CAUSAL INFERENCE IN MULTIAGENT ECONOMIES

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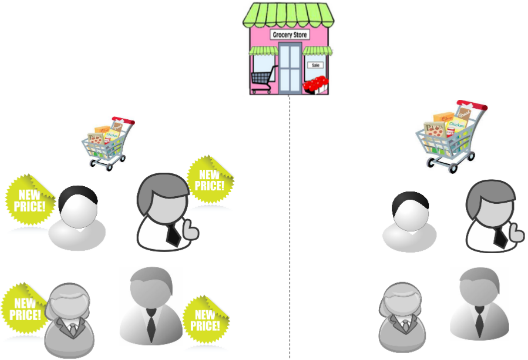
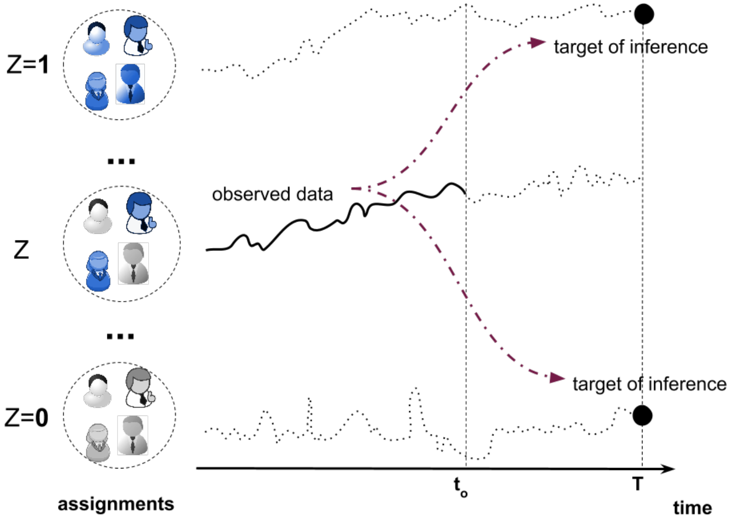
Effect of changing prices in a grocery store:

1. Measure revenue when *all* customers shop with new prices relative to revenue when all customers shop with old prices.
2. Measurement in the *long-term*.



Typically, such effects are evaluated experimentally.

1. Revenue measured when *some* customers shop (some items) with new prices, and is compared to revenue of items with old prices.
2. Measurement in short-term.


Z=1
Z
Z=0
assignments
observed data
target of inference
target of inference
t_o
T
time

NOTATION

Experiment:

- Agents $\mathcal{I}$, games $\mathcal{G}$, actions $\mathcal{A}$.
- $Z =$ assignment vector (every agent assigned to one game);
  e.g., $Z_i = j \in \mathcal{G}$ assignment of agent i to game j.

Agent actions:

- $A_{it}(Z) \in \mathcal{A} =$ action of agent i at time t under assignment Z.
- $\alpha_{j,t}(Z) \in \Delta^{|\mathcal{A}|} =$ frequency of actions in game j at time t under assignment Z. ($\Delta =$simplex).
  (e.g., if $\mathcal{A} = \{a_1, a_2\}$, then $\alpha_{j,t}(Z) = (0.2, 0.8)$. i.e., 20% agents play a_1, rest play a_2.)

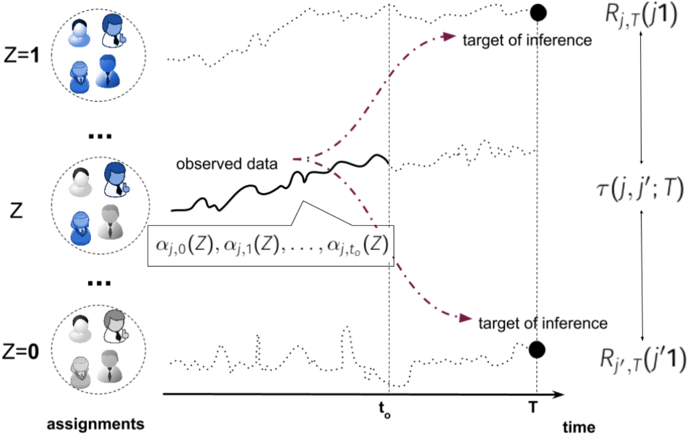
Definition

The causal effect at time t on objective R of game j over game j' is equal to the quantity

$$\tau(j, j'; t) = h(\alpha_{j,t}(j\mathbf{1})) - h(\alpha_{j',t}(j'\mathbf{1})) = \underbrace{R_{j,t}(j\mathbf{1})}_{\text{all agents play in } j} - \underbrace{R_{j',t}(j'\mathbf{1})}_{\text{all agents play in } j'} .$$

- *Long-term average causal effect* (LACE) defined at $t = T$, where T is considered long-term by the experimenter.

ILLUSTRATION



- Adopt latent-space approach, then we need:
  1. $P(\text{action}_t | \text{latent}_t)$.
  2. $P(\text{latent}_t | \text{latent}_{t-1})$.

HIGH-LEVEL IDEA

- Adopt latent-space approach, then we need:
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  2. $P(\text{latent}_t | \text{latent}_{t-1})$.

Latent state="behavior". Then, we need:

1. Behavioral model: likelihood of actions conditional on behaviors.
2. Temporal model: evolution of population behavior.

- Set of *behaviors*, $\mathcal{B} = \{b_0, b_1, \ldots\}$. (e.g., "aggressive", "passive", etc.)
- $B_{it}(Z) \in \mathcal{B}$ behavior of agent i at time t under assignment Z.

Definition

A *behavioral model* on $\mathcal{B}$ is a collection of probabilities

$$P(A_{it}(Z) = a | B_{it}(Z) = b, G_j), \tag{1}$$

for every action $a \in \mathcal{A}$ and every behavior $b \in \mathcal{B}$, where G_j indicates conditioning on a game j.

- Aggregate behavior $\beta_{j,t}(Z) \in \Delta^{|\mathcal{B}|}$ = frequency of behaviors in game j, at time t, under assignment Z.

Definition (Temporal model)

A *temporal model* for game j is a collection of parameters $\{(\phi_Z^j, \psi_Z^j)\}$, for every assignment Z, and two densities (π, f), such that, at every time t,

$$\beta_{j,0}(Z) \sim \pi(\cdot; \phi_Z^j)$$
$$\beta_{j,t}(Z)|\mathcal{F}_{t-1}, G_j \sim f(\cdot|\psi_Z^j, \mathcal{F}_{t-1}). \quad (2)$$

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($\mathcal{F}_t$=history up to t; prior π at $t = 0$, and density f are fixed, but parameters may depend on treatment assignment.)

Assumption (No anticipation)

Let $\beta_{j,-1}(Z)$ *be the aggregate behavior in game j before assignment Z happens at* $t = 0$*. Then, there exists a fixed aggregate behavior* $\beta_j^{(0)}$ *such that, for every game j and assignment Z,*

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- Agents do not anticipate Z, and thus are not affected on how they adopt their behaviors before assignment.

Assumption (Behavioral ignorability)

The treatment assignment is independent of aggregate behavior at time t, conditional on the game and the game history up to t; i.e.,

$$Z \perp\!\!\!\perp \beta_{j,t}(Z) \mid \mathcal{F}_{t-1}, G_j.$$

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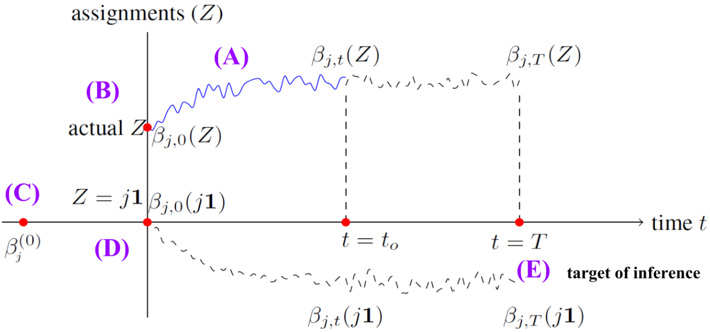
- Agent cannot distinguish between agents, e.g., adopt behavior depending on whether assigned with friends or foes.



(A) use data to estimate $(\phi^j_Z, \psi^j_Z) \equiv (\phi^j, \psi^j)$, by Assumption 2.

(B) estimate $\beta_{j,0}(Z)$ under assignment Z using $\pi(\cdot|\phi^j)$;

(C) estimate $\beta_j^{(0)}$ using $\beta_{j,0}(Z)$, and randomization of Z;

(D) set $\beta_{j,0}(j1) = \beta_j^{(0)}$ (by definition);

(E) estimate $\beta_{j,T}(j1)$ from $\beta_{j,0}(j1)$ and (ϕ^j, ψ^j) in (A).

Game	Period	A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4
1	1	0.308	0.307	0.113	0.120	0.350	0.218	0.202	0.092
1	2	0.293	0.272	0.162	0.100	0.333	0.177	0.190	01.40
1	3	0.273	0.350	0.103	0.123	0.353	0.133	0.258	0.102
1	4	0.295	0.292	0.113	0.135	0.372	0.192	0.222	0.063
2	1	0.258	0.367	0.105	0.143	0.332	0.115	0.245	0.140
2	2	0.290	0.347	0.118	0.110	0.355	0.198	0.208	0.108
2	3	0.355	0.313	0.082	0.100	0.355	0.215	0.187	0.110
2	4	0.323	0.270	0.093	0.105	0.343	0.243	0.168	0.107

- (In Table) Proportions = population actions $\alpha_{j,t}(Z)$, $t = 1, 2, 3, 4$.

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	B_1	B_2	B_3	B_4	B_5
A_1	W	L	L	L	L
A_2	L	L	W	W	W
A_3	L	W	L	L	W
A_4	L	W	L	W	L
A_5	L	W	W	L	L

- Assume objective $R_{j,t}(Z) = c'\alpha_{j,t}(Z)$.
- Two different *game designs*: $(W, L) = (\$10, -\$6)$ and $(W, L) = (\$10, -\$1)$.

"What is the long-term causal effects of design #1 over #2 on objective?".

Posterior distribution of behavioral+temporal model parameters:

$$P(\theta|\text{ actions}_{1:t_0}) = \int_{\text{behaviors}_{1:t_0}} \underbrace{P(\text{ actions}_{1:t_0}|\text{ behaviors}_{1:t_0})}_{\text{behavioral model (e.g., Ql3)}} \times$$

$$\times \prod_{t=1}^{t_0} \underbrace{P(\text{behavior}_t|\text{behavior}_{t-1}, \theta)}_{\text{temporal model (eg., VAR(1))}} \times$$

$$\times \underbrace{P(\text{ behavior}_0|\theta)}_{\text{prior for behavior } t\text{=0}}$$

$$\times \underbrace{P(\theta)}_{\text{parameter priors}}$$

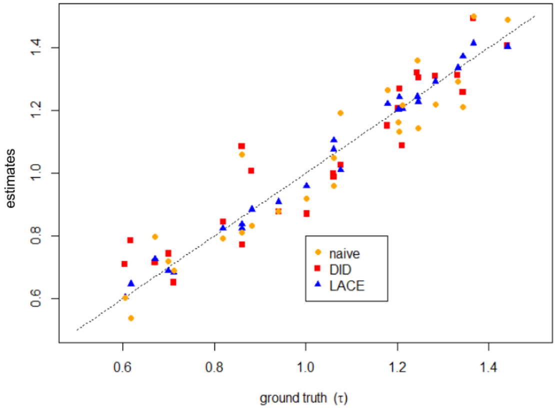

Figure 2: Random objectives. x-axis= true long-term effect. LACE=long-term effect estimates; DID=difference-in-differences; naive=use $t_0 - 1$ for estimation.

- Estimation of long-term causal effects is critical.
  - Most systems (e.g., social, economic, engineering) are dynamical.
- Need to estimate actions (1) across assignments; (2) across time.
- Our contributions
  - Formalize problem + define sufficient assumptions for identification.
  - Methodology for estimation of long-term causal effect.
  - Key idea: latent behavioral space; leverages behavioral game theory to estimate (1) and (2).
- Open issues:
  - strategic interference *between* games;
  - necessary assumptions for identification;
  - formal definition of T.

additional slides.

- **Structural approaches**, e.g., Athey et. al., (2008):
  assume observed actions are in equilibrium; no long-term effects; no behavioral modeling.
- **Directed acyclical graphs** (DAGs), e.g., Bottou et. al. (2012):
  assumes DAG is stable under treatment assignment; problematic in equilibrium systems (Dash, 2011).