it is the low correlation that weakens the price competition and drives the equilibrium prices high, whereas in our model it is the buyer’s risk aversion that provides the suppliers with the power to exploit his risk and drives the equilibrium prices high. Inspired by the context of the market for influenza vaccines Deo and Corbett (2008) formulate a two-stage model of Cournot competition among an arbitrary number of suppliers with endogenous entry to investigate how the yield uncertainty influences the firms’ entry and production strategies. Lastly, Federgruen and Yang (2009b) approximate the equilibrium behavior in a supply chain where the suppliers compete for the business with the buyers in terms of key characteristics of their yield distributions, either their means, their standard deviations or both.

Most of the literature on supply uncertainty assumes the supply chain participants are risk neutral and only care about the mean effect of their profit. The literature on supply chain models with risk aversion is quite limited and mainly focused on the reliable supply problems, e.g., Eeckhoudt
et al. (1995), Agrawal and Seshadri (2000), Schweitzer and Cachon (2000), Chen and Federgruen (2000) and Chen et al. (2007). In Eeckhoudt et al. (1995), assuming that there is no stockout cost and the risk-averse newsvendor has a second order chance if the demand exceeds his first order, they show that the optimal order quantity is less than the optimal risk-neutral order quantity and it decreases with increasing risk aversion. Schweitzer and Cachon (2000) find that for some products, decision makers exhibit risk averse behavior. Tomlin and Wang (2005) simultaneously examines mix flexibility and dual sourcing to provide insight into effective supply-chain design in the presence of Bernoulli yield and demand uncertainty. They do not consider competition and equilibrium pricing.

3. The Model

In this model we consider two unreliable suppliers and one risk-averse buyer facing deterministic demand $D > 0$ from his downstream customers. The suppliers produce perfectly substitutable products and compete to supply the buyer. We model the interaction between the suppliers and the buyer as Stackelberg games.

The suppliers start out setting their wholesale prices $w_1, w_2 > 0$ and the buyer follows by determining corresponding order quantities $Q_1, Q_2$. The market is controlled by the suppliers who can play the role of Stackelberg leaders with respect to the buyer by taking the buyer’s reaction function into consideration for their respective wholesale price decisions. Also, the supplier would have little difficulty finding out the competitor’s wholesale price on which it conditions its strategy. The suppliers start production as soon as they receive the orders and they produce the amounts ordered. We assume that a unit of product produced by the suppliers is required to produce a unit of finished product. The per unit production cost for supplier $i$ is $c_i$. The suppliers are vulnerable to supply disruptions, by which we mean with some probability they deliver an amount strictly less than the amount ordered. The supply uncertainty can be modeled in various ways. We assume a Bernoulli distributed delivery rate model where the binary delivery rate $\gamma_i$ takes value 1 with success probability $\beta_i$ and value 0 with failure probability $\bar{\beta}_i = 1 - \beta_i$. The $\gamma_i$’s are independent.
Let $p$ denote the sales price of the buyer’s finished product minus the loaded costs from other components and production cost. If the wholesale price $w_i$ is greater than the benefit from ordering from supplier $i$, the buyer will not order from this supplier. Therefore we assume that $p - w_i \geq 0$.

All supply chain participants know each other’s delivery rate distributions and cost structures and the buyer’s risk aversion level. This can be justified to a certain extent. For example, if the buyer has done business with both suppliers in the past, they can have a pretty good sense of his risk aversion and he can have a pretty good estimate of their reliability. Also, if these suppliers have been prequalified to sell to the buyer, this would certainly have improved his knowledge of their capabilities.

In the following formulation, let the subscript $S$ denote the suppliers and $-i$ denote the supplier $i$’s opponent. Also, define $(\eta)^+ \equiv \max\{0, \eta\}$ to be the positive part of $\eta$. We assume the buyer’s initial inventory is $0$, $\beta_1, \beta_2 > 0$, and $w_1, w_2 \leq p$, which will be maintained throughout the paper. We develop two models describing the buyer’s behavior. Model I analyzes a risk-neutral buyer which is used as the benchmark model to compare with the risk-averse buyer in Model II.

3.1. Model I: Risk-Neutral Buyer

As a follower the buyer chooses his order quantities to the suppliers after observing the suppliers’ wholesale prices. Let $Q, w$ be the order quantity vector and wholesale price vector, respectively.

The buyer’s profit is

$$\Pi(Q, w) = \sum_{i=1}^{2} (p - w_i) \gamma_i Q_i - p \left( \sum_{i=1}^{2} \gamma_i Q_i - D \right)^+. \quad (1)$$

The expression consists of sales revenue less acquisition costs. Thus the buyer’s problem is to choose $Q$ so as to maximize his expected profit

$$E[\Pi(Q, w)] = E_{\gamma_1, \gamma_2} \left[ \sum_{i=1}^{2} (p - w_i) \gamma_i Q_i - p \left( \sum_{i=1}^{2} \gamma_i Q_i - D \right)^+ \right]. \quad (2)$$

The expectation is taken over the random delivery rates.

As leaders the suppliers set their wholesale prices in advance by taking into account the reaction of the follower. Because the suppliers make their pricing decisions simultaneously, they choose
their wholesale price assuming that the other supplier will keep his wholesale price fixed. Thus the expected profit of supplier $i$ is

$$E[\Pi^i_S(w_i, w_{-i})] = E_{\gamma_i}[w_i\gamma_iQ^*_i(w_i, w_{-i}) - c_iQ^*_i(w_i, w_{-i})],$$

(3)

where $Q^*_i(w_i, w_{-i})$ denotes the buyer’s optimal order quantity to supplier $i$ given wholesale prices $(w_i, w_{-i})$. The expectation is taken over the random delivery rates. The buyer only pays for the products that are delivered. This assumption is adopted by the majority of the supply uncertainty literature. (e.g., Anupindi and Akella 1993, Swaminathan and Shanthikumar (1999), Federgruen and Yang (2009b), Dada et al. (2007)). Similar to Babich et al. (2007), we assume the suppliers’ production costs are incurred up front as soon as production starts.

The objective of each supply chain participant is to maximize his own expected profit. Consequently the Nash-Stackelberg game can be modeled as the following optimization problem

$$\max_{0 \leq w_i \leq p} E[\Pi^i_S(w_i, w_{-i})] \quad i = 1, 2,$$

(4)

s.t. $Q^*(w_i, w_{-i}) \in \arg \max_{Q \geq 0} E[\Pi(Q, w)].$

As we can see, the suppliers act as Stackelberg leaders with respect to the buyer, but as Nash competitors with respect to each other.

### 3.2. Model II: Risk-Averse Buyer

Model II has the same game structure as problem (4) in Model I except that the buyer is risk averse. Instead of maximizing his expected profit he wants to maximize his expected utility, that is

$$\max_{0 \leq w_i \leq p} E[\Pi^i_S(w_i, w_{-i})] \quad i = 1, 2,$$

(5)

s.t. $Q^*(w_i, w_{-i}) \in \arg \max_{Q \geq 0} E[U(\Pi(Q, w))].$

We assume the risk-averse buyer has a negative exponential utility function

$$U(x) = -\exp(-\theta x),$$
where $\theta < +\infty$ and larger value of $\theta$ connotes greater sensitivity to risk. The expected utility of the buyer is

$$E[U(\Pi(Q, w))] = E_{\gamma_1, \gamma_2} [-\exp (-\theta \Pi(Q, w))],$$

(6)

where the expectation is taken over the random delivery rates and $\Pi(Q, w)$ is shown in (1). The risk-averse buyer’s certainty equivalent is the guaranteed payoff at which he is indifferent between accepting this payoff and a higher but uncertain payoff. It is given by

$$\Pi^{CE}(Q, w) = U^{-1}(E_{\gamma_1, \gamma_2} [-\exp (-\theta \Pi(Q, w))]).$$

4. Exogenous Wholesale Prices

Here we analyze the buyer’s ordering strategy given that the suppliers’ wholesale prices are exogenously fixed. The buyer faces deterministic demand and Bernoulli distributed delivery rates of two suppliers.

4.1. The Buyer’s Ordering Strategy

Considering the wholesale prices restricted to the set $\Omega \equiv \{ w \mid 0 < w_1 < p, 0 < w_2 < p \}$ the buyer’s problem in Model I is equivalent to the following optimization problem by taking the expectation with respect to $\gamma_i, i = 1, 2$ in (2),

$$\max_{Q_1 \geq 0, Q_2 \geq 0} \beta_1 \beta_2 (pD) + [-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1)]Q_1 + [-\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2)]Q_2$$

(7a)

s.t.  

$$Q_1 + Q_2 \geq D$$

(7b)

$$Q_1 \leq D$$

(7c)

$$Q_2 \leq D.$$  

(7d)

The above statement is straightforward under the assumption that the random delivery rates are Bernoulli distributed and the buyer faces deterministic demand. There is no benefit for the buyer to order more than $D$ from one supplier because he only needs $D$ units. However he should order no less than $D$ units in total from both suppliers. If the buyer orders less than $D$ units in total, from (2) his expected profit is given by

$$E[\Pi(Q, w)] |_{Q_1 + Q_2 < D} = \beta_1 (p - w_1)Q_1 + \beta_2 (p - w_2)Q_2,$$


which is increasing in $Q_1, Q_2$. We call this feasible region in (7b-7d) the feasible order region and we restrict the following analysis to the feasible order region. In this region the quantity $-\beta_i \beta_j w_i + \beta_i \beta_j (p - w_i)$ is then the marginal profit of ordering from supplier $i$, where $\beta_i \beta_j w_i$ represents the marginal cost and $\beta_i \beta_j (p - w_i)$ represents the marginal benefit. Note that the marginal cost and benefit depend on the suppliers’ reliabilities. When both suppliers deliver the order successfully the marginal profit is negative since the total quantity delivered is more than $D$ and an extra unit ordered from either supplier would not be sold and decrease the buyer’s profit. When supplier $i$ delivers the order successfully and supplier $j$ fails to deliver, an extra unit ordered from supplier $i$ incurs positive profit to the buyer since the total quantity delivered is less than $D$ and an extra unit delivered by supplier $i$ would be sold. The marginal profit of ordering from supplier $i$ is zero if supplier $i$ fails to deliver. Facing the deterministic demand $D$, the buyer’s total order quantity received is uncertain and he might order more than $D$ in total across the suppliers to hedge against the supply uncertainty.

**Definition 1.** The ordering strategy $Q$ is called a

(i) **single sourcing strategy** if $(Q_1, Q_2) = (0, D)$ or $(Q_1, Q_2) = (D, 0)$,

(ii) **duplicate sourcing strategy** if $(Q_1, Q_2) = (D, D)$,

(iii) **diversification strategy** if $Q_1 > 0$ and $Q_2 > 0$,

(iv) **order inflation strategy** if $Q_1 + Q_2 > D$.

As we can see, whereas the single sourcing strategy is neither a diversification nor an order inflation strategy, the double sourcing strategy exhibits both.

The following proposition states the optimal ordering strategy for the buyer. The results are similar to those in Babich et al. (2007) except some model choices which we describe later. These results will be compared with those of the risk-averse case, which enables us to explain the procurement phenomenon.

**Proposition 1.** Given the exogenously fixed wholesale prices of the two suppliers $w \in \Omega$, the risk-neutral buyer’s optimal order quantities $(Q_1^*, Q_2^*)$ are given by:
\((i) (Q_1^*, Q_2^*) = (D, D) \text{ if } w_1 < \beta_2 p \text{ and } w_2 < \beta_1 p.\)

\((ii) (Q_1^*, Q_2^*) = (D, 0) \text{ if } w_2 > \beta_1 p \text{ and } \beta_1 (p - w_1) > \beta_2 (p - w_2).\)

\((iii) (Q_1^*, Q_2^*) = (0, D) \text{ if } w_1 > \beta_2 p \text{ and } \beta_1 (p - w_1) < \beta_2 (p - w_2).\)

\((iv) (Q_1^*, Q_2^*) = (\alpha D, (1 - \alpha)D), \text{ for any } \alpha \in [0, 1], \text{ if } w_1 > \beta_2 p, w_2 > \beta_1 p\)
and \(\beta_1 (p - w_1) = \beta_2 (p - w_2).\)

\((v) (Q_1^*, Q_2^*) = (D, \alpha D), \text{ for any } \alpha \in [0, 1], \text{ if } w_1 < \beta_2 p \text{ and } w_2 = \beta_1 p.\)

\((vi) (Q_1^*, Q_2^*) = (\alpha D, D), \text{ for any } \alpha \in [0, 1], \text{ if } w_2 < \beta_1 p \text{ and } w_1 = \beta_2 p.\)

\((vii) (Q_1^*, Q_2^*) = (\alpha D, \delta D), \text{ for any } \alpha, \delta \in [0, 1] \text{ and } \alpha + \delta \geq 1, \text{ if } w_1 = \beta_2 p \text{ and } w_2 = \beta_1 p.\)

Proof. See appendix. □

Proposition 1 states that if the buyer’s marginal profits of ordering from the suppliers are both positive he should order \(D\) units from both suppliers (duplicate sourcing); if the buyer’s marginal profits of ordering from suppliers are different and at least one of them is negative he should order \(D\) units from the supplier who provides higher profit margin and nothing from the other supplier (single sourcing). The rest of the cases are boundary ordering strategies and do not exhibit unique diversification or order inflation since the buyer is indifferent between these ordering strategies and single sourcing. Assuming the default events are correlated, Babich et al. (2007) have similar results. Different from us, they assume the buyer pays for the units ordered instead of the units delivered. Therefore, the buyer in their model requires lower wholesale prices to be willing to order \(D\) units from both suppliers in order to compensate the downside risk of paying for the order not delivered successfully. It will be instructive to compare the risk-neutral case with that of risk aversion.

The proposition above considers only the case when the wholesale prices are strictly less than \(p\). When \(w_i = p\) and \(w_j < p\) the optimal order quantities are \(Q_i^* = 0, Q_j^* = D\). In the case when \(w_i = w_j = p\), the buyer is indifferent among the allocations as long as the total order quantities are no greater than \(D\).

In Model II, the buyer’s profit \(\Pi(Q, w)\) is jointly concave in the order quantities and the negative exponential utility function \(U(x) = -\exp(-\theta x)\) is a concave function and increasing in \(x\). Since
an increasing concave function of a concave function is concave, \( U[\Pi(Q, w)] \) is concave in the order quantities. The quantity \( E[U(\Pi(Q, w))] \) is the expectation of a concave function, therefore \( E[U(\Pi(Q, w))] \) is also a concave function. Considering the wholesale prices restricted to the set \( \Omega \) the buyer’s problem is equivalent to the following optimization problem by taking the expectation with respect to \( \gamma_i, i = 1, 2 \) in (6),

\[
\max_{Q_1 \geq 0, Q_2 \geq 0} -\beta_1 \beta_2 \exp[-\theta (pD - w_1 Q_1 - w_2 Q_2)] - \beta_1 \beta_2 \exp[-\theta (p - w_1) Q_1] - \beta_1 \beta_2 \exp[-\theta (p - w_2) Q_2] - \beta_1 \beta_2 \exp(-\theta (p - w_1) Q_1) - \beta_1 \beta_2 \exp(-\theta (p - w_2) Q_2) - \beta_1 \beta_2
\]

\[
\text{s.t.}\quad Q_1 + Q_2 \geq D
\]

\[
Q_1 \leq D
\]

\[
Q_2 \leq D.
\]

Similar to the argument for Model I, there is no benefit for the buyer to order more than \( D \) from one supplier because he only needs \( D \) units. If the buyer orders less than \( D \) units in total, from (6) his expected utility is given by

\[
E[U(\Pi(Q, w))] |_{Q_1 + Q_2 < D} = -\beta_1 \beta_2 \exp[-\theta (p - w_1) Q_1 - \theta (p - w_2) Q_2] - \beta_1 \beta_2 \exp[-\theta (p - w_1) Q_1] - \beta_1 \beta_2 \exp[-\theta (p - w_2) Q_2] - \beta_1 \beta_2,
\]

which is increasing in \( Q_1, Q_2 \). Therefore the feasible region in Model II is the same as the feasible order region in Model I. Letting \( M_i(Q) = \partial E[U(\Pi(Q, w))] / \partial Q_i \), we have

\[
M_1(Q) = -\beta_1 \beta_2 w_1 U'(pD - w_1 Q_1 - w_2 Q_2) + \beta_1 \beta_2 (p - w_1) U'((p - w_1) Q_1),
\]

\[
M_2(Q) = -\beta_1 \beta_2 w_2 U'(pD - w_1 Q_1 - w_2 Q_2) + \beta_1 \beta_2 (p - w_2) U'((p - w_2) Q_2),
\]

where \( U'(\cdot) \) is the marginal utility of the negative exponential utility function. In the interior of the feasible order region the supplier’s marginal utility is comprised of two parts: risk-adjusted marginal cost and risk-adjusted marginal benefit. Remember in the risk-neutral case the marginal profit is simply the marginal benefit minus the marginal cost. Unlike the risk-neutral buyer, the
risk-averse buyer exhibits a wealth effect, i.e., diminishing marginal utility of wealth. This says that the more wealth (profit) the buyer has, the less he values each extra dollar he has, i.e., the utility he gets from the marginal unit of dollar diminishes. Thus the risk-averse buyer’s marginal cost and benefit are adjusted by the wealth effect through $U'()$.

Since the buyer’s problem is to maximize a concave function over a convex set, the Karush-Kuhn-Tucker conditions are necessary and sufficient for the optimal order quantities $(Q^*_1, Q^*_2)$. From (8) we know that the buyer’s problem is strictly concave in the order quantities when the wholesale prices $w \in \Omega$. Hence the optimal order quantities $(Q^*_1, Q^*_2)$ are unique. Define $d_i(\cdot), i = 1..9$ as functions of $w$ as follows:

$$d_1(w) = \frac{1}{\theta w_1} \ln \left( \frac{\beta_1(p-w_2)}{\beta_1 p} \right), \quad d_2(w) = \frac{1}{\theta w_2} \ln \left( \frac{\beta_2(p-w_1)}{\beta_2 p} \right),$$

$$d_3(w) = \frac{1}{\theta (p-w_1)} \ln \left( \frac{\beta_1 w_2}{\beta_1 (p-w_2)} \right), \quad d_4(w) = \frac{1}{\theta (p-w_1)} \ln \left( \frac{\beta_1 (\beta_2 w_2 + \beta_2 p - w_1)}{\beta_2 (p-w_2)} \right),$$

$$d_5(w) = \frac{1}{\theta (p-w_2)} \ln \left( \frac{\beta_2 w_1}{\beta_2 (p-w_1)} \right), \quad d_6(w) = \frac{1}{\theta (p-w_2)} \ln \left( \frac{\beta_2 (\beta_1 w_1 + \beta_1 p - w_2)}{\beta_1 (p-w_1)} \right),$$

$$d_7(w) = \frac{1}{\theta (p-w_2)} \ln \left( \frac{\beta_2 (p-w_1)}{\beta_1 (p-w_2)} \right), \quad d_8(w) = \frac{1}{\theta w_2 (p-w_1)} \ln \left( \frac{\beta_2 (p-w_1)}{\beta_2 w_1} \right) + \frac{p}{\theta (p-w_1)} \ln \left( \frac{\beta_1 w_2}{\beta_1 (p-w_2)} \right),$$

$$d_9(w) = \frac{1}{\theta w_1 (p-w_2)} \ln \left( \frac{\beta_1 (p-w_2)}{\beta_1 w_2} \right) + \frac{p}{\theta (p-w_2)} \ln \left( \frac{\beta_2 w_1}{\beta_2 (p-w_1)} \right).$$

Now define $\Omega_i \in \Omega, i = 1..7$ as sets of wholesale prices as follows:

$\Omega_1 \equiv \{ w \mid w_1 < \beta_3 p, w_2 < \beta_1 p \} \cap \{ w \mid D \leq d_1(w), D \leq d_2(w) \}$,

$\Omega_2 \equiv \{ w \mid w_2 > \beta_3 p, \beta_1 (p-w_1) > \beta_2 (p-w_2) \} \cap \{ w \mid w_1 < \beta_3 p, D \leq d_3(w) \text{ or } w_1 \geq \beta_3 p, D \leq d_4(w) \}$,

$\Omega_3 \equiv \{ w \mid w_1 > \beta_3 p, \beta_1 (p-w_1) < \beta_2 (p-w_2) \} \cap \{ w \mid w_2 < \beta_1 p, D \leq d_5(w) \text{ or } w_2 \geq \beta_1 p, D \leq d_6(w) \}$,

$\Omega_4 \equiv \{ w \mid w_1 > \beta_3 p, w_2 > \beta_1 p \} \cap \{ w \mid \beta_1 (p-w_1) \geq \beta_2 (p-w_2), d_4(w) < D \leq d_7(w) \text{ or } \beta_1 (p-w_1) < \beta_2 (p-w_2), d_6(w) < D \leq d_7(w) \}$,

$\Omega_5 \equiv \{ w \mid w_1 < \beta_3 p \} \cap \{ w \mid w_2 < \beta_1 p, d_1(w) < D \leq d_4(w) \text{ or } w_2 \geq \beta_1 p, d_5(w) < D \leq d_6(w) \}$,

$\Omega_6 \equiv \{ w \mid w_2 < \beta_3 p \} \cap \{ w \mid w_1 < \beta_3 p, d_2(w) < D \leq d_6(w) \text{ or } w_1 \geq \beta_3 p, d_5(w) < D \leq d_6(w) \}$,

$\Omega_7 \equiv \{ w \mid w_1 < \beta_3 p, w_2 < \beta_1 p, D > d_6(w), D > d_8(w) \text{ or } w_1 \geq \beta_3 p, w_2 < \beta_1 p, D > d_8(w) \}$,

or $w_1 < \beta_3 p, w_2 \geq \beta_1 p, D > d_6(w) \text{ or } w_1 \geq \beta_3 p, w_2 \geq \beta_1 p, D > d_7(w)$. 

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Fixing all other parameters, Proposition 2 shows the effects of the wholesale prices on the buyer’s optimal order quantities.

**PROPOSITION 2.** Given the exogenously fixed wholesale prices of the two suppliers \( w \in \Omega \), the risk-averse buyer’s optimal order quantities \((Q^*_1, Q^*_2)\) are given by:

(i) \((Q^*_1, Q^*_2) = (D, D)\) if and only if \( w \in \Omega_1 \).

(ii) \((Q^*_1, Q^*_2) = (D, 0)\) if and only if \( w \in \Omega_2 \).

(iii) \((Q^*_1, Q^*_2) = (0, D)\) if and only if \( w \in \Omega_3 \).

(iv) \(0 < Q^*_1 < D, 0 < Q^*_2 < D, Q^*_1 + Q^*_2 = D\) if and only if \( w \in \Omega_4 \).

(v) \(Q^*_1 = D, 0 < Q^*_2 < D\) if and only if \( w \in \Omega_5 \).

(vi) \(0 < Q^*_1 < D, Q^*_2 = D\) if and only if \( w \in \Omega_6 \).

(vii) \(0 < Q^*_1 < D, 0 < Q^*_2 < D, Q^*_1 + Q^*_2 > D\) if and only if \( w \in \Omega_7 \).

Proof. See appendix. □

When the optimal order quantities are in region (vii) where the buyer orders less than \( D \) from each supplier and more than \( D \) in total, by solving \( M_1(Q) = 0 \) and \( M_2(Q) = 0 \) we obtain

\[
Q^*_1 = \frac{p(p - w_2)D + \frac{p}{\theta} \ln \left[ \gamma_2 \left( \frac{p-w_1}{\gamma_2 w_1} \right) - \frac{w_2}{\theta} \ln \left[ \gamma_1 \left( \frac{p-w_2}{\gamma_1 w_2} \right) \right] \right]}{p^2 - w_1 w_2},
\]

\[
Q^*_2 = \frac{p(p - w_1)D + \frac{p}{\theta} \ln \left[ \gamma_1 \left( \frac{p-w_2}{\gamma_1 w_2} \right) - \frac{w_1}{\theta} \ln \left[ \gamma_2 \left( \frac{p-w_1}{\gamma_2 w_1} \right) \right] \right]}{p^2 - w_1 w_2}.
\]

When \( w_i = p \) and \( w_j < p \) the optimal order quantities are \( Q^*_i = 0, Q^*_j = D \). In the case when \( w_i = w_j = p \), from (6) the buyer’s expected utility is given by

\[
E[U(\Pi(Q, w))] = -\beta_1 \beta_2 \exp \left[ \theta p \left( \sum_{i=1}^{2} Q_i - D \right)^+ \right] - \beta_1 \beta_2 \exp \left[ \theta p (Q_1 - D)^+ \right],
\]

so the buyer is indifferent among the allocations as long as the total order quantity is no greater than \( D \). Considering the wholesale prices restricted in the set \( S \equiv \{ w \mid 0 < w_1 \leq p, 0 < w_2 \leq p, w \neq (p, p) \} \), the next lemma establishes the continuity of the optimal order quantities over the wholesale prices fixing all other parameters. This will prove useful in an equilibrium analysis to come.
Lemma 1. The risk-averse buyer’s optimal order quantities \((Q_1^*, Q_2^*)\) are continuous on the wholesale prices \(w \in S\).

Proof. See appendix. □

The degree of risk aversion affects the buyer’s ordering strategy. The more risk averse the buyer is the more he cares about profit variability. The diversification effect occurs when the buyer orders from both suppliers, which allows the impact of each disruption to be mitigated, resulting in a lower profit variance. As the buyer becomes more risk averse, the diversification benefits for him continue to accrue. Proposition 2 and Lemma 1 show that the risk-averse buyer implements a spectrum of distinctive diversification and order inflation strategies taking into account both the expected profit and profit variability.

In Model II high risk aversion corresponds to a large value of \(\theta\), while low risk aversion corresponds to a \(\theta\) close to zero. As the model converges to risk-neutral, that is \(\theta \to 0\), we have \(d_1(w) \to +\infty\), \(d_2(w) \to +\infty\) and \(\Omega_1 \to \{ w \in \Omega \mid w_1 < \beta_2 p, w_2 < \beta_1 p \}\). This corresponds with region (i) in Proposition 1. Similar arguments follow for region (ii) and (iii). As \(\theta \to 0\), condition \(\beta_1(p - w_1) = \beta_2(p - w_2)\) is needed for \(\Omega_4\) to be nonempty and we have \(d_7(w) \to +\infty\). Then it follows that \(\Omega_4 \Rightarrow \{ w \in \Omega \mid w_1 > \beta_2 p, w_2 > \beta_1 p\) and \(\beta_1(p - w_1) = \beta_2(p - w_2)\}\) which is the same as the conditions for region (iv) in Proposition 1. Similar arguments follow for region (v), (vi) and (vii). Thus as \(\theta \to 0\) the conditions for each ordering strategy in Model II converge to those in Model I.

The top panel of Figure 1 depicts the optimal ordering strategy of the risk-averse buyer depending on the suppliers’ wholesale prices. The boundaries of the regions are altered by the changes of the model parameters. For any two neighboring regions, whichever region has ”=” in the optimal ordering quantities while the other one does not owns the boundary between them. For example, since \(Q_2^* = D\) in region (i) and \(0 < Q_2^* < D\) in region (v), the boundary between region (i) and (v) belongs to region (i). The shaded area indicates the diversification strategies and the dotted area indicates the order inflation strategies. Regions (i), (v), (vi) and (vii) are both diversification and order inflation regions. Region (iv) is a diversification region but not an order inflation region since
Figure 1  The optimal ordering strategy of the risk-averse buyer (top) and the risk-neutral buyer (bottom) depending on the suppliers’ wholesale prices
the buyer simply splits \( D \) between two suppliers. Regions (ii) and (iii) are neither diversification regions nor order inflation regions since the buyer orders \( D \) from one of the buyer and nothing from the other one.

Notice from Proposition 2 that the wholesale prices \((w_1, w_2) = (\beta_2 p, \beta_1 p)\) are always in region (vii) regardless of the risk aversion level since we have \( d_7(w) = 0 \) and \( D > 0 \), which imply \( D > d_7(w) \). From (9) and (10) we know that when \((w_1, w_2) = (\beta_2 p, \beta_1 p)\) the marginal cost and marginal benefit are equal, and in order to satisfy the first order conditions the wealths (the profits made by the buyer) in the two cases when both suppliers deliver the order successfully and only one supplier delivers the order successfully have to be the same. The corresponding optimal order quantities are \((Q^*_1, Q^*_2) = \left( \frac{\beta_1 p_1 - \beta_2 p_2 \beta_1}{1 - \beta_1 p_2}, \frac{\beta_2 p_1 - \beta_1 p_2 \beta_2}{1 - \beta_1 p_2} \right)\). It is easy to see that \( Q^*_i \) is increasing in supplier \( i \)'s delivery rate \( \beta_i \) and decreasing in supplier \( j \)'s delivery rate \( \beta_j \).

As the buyer’s risk aversion decreases, the effects of marginal profits of ordering from the suppliers dominate the diversification effect resulting in the shrinkage of the diversification regions and order inflation regions. The bottom panel of Figure 1 represents the risk-neutral buyer’s optimal ordering strategies with only single sourcing, duplicate sourcing and various boundary ordering strategies. The risk-neutral buyer does not implement a spectrum of diversification and order inflation strategies, in contrast with a risk-averse buyer.

### 4.2. Comparative Statics for the Risk-Averse Buyer in Region (vii)

We are mostly interested in region (vii) in which the buyer orders less than \( D \) from both suppliers and orders more than \( D \) in total, because it exhibits the richest behavior. In Table 1 we present the expressions for the partial derivatives of the order quantities with respect to the parameters derived from (11) and (12). Since \( w_1 \) and \( \beta_1 \) are symmetric cases to \( w_2 \) and \( \beta_2 \), we only show the results for \( w_1 \) and \( \beta_1 \). We summarize various comparative statics of changing the parameters for the risk-averse buyer in Table 2. An increase in supplier \( i \)'s wholesale price decreases the total order quantity. The order quantity to supplier \( i \) is increasing in his reliability and decreasing in the other supplier’s reliability as the risk-averse buyer dislikes variability. The total order quantity is
decreasing in supplier $i$’s reliability since the buyer’s incentive to inflate his order to hedge risks is weakened. If two suppliers are perfectly reliable, only $D$ is needed in total to meet the deterministic demand. As $D$ increases, the order quantities to both suppliers increase to meet the higher demand.

An increase in sales price increases the total order quantity. The rest of the effects in Table 2 are not so clearly determined. The changes in these parameters influence the optimal order quantities through both marginal profit effects and wealth effects, and the net effects depend on all other parameters.

### Table 1
The Partial Derivatives of the Order Quantities with Respect to the Parameters in Region (vii)

<table>
<thead>
<tr>
<th>$\partial Q^*_1$</th>
<th>$\partial Q^*_2$</th>
<th>$\partial (Q^<em>_1 + Q^</em>_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial p}{\partial w_1}$</td>
<td>$\frac{\partial p}{\partial w_2}$</td>
<td>$\frac{\partial p}{\partial w_1}$</td>
</tr>
<tr>
<td>$\frac{\partial p}{\partial \beta_1}$</td>
<td>$\frac{\partial p}{\partial \beta_2}$</td>
<td>$\frac{\partial p}{\partial \theta}$</td>
</tr>
<tr>
<td>$\frac{\partial p}{\partial D}$</td>
<td>$\frac{\partial p}{\partial \theta}$</td>
<td>$\frac{\partial p}{\partial \theta}$</td>
</tr>
</tbody>
</table>

### Table 2
Summary Comparative Statics for the Risk-Averse Buyer in Region (vii)

<table>
<thead>
<tr>
<th>Exogenous Change</th>
<th>Change in $Q^*_1$</th>
<th>Change in $Q^*_2$</th>
<th>Change in $(Q^<em>_1 + Q^</em>_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>increase $w_1$</td>
<td>-</td>
<td>-</td>
<td>decrease</td>
</tr>
<tr>
<td>increase $w_2$</td>
<td>-</td>
<td>-</td>
<td>decrease</td>
</tr>
<tr>
<td>increase $\beta_1$</td>
<td>increase</td>
<td>decrease</td>
<td>decrease</td>
</tr>
<tr>
<td>increase $\beta_2$</td>
<td>decrease</td>
<td>increase</td>
<td>decrease</td>
</tr>
<tr>
<td>increase $\theta$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>increase $D$</td>
<td>increase</td>
<td>increase</td>
<td>increase</td>
</tr>
<tr>
<td>increase $p$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

4.3. Effects of Risk Aversion and Wholesale Prices on the Buyer’s Optimal Order Quantities

To better understand the effects that risk aversion and wholesale prices can have on the optimal order quantities in the feasible order region, consider the following numerical examples. Let the
buyer’s deterministic demand $D = 100$, and unit sales price $p = 10$. Tier 1 suppliers are those suppliers who are more reliable, while Tier 2 suppliers are less reliable. For example, in the semiconductor industry, Tier 2 suppliers are mainly Asia companies who have lower reliability and are desperate to capture the market share. Supplier 1 is a Tier 1 supplier with delivery rate $\beta_1 = 0.9$, and supplier 2 is a Tier 2 supplier with delivery rate $\beta_2 = 0.6$. Figure 2 illustrates how the risk aversion and wholesale prices affect the buyer’s optimal order quantities. We fix supplier 2’s wholesale price and see how the change of supplier 1’s wholesale price affects the optimal order quantities at a certain level of risk aversion. We consider two cases of supplier 2’s wholesale price, $w_2 = 1.5$ for all left panels and $w_2 = 6$ for all right panels. The top, middle and bottom panels illustrate the cases of high, moderate and low levels of risk aversion of the buyer with $\theta = 0.01, 0.001, 0.0001$, respectively.

The panels depict wholesale prices in different order regions, corresponding to Proposition 2.

All the panels show that the total order quantity is nonincreasing in supplier 1’s wholesale price. As risk aversion increases, the buyer has a growing tendency to diversify his portfolios. An increase in risk aversion also leads to higher (not lower) total order quantity in this numerical instance. This is opposite the results of the classical risk-averse newsvendor case with deterministic supply and stochastic demand (Eeckhoudt et al. 1995). In that setting, if a risk-averse newsvendor orders one more unit, a unit of underage cost will be saved accompanied with a decrease in the marginal utility of underage through a wealth effect because the newsvendor is richer. Similarly, a unit of overage cost will be incurred accompanied by an increase in the marginal utility of overage through a wealth effect. Therefore a risk-averse newsvendor will order less than a risk-neutral newsvendor.

In contrast, for our setting, the risk-averse buyer who is concerned about the tradeoff between the expected profit and profit variance might potentially reduce profit variability by ordering more from the reliable supplier. As our numerical results demonstrate, this can result in a higher total order quantity across suppliers.

Comparing the left and right panels we see that facing lower fixed price $w_2$ supplier 1 needs to charge lower (no higher) wholesale price to be able to capture the same market share, which reflects the price competition between the suppliers for the business with the buyer. The more
Figure 2  The optimal order quantities as functions of $w_1$ given fixed $w_2$. 

reliable supplier 1 can possibly increase his market share and hurt his competitor supplier 2 by increasing his wholesale price. The top panels show that below a certain level of $w_1$ the line of order quantity to supplier 1 is flatter than the line of the order quantity to supplier 2, which means that as $w_1$ increases the buyer reduces order quantities to both suppliers but reduces more to supplier 2. With lower risk aversion the left middle panel shows a similar result that below a certain level
of \( w_1 \) supplier 1’s market share increases in his wholesale price. This illustrates that supplier 1 does not need to compete as fiercely on price with supplier 2 since he has comparative advantage in reliability. Furthermore, he could be better off if he raises his wholesale price to a certain level. By increasing his wholesale price he makes the buyer’s procurement cost higher. The buyer would rather reduce the order quantity to the less reliable supplier 2 than to him, which leads to an increase in the more reliable supplier 1’s market share.

5. Endogenous Wholesale Prices

In this section we consider the supply chain participants’ equilibrium behavior given that the suppliers’ wholesale prices are endogenously determined. The suppliers act as leaders and set the wholesale prices simultaneously by taking into account the other supplier’s behavior and also the buyer’s best response. For the chosen wholesale prices the buyer will apply his best response by maximizing his expected utility function.

5.1. The Equilibrium Behavior of the Suppliers and the Buyer

In order to ensure that the suppliers obtain nonnegative profits, the wholesale price set by supplier \( i \) should be no less than his effective cost \( \frac{c_i}{\beta_i} \). Otherwise supplier \( i \) would be better off not having business with the buyer. The following proposition states the equilibrium for Model I.

**PROPOSITION 3.** In Model I suppose that \( \beta_1 \beta_2 p - c_1 \geq \beta_2 \beta_1 p - c_2 \).

(i) There is a unique equilibrium solution to the game between two suppliers and the equilibrium wholesale prices are:

\[
(w_1^*, w_2^*) = \begin{cases} 
(\beta_2 p, \beta_1 p) & \text{if } \beta_2 \beta_1 p - c_2 \geq 0; \\
(\beta_2 p - \frac{\beta_2 \beta_1 p - c_2}{\beta_1}, \frac{c_2}{\beta_2}) & \text{if } \beta_2 \beta_1 p - c_2 < 0.
\end{cases}
\]

(ii) The buyer’s order quantities are:

\[
(Q_1^*, Q_2^*) = \begin{cases} 
(D, D) & \text{if } \beta_2 \beta_1 p - c_2 \geq 0; \\
(D, 0) & \text{if } \beta_2 \beta_1 p - c_2 < 0.
\end{cases}
\]

Proof. See appendix. \( \square \)

From (3) we know that the marginal profit of supplier \( i \) can be denoted as \( \beta_i w_i - c_i \). The quantities \( \beta_1 \beta_2 p - c_1 \) and \( \beta_2 \beta_1 p - c_2 \) then represent the marginal profits of the suppliers in the
case when the buyer orders $D$ from both suppliers in equilibrium. Without loss of generality, we assume $\beta_1 \beta_2 p - c_1 \geq \beta_2 \beta_1 p - c_2$. When supplier 2’s effective production cost $\beta_2$ is no greater than $\beta_1 p$, that is, $\beta_2 \beta_1 p - c_2 \geq 0$, the Bertrand competition between the suppliers drives the equilibrium wholesale prices to $(w_1^*, w_2^*) = (\beta_2 p, \beta_1 p)$ under which the buyer is willing to order $D$ from both suppliers. In this case the more reliable supplier with higher delivery rate can set a higher wholesale price in equilibrium because of his reliability advantage. When supplier 2’s effective production cost is greater than $\beta_1 p$, that is, $\beta_2 \beta_1 p - c_2 < 0$, the Bertrand competition drives supplier 2 out of the market. Supplier 1 captures the whole market and sets the equilibrium wholesale price at $w_1^* = \beta_2 p - \frac{\beta_2 p - \beta_1 p - c_2}{\beta_1} \beta_1$ which is greater than $\beta_2$. In this case, as supplier 2 becomes less reliable, supplier 1 is able to set a higher wholesale price and at the same time capture the whole market.

For Model II we know that the expected profit functions of the suppliers are continuous in the wholesale price set $\mathcal{S}$ from (3) and Lemma 1. These profit functions have different functional forms in different wholesale price regions. Even in region (vii) we can see the complex nature of the suppliers’ profit functions by plugging (11) and (12) into (3). Because of the complexity of these functions, it is difficult to prove analytically the existence of a pure-strategy equilibrium for the game between the two suppliers. The main difficulty is showing conditions that ensure the quasiconcavity of the profit functions. The next two results limit the pure-strategy equilibrium in certain regions and reduce our efforts of searching the equilibrium.

Lemma 2. All the wholesale prices sets $\Omega_i, i = 1..7$ are nonempty.

Proof. See appendix. □

Proposition 4. In Model II suppose there exists a pure-strategy equilibrium for the two suppliers, then the equilibrium wholesale prices can only exist in sets $\Omega_4$, $\Omega_7$ or the boundaries between $\Omega_7$ and $\Omega_5, \Omega_6$.

Proof. See appendix. □

In contrast with the risk-neutral case, where the unique equilibrium order quantities can only be duplicate sourcing or single sourcing induced by the fierce price competition of the two suppliers, the
risk-averse case provides diversification and order inflation strategies in equilibrium. The following Proposition states the necessary conditions for the equilibrium when the equilibrium is in region (vii) and the suppliers make positive expected profits.

**Proposition 5.** In Model II if there exists a pure-strategy equilibrium in region (vii) and the suppliers make positive expected profits, then

(i) the equilibrium wholesale prices \((w_1^*, w_2^*)\) satisfy:

\[
\begin{align*}
&\frac{p(p-w_2^*)D}{\theta} + \frac{p}{\theta} \ln \left[ \frac{\beta_2(p-w_2^*)}{\beta_2w_2^*} \right] - \frac{w_2^*}{\theta} \ln \left[ \frac{\beta_1(p-w_2^*)}{\beta_1w_2^*} \right] = \frac{p^2\beta_1\beta_2^2}{\theta w_2^*} (p-w_2^*)^2 + \frac{p^2\beta_1}{\theta w_2^*} (p-w_2^*) (p-w_1^*), \\
&\frac{p(p-w_1^*)D}{\theta} + \frac{p}{\theta} \ln \left[ \frac{\beta_1(p-w_1^*)}{\beta_1w_1^*} \right] - \frac{w_1^*}{\theta} \ln \left[ \frac{\beta_2(p-w_1^*)}{\beta_2w_1^*} \right] = \frac{p^2\beta_1\beta_2^2}{\theta w_1^*} (p-w_1^*)^2 + \frac{p^2\beta_1}{\theta w_1^*} (p-w_1^*) (p-w_2^*).
\end{align*}
\]

(ii) the buyer’s equilibrium order quantities \((Q_1^*, Q_2^*)\) are:

\[
\begin{align*}
Q_1^* &= \frac{p(p-w_1^*)D + \frac{p}{\theta} \ln \left[ \frac{\beta_2(p-w_1^*)}{\beta_2w_1^*} \right] - \frac{w_1^*}{\theta} \ln \left[ \frac{\beta_1(p-w_1^*)}{\beta_1w_1^*} \right]}{p^2 - w_1^*w_2^*}, \\
Q_2^* &= \frac{p(p-w_2^*)D + \frac{p}{\theta} \ln \left[ \frac{\beta_1(p-w_2^*)}{\beta_1w_2^*} \right] - \frac{w_2^*}{\theta} \ln \left[ \frac{\beta_2(p-w_2^*)}{\beta_2w_2^*} \right]}{p^2 - w_1^*w_2^*}.
\end{align*}
\]

Proof. See appendix. □

**Corollary 1.** In Model II suppose the suppliers are identical \((c_1 = c_2 = 0, \beta_1 = \beta_2 = \beta > 0)\). If there exists a symmetric pure-strategy equilibrium in region (vii), and the suppliers make positive expected profits, then

(i) the equilibrium wholesale price \(w^*\) satisfies:

\[
\theta pD + \ln \left[ \frac{\beta(p-w^*)}{\beta w^*} \right] = \frac{p + w^*}{p - w^*},
\]

(ii) the buyer’s optimal order quantity \(Q^*\) is:

\[
Q^* = \frac{pD + \frac{1}{\theta} \ln \left[ \frac{\beta(p-w^*)}{\beta w^*} \right]}{p + w^*}.
\]

(iii) the partial derivatives of \(w^*\) and \(Q^*\) with respect to \(\theta\) satisfy:

\[
\frac{\partial w^*}{\partial \theta} > 0 \text{ and } \frac{\partial Q^*}{\partial \theta} < 0.
\]

Proof. See appendix. □

Corollary 1 indicates that in region (vii) the symmetric equilibrium wholesale price is increasing in the buyer’s risk aversion \(\theta\), whereas the symmetric equilibrium order quantity is decreasing in \(\theta\).
As the buyer becomes more risk averse, the price competition between the suppliers is weakened which drives the wholesale prices higher, and the order quantities to both suppliers are decreasing with the risk-averse buyer being more fearful of losing the procurement cost.

5.2. Numerical Results

The previous analytical results show that we can restrict our attention to certain regions for pure-strategy equilibrium. They also provide us with some insights on the equilibrium when it is in region (vii). Nevertheless, these behaviors are not universal for all possible equilibria. We rely on the numerical analysis to further investigate the effect of risk aversion on a wide range of equilibrium behavior of the supply chain participants. We find that cases of non-existence, uniqueness and multiplicity of equilibria can all possibly occur depending on the level of risk aversion.

Let the buyer’s deterministic demand $D = 100$, and unit sales price $p = 10$. Supplier 1 is a Tier 1 supplier with delivery rate $\beta_1 = 0.9$ and production cost $c_1 = 0.5$, and supplier 2 is a Tier 2 supplier with delivery rate $\beta_2 = 0.6$ and production cost $c_2 = 0.5$. Notice that $\frac{c_1}{\beta_1} < \frac{c_2}{\beta_2} < \frac{\beta_1 p}{\beta_2}$ and $\beta_1 \beta_2 - c_1 > \beta_2 \beta_1 - c_2 > 0$. From Proposition 4 we know that the pure-strategy equilibrium can only exist in regions (iv), (vii) or the boundaries between regions (vii) and (v), (vi). We compute the equilibrium wholesale prices ($w^*$), order quantities ($Q^*$), buyer’s expected utility ($E[U^*]$), certainty equivalent ($\Pi^*_{CE}$), expected profit ($E[\Pi^*]$) and variance of expected profit ($Var[\Pi^*]$), and suppliers’ expected profit ($E[\Pi^*_{S}]$) for a given $\theta$. In the following we will explore the equilibrium behavior of the supply chain participants when $\theta$ increases.

**Unique Equilibrium at the Boundary of Regions (v), (vi) and (vii).** When $\theta$ is in a very low level, the effects of marginal profits of ordering from the suppliers dominate the diversification effect. A small increase in any supplier’s wholesale price can lead to a big loss in his order quantity, so the supplier’s costs of reduced order quantities outweigh the benefits of increased wholesale price. This indicates that the buyer is very sensitive to the wholesale prices in terms of order quantities, which induces a fierce price competition between the two suppliers. Table 3 shows that with a low level of risk aversion the equilibrium wholesale prices are at the boundary of regions
Adelman and Wang: Supply Disruption with a Risk-Averse Buyer

(v), (vi) and (vii), i.e., \((Q_1^*, Q_2^*) = (D, D)\), which are similar to the risk-neutral case in the first row of Table 3 obtained from Proposition 3. Nevertheless, to compensate the more risk-averse buyer as \(\theta\) increases, the wholesale prices need to be lower in order for him to continue ordering the same quantity \(D\) from both suppliers. As a result, the equilibrium wholesale prices are decreasing in \(\theta\). The buyer benefits from the price competition and his expected profit is increasing in \(\theta\). In contrast, the suppliers’ expected profits decrease and they are worse off because of the fierce price competition. The risk-averse buyer’s certainty equivalent is less than his expected profit since he needs an additional incentive, i.e., a positive risk-premium to be willing to take on the risk. As we can see from the table, the buyer’s certainty equivalent is increasing in \(\theta\) benefiting from the price competition.

### Table 3 Equilibrium Results at the boundary of Regions (v), (vi) and (vii)

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(w_1^*)</th>
<th>(w_2^*)</th>
<th>(Q_1^*)</th>
<th>(Q_2^*)</th>
<th>(Q_1^* + Q_2^*)</th>
<th>(E[U^*])</th>
<th>(\Pi^{CE})</th>
<th>(Var[\Pi])</th>
<th>(E[\Pi_1^*])</th>
<th>(E[\Pi_2^*])</th>
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<tbody>
<tr>
<td>RN</td>
<td>4.0000</td>
<td>1.0000</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>540.00</td>
<td>21600.00</td>
<td>310.00</td>
</tr>
<tr>
<td>0.00001</td>
<td>3.9976</td>
<td>0.9964</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>-0.9946</td>
<td>540.36</td>
<td>540.43</td>
<td>21600.00</td>
<td>309.79</td>
</tr>
<tr>
<td>0.00005</td>
<td>3.9882</td>
<td>0.9822</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>-0.9733</td>
<td>541.59</td>
<td>542.13</td>
<td>21600.90</td>
<td>308.94</td>
</tr>
<tr>
<td>0.00010</td>
<td>3.9769</td>
<td>0.9648</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>-0.9471</td>
<td>543.10</td>
<td>544.20</td>
<td>21603.50</td>
<td>307.92</td>
</tr>
<tr>
<td>0.00013</td>
<td>3.9703</td>
<td>0.9545</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>-0.9317</td>
<td>543.99</td>
<td>545.41</td>
<td>21605.80</td>
<td>307.32</td>
</tr>
</tbody>
</table>

**Non-existence of Equilibrium.** As \(\theta\) increases above 0.00014 there does not exist any pure-strategy equilibrium for the supply chain. The non-existence of pure-strategy equilibrium arises from the discontinuity of the less reliable supplier’s best response function, i.e. jumping from one region to the other. Even though the profit functions of the suppliers are continuous in the wholesale prices set \(S\), there is no guarantee of quasi-concavity, therefore the best response functions might not intersect anywhere.

**Unique Equilibrium in Region (vii).** When \(\theta\) is in a higher range there exists a pure-strategy equilibrium in region (vii), which is shown in Table 4. As the buyer becomes more risk averse he is less sensitive to the wholesale prices in terms of the order quantities. Consequently the price competition between the suppliers is weakened, whereas the benefits of diversification continue to accrue. Both suppliers are able to charge higher wholesale prices as \(\theta\) increases, which decreases
the order quantities to both suppliers. Supplier 1’s expected profit decreases slightly first and then increases when the benefits of increased wholesale prices start outweighing the costs of reduced order quantities, whereas supplier 2’s expected profit increases in $\theta$. The buyer’s expected profit and certainty equivalent are decreasing in $\theta$ because of the higher wholesale prices and risk aversion.

### Table 4 Equilibrium Results in Region (vii)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$w_1^*$</th>
<th>$w_2^*$</th>
<th>$Q_1^*$</th>
<th>$Q_2^*$</th>
<th>$Q_1^* + Q_2^*$</th>
<th>$E[U^*]$</th>
<th>$\Pi^*$</th>
<th>$\text{Var}[\Pi^*]$</th>
<th>$E[\Pi^*_1]$</th>
<th>$E[\Pi^*_2]$</th>
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<tbody>
<tr>
<td>0.0014</td>
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<td>1.3960</td>
<td>99.96</td>
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<td>134.53</td>
<td>-0.4754</td>
<td>531.07</td>
<td>546.15</td>
<td>17640.60</td>
<td>295.51</td>
</tr>
<tr>
<td>0.0015</td>
<td>3.9713</td>
<td>1.4437</td>
<td>95.88</td>
<td>34.07</td>
<td>129.95</td>
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<td>533.41</td>
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<tr>
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<td>122.43</td>
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<td>510.78</td>
<td>15605.80</td>
<td>295.34</td>
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<td>1.5914</td>
<td>86.66</td>
<td>32.66</td>
<td>119.31</td>
<td>-0.4193</td>
<td>482.92</td>
<td>500.60</td>
<td>15775.30</td>
<td>296.44</td>
</tr>
</tbody>
</table>

**Multiple Equilibria in Regions (vii) and (iv).** As $\theta$ increases above a certain level multiple equilibria appear since the suppliers’ best response functions jump from region (vii) to region (iv) and the best response functions intersect in both regions. Thus there exist two pure-strategy equilibria for the supply chain, one in region (vii) and the other one in region (iv). The multiple equilibria can be observed in Table 5 for cases when $0.0019 \leq \theta \leq 0.0023$. As we can see the equilibrium wholesale prices are higher in region (iv) than in region (vii), whereas the total order quantity is lower in region (iv) than in region (vii). Comparing the equilibria in two regions, for the suppliers the benefits from higher wholesale prices outweigh the cost of lower order quantities. As a result the suppliers’ expected profits are higher in region (iv) than in region (vii). In contrast, the buyer’s certainty equivalent is lower in region (iv) than in region (vii). In both regions the suppliers expected profits are increasing in $\theta$. The suppliers benefit from exploiting the buyer’s risk and the more risk averse the buyer is the more power the suppliers have to exploit the risk.

**Unique Equilibrium in Region (iv).** Table 6 shows the equilibrium results in region (iv) for a high level of risk aversion. In this region the buyer is very risk averse and he is insensitive to the wholesale prices in terms of order quantities. As $\theta$ increases the price competition between the two suppliers is largely weakened and the benefits of diversification outweigh the benefits of competition. As we can see from the table, the equilibrium wholesale prices increase with $\theta$. Supplier
Table 5  Equilibrium Results in Regions (vii) and (iv)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$w_1^*$</th>
<th>$w_2^*$</th>
<th>$Q_1^*$</th>
<th>$Q_2^*$</th>
<th>$Q_1^* + Q_2^*$</th>
<th>$E[U^*]$</th>
<th>$\Pi^{CE*}$</th>
<th>$E[\Pi^*]$</th>
<th>$Var[\Pi^*]$</th>
<th>$E[\Pi_1^s]$</th>
<th>$E[\Pi_2^s]$</th>
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<tbody>
<tr>
<td>Region (vii)</td>
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<td></td>
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<tr>
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<td>84.32</td>
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<td>116.54</td>
<td>-0.4078</td>
<td>472.10</td>
<td>491.04</td>
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<td>297.95</td>
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<td>1.6931</td>
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<td>114.05</td>
<td>-0.3972</td>
<td>461.69</td>
<td>482.01</td>
<td>16602.90</td>
<td>299.80</td>
<td>16.40</td>
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<td>1.7447</td>
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<td>31.37</td>
<td>111.82</td>
<td>-0.3873</td>
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<td>107.97</td>
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<td>432.62</td>
<td>457.52</td>
<td>18231.40</td>
<td>306.87</td>
<td>18.63</td>
</tr>
<tr>
<td>Region (iv)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>402.67</td>
<td>427.27</td>
<td>18081.60</td>
<td>336.52</td>
<td>23.69</td>
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</tbody>
</table>

1’s equilibrium order quantity is decreasing in $\theta$ and supplier 2’s equilibrium order quantity is increasing in $\theta$, but both change at a decreasing rate. Facing very high wholesale prices set by the suppliers, a highly risk-averse buyer still diversifies his portfolio in order to reduce the profit variability. In consequence, both the buyer’s expected profit and profit variance are decreasing as he becomes more risk-averse. Even though competing with each other, the suppliers do not drive the equilibrium wholesale prices to a very low level. In contrast, the suppliers benefit from the risk exploitation by setting high wholesale prices without the fear of losing too much business, since a highly risk-averse buyer is insensitive to the wholesale prices and has strong incentives for diversification. The suppliers’ power to explore the buyer’s risk is strengthened as the buyer becomes more risk-averse.

Table 6  Equilibrium Results in Region (iv)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>$w_1^*$</th>
<th>$w_2^*$</th>
<th>$Q_1^*$</th>
<th>$Q_2^*$</th>
<th>$Q_1^* + Q_2^*$</th>
<th>$E[U^*]$</th>
<th>$\Pi^{CE*}$</th>
<th>$E[\Pi^*]$</th>
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6. Conclusions

We have investigated the question of what drives supply diversification and order inflation when buyer’s demand is deterministic under the possibility of supply disruptions. Our research differs from the random yield and supply disruption literature in two aspects: First, we uncouple the convoluted uncertainties of demand and supply which are combined to drive the diversification behavior of the buyer and shed the light on the risk aversion that drives this behavior when the buyer’s demand is deterministic under the possibility of supply disruptions. Second, we parametrically incorporate risk aversion into a supply disruption model with strategic suppliers competing to supply a buyer, where the equilibrium behavior of the supply chain participants are analyzed in a game setting. The framework proposed in this paper and the results obtained motivate a number of extensions.

- Although the exponential utility function is commonly used in economics and finance and widely applied in risk-averse decision analysis practice, it has a specific kind of curvature and implies a particular risk attitude, i.e., constant absolute risk aversion. Our model also bears the same practical challenges as other models in terms of determining the parameters for the exponential utility functions. It would be interesting to explore the impact of other risk measures such as mean-variance tradeoff analysis and Value At Risk on the equilibrium behavior of the supply chain participants.

- Another extension would be to relax our assumption of independently distributed disruptions. Babich et al. (2007) analyze the problem of multiple all-or-nothing suppliers with correlated default risk competing to supply a risk-neutral buyer. Their results show that the buyer may prefer positively correlated defaults because the price competition benefits, induced by high correlation, may more than offset the diversification benefits. It is interesting to explore how this affect will change with risk aversion.

Acknowledgments

The authors thank Rodney Parker for his insightful comments. Both authors gratefully acknowledge the financial support of the University of Chicago Booth School of Business.
Appendix

Proof of Proposition 1. From (7a) we know that the buyer’s expected profit is linear in order quantities. Then:

If \(-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1) > 0\) and \(-\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2) > 0\), that is \(w_1 < \beta_2 p\) and \(w_2 < \beta_1 p\), then the optimal order quantities are \((D, D)\).

If \(-\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2) < 0\) and \(-\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2) < -\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1)\), that is \(w_2 > \beta_1 p\) and \(\beta_1 (p - w_1) > \beta_2 (p - w_2)\), then the optimal order quantities are \((D, 0)\).

If \(-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1) < 0\) and \(-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1) < -\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2)\), that is \(w_1 > \beta_2 p\) and \(\beta_1 (p - w_1) < \beta_2 (p - w_2)\), then the optimal order quantities are \((0, D)\).

If \(-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1) = -\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2) < 0\), that is \(w_1 > \beta_2 p, w_2 > \beta_1 p\) and \(\beta_1 (p - w_1) = \beta_2 (p - w_2)\), then the optimal order quantities are \((\alpha D, (1 - \alpha) D)\) for \(\alpha \in [0, 1]\).

If \(-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1) > 0\) and \(-\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2) = 0\), that is \(w_1 < \beta_2 p\) and \(w_2 = \beta_1 p\), then the optimal order quantities are \((D, D)\) for any \(\alpha \in [0, 1]\).

If \(-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1) = 0\) and \(-\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2) > 0\), that is \(w_1 = \beta_2 p\) and \(w_2 < \beta_1 p\), then the optimal order quantities are \((\alpha D, D)\) for any \(\alpha \in [0, 1]\).

If \(-\beta_1 \beta_2 w_1 + \beta_1 \beta_2 (p - w_1) = 0\) and \(-\beta_1 \beta_2 w_2 + \beta_1 \beta_2 (p - w_2) = 0\), that is \(w_1 = \beta_2 p\) and \(w_2 = \beta_1 p\), then the optimal order quantities are \((\alpha D, \delta D)\) for any \(\alpha, \delta \in [0, 1]\) and \(\alpha + \delta \geq 1\). □

Proof of Proposition 2. In the following proof we consider all seven possible (disjoint) cases of the optimal order quantities. For each case we find a set of necessary conditions under which the order quantities are optimal. We show that the seven sets of conditions partition \(\Omega\) since they are pairwise disjoint and collectively exhaustive. Furthermore, for any fixed set of parameters the optimal order quantities \((Q_1, Q_2)\) are unique in \(\Omega\). Therefore the conditions we obtain are also sufficient.

Rewriting (9) and (10) we obtain

\[
M_1(Q) = -\theta \beta_1 \beta_2 w_1 \exp[-\theta (pD - w_1 Q_1 - w_2 Q_2)] + \theta \beta_1 \beta_2 (p - w_1) \exp[-\theta (p - w_1) Q_1],
\]

\[
M_2(Q) = -\theta \beta_1 \beta_2 w_2 \exp[-\theta (pD - w_1 Q_1 - w_2 Q_2)] + \theta \beta_1 \beta_2 (p - w_2) \exp[-\theta (p - w_2) Q_2].
\]

We solve the following KKT conditions simultaneously by considering all boundary and interior point solutions, identifying the parameter conditions for each case of order quantities.

\[Q_1 (M_1(Q) + \lambda_1 - \lambda_2) = 0, \quad M_1(Q) + \lambda_1 - \lambda_2 \leq 0, \quad (13a)\]
\[ Q_2(M_2(Q) + \lambda_1 - \lambda_3) = 0, \quad M_2(Q) + \lambda_1 - \lambda_3 \leq 0, \quad (13b) \]
\[ \lambda_1(D - Q_1 - Q_2) = 0, \quad D - Q_1 - Q_2 \leq 0, \quad (13c) \]
\[ \lambda_2(Q_1 - D) = 0, \quad Q_1 - D \leq 0, \quad (13d) \]
\[ \lambda_3(Q_2 - D) = 0, \quad Q_2 - D \leq 0, \quad (13e) \]
\[ Q_1, Q_2, \lambda_1, \lambda_2, \lambda_3 \geq 0. \quad (13f) \]

(i) If \((Q_1^*, Q_2^*) = (D, D)\), then \(\lambda_1 = 0\) from (13c). From (13a) and (13b) we have
\[
M_1(Q) - \lambda_2 = 0, \quad M_2(Q) - \lambda_3 = 0.
\]

We can always find feasible \(\lambda_2, \lambda_3\), i.e., nonnegative, provided that \(M_1(Q) \geq 0\) and \(M_2(Q) \geq 0\). Under these conditions, \(\lambda_2 = M_1(Q)\) and \(\lambda_3 = M_2(Q)\) satisfy the KKT conditions. Rewriting \(M_1(Q) \geq 0\) and \(M_2(Q) \geq 0\) they become
\[
-\theta \beta_1 \beta_2 w_1 \exp \left[ -\theta (p - w_1 - w_2) D \right] + \theta \beta_1 \beta_2 w_2 \exp \left[ -\theta (p - w_1) D \right] \geq 0, \quad (14)
\]
\[
-\theta \beta_1 \beta_2 w_2 \exp \left[ -\theta (p - w_1 - w_2) D \right] + \theta \beta_1 \beta_2 w_1 \exp \left[ -\theta (p - w_2) D \right] \geq 0. \quad (15)
\]

Simplifying (14) we have
\[
\exp [\theta w_2 D] \leq \frac{\beta_2 (p - w_1)}{\beta_2 w_1}. \quad (16)
\]
Since \(\exp [\theta w_2 D] > 1\), equation (16) implies that \(\frac{\beta_2 (p - w_1)}{\beta_2 w_1} > 1\), which is equivalent to \(w_1 < \beta_2 p\). Taking the \(\ln\) of both sides of (16), we obtain
\[
D \leq \frac{1}{\theta w_2} \ln \left[ \frac{\beta_2 (p - w_1)}{\beta_2 w_1} \right] = d_2(w).
\]

Similarly, from (15) we obtain \(w_2 < \beta_1 p\) and
\[
D \leq \frac{1}{\theta w_1} \ln \left[ \frac{\beta_1 (p - w_2)}{\beta_1 w_2} \right] = d_1(w).
\]

Thus, \((Q_1^*, Q_2^*) = (D, D) \Rightarrow w \in \Omega_1\).

(ii) If \((Q_1^*, Q_2^*) = (D, 0)\), then \(\lambda_3 = 0\) from (13e). From (13a) and (13b) we have
\[
M_1(Q) + \lambda_1 - \lambda_2 = 0, \quad M_2(Q) + \lambda_1 \leq 0.
\]
We can always find feasible $\lambda_1, \lambda_2$, i.e., nonnegative, provided that $M_2(Q^*) \leq 0$ and $M_2(Q^*) \leq M_1(Q^*)$. Under these conditions, $\lambda_1 = -M_2(Q^*)$ and $\lambda_2 = M_1(Q^*) - M_2(Q^*) \geq 0$ satisfy the KKT conditions. Rewriting and simplifying $M_2(Q^*) \leq 0$ and $M_2(Q^*) \leq M_1(Q^*)$ they become

\[
\exp[\theta(p - w_1)] \leq \frac{\beta_1 w_2}{\beta_1 (p - w_2)}, \tag{17}
\]

\[
\exp[\theta(p - w_1)] \leq \frac{\beta_1 (\beta_2 w_2 + \beta_2 p - w_1)}{\beta_1 \beta_2 (p - w_2)}. \tag{18}
\]

Since $\exp[\theta(p - w_1)] D > 1$, equations (17) and (18) imply that $\frac{\beta_1 w_2}{\beta_1 (p - w_2)} > 1$ and $\frac{\beta_1 (\beta_2 w_2 + \beta_2 p - w_1)}{\beta_1 \beta_2 (p - w_2)} > 1$ respectively, which are equivalent to $w_2 > \beta_2 p$ and $\beta_1(p - w_1) > \beta_2(p - w_2)$. Taking the ln of both sides of (17) and (18), we obtain

\[
D \leq \frac{1}{\theta(p - w_1)} \ln \left[ \frac{\beta_1 w_2}{\beta_1 (p - w_2)} \right] = d_3(w), \tag{19}
\]

\[
D \leq \frac{1}{\theta(p - w_1)} \ln \left[ \frac{\beta_1 (\beta_2 w_2 + \beta_2 p - w_1)}{\beta_1 \beta_2 (p - w_2)} \right] = d_4(w). \tag{20}
\]

From (19) and (20) we observe that if $w_1 < \beta_2 p$ we have $\frac{\beta_1 w_2}{\beta_1 (p - w_2)} > \frac{\beta_1 (\beta_2 w_2 + \beta_2 p - w_1)}{\beta_1 \beta_2 (p - w_2)}$, which means condition (19) is tighter; if $w_1 \geq \beta_2 p$ we have $\frac{\beta_1 (\beta_2 w_2 + \beta_2 p - w_1)}{\beta_1 \beta_2 (p - w_2)} \geq \frac{\beta_1 w_2}{\beta_1 (p - w_2)}$, which means condition (20) is tighter. Thus, $(Q_1^*, Q_2^*) = (D, 0) \Rightarrow w \in \Omega_2$.

(iii) Case (iii) is symmetric to case (ii). By following the same argument as above, we have $(Q_1^*, Q_2^*) = (0, D) \Rightarrow w \in \Omega_3$.

(iv) If $0 < Q_1^* < D, 0 < Q_2^* < D$ and $Q_1^* + Q_2^* = D$, then $\lambda_2 = \lambda_3 = 0$ from (13d) and (13e). From (13a) and (13b) we have

\[
M_1(Q^*) + \lambda_1 = 0,
\]

\[
M_2(Q^*) + \lambda_1 = 0.
\]

We can always find feasible $\lambda_1$, i.e., nonnegative, provided that $M_1(Q^*) \leq 0, M_2(Q^*) \leq 0, M_1(Q^*) = M_2(Q^*)$, $0 < Q_1^* < D$ and $Q_1^* + Q_2^* = D$. Under these conditions, $\lambda_1 = -M_1(Q^*) = -M_2(Q^*)$ satisfies the KKT conditions. Rewriting and simplifying $M_1(Q^*) \leq 0$ and $M_2(Q^*) \leq 0$ with $Q_1^* = D - Q_2^*$ they become

\[
\exp[\theta(p - w_2)(D - Q_1^*)] \leq \frac{\beta_2 w_1}{\beta_2 (p - w_1)}, \tag{21}
\]

\[
\exp[\theta(p - w_1)Q_1^*] \leq \frac{\beta_1 w_2}{\beta_1 (p - w_2)}. \tag{22}
\]

Since $\exp[\theta(p - w_2)(D - Q_1^*)] > 1$, equation (21) implies that $\frac{\beta_2 w_1}{\beta_2 (p - w_1)} > 1$, which is equivalent to $w_1 > \beta_2 p$. Taking the ln of both sides of (21), we obtain

\[
Q_1^* \geq D - \frac{1}{\theta(p - w_2)} \ln \left[ \frac{\beta_2 w_1}{\beta_2 (p - w_1)} \right] \tag{23}
\]
Since \( \exp[\theta(p - w_1)Q_1^*] > 1 \), equation (22) implies that \( \frac{\beta_1w_2}{\beta_1(p-w_2)} > 1 \), which is equivalent to \( w_2 > \overline{\beta}_1p \). Taking the \( \ln \) of both sides of (22), we obtain

\[
Q_1^* \leq \frac{1}{\theta(p-w_1)} \ln \left[ \frac{\beta_1w_2}{\beta_1(p-w_2)} \right].
\] (24)

From (23) and (24) follows that

\[
D \leq \frac{1}{\theta(p-w_2)} \ln \left[ \frac{\beta_2w_1}{\beta_2(p-w_1)} \right] + \frac{1}{\theta(p-w_1)} \ln \left[ \frac{\beta_1w_2}{\beta_1(p-w_2)} \right] = d_7(w).
\]

Rewriting \( f(Q_1^*) = M_1(Q^*) - M_2(Q^*) \) with \( Q_1^* = D - Q_2^* \) it becomes

\[
f(Q_1^*) = \theta\beta_1\beta_2(w_2 - w_1) \exp[-\theta(pD - w_2D - w_1Q_1^* + w_2Q_1^*)] \\
+ \theta\beta_1\overline{\beta}_2(p - w_1) \exp[-\theta(p - w_1)Q_1^*] - \theta\overline{\beta}_1\beta_2(p - w_2) \exp[-\theta(p - w_2)(D - Q_1^*)].
\] (25)

Since \( \frac{\partial f(Q_1^*)}{\partial Q_1^*} < 0 \), from \( f(Q_1^*) = M_1(Q^*) - M_2(Q^*) = 0 \) and \( 0 < Q_1^* < D \) follows that \( f(Q_1^*) |_{Q_1^*=0} > 0 \) and \( f(Q_1^*) |_{Q_1^*=D} < 0 \). Rewriting and simplifying the above inequalities they become

\[
\exp[\theta(p-w_2)D] > \frac{\beta_2(\beta_1w_1 + \overline{\beta}_1p - w_1)}{\beta_2\beta_1(p-w_1)},
\] (26)

\[
\exp[\theta(p-w_1)D] > \frac{\beta_1(\beta_2w_2 + \overline{\beta}_2p - w_1)}{\beta_1\beta_2(p-w_2)}.
\] (27)

From (26) and (27) we observe that if \( \beta_1(p-w_1) \geq \beta_2(p-w_2) \) we have \( \frac{\beta_2(\beta_1w_1 + \overline{\beta}_1p - w_1)}{\beta_2\beta_1(p-w_1)} > \frac{\beta_1(\beta_2w_2 + \overline{\beta}_2p - w_1)}{\beta_1\beta_2(p-w_2)} \), and (26) is redundant because \( \exp[\theta(p-w_2)D] > 1 \); and if \( \beta_1(p-w_1) < \beta_2(p-w_2) \) we have \( \frac{\beta_2(\beta_1w_1 + \overline{\beta}_1p - w_1)}{\beta_2\beta_1(p-w_1)} < \frac{\beta_1(\beta_2w_2 + \overline{\beta}_2p - w_1)}{\beta_1\beta_2(p-w_2)} \), and (27) is redundant because \( \exp[\theta(p-w_1)D] > 1 \). Taking the \( \ln \) of both sides of (26) and (27), we obtain

if \( \beta_1(p-w_1) \geq \beta_2(p-w_2) \),

\[
D > \frac{1}{\theta(p-w_1)} \ln \left[ \frac{\beta_1(\beta_2w_2 + \overline{\beta}_2p - w_1)}{\beta_1\beta_2(p-w_2)} \right] = d_4(w),
\]

if \( \beta_1(p-w_1) < \beta_2(p-w_2) \),

\[
D > \frac{1}{\theta(p-w_2)} \ln \left[ \frac{\beta_2(\beta_1w_1 + \overline{\beta}_1p - w_1)}{\beta_2\beta_1(p-w_1)} \right] = d_6(w).
\]

Thus, \( 0 < Q_1^* < D, 0 < Q_2^* < D \) and \( Q_1^* + Q_2^* = D \) \( \Rightarrow w \in \Omega_4 \).

(v) If \( Q_1^* = D \) and \( 0 < Q_2^* < D \), then \( \lambda_1 = \lambda_3 = 0 \) from (13c) and (13e). From (13a) and (13b) we have

\[
M_1(Q^*) - \lambda_2 = 0,
\]

\[
M_2(Q^*) = 0.
\]
We can always find feasible $\lambda_2$, i.e., nonnegative, provided that $M_1(Q^*) \geq 0$, $M_2(Q^*) = 0$ and $0 < Q_2^* < D$. Under these conditions, $\lambda_2 = M_1(Q^*)$ satisfies the KKT conditions. Rewriting and simplifying $M_1(Q^*) \geq 0$ and $M_2(Q^*) = 0$ they become

$$
\exp[\theta w_2 Q_2^*] \leq \frac{\beta_2 (p - w_1)}{\beta w_1},
$$

(28)

$$
\exp[-\theta (pD - w_1 D - pQ_2^*]) = \frac{\beta_1 (p - w_2)}{\beta w_2}.
$$

(29)

Since $\exp[\theta w_2 Q_2^*] > 1$, equation (28) implies that $\frac{\beta_2 (p - w_1)}{\beta w_1} > 1$, which is equivalent to $w_1 < \frac{\beta_2}{\beta} p$. Taking the ln of both sides of (28) and (29), we obtain

$$
Q_2^* \leq \frac{1}{\theta w_2} \ln \left[ \frac{\beta_2 (p - w_1)}{\beta w_1} \right].
$$

(30)

$$
Q_2^* = \frac{1}{\theta p} \ln \left[ \frac{\beta_1 (p - w_2)}{\beta w_2} \right] + \frac{p - w_1}{p} D.
$$

(31)

From (30) and (31) follows that

$$
D \leq \frac{p}{\theta w_2 (p - w_1)} \ln \left[ \frac{\beta_2 (p - w_1)}{\beta w_1} \right] + \frac{1}{\theta (p - w_1)} \ln \left[ \frac{\beta_1 w_2}{\beta_1 (p - w_2)} \right] = d_5(w).
$$

Plugging (31) into $Q_2^* > 0$ and $Q_2^* < D$, we obtain

$$
D > \frac{1}{\theta (p - w_1)} \ln \left[ \frac{\beta_1 w_2}{\beta_1 (p - w_2)} \right] = d_3(w),
$$

(32)

$$
D > \frac{1}{\theta w_1} \ln \left[ \frac{\beta_1 (p - w_2)}{\beta w_2} \right] = d_1(w).
$$

(33)

From (32) and (33) we observe that if $w_2 < \beta_1 p$ we have $\frac{\beta_1 w_2}{\beta_1 (p - w_2)} < 1 < \frac{\beta_1 (p - w_2)}{\beta w_2}$; and (32) is redundant because $\ln \left[ \frac{\beta_1 w_2}{\beta_1 (p - w_2)} \right] < 0$; and if $w_2 \geq \beta_1 p$ we have $\frac{\beta_1 (p - w_2)}{\beta w_2} \leq 1 \leq \frac{\beta_1 w_2}{\beta_1 (p - w_2)}$, and (33) is redundant because $\ln \left[ \frac{\beta_1 (p - w_2)}{\beta w_2} \right] \leq 0$. Thus, $Q_1^* = D$ and $0 < Q_2^* < D \Rightarrow w \in \Omega_5$.

(vi) Case (vi) is symmetric to case (v). By following the same argument as above, we have $0 < Q_1^* < D, Q_2^* = D \Rightarrow w \in \Omega_6$.

(vii) If $0 < Q_1^* < D, 0 < Q_2^* < D$ and $Q_1^* + Q_2^* > D$, then $\lambda_1 = \lambda_2 = \lambda_3 = 0$ from (13c), (13d) and (13e). From (13a) and (13b) we have $M_1(Q) = 0$ and $M_2(Q) = 0$. Solving the two equations we obtain

$$
Q_1^* = \frac{p(p - w_2) D + \frac{\theta}{\beta} \ln \left[ \frac{\beta_2 (p - w_1)}{\beta_2 w_1} \right] - \frac{p^2}{\beta_2} \ln \left[ \frac{\beta_2 (p - w_2)}{\beta_2 w_2} \right]}{p^2 - w_1 w_2},
$$

(34)

$$
Q_2^* = \frac{p(p - w_1) D + \frac{\theta}{\beta} \ln \left[ \frac{\beta_1 (p - w_2)}{\beta_1 w_2} \right] - \frac{p^2}{\beta_1} \ln \left[ \frac{\beta_1 (p - w_1)}{\beta_1 w_1} \right]}{p^2 - w_1 w_2}.
$$

(35)
Plugging (34) into $Q_1 > 0$ and $Q_1 < D$, we obtain
\[
D > \frac{w_2}{\theta p(p-w_2)} \ln \left[ \frac{\beta_1(p-w_2)}{\beta_1 w_2} \right] + \frac{1}{\theta(p-w_2)} \ln \left[ \frac{\beta_2 w_1}{\beta_2(p-w_2)} \right],
\] (36)
\[
D > \frac{p}{\theta w_1(p-w_1)} \ln \left[ \frac{\beta_1(p-w_1)}{\beta_1 w_1} \right] + \frac{1}{\theta(p-w_1)} \ln \left[ \frac{\beta_2 w_2}{\beta_2(p-w_2)} \right] = d_6(w).
\] (37)

Plugging (35) into $Q_2 > 0$ and $Q_2 < D$, we obtain
\[
D > \frac{w_1}{\theta p(p-w_1)} \ln \left[ \frac{\beta_2(p-w_1)}{\beta_2 w_1} \right] + \frac{1}{\theta(p-w_1)} \ln \left[ \frac{\beta_1 w_2}{\beta_1(p-w_2)} \right],
\] (38)
\[
D > \frac{p}{\theta w_1(p-w_2)} \ln \left[ \frac{\beta_1(p-w_2)}{\beta_1 w_2} \right] + \frac{1}{\theta(p-w_2)} \ln \left[ \frac{\beta_2 w_1}{\beta_2(p-w_1)} \right] = d_9(w).
\] (39)

Plugging (34) and (35) into $Q_1 + Q_1 > D$, we obtain
\[
D \geq \frac{1}{\theta(p-w_2)} \ln \left[ \frac{\beta_2 w_1}{\beta_2(p-w_2)} \right] + \frac{1}{\theta(p-w_2)} \ln \left[ \frac{\beta_1 w_2}{\beta_1(p-w_2)} \right] = d_7(w).
\] (40)

From (36)-(40) we observe that if $w_1 < \beta_2 p$ and $w_2 < \beta_1 p$ we have $\frac{\beta_1(p-w_1)}{\beta_1 w_1} > 1$, $\frac{\beta_2(p-w_2)}{\beta_2 w_2} > 1$, $\frac{\beta_1 w_2}{\beta_1(p-w_2)} < 1$ and $\frac{\beta_2 w_1}{\beta_2(p-w_2)} < 1$, so (40) is the redundant condition, (37) is tighter than (38) and (39) is tighter than (36) because $p > w_1, w_2$. Similarly, if $w_1 \geq \beta_2 p$ and $w_2 < \beta_1 p$ condition (39) is the tightest, if $w_1 < \beta_2 p$ and $w_2 \geq \beta_1 p$ condition (37) is the tightest, and if $w_1 \geq \beta_2 p$ and $w_2 \geq \beta_1 p$ condition (40) is the tightest. Thus, $0 < Q_1^* < D, 0 < Q_2^* < D$ and $Q_1^* + Q_2^* > D \Rightarrow w \in \Omega_7$.

In order to prove that the seven sets of conditions $(\Omega_1, \ldots, \Omega_7)$ partition $\Omega$ we consider four cases: $w_1 < \beta_2 p, w_2 < \beta_1 p; w_1 \geq \beta_2 p, w_2 < \beta_1 p; w_1 < \beta_2 p, w_2 \geq \beta_1 p; w_1 \geq \beta_2 p, w_2 \geq \beta_1 p$. In each case the seven sets of conditions are pairwise disjoint and collectively exhaustive. For the sake of brevity, we only give the proof for the case $w_1 < \beta_2 p, w_2 < \beta_1 p$ and the rest of the proof can be done in the same spirit.

When $w_1 < \beta_2 p, w_2 < \beta_1 p$, conditions sets $\Omega_2 = \Omega_3 = \Omega_4 = \emptyset$ and $w \in \Omega_1 \Leftrightarrow D \leq d_1(w), D \leq d_2(w); w \in \Omega_5 \Leftrightarrow d_1(w) < D \leq d_6(w); w \in \Omega_6 \Leftrightarrow d_2(w) < D \leq d_9(w)$ and $w \in \Omega_7 \Leftrightarrow D > d_4(w), D > d_9(w)$.

If $d_1(w) < d_2(w)$, that is $\frac{1}{\theta w_1} \ln \left[ \frac{\beta_1(p-w_2)}{\beta_1 w_2} \right] < \frac{1}{\theta w_2} \ln \left[ \frac{\beta_2(p-w_1)}{\beta_2 w_1} \right]$ we have $d_1(w) < d_8(w)$ since
\[
d_9(w) - d_1(w) = \frac{p}{\theta w_1(p-w_2)} \ln \left[ \frac{\beta_1(p-w_2)}{\beta_1 w_2} \right] - \frac{1}{\theta(p-w_2)} \ln \left[ \frac{\beta_2(p-w_1)}{\beta_2 w_1} \right] - \frac{1}{\theta w_1} \ln \left[ \frac{\beta_1(p-w_2)}{\beta_1 w_2} \right] = \frac{w_2}{\theta w_1(p-w_2)} (d_1(w) - d_2(w)) < 0.
\]

Similarly we obtain $d_2(w) < d_8(w)$. Therefore we have $d_9(w) < d_1(w) < d_2(w) < d_8(w)$, which imply that $\Omega_6 = \emptyset$ and $w \in \Omega_1 \Leftrightarrow D \leq d_1(w); w \in \Omega_5 \Leftrightarrow d_1(w) < D \leq d_8(w)$ and $w \in \Omega_7 \Leftrightarrow D > d_8(w)$. 

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If \( d_1(w) \geq d_2(w) \) we obtain \( d_6(w) \leq d_2(w) \leq d_1(w) \leq d_9(w) \), which implies that \( \Omega_5 = \emptyset \) and \( w \in \Omega_1 \Leftrightarrow D \leq d_2(w); w \in \Omega_6 \Leftrightarrow d_2(w) < D \leq d_9(w) \) and \( w \in \Omega_7 \Leftrightarrow D > d_9(w) \).

Then it follows that the seven sets of conditions are pairwise disjoint and collectively exhaustive. Moreover, we know that the optimal order quantities \( Q_1^*, Q_5^* \) exist uniquely for any given \( w \in \Omega \). Thus the necessary conditions as detailed above are also sufficient. □

Proof of Lemma 1. From Proposition 2 we know that the sets \( \Omega_1 \ldots \Omega_7 \) partition \( \Omega \). Each set is continuous on \( w_1 \) and \( w_2 \). In the following proof we show that \((Q_1^*, Q_5^*)\) are continuous on each set and for any pair of neighboring sets \((Q_1^*, Q_5^*)\) are continuous at their boundary.

It is straightforward to see that \((Q_1^*, Q_5^*)\) are continuous on set \( \Omega_1, \Omega_2 \) and \( \Omega_4 \). From (25) and \( Q_4^* + Q_5^* = D \) it follows that \((Q_1^*, Q_5^*)\) are continuous on set \( \Omega_4 \) since \( Q_1^* \) can be found by setting \( f(Q_1^*) = 0 \). Equation (31) gives the analytical solution for \( Q_5^* \) in set \( \Omega_5 \), which is a function of \( w_1 \) and \( w_2 \). Similar result can be obtained for set \( \Omega_6 \). Equation (34) and (35) give the analytical solutions for \((Q_1^*, Q_5^*)\) in set \( \Omega_7 \), which are both functions of \( w_1 \) and \( w_2 \). Hence optimal order quantities \((Q_1^*, Q_5^*)\) are continuous on each set.

For the sake of brevity, we only give the proof for set \( \Omega_1 \) and all its neighboring sets. The rest of the proof can be done in the same spirit. The neighboring sets of \( \Omega_1 \) are \( \Omega_5, \Omega_6 \) and \( \Omega_7 \). The boundary of \( \Omega_1 \) and \( \Omega_5 \) is given by \( D = d_1(w) \). Substituted in (31), the optimal order quantity \( Q_5^* \) in the closure of \( \Omega_5 \) is given by

\[
Q_5^* = \frac{w_1}{p} d_1(w) + \frac{w_2}{p} D = \frac{w_1}{p} D + \frac{w_2}{p} D = D,
\]

which is the same as \( Q_5^* \) in \( \Omega_1 \). Similarly, on the boundary of \( \Omega_1 \) and \( \Omega_6 \) we obtain \( Q_1^* = D \) in the closure of \( \Omega_6 \), which is the same as \( Q_1^* \) in \( \Omega_1 \). In the proof of region (vii) in Proposition 2 we know that condition \( d_1(w) < d_2(w) \) implies \( d_6(w) < d_1(w) < d_2(w) < d_8(w) \), and condition \( d_1(w) \geq d_2(w) \) implies \( d_9(w) \leq d_2(w) \leq d_1(w) \leq d_9(w) \). Then the boundary of \( \Omega_1 \) and \( \Omega_7 \) is given by \( D = d_2(w) = d_8(w) \) when \( d_1(w) < d_2(w) \). Substituted in (34) and (35) the optimal order quantities \((Q_1^*, Q_5^*)\) in \( \Omega_7 \) are given by

\[
Q_1^* = \frac{p(p - w_2)}{p^2 - w_1 w_2} D + \frac{w_2(p - w_1)}{p^2 - w_1 w_2} = \frac{p(p - w_2)D + w_2(p - w_1)D}{p^2 - w_1 w_2} = D,
\]

\[
Q_5^* = \frac{p(p - w_1)}{p^2 - w_1 w_2} D - \frac{p(p - w_1)D}{p^2 - w_1 w_2} = \frac{p(p - w_1)D - p(p - w_1)D + (p^2 - w_1 w_2)D}{p^2 - w_1 w_2} = D,
\]

which are the same as \((Q_1^*, Q_5^*)\) in \( \Omega_1 \). Similar results hold for the case when \( d_1(w) \geq d_2(w) \). So the optimal order quantities are continuous at the boundary of \( \Omega_1 \) and all its neighboring sets.
Similar results can be obtained for all other sets. Thus the buyer’s optimal order quantities \((Q_1^*, Q_2^*)\) are continuous on the wholesale prices \(w_1\) and \(w_2\). □

Proof of Proposition 3. Suppose that \(\beta_2 \beta_1 p - c_2 \geq 0\). We have \(\frac{\beta_1}{\beta_1} \leq \beta_2 p\) and \(\frac{\beta_2}{\beta_2} \leq \beta_1 p\). In order to ensure that the suppliers obtain nonnegative profits we need \(w_i \geq \frac{c_i}{\beta_i}\). In the bottom panel of Figure 1 we can see that any wholesale prices with \(w_1\) right to \(\frac{\beta_1}{\beta_1}\) (including \(\frac{c_1}{\beta_1}\)) and \(w_2\) above \(\frac{\beta_2}{\beta_2}\) (including \(\frac{c_2}{\beta_2}\)) are feasible. Note that \((w_1^*, w_2^*, Q_1^*, Q_2^*) = (\beta_2 p, \beta_1 p, D, D)\) is indeed a Nash equilibrium. Neither supplier can gain by deviating from these wholesale prices. By raising \(w_1^*\) supplier 1 will then make zero profit because the buyer will not order from him, and by lowering \(w_1^*\) he will then make less profit because the buyer orders the same \(D\) from him. Similar arguments can be made for supplier 2. What remains is to show that there can be no other Nash equilibrium.

If \(w_1^* < \beta_2 p\) and \(w_2^* < \beta_1 p\), i.e., the wholesale prices are in region (i), we know that \((Q_1^*, Q_2^*) = (D, D)\). Supplier \(i\) can make more profit by setting his wholesale price equal to \(w_i^* + \varepsilon\) for \(\varepsilon > 0\) while having the same order quantity \(D\). Thus, these wholesale prices choices could not constitute a Nash equilibrium.

If \(w_1^* \geq \beta_2 p\) and \(\beta_1 (p - w_1^*) < \beta_2 (p - w_2^*)\), i.e., the wholesale prices are in regions (iii) or (vi), the order quantity to supplier 2 is \(D\). In this case, supplier 2 can make more profit by setting his wholesale price equal to \(w_2^* + \varepsilon\) for \(\varepsilon > 0\) while having the same order quantity \(D\). Thus, none of these wholesale prices choices could constitute a Nash equilibrium. Similar arguments follow for the case when \(w_2^* \geq \beta_1 p\) and \(\beta_1 (p - w_1^*) > \beta_2 (p - w_2^*)\).

If \(w_1^* > \beta_2 p\), \(w_2^* > \beta_1 p\) and \(\beta_1 (p - w_1^*) = \beta_2 (p - w_2^*)\), i.e., the wholesale prices are in region (iv), the sum of the order quantities to supplier 1 and 2 is \(D\). In this case, any supplier \(i\) with order quantity lower than \(D\) can make more profit by setting his wholesale price to \(w_i^* - \varepsilon\), for \(\varepsilon > 0\) in order to have order quantity \(D\). Thus, none of these wholesale prices choices is an equilibrium.

We have just ruled out all possible wholesale prices other than \(w_1^* = \beta_2 p\) and \(w_2^* = \beta_1 p\). Thus the equilibrium wholesale prices are \((w_1^*, w_2^*) = (\beta_2 p, \beta_1 p)\) and the corresponding order quantities are \((Q_1^*, Q_2^*) = (D, D)\).

Suppose that \(\beta_2 \beta_1 p - c_2 < 0\). The feasible wholesale price \(w_2\) has to satisfy \(w_2 \geq \frac{c_2}{\beta_2} > \beta_1 p\). The intersection of \(w_2 = \frac{c_2}{\beta_2}\) and region (iv) can be found as \((w_1, w_2) = (\beta_2 p - \frac{\beta_2 p - c_2}{\beta_1}, \frac{c_2}{\beta_2})\), where supplier 1 makes nonnegative profit since \(w_1 - \frac{c_1}{\beta_1} = \beta_2 p - \frac{\beta_2 p - c_2}{\beta_1} - \frac{c_1}{\beta_1} = \beta_2 p - \beta_1 p - \frac{(\beta_2 p - c_2)}{\beta_1} \geq 0\). Note that \((w_1^*, w_2^*, Q_1^*, Q_2^*) = (\beta_2 p - \frac{\beta_2 p - c_2}{\beta_1}, \beta_1, D, 0)\) is indeed a Nash equilibrium. Neither supplier can gain by deviating from these wholesale
prices. By raising \( w_1^* \) supplier 1 will then make zero profit because the buyer will not order from him, and by lowering \( w_1^* \) he will then make less profit because the buyer orders the same \( D \) from him. By raising \( w_2^* \) supplier 2 will still make zero profit because the buyer will not order from him, and by lowering \( w_2^* \) supplier 2 will make negative profit. What remains is to show that there can be no other Nash Equilibrium.

If \( w_2^* \geq \frac{a_2}{b_2} \) and \( \beta_1(p - w_1^*) > \beta_2(p - w_2^*) \), i.e., the wholesale prices are in region (ii) and \( w_2 \) is above \( \frac{a_2}{b_2} \) (including \( \frac{a_2}{b_2} \)), the order quantity to supplier 1 is \( D \). In this case, supplier 1 can make more profit by setting his wholesale price equal to \( w_1^* + \varepsilon \) for \( \varepsilon > 0 \) while having the same order quantity \( D \). Thus, none of these wholesale prices choices could constitute a Nash equilibrium. Similar arguments follow for the case when \( w_2^* \geq \frac{a_2}{b_2} \) and \( \beta_1(p - w_1^*) < \beta_2(p - w_2^*) \).

If \( \beta_1(p - w_1^*) = \beta_2(p - w_2^*) \) and \( w_2^* \geq \frac{a_2}{b_2} \), i.e., the wholesale prices are in region (iv) and \( w_2 \) is above \( \frac{a_2}{b_2} \), the sum of the order quantities to supplier 1 and 2 is \( D \). In this case, any supplier \( i \) with order quantity lower than \( D \) can make more profit by setting his wholesale price to \( w_i^* - \varepsilon \), for \( \varepsilon > 0 \) in order to have order quantity \( D \). Thus, none of these wholesale prices choices is an equilibrium.

We have just ruled out all possible wholesale prices other than the ones satisfying \( \beta_1(p - w_1^*) = \beta_2(p - w_2^*) \) and \( w_2^* = \frac{a_2}{b_2} \). Thus the equilibrium wholesale prices are \( (w_1^*, w_2^*) = (\beta_2 p - \frac{a_2 p - \varepsilon}{\beta_2}, \frac{a_2}{b_2}) \) and the corresponding order quantities are \( (Q_1^*, Q_2^*) = (D, 0) \).

Proof of Lemma 2. We prove the Lemma by providing wholesale prices in each set. Suppose \( \varepsilon_1 \) and \( \varepsilon_2 \) are arbitrarily small numbers. Wholesale prices \( (w_1, w_2) = (\varepsilon_1, \varepsilon_2) \) are in set \( \Omega_1 \) since \( d_1(w) = \frac{1}{\theta_1} \ln \left[ \frac{\theta_1(p - \varepsilon_2)}{\beta_1 \varepsilon_2} \right] \rightarrow +\infty \) and \( d_2(w) = \frac{1}{\theta_2} \ln \left[ \frac{\theta_2(p - \varepsilon_1)}{\beta_2 \varepsilon_1} \right] \rightarrow +\infty \). Wholesale prices \( (w_1, w_2) = (\varepsilon_1, p - \varepsilon_2) \) are in set \( \Omega_2 \) since \( \beta_1(p - w_1) > \beta_2(p - w_2) \) and \( d_3(w) = \frac{1}{\theta_1(p - \varepsilon_1)} \ln \left[ \frac{\beta_1(p - \varepsilon_1)}{\beta_1 \varepsilon_1} \right] \rightarrow +\infty \). Wholesale prices \( (w_1, w_2) = (p - \varepsilon_1, \varepsilon_2) \) are in set \( \Omega_3 \) since \( \beta_1(p - w_1) < \beta_2(p - w_2) \) and \( d_5(w) = \frac{1}{\theta_1(p - \varepsilon_2)} \ln \left[ \frac{\beta_1(p - \varepsilon_2)}{\beta_1 \varepsilon_2} \right] \rightarrow +\infty \).

Wholesale prices \( (w_1, w_2) = (p - \varepsilon_1, p - \frac{a_2}{b_2} \varepsilon_1) \) are in set \( \Omega_4 \) since \( \beta_1(p - w_1) = \beta_2(p - w_2) \), \( d_4(w) = 0 \) and \( d_7(w) = \frac{\beta_2}{\theta_2(p - w_1)} \ln \left[ \frac{\beta_2(p - \varepsilon_1)}{\beta_2 \varepsilon_2} \right] + \frac{1}{\theta_1} \ln \left[ \frac{\beta_1(p - \varepsilon_1)}{\beta_1 \varepsilon_1} \right] \rightarrow +\infty \). Wholesale prices \( (w_1, w_2) = (\varepsilon_1, p) \) are in set \( \Omega_5 \) since \( d_3(w) = 0 \) and \( d_6(w) = \frac{1}{\theta_1(p - \varepsilon_1)} \ln \left[ \frac{\theta_1(p - \varepsilon_1)}{\beta_1 \varepsilon_1} \right] \rightarrow +\infty \). Wholesale prices \( (w_1, w_2) = (\beta_2 p, \varepsilon_2) \) are in set \( \Omega_6 \) since \( d_5(w) = 0 \) and \( d_6(w) = \frac{1}{\theta_2(p - \varepsilon_2)} \ln \left[ \frac{\theta_2(p - \varepsilon_2)}{\beta_2 \varepsilon_2} \right] \rightarrow +\infty \). Wholesale prices \( (w_1, w_2) = (\beta_2 p, p) \) are in set \( \Omega_7 \) since \( d_7(w) = 0 \). Therefore all the wholesale prices sets \( \Omega_i, i = 1, 7 \) are nonempty.

Proof of Proposition 4. We prove the Proposition by showing that there can be no other Nash equilibrium. By checking the definition of the sets \( \Omega_i, i = 1, 7 \) we find that they are nonempty and continuous on \( w_1 \) and \( w_2 \).
They are pairwise disjoint and collectively exhaustive. Even though the shapes of each region (corresponding to each set) change depending on the parameters, the configuration of the regions does not change which is illustrated in the top panel of Figure 1.

Suppose the equilibrium is in the interior of \( \Omega_1 \cup \Omega_5 \). Supplier 1 can make more profit by setting his wholesale price equal to \( w_1 + \varepsilon \) for \( \varepsilon > 0 \) while having the same order quantity \( D \). Similarly the equilibrium can not exist in the interior of \( \Omega_1 \cup \Omega_6 \). Suppose the equilibrium is in the closure of \( \Omega_4 \), supplier 2 makes zero profit. By lowering his wholesale price to sets \( \Omega_4, \Omega_5 \) or \( \Omega_7 \) he could make strictly positive profit and be better off. Similarly the equilibrium can not exist in the closure of \( \Omega_3 \). By eliminating all the other possibilities we conclude that the equilibrium wholesale prices can only exist in sets \( \Omega_4, \Omega_7 \) or the boundaries between \( \Omega_7 \) and \( \Omega_5, \Omega_6 \). □

Proof of Proposition 5. If there exists a pure-strategy equilibrium in region (vii), the buyer’s optimal order quantities are obtained in (11) and (12). If the suppliers make positive expected profits, the first-order condition that the solution of supplier \( i \)'s problem must satisfy is:

\[
\beta_i Q_i^* + (w_i\beta_i - c_i) \frac{\partial Q_i^*}{\partial w_i} = 0. \tag{41}
\]

Plug (11) and (12) into (41). The equilibrium wholesale prices \( (w_1^*, w_2^*) \) must satisfy:

\[
\begin{align*}
(p(p - w_2^*)D + \frac{\theta}{\beta_2} \ln \left[ \frac{\beta_2}{\beta_2 w_2^*} \right] - \frac{w_2^*}{w_2^*} \ln \left[ \frac{\beta_2(p - w_2^*)}{\beta_2 w_2^*} \right] = \frac{\beta_2}{\beta_2 w_2^*} \left( \frac{p^2(p - w_2^*)}{\beta_2(p - w_2^*)} + \frac{w_2^*}{w_2^*} \frac{p^2(p - w_2^*)}{\beta_2 w_2^*} \right), \\
(p(p - w_1^*)D + \frac{\theta}{\beta_1} \ln \left[ \frac{\beta_1}{\beta_1 w_1^*} \right] - \frac{w_1^*}{w_1^*} \ln \left[ \frac{\beta_1(p - w_1^*)}{\beta_1 w_1^*} \right] = \frac{\beta_1}{\beta_1 w_1^*} \left( \frac{p^2(p - w_1^*)}{\beta_1(p - w_1^*)} + \frac{w_1^*}{w_1^*} \frac{p^2(p - w_1^*)}{\beta_1 w_1^*} \right).
\end{align*}
\]

□

Proof of Corollary 1. Following immediately from Proposition 5 by setting \( c_1 = c_2 = 0 \), \( \beta_1 = \beta_2 = 2 \), \( w_1^* = w_2^* = w^* \) and \( Q_1^* = Q_2^* = Q^* \) we obtain (i) and (ii).

If there exists a symmetric pure-strategy equilibrium in region (vii), by Proposition 2 the wholesale prices have to satisfy \( w^* < \beta p, D > d_6(w^*) = d_9(w^*) \) or \( w^* \geq \beta p, D > d_7(w^*) \), which are equivalent respectively to

\[
\begin{align*}
\theta w^* D > \frac{1}{\beta w^*} \ln \left[ \frac{\beta(p - w^*)}{\beta w^*} \right] \quad \text{or} \quad \theta w^* D > -\frac{2}{\theta(p - w^*)} \ln \left[ \frac{\beta(p - w^*)}{\beta w^*} \right].
\end{align*}
\]

Differentiating both sides of the equation in (i) with respect to \( \theta \), where \( w^* \) is an implicit function of \( \theta \), and solving for \( \partial w^*/\partial \theta \), yields

\[
\frac{\partial w^*}{\partial \theta} = \frac{D w^*(p - w^*)^2}{p + w^*} > 0. \tag{42}
\]
Differentiating both sides of the equation in (ii) with respect to $\theta$, we obtain
\[
\frac{\partial Q^*}{\partial \theta} = -\left( \frac{p(p + w^* + (p - w^*)\theta w^*)}{\theta w^*(p - w^*)(p + w^*)^2} \right) \frac{\partial w^*}{\partial \theta} - \frac{\theta (\theta w^*)}{\theta (p + w^*)^2} \ln \left[ \frac{(p - w^*)}{\theta w^*} \right].
\]

If $w^* < \beta p$, then $\frac{(p - w^*)}{\theta w^*} > 1$. We have $\ln \left[ \frac{(p - w^*)}{\theta w^*} \right] > 0$. As $\frac{\partial w^*}{\partial \theta} > 0$, it follows that $\frac{\partial Q^*}{\partial \theta} < 0$. If $w^* \geq \beta p$, then $D > -\frac{2}{\theta (p - w^*)} \ln \left[ \frac{(p - w^*)}{\theta w^*} \right]$, which are equivalent to $w^* \geq \beta p$, $-\ln \left[ \frac{(p - w^*)}{\theta w^*} \right] < \frac{D}{2}$. We have
\[
\frac{\partial Q^*}{\partial \theta} < -\left( \frac{p(p + w^* + (p - w^*)\theta w^*)}{\theta w^*(p - w^*)(p + w^*)^2} \right) \frac{\partial w^*}{\partial \theta} + \frac{\theta (\theta w^*)}{\theta (p + w^*)^2} \left( \frac{\theta D(p - w^*)}{2} \right). \tag{43}
\]

Plugging (42) into (43) we have
\[
\frac{\partial Q^*}{\partial \theta} < -\frac{D(p - w^*)^2(1 + \theta w^*)}{2\theta(p + w^*)^2} < 0.
\]

\[
\square
\]

References


