Increasing the Frequency of Financial Reporting:  
An Equilibrium Analysis of Costs and Benefits

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1. Introduction

How frequently should publicly traded firms be required to report the results of their operations to the capital market? This is an important policy question that accounting regulators must grapple with. In the United States the frequency of mandatory reporting has risen from annual reporting to semi-annual reporting to quarterly reporting. With the current regulatory environment calling for greater accountability and higher transparency of financial information, it is likely there will be pressure on regulators to require firms to report even more frequently. In the absence of a clear sense of the potential costs and benefits associated with the frequency of financial reporting, the knee jerk reaction will be the more the better, since usually more information is preferred to less. In this paper we develop one plausible approach to studying the costs and benefits associated with this frequency issue and study the nature of their tradeoff.

More frequent reporting will usually provide more timely and more disaggregate information on a firm’s performance, thus enhancing the information that is impounded in stock prices. More informed pricing is believed to be efficiency enhancing because such prices would better guide the flow of resources to competing investment projects. Our analysis explicitly captures this benefit from increased frequency of financial reporting. There are other potential benefits that we do not explicitly consider. Perhaps, more frequent reports would decrease informational differences across traders in the capital market thus increasing market liquidity and, perhaps more frequent reports would facilitate corporate governance. While the benefits are not difficult to imagine, the costs associated with increasing the frequency of reporting (apart from increased compliance costs) are more subtle and less apparent. In the voluntary disclosure literature, proprietary costs arising from information leakage to competing firms is commonly believed to be a potent force that limits disclosure.\footnote{For examples, see Dye [1986], Darrough and Stoughton [1990], and Gigler [1994].} However, if disclosure triggers proprietary costs that damage the cash flows of the disclosing firm, such disclosures enhance the cash flows...
of competing firms, so the social costs to such disclosure could be small or even non-existent. Thus a regulator, concerned with social welfare and overall economic efficiency, is unlikely to be swayed by proprietary cost arguments. Gigler and Hemmer [2001] argue that frequent reporting is costly because moral hazard problems arising from the unobservable effort of a firm’s manager become more severe if reporting frequency is increased.

Neither proprietary costs nor moral hazard costs arise directly from disclosure to the capital market. They arise because disclosure to capital markets is equated with disclosure to other parties. One plausible cost that could arise directly from capital market pricing comes from the fact that accounting measurement errors would become more severe if the measurement window is shortened due to more frequent disclosure requirements. Kanodia and Mukherji [1996], and Kanodia, Sapra and Venugopalan [2004] show how such measurement errors distort market pricing and create price pressure to forego or decrease investments that are not directly observable.

But there could be another potentially more serious cost associated with the frequency of financial reporting that is suggested by the recent debate in Europe, Singapore and Australia surrounding the proposal to mandate quarterly reporting. Bhojraj and Libby [2005] report the following excerpts from the popular press:

“Some of Europe’s most powerful investors are calling on the European Commission to drop plans to introduce mandatory quarterly reporting for companies….it (quarterly reporting) has not helped prevent corporate scandals in the U.S., and there is risk that it will encourage short-termism.” (Financial Times, January 27, 2003)

“Hong Kong says no to quarterly reporting …..Critics say an unintended consequence will be short-termism in the market, with investors focused on seasonal profits rather than long-term earnings growth.” (Investor Relations Magazine, November 15, 2002)
Rahman, Tay, Ong and Cai [2007] summarize the European debate as follows:

“In the debate over quarterly reporting, those in favor believe that the more frequent reporting of earnings increases analyst following of firms, improves timeliness of earnings, and improves stock trading. Those in opposition argue that it encourages short-termism, which can lead to earnings management and stock price volatility.”

They also report that: “In the United Kingdom, Chartered Institute of Management Accountants (CIMA) warned that without the conclusion of enough management commentary on business outlook in quarterly reports, companies ran the risk of making short-term decisions to make the bottom-line numbers attractive to investors.”

In lieu of these concerns, in 2004, the European Union Parliament rejected the proposal to mandate quarterly reporting, and in Singapore, the Council on Corporate Disclosure and Governance recommended that companies with a market capitalization of less than $75 million should be exempt from quarterly reporting. A similar example in the U.S. received much publicity. During Google’s IPO offering in 2004, the management of Google explicitly declined to provide frequent earnings guidance to analysts, saying that it did not want to lose focus on its long-term goals.

The above excerpts from the popular press suggest that a broad spectrum of practitioners intuitively feel that if firms are required to report, or forecast, the results of their operations too frequently, managers would become overly focused on short-term goals that are not in the best interests of the firm. Importantly, the intuition is that this short-termism is an optimal response to price pressure from the capital market, rather than an outcome of managerial career concerns or the result of poorly designed performance measures. If the short-termism hypothesis is true, the costs of requiring ever more frequent disclosure could become formidable. In this paper, we flesh out this managerial short-termism /myopia hypothesis, and develop plausible conditions under
which an increase in the frequency of financial reporting would precipitate managerial short-termism as an equilibrium response solely to price pressure from the capital market.

Thus, as analyzed in this paper, there is clear tradeoff between the costs and benefits of increasing the frequency of mandatory financial reporting. The benefit from increasing the frequency of financial reporting is that it increases the amount of firm specific information that is impounded in stock prices which, in turn, provides better *ex ante* incentives for investment. The cost of increased frequency is that it increases the probability of inducing managerial short-termism. We tradeoff these costs and benefits and develop conditions under which greater frequency is desirable and conditions under which it is not.

The layman’s perception of corporate myopia is that it is caused by impatient traders in the capital market who hold the firm only for short-term capital gains and consequently demand quick returns to managerial actions. We show that this popular intuition is incomplete. Impatience in the capital market, while necessary, is insufficient create the kind of price pressure that would sustain managerial myopia. Since markets are forward looking, any actions that favor the short-term at the expense of greater long-term value creation would be swiftly punished by lower capital market prices. Managerial myopia is sustainable only if there are gaps between the information in the capital market and the information possessed by corporate managers, leading to market inferences from noisy summary statistics of the sort typically reported by periodic accounting statements. Given informational differences between corporate managers and the capital market, we study equilibrium pricing and investment strategies in two accounting regimes that differ only in the frequency with which firms are required to report the results of operations. In each regime, capital markets are “efficient” in the sense that market participants make rational Bayesian inferences from accounting reports regarding variables that affect the future profitability of the firm, their inferences are consistent with the optimizing strategy of the firm, and market prices fully reflect these rational inferences. We show that frequent reporting results in rational inferences and price pressures that are analogous to the pressure caused by the premature
evaluation of any action whose value is probabilistically manifested only over the long run. Thus, frequent reports magnify the attraction of managerial actions that are more likely to produce quick bottom line results. These premature evaluations are tempered by subsequent evaluations, but the damage caused by early evaluations cannot be overcome when shareholders are sufficiently impatient. Such pressures disappear when the reporting frequency is decreased. Thus, infrequent could better guide the firm’s investment even though they provide less information to the capital market.

Managerial myopia has been studied in other contexts. Stein [1988] found that corporate takeover threats induce managers to signal the hidden true value of their firms by prematurely selling off assets at prices lower than the benefits they would yield to the firm over the long-term. Bebchuk and Stole [1993] develop informational conditions under which managers would over-invest and conditions under which they would under-invest in long-term projects relative to short-term projects. Narayanan [1985] shows that labor market reputational concerns could induce managers to make decisions that yield short-term personal gains to the manager at the expense of the long-term interests of shareholders. Stein [1989] showed that capital market price pressure could induce firms to borrow earnings from the future at unfavorable terms in order to boost their current period price. None of these studies are concerned with the frequency of financial reporting, which is the main object of study in the current paper. However, all share the feature that managerial myopia is caused by inferences that outsiders are forced to make when they know less than the firm’s manager. In a different kind of model, Dye [2008] showed that managers who gradually divest their shares over time would prefer rules that allow bunching of disclosures at a single point in time, rather than rules that require continuous dissemination of information over the disclosure horizon.

Bhojraj and Libby [2005] manipulated reporting frequency and price pressure in a laboratory experiment, with experienced financial managers from publicly traded corporations, and empirically demonstrated that corporate managers become myopic when faced with intense
price pressure and high reporting frequency. These results were obtained in the absence of any agency frictions and even when managers had the opportunity to make voluntary disclosures. The results of our analysis are fully consistent with the empirical findings reported in Bhojraj and Libby.

2. The Model

Consider a setting where the returns to investment by a publicly traded firm depend stochastically upon one of two possible states of nature, state $G$ (good) or state $B$ (bad). Investment is desirable in state $G$ but not in state $B$, in a sense to be described below. The state itself is not observable to any agent in the economy, but can be probabilistically inferred from observable outcomes and signals. The firm’s manager observes a noisy signal $\tilde{S}$ that is informative about the state, before she makes the investment decision. We refer to the state generically as $\sigma$, so $\sigma \in \{G, B\}$ and denote the prior probability that $\sigma = G$ as $\lambda$. The signal $S$ has fixed support on the interval $[\tilde{S}, \tilde{S}]$ regardless of whether the state is $G$ or $B$ and the signal is generated through the conditional density functions $\xi(S | G)$ and $\xi(S | B)$. We assume these conditional densities satisfy the strict monotone likelihood ratio property (MLRP), so that higher values of $S$ represent good news. We further assume that the signal becomes perfectly informative in the limit. More explicitly, we assume that:

\[
\frac{\xi(S | G)}{\xi(S | B)} \text{ is strictly increasing in } S
\] (1)

Also, $\frac{\xi(S | G)}{\xi(S | B)} \to \infty$ as $S \to \tilde{S}$ and $\to 0$ as $S \to \tilde{S}$. These assumptions imply that $\text{Prob}(G | S)$ is strictly increasing in $S$, that $\lim_{S \to \tilde{S}} \text{Prob}(G | S) = 1$ and $\lim_{S \to \tilde{S}} \text{Prob}(G | S) = 0$.

The manager chooses whether or not to invest after observing the signal $S$ and, if she chooses to invest, she chooses between a short-term and a long-term project (projects $M$ and $L$
respectively). The investment choice is therefore \( I \in \{\emptyset, M, L\} \), were the choice of \( \emptyset \) indicates the manager does not invest. Cash flows are normalized such that if no investment is made, all period cash flows are identically zero. For reasons that will become apparent later, we assume that projects \( M \) and \( L \) require the same initial investment of \( K \). Investment outlays occur at date zero, and the chosen project, either \( M \) or \( L \), yields stochastic cash inflows in periods 1 through \( N \), with \( N > 2 \). Let:

\[
\tilde{x}_t = \text{ the stochastic t-period cash inflow from a project, } t = 1,2,\ldots,N, \text{ and }
\]

\[
\tilde{y}_t = \sum_{r=1}^{t} \tilde{x}_r = \text{ the stochastic cumulative cash inflow through period } t, t = 1,2,\ldots,N.
\]

Since cash flows are jointly affected by the project choice and the underlying state of nature, we represent the probability density of the period \( t \) cash flow conditional on each state (\( \sigma = G \) or \( B \)) and each project (\( I = L \) or \( M \)) as \( f_t(x_t \mid \sigma, I) \). We assume that, conditional on the state and the project, cash flows are inter-temporally independent and are likewise conditionally independent of the signal \( S \). Inter-temporal independence is assumed for simplicity, while the conditional independence of \( S \) and \( x_t \) captures the idea that the return on investment is a function of the state of nature, not of the signal per se. Every period’s cash inflow from each of the two projects is stochastically smaller in state \( B \) than in state \( G \), and each period’s cash inflow satisfies strict monotone likelihood ordering:

\[
\frac{f_t(x_t \mid G, I)}{f_t(x_t \mid B, I)} \text{ is strictly increasing in } x_t \text{ for all } t \text{ and for each } I \in \{M, L\}. \quad (2)
\]

Thus, observed cash flows are informative about the state with \( \text{Prob}(G \mid x_t, I) \) strictly increasing in \( x_t \), regardless of which of the two projects has been chosen. These likelihood ratios are
assumed to satisfy boundary conditions similar to those of the signal $S$ in that

$$
\frac{f_t(x_t \mid G, I)}{f_t(x_t \mid B, I)} \to \infty
$$

as $x_t \to \infty$ and $\to 0$ as $x_t \to -\infty$ for all $t$ and for each $I \in \{M, L\}$.

For each $t = 0, 1, \ldots, N - 1$, let $V_t(\sigma, I)$ denote the expectation of the sum of future cash flows from date $t$ onwards, under each project and each state, i.e.,

$$
V_t(\sigma, I) = E_t(\tilde{x}_{t+1} + \tilde{x}_{t+2} + \ldots + \tilde{x}_N \mid \sigma, I), \sigma \in \{G, B\}, I \in \{M, L\}.
$$

Our previous assumptions regarding the nature of good and bad states imply that

$$
V_t(G, I) > V_t(B, I), \forall t, \forall I \in \{M, L\}. \quad (3)
$$

The key differences between the short-term and the long-term project are as follows. Looking forward from any date, the long term project has a higher present value of expected future cash flows than the short-term project in each of the two states. More precisely,

$$
V_t(\sigma, L) > V_t(\sigma, M), \forall t, \forall \sigma \in \{G, B\} \quad (4)
$$

Thus the long-term project is superior to the short-term project in a very strong sense, and the choice of project $M$ rather than project $L$ clearly represents dysfunctional myopia. The only possible attraction for the short-term project is that, in early periods, the short-term project produces stochastically bigger cash flows than the long-term project, in each of the two states. Consistent with this idea, we assume there exists a date $t^* < N$ such that, in each state, project $M$ produces stochastically bigger cumulative cash flows $\tilde{y}_t$ at each $t < t^*$, but stochastically smaller

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2 We ignore discounting of future cash flows and assume risk-neutral pricing, as discounting and risk aversion are immaterial to our arguments.

3 The notation $E_t$ means the expectation conditional on the information in period $t$. 
cumulative cash flows at each $t \geq t^*$. Throughout our analysis we assume that $t^* = 2$, so that the tradeoff between short-term and long-term returns can be captured in a relatively simple two period setting. This simplification gives us the following representation of short v. long-term investment: For each $\sigma \in \{G, B\}$,

$$\tilde{x}_1 | \sigma, M \text{ is stochastically bigger than } \tilde{x}_1 | \sigma, L \text{, } (5)$$

$$\tilde{x}_2 | \sigma, M \text{ is stochastically smaller than } \tilde{x}_2 | \sigma, L \text{, } (6)$$

$$\tilde{x}_1 + \tilde{x}_2 | \sigma, M \text{ is stochastically smaller than } \tilde{x}_1 + \tilde{x}_2 | \sigma, L \text{. } (7)$$

Consistent with the idea that investment is desirable in the good state, but undesirable in the bad state, we assume:

$$V_0(G, L) > V_0(G, M) > K \text{, and } (8)$$

$$V_0(B, M) < V_0(B, L) < K \text{. } (9)$$

We assume that the prior probability of the good state is sufficiently small so that in the absence of sufficiently good news it is undesirable to invest, i.e.,

$$\lambda V_0(G, L) + (1 - \lambda)V_0(B, L) - K < 0 \text{.}^4 (10)$$

The firm outlives its current shareholders, and all cash inflows are retained in the firm until the terminal date. Thus, current shareholders derive their returns entirely through the pricing of the firm in the capital market. This last assumption is essential to the existence of “price

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^4 This assumption is without loss of generality. The tradeoffs we wish to capture are essentially unaffected if the inequality in (10) is reversed.
pressure.” If current shareholders hold the firm until the terminal date and obtain their returns from a liquidating dividend, market pricing becomes irrelevant and there is no scope for price pressure.\(^5\) In our two-period representation, current shareholders (i.e., date 0 shareholders) are therefore one of two types: long-term investors who sell at date 2 or short-term investors who sell at date 1. The proportion (or, equivalently, the probability) of short-term investors is assumed common knowledge and is parameterized by \(\alpha \in [0,1]\). Thus, ex ante, before a shareholder knows his type, he would like the firm to choose its investment strategy to maximize:

\[
\max_I \left[ \alpha E_0(P_1 | S, I) + (1 - \alpha) E_0(P_2 | S, I) \right],
\]

where \(P_1\) and \(P_2\) are equilibrium capital market prices of the firm at dates 1 and 2, respectively.

In order to focus the analysis solely on price pressure, we assume the manager is benevolent and imbibes the preferences of the current shareholders. Thus, in our model, there are no conflicts of interest between corporate managers and their shareholders, no managerial career concerns, and therefore no incentive issues that would generate a demand for compensation contracts.

By assuming that the firm’s objective function incorporates capital market valuations only at dates 1 and 2 even though the cash flows from investment occur over \(N > 2\) periods, we have operationalized the layman’s intuition that impatience in the capital market is an important factor underlying managerial myopia. Increases in the parameter \(\alpha\) represent increased impatience in the capital market. We show, however, that the layman’s intuition is incomplete. Impatience in the capital market, no matter how extreme, cannot by itself produce the kind of

\(^5\) Realistically, publicly traded firms do not have well defined terminal dates and do not pay liquidating dividends unless they go into bankruptcy. Also the composition of its shareholders is continuously changing, as witnessed by the enormous volume of trading in the capital market. So the assumption that shareholders obtain their returns through market pricing is much more realistic than the more commonly made assumption that shareholders obtain their returns from terminal dividends.
price pressure that would induce managerial myopia. Informational imperfections in the capital market and the frequency of financial reporting play critical roles.

3. Equilibrium when Capital Markets are fully Informed: The First Best Benchmark

In the first best world, the capital market observes everything the manager observes. Specifically, the capital market observes all realizations of cash flows, observes the manager’s signal $S$ about the state, observes whether or not the manager has invested and, if she has, whether she has chosen the short-term or the long-term project. Hence, in the first best world, equilibrium prices are:

$$P_1(S, I, x_1) = x_1 + \text{Prob}(G \mid S, I, x_1)V_1(G, I) + \text{Prob}(B \mid S, I, x_1)V_1(B, I) - K$$

and

$$P_2(S, I, x_1, x_2) = x_1 + x_2 + \text{Prob}(G \mid S, I, x_1, x_2)V_2(G, I) + \text{Prob}(B \mid S, I, x_1, x_2)V_2(B, I) - K$$

for each $I \in \{M, L\}$. (12)

And clearly $P_1(S, \emptyset) = P_2(S, \emptyset) = 0$, given that zero investment generates zero cash flow.

Let $I(S, \alpha)$ denote the firm’s equilibrium investment strategy. We now examine how impatience in the capital market affects the firm’s equilibrium investment strategy when markets are fully informed.

**Proposition 1**: When markets are fully informed, the firm’s equilibrium investment strategy is:

$$I(S, \alpha) = L \text{ when } S \geq S^* \text{ and } I(S, \alpha) = \emptyset \text{ when } S < S^*, \ \forall \alpha \in [0,1]$$

where $S^*$ is characterized by:

$$\text{Prob}(G \mid S^*)V_0(G, L) + \text{Prob}(B \mid S^*)V_0(B, L) - K = 0.$$
**Proof:** Given the price function in (12):

\[ E_0[P_1 \mid S, I] = E_0(x_1 \mid S, I) + E_0[\text{Prob}(G \mid S, I, x_1) \mid S, I]V_1(G, I) + E_0[\text{Prob}(B \mid S, I, x_1) \mid S, I]V_1(B, I) - K \]

But,

\[ E_0(x_1 \mid S, I) = \text{Prob}(G \mid S)E_0(x_1 \mid G, I) + \text{Prob}(B \mid S)E_0(x_1 \mid B, I) \]

and, from the law of iterated expectations,

\[ E_0[\text{Prob}(G \mid S, I, x_1) \mid S, I] = \text{Prob}(G \mid S). \]

Therefore,

\[
E_0[P_1 \mid S, I] = \text{Prob}(G \mid S)[E_0(x_1 \mid G, I) + V_1(G, I)] + \text{Prob}(B \mid S)[E_0(x_1 \mid B, I) + V_1(B, I)] - K
\]

\[ = \text{Prob}(G \mid S)V_0(G, I) + \text{Prob}(B \mid S)V_0(B, I) - K. \]

Also, using the price function in (13),

\[
E_0[P_2 \mid S, I] = E_0(x_1 + x_2 \mid S, I) + E_0[\text{Prob}(G \mid S, I, x_1, x_2) \mid S, I]V_2(G, I)
\]

\[ + E_0[\text{Prob}(B \mid S, I, x_1, x_2) \mid S, I]V_2(B, I) - K. \]

Then, using

\[ E_0(x_1 + x_2 \mid S, I) = \text{Prob}(G \mid S)E_0(x_1 + x_2 \mid G, I) + \text{Prob}(B \mid S)E_0(x_1 + x_2 \mid B, I). \]

and, \( E_0[\text{Prob}(G \mid S, I, x_1, x_2) \mid S, I] = \text{Prob}(G \mid S), \) (from the law of iterated expectations), yields:

\[
E_0[P_2 \mid S, I] = \text{Prob}(G \mid S)[E_0(x_1 + x_2 \mid G, I) + V_2(G, I)] + \text{Prob}(B \mid S)[E_0(x_1 + x_2 \mid B, I) + V_2(B, I)] - K
\]

\[ = \text{Prob}(G \mid S)V_0(G, I) + \text{Prob}(B \mid S)V_0(B, I) - K. \]

Therefore for every value of \( \alpha \in [0,1], \)

\[
\max_{I} \left[ \alpha E_0(\tilde{P}_1 \mid S, I) + (1-\alpha)E_0(\tilde{P}_2 \mid S, I) \right] = \max_{I} \text{Prob}(G \mid S)V_0(G, I) + \text{Prob}(B \mid S)V_0(B, I) - K.
\]

By (8) and (9) project \( L \) is preferred to \( M \) for all \( S \), and \( L \) is preferred to \( \emptyset \) for only those values of \( S \) that satisfy:
\[ \text{Prob}(G \mid S) V_0(G, L) + \text{Prob}(B \mid S) V_0(B, L) - K \geq 0. \]

Therefore, regardless of the degree of impatience in the capital market, the manager chooses to invest in the long-term project whenever she observes \( S \geq S^* \), and chooses not to invest when she observes \( S < S^* \). Also, \( S^* \) is a unique interior threshold because \( \text{Prob}(G \mid S) \) is strictly increasing in \( S \) and has the limit properties assumed in (1).

\[ Q.E.D. \]

The above result establishes that if all knowable information is impounded in capital market valuations, managerial short-termism cannot be caused by price pressure, no matter how impatient the firm’s current shareholders are. This important result is due to the fact that capital market prices anticipate all future cash flows, so when the market is fully informed, the cost of any myopic behavior is fully internalized by the firm’s current shareholders. They cannot possibly gain by producing attractive short-term cash flows at the expense of long-term cash flows.

4. Information Asymmetry between the Firm and the Market: The Importance of Performance Reporting

We believe the assumption that at every point of time the outside world has the same information as the firm’s manager is highly unrealistic. Managers make choices based on large amounts of detailed information much of which is soft, sensitive and unverifiable. This kind of detailed information cannot and is not disclosed in mandatory financial statements (or in voluntary disclosures) that are disseminated to the world at large. What \textit{is} disclosed is aggregated information on managerial choices and verifiable information on the periodic results of operations. Consistent with these observations, we make the following informational assumptions. First, we assume the capital market cannot observe the manager’s information, \( S \), in
the light of which she makes her choice. Second, we assume that, while the market can observe whether the firm invested in a new project and the amount of such investment, it cannot discern whether the project chosen was the short-term project or the long-term project. Third, we assume that accounting reports regarding the results of operations consist of reporting the periodic cash inflows, $x_1, x_2, x_3, \ldots$, or the cumulative cash flows $y_1, y_2, y_3, \ldots$. In our simple setting with intertemporal independence of cash flows and perfect measurement of investment outlays, there is no scope for informative accounting accruals. Such accruals arise in more complex settings and cause measurement difficulties and measurement errors of the kind studied in Kanodia and Mukherji [1996] and Kanodia, Sapra and Venugopalan [2004].

In such asymmetric information settings, periodic performance reports play a vital role similar to that described in Kanodia and Lee [1998]. In order to establish this beneficial role of performance reporting, we will show that if there are no performance reports at all, the firm would be trapped in a very bad equilibrium where no investment can possibly occur no matter how favorable the signal that the manager receives. This result establishes that the benefits of performance reporting are so large that such reports are indispensable, but it doesn’t answer the main issue to be examined in this paper: How frequently should the firm be required to release performance reports to the capital market? This latter issue will be examined in the next three sections.

If there are no performance reports, the date 2 price in the capital market must equal the date 1 price since no new information arrives at date 2. Both prices must depend only upon whether the firm invested or did not invest. If the firm does not invest, both prices are zero. If the firm does invest, both prices must depend upon a belief of whether the manager chose project M or project L and an inference about the signal S that the manager must have observed, an inference that is based solely on the observation that the manager invested. 

14
Suppose that, upon observing that the firm invested, the market makes the inference that $S \geq S^0$ for some threshold signal $S^0 > S$. If $S^0$ is such that $\text{Prob}(G \mid S \geq S^0) > \text{Prob}(G \mid S^*),$ and the market believes that any investing type invests in the long-term project, $L$, then both date 1 and date 2 prices will be strictly greater than zero. This being the case all firm types, including types $S < S^0$ will also invest, thus disconfirming the market’s inference. On the other hand, if $S^0$ is such that $\text{Prob}(G \mid S \geq S^0) < \text{Prob}(G \mid S^*),$ then no firm type would invest since investment would result in negative date 1 and date 2 prices. Lastly, if $\text{Prob}(G \mid S \geq S^0) = \text{Prob}(G \mid S^*),$ then investment would result in prices of zero, making the firm indifferent between investing and not investing. In this case either all types invest or no type invests, so once again the market’s inference cannot be sustained. These arguments do not depend upon whether the market believes that an investing type invests in project $L$ or invests in project $M$. In either case the market’s inferred threshold will unravel. The only sustainable inference, upon seeing investment, is that $S \geq S$. But, since $\text{Prob}(G \mid S \geq S) = \lambda$, investment will be priced using this prior probability of $\lambda$ and therefore the equilibrium date 1 and date 2 prices resulting from investment must both be negative. Thus, in equilibrium, the manager will prefer no investment over investment in either $L$ or $M$. We have established the following proposition, albeit informally.\footnote{A formal proof involving specification of off-equilibrium beliefs is available, upon request, from the authors.}

\textbf{Proposition 2:} The equilibrium investment strategy without periodic performance reporting is $I(S, \alpha) = \emptyset, \forall S, \forall \alpha.$
investment and, therefore, no way to credibly communicate any information about the probability of state $G$. Consequently, a value maximizing manager is unable to make use of her private information and simply makes the investment choice that maximizes the \textit{ex ante} value of the firm.\footnote{See Brandenburger and Polak [1996] for a generalization of such phenomena.} This results in underinvestment relative to first best whenever $S > S^*$. (Notice that if we had assumed the reverse inequality in (10), the manager would always invest in $L$, leading to overinvestment when $S < S^*$.)

Next we illustrate how reporting the results of operations mitigates this underinvestment problem and study regimes with more frequent vs. less frequent performance reporting. In the frequent reporting regime, operating results are disclosed at both dates 1 and 2, while in the infrequent reporting regime there is no report at date 1 and the date 2 report discloses the cumulative result of operations up to date 2. In both regimes, at date zero, the market observes only whether the firm has invested or not invested. More precisely:

Frequent Reporting: date 1 report = $\{x_1\}$, date 2 report = $\{y_2 = x_1 + x_2\}$,

Infrequent Reporting: no report at date 1, date 2 report = $\{y_2 = x_1 + x_2\}$.

Note that while frequent reporting obviously provides more information at date 1, in principle it could also provide more information at date 2 than infrequent reporting. In the frequent reporting regime the market can calculate $x_2$ from the date 2 and the date 1 reports, so that the information in the market at date 2 is $\{x_1, x_2\}$. However, in the infrequent reporting regime, information about $x_1$ is lost because of the failure to measure it at date 1, so that at date 2 the market learns only the \textit{aggregate} two-period cash flow, $y_2 = x_1 + x_2$. Thus, in principle, frequent reporting could provide more timely and less aggregated information than infrequent reporting. However, in order to simplify the analysis, we make the following assumption:

$$y_2 \equiv x_1 + x_2 \text{ is a sufficient statistic for } \{x_1, x_2\}.$$
5. Equilibrium with Infrequent Reporting

Recall that at date 0, the capital market learns only whether or not the manager has invested. If the market observes that the firm has not invested, period 1 and the period 2 prices are identically zero. If the market observes that the firm has invested, it still does not know whether the firm has invested in project $L$ or project $M$. Since there is no performance report at date 1, the date 1 price, conditional on investment, must reflect only an inference about $S$ based on the fact that the firm has invested and the market’s anticipation of which project the manager would choose. At date 2, however, the realized cumulative cash inflow, $y_2$, is reported, so the date 2 price must reflect the information contained in $y_2$. In contrast to the no performance reporting scenario, we will show that the anticipation of the date 2 performance report disciplines the manager’s ex ante incentives in such a manner that: (i) There is a lower interval of firm types that would choose not to invest. The threshold signal defining this lower interval depends on the known parameter $\alpha$. (ii) All firm types above the threshold would invest in project $L$ rather than project $M$.

We first establish result (ii), then go on to prove (i) and establish the properties of the lower interval of firm types that choose not to invest. In order to distinguish equilibrium prices in this setting from prices in other settings we use the notation $\varphi$ to represent prices in this setting.

Let $S_I(\alpha)$ be the threshold value of the signal so that only types $S \geq S_I(\alpha)$ are believed to invest. Then investment in either $L$ or $M$ conveys the information $S \geq S_I(\alpha)$. If the market believes that the manager invested in project $I \in \{L, M\}$ the date 1 price is:

$$\varphi_1(\alpha) = \text{Prob}(G \mid S \geq S_I(\alpha))V_0(G, I) + \text{Prob}(B \mid S \geq S_I(\alpha))V_0(B, I) - K,$$

and the date 2 price, given observation of $y_2$ is:

$$\varphi_2(\alpha, y_2) = y_2 + \text{Prob}(G \mid S \geq S_I(\alpha), y_2, I)V_2(G, I) + \text{Prob}(B \mid S \geq S_I(\alpha), y_2, I)V_2(B, I) - K.$$

(15)
We have earlier assumed that each period’s cash flow satisfies MLRP. Given, the additional assumption that \( y_2 \equiv x_1 + x_2 \) is a sufficient statistic for \( \{x_1, x_2\} \), \( y_2 \) inherits the joint likelihood properties of \( \{x_1, x_2\} \). Thus, using \( h(.) \) as the density function of \( y_2 \):

\[
\frac{h(y_2 \mid G, I)}{h(y_2 \mid B, I)} \text{ is increasing in } y_2 \text{ for each } I \in \{M, L\}.
\]  

(16)

Note that, as assessed by the manager, the distribution of \( y_2 \) conditional on signal \( S \) and any project \( I \) is defined by the following mixture of distributions:

\[
h(y_2 \mid S, I) = \text{Prob}(G \mid S)h(y_2 \mid G, I) + \text{Prob}(B \mid S)h(y_2 \mid B, I).
\]  

(17)

Because \( \text{Prob}(G \mid S) \) is strictly increasing in \( S \), and the distribution of \( y_2 \) conditional on \( (G, I) \) first order dominates the distribution of \( y_2 \) conditional on \( (B, I) \), higher values of \( S \) will cause the distribution of \( y_2 \) conditional on \( (S, I) \) to shift to the right. Also, because \( y_2 \) is stochastically bigger in each state under project \( L \) than under project \( M \), \( h(y_2 \mid S, L) \) first order dominates \( h(y_2 \mid S, M) \), \( \forall S \).

Now, consider the manager’s choice between projects \( L \) and \( M \) given observation of some signal value \( S \). In making this choice the manager takes the market’s pricing rules, described in (14) and (15) as given and beyond her control. Specifically, the threshold \( S_I(\alpha) \) and the market’s belief about which project is chosen if investment is observed to occur are taken as givens. Given this price taking behavior, all the manager can do, by choosing between projects \( L \) and \( M \), is to influence the distribution from which \( y_2 \) is drawn. This implies that the date 1 price, \( \varphi_1(\alpha) \), as described in (14) is a constant, in the sense that it does not change with the project that is actually chosen by the manager. Therefore, for every \( \alpha \), the manager’s objective function:

\[
\max_{I \in \{L, M\}} [\alpha E_0(\varphi_1 \mid S, I) + (1 - \alpha) E_0(\varphi_2 \mid S, I)]
\]

is equivalent to

\[
\max_{I \in \{L, M\}} [E_0(\varphi_2 \mid S, I)].
\]
Proposition 3: Given any threshold $S_f(\alpha)$, and given any conjectured investment $I \in \{M,L\}$ that is incorporated in stock prices, the manager strictly prefers project $L$ to project $M$ at every signal $S$ that she may observe and at every value of $\alpha$.

Proof: The manager’s expectation of the date 2 price if she chooses project $L$ is

$$E_0[\varphi_2 \mid S, L] = E_0(y_2 \mid S, L) + E_0[\text{Prob}(G \mid S \geq S_f(\alpha), y_2, I) \mid S, L]V_2(G, I)$$

$$+ E_0[\text{Prob}(B \mid S \geq S_f(\alpha), y_2, I) \mid S, L]V_2(B, I) - K. \tag{18}$$

and the same expectation if she chooses project $M$ is:

$$E_0[\varphi_2 \mid S, M] = E_0(y_2 \mid S, M) + E_0[\text{Prob}(G \mid S \geq S_f(\alpha), y_2, I) \mid S, M]V_2(G, I)$$

$$+ E_0[\text{Prob}(B \mid S \geq S_f(\alpha), y_2, I) \mid S, M]V_2(B, I) - K.$$ Since $h(y_2 \mid S, L)$ first order dominates $h(y_2 \mid S, M), \forall S$,

$$E_0(y_2 \mid S, L) > E_0(y_2 \mid S, M). \tag{19}$$

Additionally, $\text{Prob}(G \mid S \geq S_f(\alpha), y_2, I)$ is strictly increasing in $y_2$ for each $I$ because of the strict MLRP property described in (16). Therefore, stochastic dominance implies:

$$E_0[\text{Prob}(G \mid S \geq S_f(\alpha), y_2, I) \mid S, L] > E_0[\text{Prob}(G \mid S \geq S_f(\alpha), y_2, I) \mid S, M]. \tag{20}$$

From (19) and (20), and from the fact that $V_2(G, I) > V_2(B, I)$ for all $I$, it follows that

$$E_0(\varphi_2 \mid S, L) > E_0(\varphi_2 \mid S, M)$$

for all $S$, which implies that the manager would prefer project $L$ to project $M$ regardless of the $S$ she observes. Q.E.D.

Proposition 3 implies that managerial short-termism cannot occur in the infrequent reporting regime, no matter how impatient the firm’s current shareholders are. The only reason why the short-term project could be attractive is that it could boost the date 1 price. But, since there is no performance report at date 1, the date 1 price becomes an exogenous constant. It
cannot be boosted by producing attractive short-term cash flows via the choice of the short-term project.

Proposition 3 also implies that the only sustainable belief for the capital market is that whenever the firm chooses to invest, it invests in project \( L \) rather than project \( M \). The equilibrium pricing rules in the capital market must reflect this fact, and therefore in equations (14) and (15) the undefined project \( I \) must be replaced by the known project \( L \). We use this fact in the remainder of the analysis.

We now establish that the set of types that invest is an upper interval of the support \([S, \overline{S}]\) and establish the properties of this interval. Consider the firm’s choice between not investing and investing in project \( L \), given that it has observed a signal value \( S \). If the firm does not invest its expected payoff is zero. If it invests in project \( L \), the firm’s expected payoff is

\[
\alpha \varphi_1(\alpha) + (1 - \alpha) E_0[\varphi_2(\alpha, y_2) | S, L],
\]

where these prices are defined in (14) and (15), respectively, with the undefined project \( I \) replaced by project \( L \). Now, \( \varphi_1(\alpha) \) is a constant, in the sense that it does not vary with \( S \), and

\[
E_0[\varphi_2(\alpha, y_2) | S, L] = E_0(y_2 | S, L) + E_0[\text{Prob}(G | S \geq S_f(\alpha), y_2, L) | S, L] V_2(G, L)
+ E_0[\text{Prob}(B | S \geq S_f(\alpha), y_2, L) | S, L] V_2(B, L) - K.
\]

Now, \( E_0(y_2 | S, L) \) is strictly increasing in \( S \). Additionally, since \( \text{Prob}(G | S \geq S_f(\alpha), y_2, L) \) is strictly increasing in \( y_2 \), \( E_0[\text{Prob}(G | S \geq S_f(\alpha), y_2, L) | S, L] \) is also strictly increasing in \( S \).

Consequently \( E_0[\varphi_2 | S, L] \) and the entire expected payoff from investing in project \( L \) are also strictly increasing in \( S \).

The above analysis implies that if any type \( S' \) prefers investing in \( L \) to not investing, then all types \( S'' > S' \) will also invest in \( L \). Thus, the set of types who invest in \( L \) must be an upper interval of the form \([S_f(\alpha), \overline{S}]\). The marginal type \( S_f(\alpha) \) must be indifferent between investing and not investing. This implies that \( S_f(\alpha) \) must satisfy:
Lemma 1: In the infrequent reporting regime the equilibrium date 1 price is strictly bigger than the expectation of the equilibrium date 2 price, conditional on the marginal type $S_f(\alpha)$, i.e. 

$$\phi_1(\alpha) > E_0[\phi_2(\alpha, y_2) \mid S_f(\alpha), L], \forall \alpha.$$ 

Proof: Since $V_0(\sigma, L) = E(y_2 \mid \sigma, L) + V_2(\sigma, L)$, $\forall \sigma \in \{G, B\}$, $\phi_1(\alpha)$, as described in (14), can be expressed as:

$$\phi_1(\alpha) = E(y_2 \mid S \geq S_f(\alpha), L)$$

$$+ \operatorname{Prob}(G \mid S \geq S_f(\alpha))V_2(G, L) + \operatorname{Prob}(B \mid S \geq S_f(\alpha))V_2(B, L) - K$$

and from (15),

$$E_0[\phi_2 \mid S_f(\alpha), L] = E_0(y_2 \mid S_f(\alpha), L) + E_0[\operatorname{Prob}(G \mid S \geq S_f(\alpha), y_2, L) \mid S_f(\alpha), L]V_2(G, L)$$

$$+ E_0[\operatorname{Prob}(B \mid S \geq S_f(\alpha), y_2, L) \mid S_f(\alpha), L]V_2(B, L) - K.$$ 

Now,

$$E(y_2 \mid S \geq S_f(\alpha), L) > E(y_2 \mid S_f(\alpha), L),$$

and

$$\operatorname{Prob}(G \mid S \geq S_f(\alpha)) = E_0[\operatorname{Prob}(G \mid S \geq S_f(\alpha), y_2, L) \mid S \geq S_f(\alpha), L]$$

$$> E_0[\operatorname{Prob}(G \mid S \geq S_f(\alpha), y_2, L) \mid S_f(\alpha), L]$$

where, the equality in the last step is due to the law of iterated expectations and the inequality is due to the fact that $S \geq S_f(\alpha)$ is a more favorable event than $S = S_f(\alpha)$. These inequalities together with $V_2(G, L) > V_2(B, L)$ yield the desired result. 

Q.E.D.

Lemma 2: $S_f(\alpha)$ is strictly decreasing in $\alpha$.

Proof: Since $S_f(\alpha)$ must satisfy (22) for every $\alpha$.

21
\[
\frac{\partial}{\partial \alpha} \{\alpha \varphi_1(\alpha) + (1-\alpha)E_0[\varphi_2(\alpha, y_2) \mid S_I(\alpha), L]\} = 0
\]

where \( \varphi_1 \) is described in (14) and \( E_0(\varphi_2) \) is described in (23). Carrying out the differentiation yields,

\[
\varphi_1 - E_0(\varphi_2) + \frac{dS_I}{d\alpha} \left[ \alpha \frac{\partial \varphi_1}{\partial S_I} + (1-\alpha) \frac{\partial E_0(\varphi_2)}{\partial S_I} \right] = 0
\]

(24)

Now, from Lemma 1, \( \varphi_1 - E_0(\varphi_2) > 0, \forall \alpha \). Also, since \( \text{Prob}(G \mid S \geq S_I) \) is strictly increasing in \( S_I, \frac{\partial \varphi_1}{\partial S_I} > 0 \). Additionally, note that in equation (23) \( S_I \) appears only as a conditioning argument in each of the expected value expressions. Since both \( E(y_2 \mid S_I, L) \) and \( E_0[\text{Prob}(G \mid S \geq S_I, y_2, L) \mid S_I, L] \) are strictly increasing in \( S_I \), it follows that \( \frac{\partial E_0(\varphi_2)}{\partial S_I} > 0 \).

Therefore, (24) implies that \( \frac{dS_I}{d\alpha} < 0 \).

Q.E.D.

Lemma 3: The set of types who invest is a strict subset of \( [\underline{S}, \bar{S}] \). Specifically,

\( \underline{S} < S_I(\alpha) < S^*, \forall \alpha \).

Proof: Since \( S_I(\alpha) \) is strictly decreasing, we need only establish that \( S_I(\alpha = 0) < S^* \) and \( S_I(\alpha = 1) > \underline{S} \). Consider \( \alpha = 0 \). At \( \alpha = 0 \) the ex post payoff to the firm’s current shareholders is simply the date 2 price. Therefore \( S_I(\alpha = 0) \) must satisfy:

\[
E_0(y_2 \mid S_I(0), L) + E_0[\text{Prob}(G \mid S \geq S_I(0), y_2, L) \mid S_I(0), L]V_2(G, L) +
\]

\[
E_0[\text{Prob}(B \mid S \geq S_I(0), y_2, L) \mid S_I(0), L]V_2(B, L) - K
\]

\[
= \text{Prob}(G \mid S^*)V_0(G, L) + \text{Prob}(B \mid S^*)V_0(B, L) - K = 0.
\]

(25)

But,
\[
\text{Prob}(G \mid S^*) V_0(G, L) + \text{Prob}(B \mid S^*) V_0(B, L) =
\]

\[
\text{Prob}(G \mid S^*) [E_0(y_2 \mid G, L) + V_2(G, L)] + \text{Prob}(B \mid S^*) [E_0(y_2 \mid B, L) + V_2(B, L)]
\]

\[
= E_0(y_2 \mid S^*, L) + \text{Prob}(G \mid S^*) V_2(G, L) + \text{Prob}(B \mid S^*) V_2(B, L).
\]

We show that at \( S_f(0) = S^* \) the left hand side of (25) is strictly greater than the right hand side of (25), from which it follows that \( S_f(0) < S^* \). At \( S_f(0) = S^* \) the left hand side of (25) is:

\[
E_0(y_2 \mid S^*, L) + E_0[\text{Prob}(G \mid S \geq S^*, y_2, L) \mid S^*, L] V_2(G, L)
\]

\[
+ E_0[\text{Prob}(B \mid S \geq S^*, y_2, L) \mid S^*, L] V_2(B, L) - K
\]

which is strictly greater than the right hand side of (25) because,

\[
E_0[\text{Prob}(G \mid S \geq S^*, y_2, L) \mid S^*, L] > E_0[\text{Prob}(G \mid S^*, y_2, L) \mid S^*, L] = \text{Prob}(G \mid S^*)
\]

Now consider \( \alpha = 1 \). In this case since all the weight is on the first period price, \( S_f(\alpha = 1) \)

must satisfy:

\[
\text{Prob}(G \mid S \geq S_f(1)) V_0(G, L) + \text{Prob}(B \mid S \geq S_f(1)) V_0(B, L) =
\]

\[
\text{Prob}(G \mid S^*) V_0(G, L) + \text{Prob}(B \mid S^*) V_0(B, L).
\]

But this equality can only hold if \( \text{Prob}(G \mid S \geq S_f(1)) = \text{Prob}(G \mid S^*) \) which, in turn, implies that

\( S_f(1) > \underline{S} \) because \( \text{Prob}(G \mid S \geq \underline{S}) = \lambda < \text{Prob}(G \mid S^*) \).

\( Q.E.D. \)

Intuitively, the reason why sufficiently low types find investment unattractive is that the presence of a performance report disciplines their incentives. To see this disciplining effect more clearly, suppose that the market believes that the set of types who invest are types contained in some interval \([\hat{S}, \overline{S}]\), where \( \hat{S} > \underline{S} \). Then the very act of investment conveys the information that \( S \geq \hat{S} \), which results in a revision of the prior probability of state \( G \) from \( \lambda \) to the higher number \( \text{Prob}(G \mid S \geq \hat{S}) \). This new prior is additionally updated into a posterior probability upon
observation of the performance report $y_2$, and this posterior probability is strictly increasing in $y_2$. Now, define $S^0$ by the equation $\text{Prob}(G \mid S \geq \hat{S}) = \text{Prob}(G \mid S = S^0)$. Clearly $S^0 > \hat{S}$. Then for every $S < S^0$, $E_0[\text{Prob}(G \mid S \geq \hat{S}, y_2, L) \mid S, L] < \text{Prob}(G \mid S \geq \hat{S})$. Thus if any type lower than $S^0$ invests, that type expects that the prior $\text{Prob}(G \mid S \geq \hat{S})$ will be downgraded upon observation of the performance report, with lower types expecting even greater degrees of downgrading. This is because lower types expect lower values of $y_2$. Thus, as in Kanodia and Lee [1998], the presence of a performance report disciplines their incentives. This discipline was missing in the previous setting where there are no performance reports. So, in the previous setting if any single type found investment attractive then all types below that type would also find investment attractive.

The result that $S_1(\alpha) < S^*, \forall \alpha$, implies that, in equilibrium, there are some low types that invest even though they privately know that the project has negative expected net present value. These types get pooled with higher types whose expected net present value is positive, resulting in an expected valuation greater than zero. In this sense, there is over-investment consistent with the general result that in any signaling equilibrium there is over-investment in the signal. Lemma 2 indicates that the region of over-investment is larger when there is greater impatience in the capital market ($\alpha$ is larger).

The following proposition summarizes the salient properties of equilibrium in the infrequent reporting regime.

**Proposition 4:** In the infrequent reporting regime, managerial short-termism does not occur. The firm invests in the long-term project if it receives sufficiently high signals and does not invest if it receives sufficiently low signals. The discipline imposed by performance reporting is not perfect. There is a region of inefficient over-investment and this region expands with the degree of impatience in the capital market.
Figure 1 below is a pictorial representation of the equilibrium.

![Diagram showing equilibrium investment strategy in the infrequent reporting regime](image)

**6. Equilibrium with Frequent Reporting**

The only difference between the frequent and infrequent reporting regimes is that in the frequent reporting regime there is a performance report at date 1 that reveals the first period cash inflow $x_1$, in addition to the previous performance report at date 2. Therefore, the date 1 price in the capital market is not a constant, rather it is a function of $x_1$. Given the assumption that the cumulative cash flow $y_2 = x_1 + x_2$ is a sufficient statistic for $(x_1, x_2)$ the date 2 price is a function of $y_2$. As in the infrequent reporting regime, the performance reports in the frequent reporting regime discipline the firm’s investment so that investment occurs only when the signal $S$ is above some critical threshold, which we denote $S_F(\alpha)$. It will turn out to be the case that the equilibrium date 1 price is strictly increasing in $x_1$ and the equilibrium date 2 price is strictly increasing in $y_2$. Since the short-term project produces a stochastically bigger $x_1$ while the long-
term project produces a stochastically bigger $y_2$, the manager faces a non-trivial tradeoff when choosing between the short and long-term projects. Naturally, this tradeoff depends upon the weight, $\alpha$, on the first period price, i.e., on the degree of impatience in the capital market. But, in general, the tradeoff also depends upon the signal $S$ that the manager observes. In order to insure that the tradeoff between the first and second period prices is monotone in $S$, we need to make some additional assumptions.

Let $F_1(x_1 \mid \sigma, I)$ and $F_2(x_2 \mid \sigma, I)$ be the cumulative distribution functions of the first and second period cash flow, conditional on each state and each project, and let $H(y_2 \mid \sigma, I)$ be the corresponding cumulative distribution function of $y_2$. We assume,

$$F_1(x_1 \mid G, L) F_2(x_2 \mid G, L) - F_1(x_1 \mid B, L) F_2(x_2 \mid B, L) \leq F_1(x_1 \mid G, M) F_2(x_2 \mid G, M) - F_1(x_1 \mid B, M) F_2(x_2 \mid B, M), \forall x_1, x_2. \tag{26}$$

By letting $x_2 \to \infty$, it can be seen that (26) implies:

$$F_1(x_1 \mid G, L) - F_1(x_1 \mid B, L) \leq F_1(x_1 \mid G, M) - F_1(x_1 \mid B, M), \forall x_1. \tag{27}$$

Additionally, because $H(y_2 \mid \sigma, I) = \int F_1(y_2 - x_2 \mid \sigma, I) F_2(x_2 \mid \sigma, I) dx_2$ (26) implies:

$$H(y_2 \mid G, L) - H(y_2 \mid B, L) \leq H(y_2 \mid G, M) - H(y_2 \mid B, M), \forall y_2. \tag{28}$$

In order to interpret (26), note that (27) implies:

$$E(x_1 \mid G, L) - E(x_1 \mid B, L) \geq E(x_1 \mid G, M) - E(x_1 \mid B, M),$$

and (28) implies:

$$E(x_1 + x_2 \mid G, L) - E(x_1 + x_2 \mid B, L) \geq E(x_1 + x_2 \mid G, M) - E(x_1 + x_2 \mid B, M).$$

Our earlier assumption, that a switch from state $G$ to state $B$ stochastically decreases each project’s cash flow in every period, implies that both sides of (26) are negative. Assumption (26)

---

8 See Shaked and Shantikumar [2007], Theorem 6.B.16.
additionally says that, in a probabilistic sense, the damage to the long term project is at least as large as the damage to the short-term project.

The nature of the equilibrium depends on whether (26) holds as an equality or as a strict inequality. We label these two cases as Case (I) and Case (II). Case (I) is discussed here and Case (II) is analyzed in the Appendix. When (26) holds as an equality, the tradeoff between the short-term and long-term project does not depend on $S$, yielding a simple and intuitive characterization of the equilibrium. When (26) holds as a strict inequality, this tradeoff depends on both $\alpha$ and $S$, and the resulting equilibrium is significantly more complex. However, managerial short-termism is present in both cases.

**Case (I):**

When (26) holds with equality, the equilibrium is as depicted in Figure 2 below.

![Fig. 2 – equilibrium investment strategy in the frequent reporting regime Case (I)](image-url)
The salient properties of this equilibrium are described in Proposition 5 below:
Proposition 5: Assume that (26) holds with equality everywhere. Then there exists \( \alpha^* \in (0,1) \), and a schedule \( S_F(\alpha) \) such that:

\[
\begin{align*}
(i) & \quad I(S, \alpha) = L, \forall \alpha \leq \alpha^*, S \geq S_F(\alpha) \\
(ii) & \quad I(S, \alpha) = M, \forall \alpha > \alpha^*, S < S_F(\alpha) \\
(iii) & \quad I(S, \alpha) = \emptyset, \forall \alpha, S < S_F(\alpha) \\
(iv) & \quad S_F(\alpha) \text{ is strictly decreasing in } \alpha \\
(v) & \quad S < S_F(\alpha) < S^*, \forall \alpha
\end{align*}
\]

Proposition 5 says that frequent reporting will induce managerial short-termism, with probability one, if the degree of impatience in the capital market is sufficiently high. This result is consistent with the intuition expressed in the European debate, described earlier. Since managerial short-termism never occurs in the infrequent reporting regime, but does occur in the frequent reporting regime, Proposition 5 identifies a potent endogenous cost that could be precipitated when the frequency of financial reporting is increased. But there is also a benefit from increasing the frequency of financial reporting. As in the infrequent reporting regime, in the frequent reporting regime too, there is over-investment for sufficiently low values of \( S \) and the probability of over-investment increases with the degree of impatience in the capital market. We show later that the discipline is stronger in the frequent reporting regime causing the over-investment region to shrink relative to the infrequent reporting regime.

We establish Proposition 5 through a series of lemmas. Suppose the market believes that the firm’s investment strategy is as described in Proposition 5 and Figure 2. Then the date 1 and date 2 prices in the capital market, conditional on observing investment, are:

at each \( \alpha \leq \alpha^* \):

\[
P_1(\alpha, x_1) = x_1 + \text{Prob}(G \mid S \geq S_F(\alpha), x_1, L) V_1(G, L)
\]

\[+ \text{Prob}(B \mid S \geq S_F(\alpha), x_1, L) V_1(B, L) - K
\]

\[\text{(30)}\]
and,

\[
P_2(\alpha, x_1, x_2) = P_2(\alpha, y_2) = y_2 + \text{Prob}(G \mid S \geq S_F(\alpha), y_2, L)V_2(G, L) \\
+ \text{Prob}(B \mid S \geq S_F(\alpha), y_2, L)V_2(B, L) - K.
\]  (31)

At each \( \alpha > \alpha^* \):

\[
P_1(\alpha, x_1) = x_1 + \text{Prob}(G \mid S \geq S_F(\alpha), x_1, M)V_1(G, M) \\
+ \text{Prob}(B \mid S \geq S_F(\alpha), x_1, M)V_1(B, M) - K 
\]  (32)

and,

\[
P_2(\alpha, y_2) = y_2 + \text{Prob}(G \mid S \geq S_F(\alpha), y_2, M)V_2(G, M) \\
+ \text{Prob}(B \mid S \geq S_F(\alpha), y_2, M)V_2(B, M) - K. 
\]  (33)

Equations (32) and (33) are identical to (30) and (31) except that project L is replaced by project M, consistent with the market belief that the firm switches to M when \( \alpha > \alpha^* \).

Lemma 4: Regardless of whether \( \alpha \leq \alpha^* \) or \( \alpha > \alpha^* \):

(i) The date 1 price exerts pressure on the manager to choose project M rather than project L, i.e. \( E_0[P_1(\alpha, x_1) \mid S, M] > E_0[P_1(\alpha, x_1) \mid S, L], \forall S \), and

(ii) The date 2 price pressures the manager to choose project L rather than project M, i.e., \( E_0[P_2(\alpha, y_2) \mid S, L] > E_0[P_2(\alpha, y_2) \mid S, M], \forall S \).

Proof: Given any \( I \in \{L, M\} \), \( P_1(\alpha, x_1) \) is strictly increasing in \( x_1 \) if \( \text{Prob}(G \mid S \geq S_F(\alpha), x_1, I) \) is increasing in \( x_1 \), where,

\[
\text{Prob}(G \mid S \geq S_F(\alpha)) = \frac{\text{Prob}(G \mid S \geq S_F(\alpha))f_1(x_1 \mid G, I)}{\text{Prob}(G \mid S \geq S_F(\alpha))f_1(x_1 \mid G, I)+\text{Prob}(B \mid S \geq S_F(\alpha))f_1(x_1 \mid B, I)}. 
\]  (34)
The MLRP assumption (2) guarantees that this probability is indeed increasing in $x_1$. Then, since $f_1(x_1 \mid S, M)$ is to the right of $f_1(x_1 \mid S, L)$ in the sense of first order stochastic dominance,

$$\int P_1(\alpha, x_1) f_1(x_1 \mid S, M) dx_1 > \int P_1(\alpha, x_1) f_1(x_1 \mid S, L) dx_1 \quad \text{for all } S.$$ 

This establishes (i) of the Lemma. The proof of (ii) is virtually identical to the proof in Proposition 3, so it is omitted. \[ Q.E.D. \]

Given $\alpha$ and $S$, the manager chooses project $L$ if,

$$\alpha \{ E_0[P_1(\alpha, x_1) \mid S, M] - E_0[P_1(\alpha, x_1) \mid S, L] \} < (1 - \alpha) \{ E_0[P_2(\alpha, y_2) \mid S, L] - E_0[P_2(\alpha, y_2) \mid S, M] \}.$$ 

(35)

The manager chooses project $M$ if the inequality in (35) is reversed. If (35) holds with equality, the manager is indifferent between projects $L$ and $M$, and we assume that in the case of indifference the manager chooses project $L$. Lemma 4 established that both sides of (35) are strictly greater than zero. The left hand side of (35) represents the expected gain from producing attractive cash flows in period 1 by choosing the short-term project, while the right hand side represents the expected loss from producing lower cumulative cash flows over two periods. Obviously, the value of $\alpha$ (representing the degree of impatience in the capital market) is one important determinant of whether or not (35) is satisfied. Below, we show how the relationships in (35) depend on $S$. Let,

$$\Delta_1(S) \equiv E_0[P_1(\alpha, x_1) \mid S, M] - E_0[P_1(\alpha, x_1) \mid S, L], \quad \text{and}$$

$$\Delta_2(S) \equiv E_0[P_2(\alpha, y_2) \mid S, L] - E_0[P_2(\alpha, y_2) \mid S, M].$$
Lemma 5: If (26) holds as an equality everywhere (implying that (27) and (28) hold with equality everywhere):

\[ \frac{\partial}{\partial S} \{\Delta_1(S)\} = \frac{\partial}{\partial S} \{\Delta_2(S)\} = 0 \quad \text{for all } \alpha. \]

If (26) holds as a strict inequality:

\[ \frac{\partial}{\partial S} \{\Delta_1(S)\} < 0 \quad \text{and} \quad \frac{\partial}{\partial S} \{\Delta_2(S)\} > 0 \quad \text{for all } \alpha. \]

Proof:

\[ \frac{\partial}{\partial S} \{\Delta_1(S)\} = \int \left( P_1(\alpha, x_1) \frac{\partial}{\partial S} f_1(x_1 | S, M) \right) dx_1 - \int \left( P_1(\alpha, x_1) \frac{\partial}{\partial S} f_1(x_1 | S, L) \right) dx_1 \]

But,

\[ \frac{\partial}{\partial S} f_1(x_1 | S, M) = \left( \frac{\partial}{\partial S} \text{Prob}(G | S) \right) [f_1(x_1 | G, M) - f_1(x_1 | B, M)]. \]

Similarly,

\[ \frac{\partial}{\partial S} f_1(x_1 | S, L) = \left( \frac{\partial}{\partial S} \text{Prob}(G | S) \right) [f_1(x_1 | G, L) - f_1(x_1 | B, L)]. \]

Therefore,

\[ \frac{\partial}{\partial S} \{\Delta_1(S)\} = \left( \frac{\partial}{\partial S} \text{Prob}(G | S) \right) \left[ \int P_1(\alpha, x_1) [f_1(x_1 | G, M) - f_1(x_1 | B, M)] - f_1(x_1 | G, L) + f_1(x_1 | B, L) \right] dx_1 \]

Evaluating the integral by parts and cancelling common terms gives,

\[ \frac{\partial}{\partial S} \{\Delta_1(S)\} = \]
\[
\left( \frac{\partial}{\partial S} \text{Prob}(G \mid S) \right) \left[ \int \frac{\partial P}{\partial x_1} [F_1(x_1 \mid B, M) - F_1(x_1 \mid G, M) + F_1(x_1 \mid G, L) - F_1(x_1 \mid B, L)] \, dx_1 \right].
\]

Clearly, \( \frac{\partial}{\partial S} \{\Delta_1(S)\} = 0 \) when (27) holds with equality. Also, since \( \frac{\partial}{\partial S} \{\text{Prob}(G \mid S)\} > 0 \)

and \( \frac{\partial P}{\partial x_1} > 0, \quad \frac{\partial}{\partial S} \{\Delta_1(S)\} < 0 \) when (27) holds as a strict inequality.

Using exactly the same analysis as above, it can be shown that,

\[
\frac{\partial}{\partial S} \{\Delta_2(S)\} =
\int \left( \frac{\partial P}{\partial y_2} [H(y_2 \mid B, L) - H(y_2 \mid G, L) - H(y_2 \mid B, M) + H(y_2 \mid G, M)] \right) \, dy_2.
\]

Therefore \( \frac{\partial}{\partial S} \{\Delta_2(S)\} = 0 \) when (28) holds with equality, and \( \frac{\partial}{\partial S} \{\Delta_2(S)\} > 0 \) when (28) holds as a strict inequality. \( Q.E.D \)

Lemma 5 shows that when the bad state equally damages (in a stochastic sense) the cash flows of the short-term and long-term projects, the tradeoff between these projects is independent of the manager’s private signal \( S \). More precisely, \( \Delta_1(S) = \Delta_1 > 0, \forall S \) and \( \Delta_2(S) = \Delta_2 > 0, \forall S \).

Then there is a unique value of \( \alpha \), say \( \alpha^* \), satisfying the equation: \( \alpha^* \Delta_1 = (1 - \alpha^*) \Delta_2 \), i.e.,

\[
\alpha^* = \frac{\Delta_2}{\Delta_1 + \Delta_2}.
\]

At \( \alpha = \alpha^* \), the manager is indifferent between projects \( L \) and \( M \), at \( \alpha < \alpha^* \) the manager strictly prefers \( L \) to \( M \) and at \( \alpha > \alpha^* \) the manager strictly prefers \( M \) to \( L \), for every value of \( S \). This establishes claims (i) and (ii) of Proposition 5.

At any \( \{\alpha < \alpha^*, S\} \) the manager chooses not to invest, i.e. the manager prefers \( \emptyset \) to \( L \) if:

\[
\alpha \, E_0[P_1(\alpha, x_1) \mid S, L] + (1 - \alpha) \, E_0[P_2(\alpha, y_2) \mid S, L] < 0.
\]
Since the left hand side is strictly increasing in $S$, the set of $S$ values at which there is no investment must be a lower interval of the support $[S, S]$. The boundary of this interval, which we denote $S_F(\alpha)$, must satisfy:

$$aE_0[P_1(\alpha, x_1) | S_F(\alpha), L] + (1-\alpha)E_0[P_2(\alpha, y_2) | S_F(\alpha), L] = 0$$

(36)
i.e.,

$$aE_0(x_1 | S_F(\alpha), L) + E_0[\text{Prob}(G | S \geq S_F(\alpha), x_1, L) | S_F(\alpha), L][V_1(G, L) - V_1(B, L)] +$$

$$(1-\alpha)E_0(y_2 | S_F(\alpha), L) + E_0[\text{Prob}(G | S \geq S_F(\alpha), y_2, L) | S_F(\alpha), L][V_2(G, L) - V_2(B, L)]$$

$$+ \alpha V_1(B, L) + (1-\alpha)V_2(B, L) - K = 0.$$  

(37)

We now establish the salient properties of the schedule $S_F(\alpha)$.

**Lemma 6:** For any $S_F$,

$$E_{x_1}[\text{Prob}(G | S \geq S_F, x_1, L) | S_F, L] > E_{y_2}[\text{Prob}(G | S \geq S_F, y_2, L) | S_F, L].$$

(38)

**Proof:** We first establish that the desired inequality is generally true without using the sufficiency of $y_2$ for $\{x_1, x_2\}$, then show that the inequality continues to remain valid when $y_2$ is a sufficient statistic, as assumed in the remainder of our analysis. From the law of iterated expectations:

$$E_{x_1}[E_{x_2}[\text{Prob}(G | S \geq S_F, x_1, x_2, L) | S_F, x_1, L] | S_F, L].$$

Also,

$$E_{x_1, x_2}[\text{Prob}(G | S \geq S_F, x_1, x_2, L) | S_F, L] =$$

$$E_{x_1}[E_{x_2}[\text{Prob}(G | S \geq S_F, x_1, x_2, L) | S_F, x_1, L] | S_F, L].$$

But, because $S \geq S_F$ is a more favorable event for state $G$ than $S = S_F$,

$$E_{x_2}[\text{Prob}(G | S \geq S_F, x_1, x_2, L) | S \geq S_F, x_1, L].$$
Therefore,

$$E_{x_2} [\text{Prob}(G \mid S \geq S_F, x_1, x_2, L) \mid S_F, x_1, L], \forall x_1.$$  

(39)

Now consider the case where \( y_2 \) is sufficient for \( \{x_1, x_2\} \). Then, express the right hand side of (39) as:

$$E_{x_1, x_2} [\text{Prob}(G \mid S \geq S_F, x_1, x_2, L) \mid S_F, L] = E_{x_1, y_2} [\text{Prob}(G \mid S \geq S_F, x_1, y_2 - x_1, L) \mid S_F, L].$$

The sufficiency of \( y_2 \) for \( \{x_1, x_2\} \) implies that \( \text{Prob}(G \mid S \geq S_F, x_1, y_2 - x_1, L) \) is a constant with respect to variations in \( x_1 \), so that,

$$E_{x_1, x_2} [\text{Prob}(G \mid S \geq S_F, x_1, x_2, L) \mid S_F, L] = E_{y_2} [\text{Prob}(G \mid S \geq S_F, y_2, L) \mid S_F, L].$$

Then (39) implies (38).

Q.E.D.

**Lemma 7:** In the frequent reporting regime the expectation of the equilibrium date 1 price is strictly bigger than the expectation of the equilibrium date 2 price, conditional on the marginal type \( S_F(\alpha) \), i.e.

$$E_0[P_1(\alpha, x_1) \mid S_F(\alpha), L] > E_0[P_2(\alpha, y_2) \mid S_F(\alpha), L], \forall \alpha \leq \alpha^*.$$  

**Proof:** Using the fact that for each \( \sigma \in (G, B) \), \( V_1(\sigma, L) = E(x_2 \mid \sigma, L) + V_2(\sigma, L) \), the expectation of the date 1 price can be expressed as:

$$E_0[P_1(\alpha, x_1) \mid S_F(\alpha), L] = E_0(x_1 \mid S_F(\alpha), L) +$$

$$E_0[\text{Prob}(G \mid S \geq S_F(\alpha), x_1 \mid S_F(\alpha), L) [E_0(x_2 \mid G, L) - E_0(x_2 \mid B, L)] +$$

$$E_0[\text{Prob}(G \mid S \geq S_F(\alpha), x_1 \mid S_F(\alpha), L) [V_2(G, L) - V_2(B, L)] +$$

$$E_0(x_2 \mid B, L) + V_2(B, L) - K.$$  

The expectation of the date 2 price can be expressed as:

$$E_0(P_2(\alpha, y_2) \mid S_F(\alpha), L) = E_0(x_2 \mid y_2, L) +$$

$$E_0[\text{Prob}(G \mid S \geq S_F(\alpha), y_2 \mid S_F(\alpha), L) [E_0(x_2 \mid G, L) - E_0(x_2 \mid B, L)] +$$

$$E_0[\text{Prob}(G \mid S \geq S_F(\alpha), y_2 \mid S_F(\alpha), L) [V_2(G, L) - V_2(B, L)] +$$

$$E_0(x_2 \mid B, L) + V_2(B, L) - K.$$
\[ E_0[P_2(\alpha, y_2) \mid S_F(\alpha), L] = E_0(x_1 \mid S_F(\alpha), L) + \]

\[ \text{Prob}(G \mid S_F(\alpha)) \left[ E_0(x_2 \mid G, L) - E_0(x_2 \mid B, L) \right] + \]

\[ E_0[\text{Prob}(G \mid S \geq S_F(\alpha), y_2 \mid S_F(\alpha), L) \left[ V_2(G, L) - V_2(B, L) \right]] + \]

\[ E_0(x_2 \mid B, L) + V_2(B, L) - K. \]

Lemma 7 then follows from Lemma 6 and the fact that:

\[ E_0[\text{Prob}(G \mid S \geq S_F(\alpha), x_1 \mid S_F(\alpha), L)] > E_0[\text{Prob}(G \mid S_F(\alpha), x_1 \mid S_F(\alpha), L)] = \text{Prob}(G \mid S_F(\alpha)). \quad Q.E.D.\]

We have proved Lemmas 6 and 7 for the region \( \alpha \leq \alpha^* \). By replacing project \( L \) by project \( M \), it can be seen that these results extend to the region \( \alpha > \alpha^* \). Given the result that the expectation of the date 1 price is strictly bigger than the expectation of the date 2 price, conditional on the marginal type \( S_F(\alpha) \), it is easy to see that \( S_F(\alpha) \) must be decreasing in \( \alpha \). A formal proof is omitted because it would follow virtually the same reasoning developed in Lemma 2 for the infrequent reporting regime. The claim that \( S_F(0) < S^* \) is identical to the similar claim for the infrequent reporting regime, which was proved in Lemma 3. In fact, \( S_F(0) = S_1(0) \). We show, in Proposition 6, that \( S_F(\alpha) > S_1(\alpha) \), \( \forall \alpha > 0 \), hence the proof that \( S_F(1) > S \) being subsumed by this result, is not presented here. This completes our description of the equilibrium for Case (I).

**Case (II)**

We now turn to the case where (26), and hence (27) and (28), hold with strict inequality. We have previously shown, in Lemma 5, that in this case,

\[ E_0[P_1(\alpha, x_1) \mid S, M] - E_0[P_1(\alpha, x_1) \mid S, L] \text{ is strictly decreasing in } S \text{ while} \]

\[ E_0[P_2(\alpha, y_2) \mid S, L] - E_0[P_2(\alpha, y_2) \mid S, M] \text{ is strictly increasing in } S. \] Therefore in the region
\( \alpha < \alpha^* \) (depicted in Figure 2), where project \( L \) was strictly preferred to project \( M \) for every \( S \), strict preference for project \( L \) continues to hold. However, in the region \( \alpha > \alpha^* \), strict preference for project \( M \) cannot be sustained for every \( S \). Given any \( \alpha > \alpha^* \), project \( L \) becomes more attractive in terms of the date 2 price, while project \( M \) becomes less attractive in terms of the date 1 price, as \( S \) increases. Therefore, beyond some critical value of \( S \) (which we denote \( S_L(\alpha) \)), project \( L \) will become more attractive than project \( M \), and below this critical value of \( S \) project \( M \) will be more attractive. This implies that at any \( \alpha > \alpha^* \), there will be some equilibrium probability that the firm has invested in project \( L \) and some probability that the firm has invested in project \( M \). The equilibrium price in the capital market must reflect an assessment of these probabilities conditional on observables.

We will show that the firm’s equilibrium investment policy is as depicted in Figure 3 below.
The proof of the claims made in Figure 3, and the technical details underlying the equilibrium are contained in the Appendix.

7. Is Frequent Reporting Socially Desirable?

We examine social welfare in terms of the size of the expected total pie over the entire investment horizon (N periods). Thus, we are concerned with economic efficiency as viewed by an outside observer, such as a regulator, who is empowered to choose the reporting frequency and implement it by fiat. We do not contrast the payoffs to current vs. prospective shareholders across the two regimes.

We first compare the equilibrium for the infrequent reporting regime to the equilibrium described in Case (I) of the frequent reporting regime. In the Appendix we extend the analysis to Case (II) of the frequent reporting regime. For Case (I), the following proposition captures the benefits that are obtained by increasing the reporting frequency.

**Proposition 6:** There is less overinvestment with frequent reporting than with infrequent reporting, i.e., \( S_I(\alpha) < S_F(\alpha) < S^* \) at all \( \alpha > 0 \), and \( S_I(\alpha) = S_F(\alpha) < S^* \) at \( \alpha = 0 \).

**Proof:** First consider the region \( \alpha \leq \alpha^* \). In this region the firm invests in \( L \) in both the infrequent and frequent reporting regimes, whenever it decides to invest. Therefore, \( S_I(\alpha) \) and \( S_F(\alpha) \) must satisfy:

\[
\alpha \varphi_1(\alpha) + (1 - \alpha) E_0[\varphi_2(\alpha, y_2) | S_I(\alpha), L] = aE_0[P_1(\alpha, x_1) | S_F(\alpha), L] + (1 - \alpha) E_0[P_2(\alpha, y_2) | S_F(\alpha), L] = 0
\]

where,

\[
\varphi_1(\alpha) = \text{Prob}(G | S \geq S_I(\alpha))V_0(G, L) + \text{Prob}(B | S \geq S_I(\alpha))W_0(B, L) - K
\]
\[ E_0[\varphi_2(\alpha, y_2) \mid S_f(\alpha), L] = E_0(y_2 \mid S_f(\alpha), L) + V_2(B, L) - K + \]

\[ E_0[\text{Prob}(G \mid S \geq S_f(\alpha), y_2, L) \mid S_f(\alpha), L][V_2(G, L) - V_2(B, L)], \]

Now consider the possibility that \( S_F(\alpha) = S_I(\alpha) \). Then the expression in (44) equals the expression in (42) and the expression in (41) is greater than the expression in (43) because:

\[ \alpha \varphi_f(\alpha) + (1 - \alpha)E_0[\varphi_2(\alpha, y_2) \mid S_f(\alpha), L] = \]

\[ \alpha \varphi_f(\alpha) + (1 - \alpha)E_0[y_2 \mid S_f(\alpha), L] + V_2(B, L) - K + \]

\[ E_0[\text{Prob}(G \mid S \geq S_f(\alpha), x_1, L) \mid S_f(\alpha), L][V_1(G, L) - V_1(B, L)], \]

Therefore the left hand side of (40) exceeds the right hand side of (40) when \( S_F(\alpha) = S_I(\alpha) \) and \( \alpha > 0 \). Since the right hand side of (40) is strictly increasing in \( S_f(\alpha) \), the satisfaction of (40) requires \( S_F(\alpha) > S_I(\alpha), \forall \alpha \in (0, \alpha^*], \) and \( S_F(0) = S_I(0). \)

Now consider the region \( \alpha > \alpha^* \). In this region, the firm invests in \( L \) in the infrequent reporting regime, but invests in \( M \) in the infrequent reporting regime, whenever it does invest.

Therefore, \( S_f(\alpha) \) and \( S_F(\alpha) \) must satisfy:

\[ \alpha \varphi_f(\alpha) + (1 - \alpha)E_0[\varphi_2(\alpha, y_2) \mid S_f(\alpha), L] = \]
\[ aE_0[P_1(\alpha, x_1) \mid S_F(\alpha), M] + (1 - \alpha)E_0[P_2(\alpha, y_2) \mid S_F(\alpha), M] = 0. \]  

(45)

Again, consider the possibility that \( S_F(\alpha) = S_I(\alpha) \). Because \( V_0(G, L) > V_0(G, M) \) and \( V_0(B, L) > V_0(B, M) \),

\[ \phi_1(\alpha) > \text{Prob}(G \mid S \geq S_I(\alpha))V_0(G, M) + \text{Prob}(B \mid S \geq S_I(\alpha))V_0(B, M) - K \]

\[ = E_0(x_1 \mid S \geq S_I(\alpha), M) + E_0(\text{Prob}(G \mid S \geq S_I(\alpha), x_1, M) \mid S_I(\alpha), M) \]

\[ + E_0(\text{Prob}(B \mid S \geq S_I(\alpha), x_1, M) \mid S_I(\alpha), M) V_1(G, M) \]

\[ > E_0(x_1 \mid S_I(\alpha), M) + E_0(\text{Prob}(G \mid S \geq S_I(\alpha), x_1, M) \mid S_I(\alpha), M) V_1(G, M) \]

\[ + E_0(\text{Prob}(B \mid S \geq S_I(\alpha), x_1, M) \mid S_I(\alpha), M) V_1(B, M) - K \]

\[ = E_0[P_1(\alpha, x_1) \mid S_I(\alpha), M]. \]

Now compare the expectation of the date 2 prices across regimes. \( E_0[P_2(\alpha, y_2) \mid S_I(\alpha), L] = E_0(\varphi_2(\alpha, y_2) \mid S_I(\alpha), L) > E_0(\varphi_2(\alpha, y_2) \mid S_I(\alpha), M) \) where the inequality was established in Proposition 3. Therefore the left hand side of (45) exceeds the right hand side of (45) when \( S_F(\alpha) = S_I(\alpha) \). Since the right hand side of (45) is strictly increasing in \( S_F(\alpha) \), the satisfaction of (45) requires \( S_F(\alpha) > S_I(\alpha), \forall \alpha > \alpha^* \).  

Q.E.D.

Notice that when \( \alpha \leq \alpha^* \), the firm invests in the long-term project in both the infrequent and frequent reporting regimes whenever \( S \geq S^* \) (i.e. when investment has positive net present value) and that there is less incentive to choose negative net present value projects in the frequent reporting regime than in the infrequent reporting regime when \( S < S^* \). Thus when \( \alpha \leq \alpha^* \), there are strict benefits from increasing the reporting frequency and no costs, giving immediate rise to our next proposition.

**Proposition 7:** If the firm’s current shareholders are sufficiently patient, frequent reporting of the results of operations dominates infrequent reporting.
However when $\alpha > \alpha^*$, i.e. when the firm’s current shareholders are sufficiently impatient, there is a clear tradeoff between the benefits and costs associated with increasing the reporting frequency. The benefit arises from the increased discipline on the firm’s investment decision that causes the over-investment region to shrink, as described in Proposition 6. The cost is due to the result that frequent reporting precipitates managerial short-termism. We study these costs and benefits below.

Let $\xi(S) = \lambda \xi(S \mid G) + (1 - \lambda) \xi(S \mid B)$ be the unconditional density of the manager’s private signal. Let $\Gamma_I(\alpha)$ be the efficiency loss in the infrequent reporting regime relative to first best, and let $\Gamma_F(\alpha)$ be the corresponding efficiency loss in the frequent reporting regime. These efficiency losses can be read from the table in Figure 4 below.

![Fig. 4 – welfare comparison when $\alpha > \alpha^*$ Case (I)](image)

As indicated in Figure 4, at each $\alpha > \alpha^*$:

$$
\Gamma_I(\alpha) = \int_{S_{I}(\alpha)}^{S^*} \left[ \xi(s)[K - (\text{Prob}(G \mid S)V_0(G, L) + \text{Prob}(B \mid S)V_0(B, L))] \right] ds.
$$

(46)
Note that $\Gamma_I(\alpha) > 0$ because at each $S < S^*$, the long-term project has negative net present value. The calculation of $\Gamma_F(\alpha)$ reflects the fact that whenever the firm invests, it invests in the short-term project rather than the long-term project.

$$\Gamma_F(\alpha) = \int_{S_F(\alpha)}^{S^*} \xi(s) [K - (\text{Prob}(G \mid S)V_0(G, M) + \text{Prob}(B \mid S)V_0(B, M))] ds + \int_{S^*}^{S} \xi(s) (\text{Prob}(G \mid S)[V_0(G, L) - V_0(G, M) + \text{Prob}(B \mid S)[V_0(B, L) - V_0(B, M)])] ds. \quad (47)$$

Subtracting (46) from (47) gives,

$$\Gamma_F(\alpha) - \Gamma_I(\alpha) = \int_{S_F(\alpha)}^{S^*} \xi(s) [K - (\text{Prob}(G \mid S)V_0(G, M) + \text{Prob}(B \mid S)[V_0(B, L) - V_0(B, M)])] ds - \int_{S^*}^{S} \xi(s) [K - (\text{Prob}(G \mid S)V_0(G, L) + \text{Prob}(B \mid S)V_0(B, L)])] ds. \quad (48)$$

By substituting $S$ in place of $S_F(\alpha)$ in the lower limit of the second integral in (48), we obtain a lower bound on $\Gamma_F(\alpha) - \Gamma_I(\alpha)$. Call this lower bound $\underline{\Gamma}(\alpha)$. Then because $S_F(\alpha)$ is strictly decreasing in $\alpha$, $\underline{\Gamma}(\alpha)$ is strictly increasing in $\alpha$. The next proposition follows from this fact.

**Proposition 8:** In the region $\alpha > \alpha^*$, if there exists $\alpha^0 < 1$ such that $\underline{\Gamma}(\alpha^0) = 0$, then infrequent reporting dominates frequent reporting at every $\alpha \geq \alpha^0$.

We now derive a sufficient condition, stated entirely in terms of exogenous parameters, under which infrequent reporting will dominate frequent reporting at every $\alpha > \alpha^*$. The sufficient condition is obtained by replacing $S_F(\alpha)$ by $S^*$ in the expression that defines $\underline{\Gamma}(\alpha)$, thus yielding a lower bound to $\Gamma_F(\alpha) - \Gamma_I(\alpha)$ that is independent of $\alpha$.
**Proposition 9:** If the firm’s current shareholders are sufficiently impatient, i.e. if $\alpha > \alpha^*$, infrequent reporting dominates frequent reporting if:

$$
\int_{S^*}^{\bar{S}} \xi(S) \left[ \text{Prob}(G \mid S)V_0(G, L) + \text{Prob}(B \mid S)V_0(B, L) \right] dS - \\
\int_{S^*}^{\bar{S}} \xi(S) \left[ \text{Prob}(G \mid S)V_0(G, M) + \text{Prob}(B \mid S)V_0(B, M) \right] dS \geq \\
\int_{S^*}^{\bar{S}} \xi(s) \left[ K - (\text{Prob}(G \mid S)V_0(G, L) + \text{Prob}(B \mid S)V_0(B, L)) \right] dS.
$$

The expression preceding the inequality in (49) is a lower bound to the gain from infrequent reporting vis-à-vis frequent reporting arising from the choice of the long-term rather than the short-term project, while the right hand side is an upper bound to the loss due to infrequent reporting arising from investment in negative net present value projects when $S < S^*$.

7. Discussion and Conclusion

In research organizations, it is common wisdom that premature evaluations of research based solely on immediately observable outcomes are dysfunctional. We have shown that the same wisdom applies when uninformed capital markets price the firm solely in the light of observed cash flows from investment projects. When the results of operations are reported too frequently, capital market pricing becomes equivalent to premature evaluation of managerial actions whose benefits arrive mostly in later periods. Consequently, actions that produce large immediate (short-term) benefits become more attractive and actions that do not immediately produce such benefits but would ultimately create more value for the firm become less attractive. Thus, because frequent reporting triggers managerial short-termism, frequent reporting could be dysfunctional even though such reporting provides more information to the capital market.
The policy implications we have derived from a study of the *real* effects of disclosure stand in strong contrast to the policy implications one would derive from the study of pure trade economies. In pure trade economies with risk aversion, the only effect of disclosure is to decrease the residual uncertainty of exogenous liquidating dividends thus decreasing the risk premium incorporated in equilibrium capital market prices. Thus, greater frequency of disclosure would *always* be good, and the only cost that would prevent disclosure frequency from degenerating into weekly or even daily reports would be the legal and book-keeping costs of compliance. Our study also illustrates the importance of distinguishing “price efficiency” from “economic efficiency.” There is an unstated assumption in the accounting literature that any new disclosure mandate that adds information to the capital market and thus makes prices “more efficient” must promote social welfare. Such a result always holds in a first best world or when enhanced disclosure is so rich that it moves the economy to a first best world. However, when a first best world is unattainable, the provision of new information to the capital market could motivate firms to change their business decisions in such a way that economic efficiency suffers even though price efficiency is enhanced. By explicitly analyzing such *real* effects, we have shown that infrequent reporting could provide better incentives for investment by destroying information. This result may seem counterintuitive in the light of Blackwell’s theorem, but begins to make sense when we take into account that information has strategic consequences, i.e., it changes the world that is being assessed.

We have chosen to model the benefits to periodic performance reporting in terms of the *ex ante* discipline they impose on managerial decisions, as first documented in Kanodia and Lee [1998]. An alternative, and more popular, view is that such reports guide *subsequent* investment decisions that new investors intend to make after buying into the firm. Our view is that such a situation is analogous to an IPO or a seasoned equity offering. Not only are such offerings rare, 

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9 See Verrecchia [2001]
but they are always accompanied by a detailed prospectus and forecasts by underwriters and analysts that contain much more information than is typically contained in periodic performance reports. We have tried to capture the benefits of performance reports that are *routinely* disseminated to the entire capital market, regardless of whether the firm intends to raise new capital in the market. We think the debate on disclosure frequency is more concerned with such routine mandatory reports that are disseminated at prespecified regular intervals.

It may seem that the inefficiency caused by frequent disclosure could be easily mitigated by appropriately designed managerial compensation contracts. Surely, any compensation contract that rewards the manager solely on the basis of cumulative two-period cash flow would induce her to choose the long-term project, rather than the short-term project. It may seem that such a contract would also benefit the firm’s current shareholders, since capital market valuations would improve. However, the benefit to current shareholders comes not from aligning the incentives of the manager with that of current shareholders – by assumption, their incentives are already perfectly aligned. The benefits come from signalling *future* shareholders that the manager, acting in the best interests of current shareholders, will not behave opportunistically. But having so convinced future shareholders, current shareholders would want the manager to behave in an opportunistic way, so any such contract would quickly unravel.\(^\text{10}\) What is needed is a contract between current and future shareholders which binds current shareholders to not demand opportunistic behavior from their manager. Such a contract is problematic because future shareholders constitute an unidentifiable faceless crowd in the capital market. We think that mandatory corporate disclosure *is* the principal mechanism favored by regulators for mediating the tension between current and future shareholders. In previous work, we have demonstrated repeatedly that the effectiveness of such disclosure mechanisms depends critically

\(^{10}\) See Persons [1994] for a complete articulation of such an argument.
In the present paper, we have shown that the frequency of disclosure is also a critical policy choice available to regulators and we have demonstrated that a judicious choice of disclosure frequency could help to curb managerial opportunism.

11 See Kanodia [2006] for a survey of research that documents the effect of accounting measurement rules on corporate decisions through the interaction of those decisions with market pricing.
APPENDIX

In this Appendix we derive the equilibrium depicted in Figure 3 for the case where (26) is satisfied with strict inequality, i.e. for Case (II). We will use the following three additional assumptions:

\[
\frac{f_1(x_1 \mid G, M)}{f_1(x_1 \mid G, L)} \text{ is increasing in } x_1 \quad (50)
\]

\[
\frac{f_1(x_1 \mid G, L)}{f_1(x_1 \mid B, M)} \text{ is increasing in } x_1 \quad (51)
\]

\[
\frac{h(y_2 \mid G, M)}{h(y_2 \mid B, L)} \text{ is increasing in } y_2 \quad (52)
\]

Assumption (50) is only a slight strengthening of our previous assumption that the short-term project produces stochastically larger first period cash flows than the long term project in each state. Our previous assumptions provided likelihood ratio orderings across states G and B for the same investment project. Unlike these previous likelihood ratio orderings, assumptions (51) and (52) are likelihood ratio orderings across projects in different states. Assumption (51) says that higher first period cash flows increase the likelihood that the cash flow was generated by an undamaged long-term project than by a damaged short-term project. Assumption (52) says that higher cumulative two-period cash flows increase the likelihood that the cash flow was generated by an undamaged short-term project than by a damaged long-term project.

Proposition 10: Assume (50) through (52) and assume that (26) holds with strict inequality.

Then there exists \( \alpha^* \in (0,1) \) and two schedules \( S_F(\alpha) \) and \( S_L(\alpha) \) such that:

(i) \( I(S, \alpha) = L, \forall \alpha \leq \alpha^*, S \geq S_F(\alpha) \)

(ii) \( I(S, \alpha) = L, \forall \alpha > \alpha^*, S \geq S_L(\alpha) \)
(iii) \( I(S, \alpha) = M, \forall \alpha > \alpha^*, \forall S \in [S_F(\alpha), S_L(\alpha)) \)

(iv) \( I(S, \alpha) = \emptyset, \forall \alpha, S < S_F(\alpha) \)

(v) \( S_L(\alpha) \) is strictly increasing with \( S_L(1) = \bar{S} \)

(vi) \( S_F(\alpha) \) is strictly decreasing in \( \alpha \)

(vii) \( \underline{S} < S_F(\alpha) < S^*, \forall \alpha \)

The proof of Proposition 10 follows the same steps used to prove Proposition 5, but the construction of the equilibrium is considerably complicated by the fact that the market is uncertain about which project the firm has undertaken when \( \alpha > \alpha^* \). Let \( \psi_1 \) and \( \psi_2 \) be the date 1 and date 2 prices in this new equilibrium, conditional on observing that investment has occurred. In the region \( \alpha \leq \alpha^* \), these prices are identical to that described in (30) and (31), respectively, and all of the analysis is identical to that in the previous equilibrium characterized in Proposition 5. However, as depicted in Figure 3, when \( \alpha > \alpha^* \) the firm could have invested in either project L or project M, depending on the value of \( S \) that it privately observed. Hence, when \( \alpha > \alpha^* \),

\[
\psi_i(\alpha, x_i) = x_i + \text{Prob}(\{G, L\} \mid S \geq S_F(\alpha), x_i)V_1(G, L) + \\
\text{Prob}(\{G, M\} \mid S \geq S_F(\alpha), x_i)V_1(G, M) + \text{Prob}(\{B, L\} \mid S \geq S_F(\alpha), x_i)V_1(B, L) + \\
\text{Prob}(\{B, M\} \mid S \geq S_F(\alpha), x_i)V_1(B, M) - K .
\]

Given the firm’s equilibrium investment strategy, the probabilities incorporated in (53) are:

\[
\text{Prob}(\{G, L\} \mid S \geq S_F(\alpha), x_i) = \text{Prob}(G, S \geq S_L(\alpha) \mid S \geq S_F(\alpha), x_i) = \\
\text{Prob}(S \geq S_L(\alpha) \mid G, S \geq S_F(\alpha)) \text{Prob}(G \mid S \geq S_F(\alpha), x_i) \\
\text{Prob}(\{G, M\} \mid S \geq S_F(\alpha), x_i) = \text{Prob}(S < S_L(\alpha) \mid G, S \geq S_F(\alpha)) \text{Prob}(G \mid S \geq S_F(\alpha), x_i),
\]

48
\[
\text{Prob}(\{B,L\} \mid S \geq S_F(\alpha), x_1) = \text{Prob}(S \geq S_L(\alpha) \mid B, S \geq S_F(\alpha)) \text{ Prob}(B \mid S \geq S_F(\alpha), x_1),
\]

\[
\text{Prob}(\{B,M\} \mid S \geq S_F(\alpha), x_1) = \text{Prob}(S < S_L(\alpha) \mid B, S \geq S_F(\alpha)) \text{ Prob}(B \mid S \geq S_F(\alpha), x_1)
\]

where:

\[
\text{Prob}(S \geq S_L(\alpha) \mid G, S \geq S_F(\alpha)) \equiv \gamma_1 = \frac{\int_{S_{L}(\alpha)}^{S} \xi(S \mid G)ds}{\int_{S_{F}(\alpha)}^{S} \xi(S \mid G)ds}
\]

(54)

\[
\text{Prob}(S < S_L(\alpha) \mid G, S \geq S_F(\alpha)) \equiv \gamma_2 = \frac{\int_{S_{L}(\alpha)}^{S} \xi(S \mid G)ds}{\int_{S_{F}(\alpha)}^{S} \xi(S \mid G)ds}
\]

(55)

\[
\text{Prob}(S \geq S_L(\alpha) \mid B, S \geq S_F(\alpha)) \equiv \gamma_3 = \frac{\int_{S_{L}(\alpha)}^{S} \xi(S \mid B)ds}{\int_{S_{F}(\alpha)}^{S} \xi(S \mid B)ds}
\]

(56)

\[
\text{Prob}(S < S_L(\alpha) \mid B, S \geq S_F(\alpha)) \equiv \gamma_4 = \frac{\int_{S_{L}(\alpha)}^{S} \xi(S \mid B)ds}{\int_{S_{F}(\alpha)}^{S} \xi(S \mid B)ds}
\]

(57)

Note that \(\gamma_2 = 1 - \gamma_1\), and \(\gamma_4 = 1 - \gamma_3\). In order to calculate \(\text{Prob}(G \mid S \geq S_F(\alpha), x_1)\) the distribution of \(x_1\) must be specified. Since outsiders do not know for certain whether the firm has invested in L or M, the assessment of this distribution must incorporate the equilibrium probabilities of either investment, conditional on states G and B. Given the firm’s equilibrium investment strategy, if the state is G the firm has invested in L with probability \(\gamma_1\) and has invested in M with probability \(\gamma_2\) as specified in (54) and (55), respectively. If the state is B the probability of L is \(\gamma_3\) and the probability of M is \(\gamma_4\) as specified in (56) and (57), respectively. Hence,
\[
\Pr( G | S \geq S_F(\alpha), x_i ) = \frac{\Pr(G | S \geq S_F(\alpha) \cdot \gamma_1 f_1(x_i | G, L) + \gamma_2 f_1(x_i | G, M))}{f_1(x_i | S \geq S_F(\alpha))} \quad (58)
\]

where,
\[
f_1(x_i | S \geq S_F(\alpha)) = \Pr(G | S \geq S_F(\alpha) [\gamma_1 f_1(x_i | G, L) + \gamma_2 f_1(x_i | G, M)] + \Pr(G | S \geq S_F(\alpha) [\gamma_3 f_1(x_i | B, L) + \gamma_4 f_1(x_i | B, M)] \quad (59)
\]

Also,
\[
\Pr(G | S \geq S_F(\alpha), x_i) = 1 - \Pr(G | S \geq S_F(\alpha), x_i) \quad (60)
\]

**Lemma 8:** The date 1 price \( \psi_1(\alpha, x_i) \) is strictly increasing in \( x_i \) at every value of \( \alpha \).

**Proof:** Consider \( \alpha > \alpha^* \). Given the probability calculations in (54) though (60) the date 1 price can be expressed as:

\[
\psi_1(\alpha, x_i) = x_i + \Pr(G | S \geq S_F(\alpha), x_i) [\gamma_1 V_1(G, L) + (1 - \gamma_1) V_1(G, M)] + [1 - \Pr(G | S \geq S_F(\alpha), x_i)] [\gamma_3 V_1(B, L) + (1 - \gamma_3) V_1(B, M)] - K = \]

\[
x_i + \Pr(G | S \geq S_F(\alpha), x_i) [\gamma_1 V_1(G, L) + (1 - \gamma_1) V_1(G, M) - (\gamma_3 V_1(B, L) + (1 - \gamma_3) V_1(b, M))] + \gamma_3 V_1(B, L) + (1 - \gamma_3) V_1(B, M) - K
\]

Clearly \( \psi_1 \) is strictly increasing in \( x_i \) if \( \Pr(G | S \geq S_F(\alpha), x_i) \) is increasing in \( x_i \) and the term multiplying this probability is strictly greater than zero. Now \( \gamma_1 \) as defined in (54) is greater than \( \gamma_3 \) as defined in (56) because both expressions are hazard rates and likelihood ratio ordering
implies hazard rate ordering. Additionally, because $V_1(G,L) > V_1(B,L)$ and 

$$V_1(G,M) > V_1(B,M),$$

$$\gamma_1 V_1(G,L) + (1 - \gamma_1) V_1(G,M) - \{\gamma_3 V_1(B,L) + (1 - \gamma_3) V_1(B,M)\} > 0$$

is increasing in $x_1$. We claim that this follows from assumptions (50) through (52) and our earlier likelihood ratio ordering assumptions.\(^{13}\)

**Q.E.D.**

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\(^{12}\) See Shaked and Shantikumar, Theorem 1.C.1

\(^{13}\) Let $Y$, $W$, and $Z$ be three random variables with common support, and let $Y \geq_{LR} W$ denote that $Y$ is bigger than $Z$ in the likelihood ratio order. The proof of our claim follows from the following 3 facts, each of which can be easily verified:

(i) $Y \geq_{LR} W \geq_{LR} Z \Rightarrow Y \geq_{LR} Z$

(ii) If $Y \geq_{LR} W$, and $Y \geq_{LR} Z$ then $Y \geq_{LR} tW + (1 - t)Z$, $\forall t \in [0,1]$

(iii) If $Y \geq_{LR} Z$ and $W \geq_{LR} Z$ then $tY + (1 - t)W \geq_{LR} Z$, $\forall t \in [0,1]$

Let $X_1(G,L)$ denote the random variable whose density function is $f_1(x_1 | G,L)$ and analogously define the remaining random variables used below. Since $X_1(G,L) \geq_{LR} X_1(B,L)$ and 

$$X_1(G,L) \geq_{LR} X_1(B,M),$$

as assumed in (51), then $X_1(G,L) \geq_{LR} \gamma_3 X_1(B,L) + (1 - \gamma_3) X_1(B,M)$ from fact (i). Finally, from fact (iii),

$$\gamma_1 X_1(G,L) + (1 - \gamma_1) X_1(G,M) \geq_{LR} \gamma_3 X_1(B,L) + (1 - \gamma_3) X_1(B,M)$$

as is to be proved.
Lemma 9: The date 2 price $\psi_2(\alpha, y_2)$ is strictly increasing in $y_2$ at every value of $\alpha$.

Proof: (sketch)
Consider $\alpha > \alpha^*$. The date 2 price has exactly the same structure as the date 1 price, except that $x_1$ is replaced by $y_2$ and $V_1(\cdot, \cdot)$ is replaced by $V_2(\cdot, \cdot)$. Therefore, following exactly the same reasoning as in Lemma 8, the only new result to be established is that the likelihood ratio:

$$\frac{\gamma_1 h(y_2 \mid G, L) + (1 - \gamma_1) f_1(y_2 \mid G, M)}{\gamma_3 h(y_2 \mid B, L) + (1 - \gamma_3) h(y_2 \mid B, M)}$$

is increasing in $y_2$. The only new assumption required to establish this claim is that specified in (52).

Lemma 10:

(i) $E_0[\psi_1(\alpha, x_1) \mid S, M] > E_0[\psi_1(\alpha, x_1) \mid S, L], \forall S$

(ii) $\frac{\partial}{\partial S} \{ E_0[\psi_1(\alpha, x_1) \mid S, M] - E_0[\psi_1(\alpha, x_1) \mid S, L] \} < 0$

(iii) $E_0[\psi_2(\alpha, y_2) \mid S, L] > E_0[\psi_2(\alpha, y_2) \mid S, M], \forall S$

(iv) $\frac{\partial}{\partial S} \{ E_0[\psi_2(\alpha, y_2) \mid S, L] - E_0[\psi_2(\alpha, y_2) \mid S, M] \} > 0$

Proof Identical to that contained in Lemmas 4 and 5.

---

14 Since $\frac{\partial}{\partial S} \{ E_0[\psi_1(\alpha, x_1) \mid S, M] - E_0[\psi_1(\alpha, x_1) \mid S, L] \} < 0$ and since $\frac{\partial}{\partial S} \{ E_0[\psi_2(\alpha, y_2) \mid S, L] - E_0[\psi_2(\alpha, y_2) \mid S, M] \} > 0$ as assumed in (52),

$\gamma_1 y_2(G, M) \geq_{LR} \gamma_3 y_2(B, L) + (1 - \gamma_3) y_2(B, M)$.

Then it follows that

$\gamma_1 y_2(G, M) + (1 - \gamma_1) y_2(G, M) \geq_{LR} \gamma_3 y_2(B, L) + (1 - \gamma_3) y_2(B, M)$ as is to be proved.
Lemma 10, (ii) and (iv) imply that if the firm prefers project L to M at some value of S, then it must continue to have this preference at all higher values of S. Also, if the firm prefers project M to L at some value of S, then it must continue to have this preference for all lower values of S. Therefore, the set of S values at which the firm invests in L must be an upper interval of its support, while the set of S values at which the firm invests in M must be a lower interval, as claimed in Proposition 10. The boundary of these two intervals, which we have denoted $S_L(\alpha)$ must be upward sloping as depicted in Figure 3. This is because an increase in $\alpha$ tilts the firm’s preference towards the short-term project, so $S_L$ must be increased to restore indifference between projects L and M. Additionally, Lemma 10 (i) indicates that the firm will surely choose project M if all of the weight is on the date 1 price, so $S_L(\alpha) \rightarrow \overline{S}$ as $\alpha \rightarrow 1$.

Finally, we establish that $S_F(\alpha)$ has the properties claimed in Proposition 10. For this purpose, we need to compare the expectation of the date 1 and date 2 prices, conditional on the marginal type $S_F(\alpha)$ at which the firm is indifferent between investing in project M and not investing. In order to do this, we first establish:

**Lemma 11:** At each $\alpha > \alpha^*$,

$$E_0[\text{Prob}(G \mid S \geq S_F(\alpha), x_1) \mid S_F(\alpha), M] > E_0[\text{Prob}(G \mid S \geq S_F(\alpha), y_2) \mid S_F(\alpha), M]$$

**Proof:**

First, we claim that:

$$\text{Prob}(G \mid S \geq S_F, x_1) = E_{x_2}[\text{Prob}(G \mid S \geq S_F, x_1, x_2) \mid S \geq S_F, x_1]$$

(61)

Although this equality follows from the law of iterated expectations, it is instructive to see how it works out even when the market is uncertain about the project that the firm has undertaken. The probability density of $x_2$ used in the right hand side of (61) is:
\[ f_2(x_2 \mid S \geq S_F, x_1) = \text{Prob}(G \mid S \geq S_F, x_1) \left[ \gamma_1 f_2(x_2 \mid G, L) + (1 - \gamma_1) f_2(x_2 \mid G, M) \right] + \]
\[ \text{Prob}(B \mid S \geq S_F, x_1) \left[ \gamma_3 f_2(x_2 \mid B, L) + (1 - \gamma_3) f_2(x_2 \mid B, M) \right], \]

where \( \text{Prob}(G \mid S \geq S_F, x_1) \) is defined in (58) and (59). Then:

\[
\text{Prob}(G \mid S \geq S_F, x_1, x_2) = \frac{\text{Prob}(G \mid S \geq S_F, x_1)[\gamma_1 f_2(x_2 \mid G, L) + (1 - \gamma_1) f_2(x_2 \mid G, M)]}{f_2(x_2 \mid S \geq S_F, x_1)}
\]

and,

\[
E_{x_1}[\text{Prob}(G \mid S \geq S_F, x_1, x_2) \mid S \geq S_F, x_1] = \int \left( \frac{\text{Prob}(G \mid S \geq S_F, x_1)[\gamma_1 f_2(x_2 \mid G, L) + (1 - \gamma_1) f_2(x_2 \mid G, M)]}{f_2(x_2 \mid S \geq S_F, x_1)} \right) f_2(x_2 \mid S \geq S_F, x_1) \, dx_2
\]

Next, because \( y_2 \) is sufficient for \( \{x_1, x_2\} \):

\[
E_y[\text{Prob}(G \mid S \geq S_F, y_2) \mid S_F, M] = E_{x_2}[E_{x_1}[\text{Prob}(G \mid S \geq S_F, x_1, x_2) \mid S_F, x_1, M] \mid S_F, M]
\]

The inside expectation is

\[
E_{x_1}\{\text{Prob}(G \mid S \geq S_F, x_1, x_2) \mid S_F, x_1, M\} = \int \left( \frac{\text{Prob}(G \mid S \geq S_F, x_1)[\gamma_1 f_2(x_2 \mid G, L) + (1 - \gamma_1) f_2(x_2 \mid G, M)]}{f_2(x_2 \mid S \geq S_F, x_1)} \right) f_2(x_2 \mid S_F, x_1, M) \, dx_2
\]

where,

\[
f_2(x_2 \mid S_F, x_1, M) = \text{Prob}(G \mid S_F, x_1, M) f_2(x_2 \mid G, M) + \text{Prob}(B \mid S_F, x_1, M) f_2(x_2 \mid B, M)
\]
\[ f_2(x_2 \mid x_1, S \geq S_F) = \text{Prob}(G \mid S \geq S_F, x_1) \cdot f_2(x_2 \mid x_1, G, S \geq S_F) + \text{Prob}(B \mid S \geq S_F, x_1) \cdot f_2(x_2 \mid x_1, B, S \geq S_F) \]

\[ f(x_1, x_2 \mid G, S \geq S_F) = \gamma_1 f_1(x_1 \mid G, L) f_2(x_2 \mid G, L) + (1 - \gamma_1) f_1(x_1 \mid G, M) f_2(x_2 \mid G, M) , \] (60)

\[ f(x_1, x_2 \mid B, S \geq S_F) = \gamma_2 f_1(x_1 \mid B, L) f_2(x_2 \mid B, L) + (1 - \gamma_2) f_1(x_1 \mid B, M) f_2(x_2 \mid B, M) , \] (61)

\[ f(x_1, x_2 \mid S \geq S_F) = \text{Prob}(G \mid S \geq S_F) \cdot f(x_1, x_2 \mid G, S \geq S_F) + \text{Prob}(B \mid S \geq S_F) \cdot f(x_1, x_2 \mid B, S \geq S_F) \] (62)

Equation (60) is the joint probability density of the period 1 and period 2 cash flow, conditional on state G and conditional on the observation that investment has occurred; Equation (61) is the density conditional on state B, and equation (62) is the unconditional density. Let:

\[ f_1(x_1 \mid G, S \geq S_F) = \gamma_1 f_1(x_1 \mid G, L) + (1 - \gamma_1) f_1(x_1 \mid G, M) , \] (63)

\[ f_1(x_1 \mid B, S \geq S_F) = \gamma_2 f_1(x_1 \mid B, L) + (1 - \gamma_2) f_1(x_1 \mid B, M) \] (64)

Equations (63) and (64) are the probability densities of first period cash flow conditional on the state and conditional on the observation that investment has occurred. All of these probability densities incorporate the fact that the market is uncertain about which project the firm has undertaken. Next we specify the probability densities of the second period cash flow conditional on the information contained in the first period cash flow, the state, and the observation that investment has occurred.
where,

\[
\Pr ob(S \geq S_L \mid x_1, G, S \geq S_F) = \frac{\Pr ob(S \geq S_L \mid G, S \geq S_F)f_1(x_1 \mid G, L) + \Pr ob(S < S_L \mid G, S \geq S_F)f_1(x_1 \mid G, M)}{\Pr ob(S \geq S_L \mid G, S \geq S_F)f_1(x_1 \mid G, L) + \Pr ob(S < S_L \mid G, S \geq S_F)f_1(x_1 \mid G, M)}
\]  

(65)

Inserting (66) into (65) yields,

\[
f_2(x_2 \mid x_1, G, S \geq S_F) = \frac{\gamma_1 f_1(x_1 \mid G, L)f_2(x_2 \mid G, L) + (1 - \gamma_1)f_1(x_1 \mid G, M)f_2(x_2 \mid G, M)}{\gamma_1 f_1(x_1 \mid G, L) + (1 - \gamma_1)f_1(x_1 \mid G, M)}
\]  

(67)

Similarly,

\[
f_2(x_2 \mid x_1, B, S \geq S_F) = \frac{\gamma_3 f_1(x_1 \mid B, L)f_2(x_2 \mid B, L) + (1 - \gamma_3)f_1(x_1 \mid B, M)f_2(x_2 \mid B, M)}{\gamma_3 f_1(x_1 \mid B, L) + (1 - \gamma_3)f_1(x_1 \mid B, M)}
\]  

(68)

and, the unconditional density of second period cash flow is:

\[
f_2(x_2 \mid x_1, S \geq S_F) = \Pr ob(G \mid S \geq S_F, x_1)f_2(x_2 \mid x_1, G, S \geq S_F) + \Pr ob(B \mid S \geq S_F, x_1)f_2(x_2 \mid x_1, B, S \geq S_F)
\]  

(69)

Now,

\[
\Pr ob(G \mid S \geq S_F, x_1) = \frac{\Pr ob(G \mid S \geq S_F)f_1(x_1 \mid G, S \geq S_F)}{\Pr ob(G \mid S \geq S_F)f_1(x_1 \mid G, S \geq S_F) + \Pr ob(B \mid S \geq S_F)f_1(x_1 \mid B, S \geq S_F)}
\]  

(70)

and,

\[
\Pr ob(G \mid S \geq S_F, x_1, x_2) = \frac{\Pr ob(G \mid S \geq S_F)f(x_1, x_2 \mid G, S \geq S_F)}{f(x_1, x_2 \mid S \geq S_F)}
\]  

(71)

Therefore:
where, \( f_2(x_2 \mid x_1, S \geq S_F) \) is specified in (69). Inserting (70) into (69) and using the fact that

\[
f_1(x_1 \mid G, S \geq S_F) \ f_2(x_2 \mid x_1, G, S \geq S_F) = \ f(x_1, x_2 \mid G, S \geq S_F),
\]

(73)
gives:

\[
f_2(x_2 \mid x_1, S \geq S_F) = \frac{\text{Prob}(G \mid S \geq S_F) f_1(x_1 \mid G, S \geq S_F) + \text{Prob}(B \mid S \geq S_F) f_1(x_1 \mid B, S \geq S_F)}{\text{Prob}(G \mid S \geq S_F) f_1(x_1 \mid G, S \geq S_F) + \text{Prob}(B \mid S \geq S_F) f_1(x_1 \mid B, S \geq S_F)} \ f_2(x_2 \mid x_1, G, S \geq S_F) dx_2
\]

(74)

Inserting (74) into (72) and using (73), yields:

\[
E_{x_2}[\text{Prob}(G \mid S \geq S_F, x_1, x_2) \mid S \geq S_F, x_1] = \int \left( \frac{\text{Prob}(G \mid S \geq S_F) f_1(x_1 \mid G, S \geq S_F)}{\text{Prob}(G \mid S \geq S_F) f_1(x_1 \mid G, S \geq S_F) + \text{Prob}(B \mid S \geq S_F) f_1(x_1 \mid B, S \geq S_F)} \right) f_2(x_2 \mid x_1, G, S \geq S_F) dx_2 = \text{Prob}(G \mid S \geq S_F, x_1)
\]

This completes the proof of our claim. Next, we observe that,

\[
E_0[\text{Prob}(G \mid S \geq S_F, y_2) \mid S_F, M] = E_{x_2}[E_{x_1}[\text{Prob}(G \mid S \geq S_F, x_1, x_2) \mid S_F, x_1, M] \mid S_F, M]
\]
References


