Bayesian Modeling of Economies and Data Requirements*

by

Arnold Zellner and Bin Chen

U. of Chicago

Abstract

In previous work, we have used Bayesian methods in the analysis of various models to explain past variation and forecast future values of the rates of growth of real GDP for 18 industrialized countries. Using these models, point and turning point forecasts were calculated and found to be reasonably accurate compared to those of benchmark and other models' forecasts. In this paper, Marshallian demand, supply and entry models are employed for major sectors of an economy that can be combined with factor market models for money, labor, capital and bonds to provide a Marshallian macroeconomic model (MMM). Herein, the sectoral models are used to produce sectoral output forecasts which are summed to provide forecasts of annual growth rates of U.S. real gross domestic product (GDP). These disaggregative forecasts are compared to forecasts derived from models implemented with aggregate data. The empirical evidence indicates that it pays to disaggregate, particularly when employing Bayesian shrinkage forecasting procedures. Further, some considerations bearing on alternative model-building strategies will be presented using the MMM as an example and describing its general properties. Last, data requirements for implementing MMMs are discussed.

1. Introduction

For many years, theoretical and empirical workers have tried to model national economies in order to (1) understand how they operate, (2) forecast future outcomes and (3) evaluate alternative economic policies. While much progress has been made in the decades since Tinbergen's pioneering work, it is the case that no generally accepted model has as yet appeared. On the theoretical side, there are monetary, neo-monetary, Keynesian, neo-Keynesian, real business cycle, generalized real business cycle and other theoretical models; see, Belongia and Garfinkel (1992) for an excellent review of many of these models and Min (1992) for a description of a generalized real business cycle model. Some empirical testing of alternative models has appeared in the literature.

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However, in Fair (1992) and Zellner (1992), invited contributions to a St. Louis Federal Reserve Bank conference on alternative macroeconomic models, it was concluded that there is a great need for additional empirical testing of alternative macroeconomic models and production of improved models.

Over the years many structural econometric and empirical statistical models have been constructed and used. These include large structural econometric models, e.g. the Tinbergen, Klein, Brookings-SSRC, Federal Reserve-MIT-PENN, OECD, Project Link and other models. While progress has been made, there does not yet appear to be a structural model that performs satisfactorily in point and turning point forecasting. Indeed, the forecasting performance of some of these models is not as good as that of simple benchmark models, e.g., random walk, autoregressive, Box-Jenkins univariate ARIMA and autoregressive-leading indicator (ARLI) models; see, e.g., Cooper (1972), Garcia-Ferrer et al. (1987), Hong (1989), and Nelson and Plosser (1982). Further, some have implemented vector autoregressive (VAR) and Bayesian VAR models in efforts to obtain improved forecasts; see, e.g. Litterman (1986) and McNees (1986). However these VARs have not in general been successful in point and turning point forecasting performance as noted by Zarnowitz (1986) and McNees (1986). See also the simulation experiments performed by Adelman and Adelman (1959) and Zellner and Peck (1987) that revealed some rather unusual properties of two large scale econometric models.

Given the need for improved models, in Garcia-Ferrer et al. (1987) an empirical implementation of the structural econometric-time series analysis (SEMTSA) approach of Zellner and Palm (1974, 1975), Palm (1976, 1977, 1983) and Zellner (1979,1994) was reported. In line with the SEMTSA general approach, relatively simple forecasting equations, autoregressive leading indicator (ARLI) models were formulated and tested in forecasting output growth rates for nine industrialized countries with some success. In later work, the sample of countries was expanded to eighteen and the forecast period extended to include more out of sample growth rates of real GDP to be forecast. Building on work of Wecker (1979) and Kling (1987), a Bayesian decision theoretic procedure for forecasting turning points was formulated and applied that yielded correct forecasts in about 70% of 211 turning point episodes; see Zellner and Min (1999), Zellner, Tobias and Ryu (1999) and the references cited in these papers. Further, the ARLI models were shown to be compatible with certain aggregate supply and demand, Hicksian "IS-LM" and generalized real business cycle models in Hong (1989), Min (1992) and Zellner (1999).

In a continuing effort to improve our models, in the present paper, we use a relatively simple, Marshallian model in Section 2 that features demand, supply and entry equations for each sector of an economy; see Veloce and Zellner (1985) for a derivation of this model and an application of it in the analysis of data for a Canadian industry. The model is solved to produce a sectoral relation that can be employed to forecast sectoral output. These sectoral output forecasts are summed to produce forecasts of total output that are compared to forecasts derived from models implemented with aggregate data. Some possible advantages of disaggregation have been discussed earlier by Orcutt et al. (1961), Espasa (1994), Espasa and Matea (1990), and de Alba and Zellner (1991), among others. Actual comparisons of such forecasts for U.S. annual real GDP growth rates, 1980-1997 will be reported in Section 4 after statistical estimation and forecasting techniques, employed to implement the MMM are presented in Section 3. In Section 4, the data used in our empirical forecasting work are described and forecasting results using the MMM and other models with no disaggregation and with disaggregation are reported . Also, MMM models'forecast performance is compared to that of various benchmark and ARLI model. In Section 5, some comments on data requirements, a summary of conclusions and remarks on future research are presented.

2. The Marshallian Macroeconomic Model (MMM)

In the MMM, we have three basic rather well known equations, described and applied in Veloce and Zellner (1985), namely the usual (1) demand for output, (2) supply of output and (3) entry equations encountered in Marshall's famous economic analyses of the behavior of industries. While many macro models have included demand and supply equations, they have not included an entry equation. For example, in some models there is just a representative firm and one wonders what happens when the representative firm shuts down. In our MMM model, supply depends on the number of firms in operation and thus an equation governing the number of firms in operation, an entry equation, is introduced.

We shall use two variants of the MMM model, namely an aggregate, reduced form variant and a disaggregated structural equation variant. In the aggregate variant, we shall adopt a "one-sector" view of an economy while in the disaggregated variant, we adopt a multi-sector view of an economy. As regards the multi-sectoral view, many assumed structures are possible, all the way from the multi-sectoral view of traditional Leontieff input-output analysis to the simple view that we shall employ, namely an economy in which each sector sells in a final product market. Herein, we do not take up the interesting problem of classifying economies by the nature of their sectoral interrelations. However, we shall show that by adopting our sectoral view, we are able to improve forecasts of aggregate output growth rates since disaggregation provides more observations to estimate relationships and permits use of sectoral specific variables to help improve forecasts. Of course, if the disaggregated relations are misspecified and/or the disaggregated data are faulty, then there may be no advantages and perhaps some disadvantages in using disagregated data, as is evident. Also, there are some circumstances even when data are good and relations are well formulated when disaggregation does not lead to improved forecasts. However the issue can not be completely settled theoretically and hence our current empirical work.

As explained in Veloce and Zellner (1985), the equations for a sector that we use are a demand equation for output, an industry supply equation for output and a firm entry equation. While we could elaborate the system in many ways, we shall go forward to determine how well this simplest system peforms empirically, our “Model T” that can be improved in many different ways in the future. When these three equations are solved for the implied equation for the sectoral output growth rate, see Veloce and Zellner (1985) for details, the result is the following differential equation for total industry sales, denoted by S = S(t):

$$(1/S)dS/dt = a(1-S/F) + g \qquad (2.1)$$

where a and F are postive parameters and g is a linear function of the growth rates of the wage rate, the price of capital, and of demand shifters such as real income, real money balances, etc. If g = 0 or g = c, a positive constant, it is the case that (2.1) is the differential equation with a logistic curve solution that is employed in many sciences, including economics. Also, note that (2.1) incorporates both the rate of change of S and the level of S, a “cointegration” effect. Also, see Veloce and Zellner (1985, p.463), for analysis of (2.1) when g = g(t), a special form of Bernouilli’s differential equation and its solution.

In our empirical work we shall use the discrete approximations to equation (2.1) shown in Table 1[1]and denoted by MMM(DA)I-IV. In these equations, the rate of growth of S, real output, is related to lagged levels of S, lagged rates of change of real stock prices, SR, and real money, m, and current rates of change of real wage rates, W, and real GDP, Y. The variables m and Y are “demand shifters” while W is the price of labor and SR is related to the price of capital. As noted in the literature and in our past work, the rates of change of m and SR are effective leading indicator variables in a forecasting context and their use has led to improved forecasts in our past work; see, references cited above for empirical evidence.

Shown under Sectoral Forecasting Equations in Table 1 are three benchmark models that will be used to produce sectoral one year ahead forecasts of the rates of change of output for each of our 11 sectors. The first is an AR(3) that has been used in many earlier studies as a benchmark model. The second is an AR(3) that incorporates lagged leading indicator variables and current values of W and Y but no lagged level variables. The third “Distributed Lag” model is like the second except for the inclusion of lagged rates of change of W and of Y.

At the top of Table 1, under Reduced Form Equations, are shown reduced form equations for the rate of change of Y, annual real GDP. The first is a benchmark AR(3) model. The second is an AR(3) with lagged leading indicator variables that is denoted by AR(3)LI. The third model, denoted MMM(A) is the same as the AR(3)LI model except for the inclusion of two lagged Y variables, where Y =real GDP, and a time trend, t.

For our aggregate analyses, we use the Reduced Form Equations in Table 1 to produce one year ahead forecasts of the rate of change of real GDP, Y, that we refer to as

[1] All tables and figures are at the end of the paper.

"aggregate forecasts." These are means of diffuse prior Bayesian predictive densities for each model that are simple one year ahead least squares forecasts. As explained below, the MMM(A) reduced form equations for the rates of change of Y and of W will be employed in the estimation of the Sectoral Forecasting Equations and in computing one year ahead forecasts of sectoral outputs growth rates. These sectoral growth rate forecasts are transformed into forecasts of levels, added across the sectors and converted into a forecast of the rate of change of real GDP, Y. Root mean squared errors (RMSEs) and mean absolute errors (MAEs) are computed for each forecasting procedure and are shown in tables below.

3. Estimation and Forecasting Methods

3.1 Notation and Equations

In what follows, we shall use the following notation. For each sector, we have:

1. Endogenous or Random Current Exogenous Variables:

$$y_{1t} = (1-L)\log S_t;\;\; y_{2t} = (1-L)\log W_t;\;\; y_{3t} = (1-L)\log Y_t$$

where S_t = sectoral real output, W_t = national real wage rate, and Y_t = real GDP.

2. Predetermined Variables:

$$x'_{1t} = (1,\; S_{t-1}, S_{t-2}, S_{t-3},\; (1-L)\log SR_{t-1}, (1-L)\log m_{t-1})$$

where SR_t = real stock price and m_t = real money.

We use these variables to form the following structural equation for each sector:

$$y_{1t} = y_{2t}g_{21} + y_{3t}g_{31} + x'_{1t}b_1 + u_{1t} \qquad t = 1,2,.., T$$

or

$$y_1 = Y_1 g_1 + X_1 b_{1+} u_1 \qquad (3.1)$$

where the vectors y_1 and u_1 are $Tx1$, Y_1 is $Tx2$ and X_1 is $Tx5$ and $d_1' = (g_1', b_1')$ is a vector of structural parameters.

The MMM unrestricted reduced form equations, shown in Table 1, are denoted by:

$$y_1 = X p_1 + v_1 \qquad (3.2a)$$

and

$$Y_1 = X\Pi_1 + V_1 \qquad (3.2b)$$

where X = (X_1, X_0) with X_0 containing predetermined variables in the system that are not included in equation (3.1).

By substituting from (3.2b) in (3.1), we obtain the following well known restricted reduced form equation for y_1 :

$$y_1 = X\Pi_1 g_1 + X_1 b_1 + v_1 \qquad (3.3a)$$

$$= \bar{Z} d_1 + v_1 \qquad (3.3b)$$

where $\bar{Z} = (X\Pi_1, X_1)$, that is assumed of full column rank.

Further, if we consider the regression of v_1 on V_1,

$$v_1 = V_1 h_1 + e_1 = (Y_1 - X\Pi_1)h_1 + e_1 \qquad (3.4)$$

we can substitute for v_1 in (3.3) to obtain:

$$y_1 = X\Pi_1 g_1 + X_1 b_1 + (Y_1 - X\Pi_1)h_1 + e_1 \qquad (3.5)$$

In (3.5), for given Π_1, we have a regression of y_1 on $X\Pi_1, X_1$ and $Y_1 - X\Pi_1$. Given that e_1 is uncorrelated with the the elements of V_1, the system (3.2b) and (3.5) is a nonlinear in the parameters SUR system with an error covariance matrix restriction. Pagan (1979) has earlier recognized a connection of the above model in (3.1) and (3.2b) to the SUR model given the "triangularity" of the system and reported an iterative computational procedure for obtaining maximum likelihood estimates of the structural coefficients. In our case, we shall use (3.2b) and (3.5) as a basis for producing a convenient algorithm for computing posterior and predictive densities.

Note further that if $g_1 = h_1$, (3.5) becomes:

$$y_1 = Y_1 g_1 + X_1 b_1 + e_1 \qquad (3.6)$$

the same as (3.1) except for the error term. It is possible to view (3.6) as a regression with Y_1 containing observations on stochastic independent variables given that the elements of e_1 and V_1 are uncorrelated. The above restriction however may not hold in general. Another interpretation that permits (3.6) to be viewed as a regression with stochastic input variables is that the variables y_{2t} and y_{3t} are stochastic exogenous variables vis a vis the sectoral model. In such a situation, (3.1) can be treated as a regression equation with stochastic independent variables. However, we are not sure that this exogeneity assumption is valid and thus will use not only least squares techniques to estimate (3.1) but also special simultaneous equations techniques.

3.2 Estimation Techniques

The sampling theory estimation techniques that we shall employ in estimating the parameters of (3.1) are the well known "ordinary least squares" (OLS) and "two-stage least squares"(2SLS) methods. As shown in Zellner (1998), in very small samples, but not in large samples, the OLS estimate is an optimal Bayesian estimate relative to a generalized quadratic "precision of estimation" loss function when diffuse priors are employed. Also, the 2SLS estimate has been given an interpretation as a conditional Bayesian posterior mean using (3.3) conditional on $\Pi_1 = \hat{\Pi}_1 = (X'X)^{-1}X'Y_1$, a normal likelihood function and diffuse priors for the other parameters of (3.3). A similar conditional result is obtained without the normality assumption using the assumptions of the Bayesian method of moments (BMOM) approach; see, e.g. Zellner (1997, 1998). Since the "plug in" assumption $\Pi_1 = \hat{\Pi}_1$, does not allow appropriately for the uncertainty regarding Π_1 's value, the 2SLS estimate will not be optimal in small samples; see, e.g. Monte Carlo experiments reported by Tsurumi (1990), Park (1982) and Lahiri and Gao (1999). However, since OLS and 2SLS are widely employed methods, we shall employ them in our analyses of the models for individual sectors.

In the Bayesian approach, we decided to use the "ExtendedMinimum Expected Loss" (EMELO) optimal estimate put forward in Zellner(1986, 1998) that has performed well in Monte Carlo experiments of Tsurumi (1990) and Gao and Lahiri (1999). It is the estimate that minimizes the posterior expectation of the following extended or balanced loss function:

$$\begin{aligned} L(d_1,\hat{d}_1) &= w(y_1 - \bar{Z}\hat{d}_1)'(y_1 - \bar{Z}\hat{d}_1) + (1-w)(d_1 - \hat{d}_1)'\bar{Z}'\bar{Z}(d_1 - \hat{d}_1) \\ &= w(y_1 - \bar{Z}\hat{d}_1)'(y_1 - \bar{Z}\hat{d}_1) + (1-w)(Xp_1 - \bar{Z}\hat{d}_1)'(Xp_1 - \bar{Z}\hat{d}_1) \end{aligned} \qquad (3.7)$$

where w has a given value in the closed interval 0 to 1, $\hat{d}_1$ is some estimate of d_1, and in going from the first line of (3.7) to the second, the identifying restrictions, multiplied on the left by X, namely $Xp_1 = \bar{Z}d_1$ have been employed.

Relative to equation (3.3), the first term on the right side of (3.7) reflects goodness of fit while the second reflects precision of estimation or from the second line of (3.7), the extent to which the identifying restrictions are satisfied when an estimate of d_1 is employed. When the posterior expectation of the loss function in (3.7) is minimized with respect to $\hat{d}_1$ the minimizing value is:

$$\hat{d}_1 = \left(E\overline{Z}'\overline{Z}\right)^{-1}\left[wE\overline{Z}'y_1 + (1-w)E\overline{Z}'Xp_1\right] \tag{3.8}$$

On evaluation of the moments on the rhs of (3.8), we have an explicit value for the optimal estimate. For example, with the assumption that for the unrestricted reduced form system in (3.2), the rows of (v_1, V_1), are iid N(0, Ω), where Ω is a pds covariance matrix, combining a standard diffuse prior for the reduced form parameters with the normal likelihood function yields a marginal matrix t density for the reduced form coefficients. Thus the moments needed to evaluate (3.8) are readily available; see Zellner (1986) for details, and the result is surprisingly in the form of a double K class estimate shown in (3.9):

$$\hat{d}_1 = \begin{bmatrix} \hat{g}_1 \\ \\ \hat{b}_1 \end{bmatrix} = \begin{bmatrix} Y_1'Y_1 - K_1\hat{V}_1'\hat{V}_1 & Y_1'X_1 \\ \\ X_1'Y & X_1'X_1 \end{bmatrix}^{-1} \begin{bmatrix} \left(Y_1 - K_2\hat{V}_1\right)'y_1 \\ \\ X_1'y_1 \end{bmatrix} \tag{3.9}$$

with $\hat{V}_1 = Y_1 - X\Pi_1, \hat{\Pi}_1 = (X'X)^{-1}X'Y_1$ and

$$K_1 = 1 - k/(T - k - m - 2) \text{ and } K_2 = K_1 + wk/(T - k - m - 2) \tag{3.10}$$

K-class and double K-class estimates are discussed in most econometrics texts; see, e.g. Judge, et al (1987) and the choice of optimal values for the K's has been the subject of much sampling theory research. The Bayesian approach provides optimal values of these parameters quite directly on use of goodness of fit, precision of estimation or balanced loss functions.

When the form of the likelihood function is unknown and thus a traditional Bayesian analysis is impossible, we used the Bayesian method of moments (BMOM) approach in Zellner (1998) to obtain a postdata maxent density for the elements of $\Pi = (\Pi_1 p_1)$ that was used to evaluate the expectation of the balanced loss function in (3.7) and derive an optimal value of $\hat{d}_1$ that is also in form of a double K class estimate, shown in (3.9) but with slightly different values of the K parameters, namely $K_1 = 1 - k/(T - k)$ and $K_2 = K_1 + wk/(T - k)$. In our calculations based on the extended MELO estimate, we used the BMOM K values and w = 0.75, the value used by Tsurumi (1990) in his Monte Carlo experiments.

SUR estimates for the system were computed by assuming that the y_2 and y_3 variables in (3.1) are stochastic exogenous variables for each sector and treating the 11 sectoral equations as a set of seemingly unrelated regression equations. We estimated the parameters by "feasible" generalized least squares. The parameter estimates so obtained are means of conditional posterior densities in traditional Bayesian and BMOM approaches.

Complete shrinkage estimation utilized the assumption that all sectors' parameter vectors are the same. Under this assumption and the assumption that the y_2 and y_3 variables are stochastic exogenous variables, estimates of the restricted parameter vector were obtained by least squares that are also posterior means in Bayesian and BMOM approaches.

Exact posterior densities for the structural parameters in (3.5) can readily be calculated in the Bayesian approach by using diffuse priors for the parameters of (3.5) given Π_1, that is, a uniform prior on elements of $d_1, b_1, h_1,$ and log s_e, where s_e is the standard deviation of each element of e . Further, the usual diffuse priors are employed for Π_1 and Ω_1, a marginal uniform prior on the elements of the reduced form matrix Π_1 in (3.2) and a diffuse prior on Ω_1 , the covariance matrix for the independent, zero mean, normal rows of V_1. With use of these priors, the usual normal likelihood function for the system and Bayes' Theorem, we obtain the following joint posterior density for the parameters, where D denotes the given data; see Zellner, Min, Dallaire and Currie (1994) and Currie (1996):

$$\underbrace{f\left(g_1, b_1, h_1 \middle| s_e, \Pi_1, D\right)}_{\text{MVN}} \underbrace{g\left(s_e \middle| \Pi_1, D\right)}_{\text{IG}} \underbrace{h\left(\Pi_1 \middle| \Omega_1, D\right)}_{\text{MVN}} \underbrace{j\left(\Omega_1 \middle| D\right)}_{\text{IW}} \qquad (3.11)$$

where MVN denotes a multivariate normal density, IG an inverted gamma density and IW an inverted Wishart density. A similar factorization of the joint BMOM post data density is available; see Zellner (1997).

Given (3.11), we can draw from the IW density and insert the drawn values in h and make a draw from it. The Π_1 value so drawn is then inserted in g and a draw from it and the drawn values of s_e and Π_1 are inserted in f and a draw of the structural coefficients in f is made. This direct Monte Carlo procedure can be repeated many times to yield moments, fractiles and marginal densities for all parameters appearing in (3.11). Also, a similar approach, described below can be employed to compute predictive densities. Some of these calculations have been performed using sectoral models and data that will be reported in a future paper.

3.3 Forecasting Techniques

For one year ahead forecasts of the rates of growth of real GDP using the aggregate models in Table 1, we employed least squares forecasts that are means of Bayesian predictive densities when diffuse priors are employed and the usual normal likelihood functions are employed. Predictive means are optimal in terms of providing minimal expected loss vis a vis squared error predictive loss functions. Further, since these predictive densities are symmetric, the predictive mean is equal to the predictive median that is optimal relative to an absolute error predictive loss function.

One year ahead forecasts for the sectoral models in Table 1 were made using one year ahead MMM(A) reduced form forecasts of the y_{2T+1} and y_{3T+1} variables on the right hand side of equation (3.1) and using the parameter estimates provided by the methods described above. That is, the one year ahead forecast is given by:

$$\hat{y}_{1T+1} = \hat{y}_{2T+1}\hat{g}_{21} + \hat{y}_{3T+1}\hat{g}_{31} + x'_{1T+1}\hat{b}_1 \qquad (3.12)$$

The "eta" shrinkage technique, derived and utilized in Zellner and Hong (1989) involves shrinking a sector's forecast toward the mean of all eleven sectors' forecasts by averaging a sector's forecast with the mean of all sectors' forecasts as follows: $\hat{y}_{1T+1} = h\,\hat{y}_{1T+1} + (1-h)\bar{y}_{1T+1}$, where $\hat{y}_{1T+1}$ is the sector forecast, $\bar{y}_{T+1}$ is the mean of all the sectors' forecasts and h is assigned a value in the closed interval zero to 1.

Gamma shrinkage, discussed and applied in Zellner and Hong (1989), involves assuming that the individual sector's coefficient vectors are distributed about a mean, say q, and then using an average of an estimate of the sector's coefficient vector with an estimate of the mean q of the parameter vectors. That is,

$$\hat{d}_h = \left(\hat{d}_1 + g\hat{q}\right)/\left(1 + g\hat{q}\right) \qquad (3.13)$$

with $0 < g < \infty$. This coefficient estimate can be employed to produce one year ahead forecasts using the structural equations for each sector and MMM(A) reduced form forecasts of the endogenous variables (1-L)logW_{T+1} and (1-L) logY_{T+1}. Various values of h and g will be employed in forecasting sectoral growth rates that are used to construct an aggregate forecast of the growth rate of real GDP.

We can also compute a predictive density for a sector's one year ahead growth rate as follows. From (3.5), we can form the conditional density $q\left(y_{1T+1} \middle| \Pi_1, g_1, b_1, h_1, s_e, y_{2T+1}, y_{3T+1}, D\right)$, that will be in a normal form given error term normality. Thus, each draw from (3.11) and a draw from the predictive density for $\left(y_{2T+1}, y_{3T+1}\right)$ can be inserted in q and and a value of y_{1T+1} drawn from q. Repeating the process will produce a sample of draws from q from which the complete predictive density, its moments, etc. can be computed. Shown in Fig. 4 are two such predictive densities, one for the durables sector and the other for the services sector. The densities are slightly skewed to the left and rather spread out. However, the means that are optimal relative to squared error loss are not too far from the actual values being forecasted. Also, these densities are valuable in making probability statements about future outcomes, including turning point forecasts.

With this said about estimation and forecasting methods, we now turn to consider plots of the data and reports of forecasting results in the next section.

4. Discussion of Data and Forecasting Results

In Figure 1a are shown plots of the rates of growth of real GDP, real M1, real currency, real stock prices and real wage rates, 1949-1997. Peaks and troughs in the plots occur roughly at about 4 to 6 year intervals. Note the sharp declines in real GDP growth rates in 1974 and 1982 and a less severe drop in 1991. The money and stock price growth rate variables tend to lead the real GDP growth rate variable, as observed in earlier work of many. While the two money growth rate variables show similar patterns before the 1990s, in the 1990s their behavior is somewhat different for some unknown (to us) reason. In our forecasting results, we find that use of the currency variable yields somewhat better results than use of the M1 variable.

Figure 1b presents a plot of the output growth rates for 11 sectors of the U.S. economy. It is seen that except for the agriculture and mining sectors, the sectoral output growth rates tend to move together over the business cycle, while the agricultural and mining sectors show extreme variation. In contrast, the other sectors have much smaller inter-quartile ranges and fewer outlying growth rates. See also the boxplot for the sectoral growth rates in Fig. 1c.

In Figure 2a are shown the one year ahead, aggregate forecasts plotted as solid lines and the actually observed rates of growth plotted as circles. In the first panel of Fig. 2a, labeled AR(3), an aggregate AR(3) model for the real GDP growth rates, see Table 1, was employed to generate one year ahead forecasts year by year, 1980-1997, with estimates being updated each year. The plot shows dramatically the failure of the AR(3) model to forecast turning points successfully. Very large errors occurred in 1982 and 1991. Use of the AR(3)LI model, see Table 1, that incorporates two lagged leading indicator variables, the rates of growth of real currency and of real stock prices, produced the forecasts shown by the solid lines in the second panel of Fig. 2a. There are improvements in forecasts for 1982 and 1984 vis a vis use of the AR(3) model. However, there is still a large error in the 1991 forecast. Use of the MMM(A) model, see Table 1, that incorporates two lagged level GDP variables and a linear time trend in the AR(3)LI model produced the forecasts shown in the third panel of Fig. 2a. Here there are improvements, as compared to the use of the AR(3) model in most years, especially 1982, 1990 and 1991. Similar use of the MMM(A) model led to improved forecasts as compared to those provided by the AR(3)LI, especially in the 1990s.

In Fig. 2b are shown the disaggregated, one year ahead forecasts, plotted as solid lines and the observed real GDP growth rate data, plotted as circles. Here, for each year, the 11 sectoral forecasts are employed to generate a forecast of the growth rate of aggregate real GDP using annually updated estimates of relations. Again, even though the sectoral AR(3) forecasts were employed, there is little improvement as compared with the aggregate AR(3) forecasts, shown in Fig. 2a. The AR(3)LI and Distributed Lag(DL) models were used to generate forecasts for each of the 11 sectors and these were employed to calculate a forecast of the annual growth rates of real GDP with results shown in the second panel of the first column of Fig. 2b. The forecast performance of the DL model is seen to be better than that of the AR(3) model and about the same as that of the AR(3)LI model. With use of disaggregation and of the MMM(DA) models, I-IV, see Table 1 for their definitions, that include lagged level variables, the forecasting results shown in Fig. 2b were obtained. The MMM(DA) models outperformed the AR(3) model by a wide margin and the disaggregated Distributed Lag and AR(3)LI models by smaller margins. Also, from a comparison of Figures 2a and 2b, the MMM(DA) models performed better than all the aggregate models.

With respect to the four MMM(DA) models, it appears that MMM(DA)III has a slight edge on the other three MMM(DA) models. It caught the 1982 downturn and subsequent upturn rather well and its performance in later years, particularly the 1990s is slightly better than that of the alternative models considered in Figures 2a-2b. However, it missed the 1991 trough growth rate.

When the lagged rate of growth of real M1 is used as a leading indicator variable, rather than the lagged rate of growth of real currency, the results in Figs. 3a-b were obtained. The results in Fig. 3a are similar to those in Fig. 2a in that both the AR(3)LI and MMM(A) models' forecasting performance was much better than that of the AR(3) model. Use of the M1 variable rather than the currency variable led to a slight deterioration of the forecasting performance of the MMM(A). To a lesser degree, the same conclusion holds for the AR(3)LI model's performance. In Fig. 3b, the use of the M1 variable produced results similar to those reported in Fig. 2b. Note however, that use of M1 and the models other than the AR(3) led to better forecasts of the low 1991 real GDP growth rate and slightly worse forecasts of the low 1982 growth rate.

Shown in Table 2 are the RMSEs and MAEs associated with various models' one year ahead forecasts of annual real GDP growth rates, 1980-1997, using data 1952-1979 to estimate models which were then re-estimated year by year in the forecast period. Currency was used as the money variable. From the Aggregate Forecast part of the table, it is seen that the MMM(A) model has a RMSE = 1.72 and a MAE = 1.48, lower than those associated with the AR(3) and AR(3)LI models. For the rates of change of the real wage rate, the AR(3) model's RMSE = 1.43 and MAE= 0.98 are somewhat smaller than those of the MMM(A) and AR(3)LI models. These results indicate that the MMM(A) model for the growth rate of the real wage needs improvement, perhaps by inclusion of demographic and other variables.

As regards the disaggregated forecasts for the rate of growth of real GDP, shown in the second part of Table 2, it is seen that all the disaggregated forecasts have smaller RMSEs and MAEs than those for the aggregate and disaggregated AR(3) model. For example, the disaggregated AR(3) model has RMSE = 2.26 and MAE= 1.65, whereas the disaggregated AR(3)LI, Distributed Lag and MMM(DA) models have RMSEs ranging from 1.40 to 1.98 and MAEs ranging from 1.17 to 1.62. . As regards just the MMM(DA) models shown in Table 2, their associated RMSEs and MAEs ranged from 1.40 to 1.92 and 1.17 to 1.62, respectively. The lowest RMSE and MAE are encountered for the MMM(DA) III model fitted using the SUR approach, namely RMSE = 1.40 and MAE= 1.17. However, quite a few other MMM(DA) models had RMSEs in the 1.4-1.5 range and MAEs in the 1.2-1.4 range.

In Table 3 results similar to those presented in Table 2 are shown for models incorporating a lagged rate of change of real M1 rather than the real currency variable. In general the use of the M1 variable resulted in a generally small deterioration in forecasting precision for all the models. However, again disaggregation led to improved forecasting precision for the AR(3)LI and MMM models in all cases. Use of the MMM(DA)III model generally led to slightly lower RMSEs and MAEs than other MMM(DA) models. The lowest RMSE and MAE, namely 1.84 and 1.52, respectively are associated with the use of MMM(DA)III and h shrinkage with a value of $h = 0.50$ that can be compared to the aggregate MMM(A) model's RMSE = 2.23 and MAE = 1.90 and the aggregate AR(3)LI model's RMSE = 2.32 and MAE = 1.98. The RMSE and MAE for the aggregate AR(3) benchmark model are 2.32 and 1.71. Clearly use of disaggregation has led to improved forecasting performance again, about a 20% reduction in both RMSE and MAE.

In Tables 2 and 3, use of alternative methods of estimation, OLS, Extended MELO and 2SLS did not have much influence on the precision of forecasts. It may be that for the present model, the rates of change of real income and of the real wage rate are stochastic exogenous or independent variables in the sector models and thus endogeneity is not a problem. However, these two variables must be forecast in order to forecast sectoral output growth rates and thus there is a need for the reduced form equations shown in Table 1 whether these variables are stochastic exogenous or endogenous variables.

For the MMM(DA)III model, predictive densities for the sectoral output growth rates for the Services and Durables sectors were calculated for 1980 and are shown in Fig. 4 . Plotted are 1,000 draws from the BMOM predictive density made using the methods described in Section 3. The Services predictive density has a mean equal to 3.14 percentage points and a standard deviation equal to 2.05 percentage points. The actual growth rate for the Services sector's output in 1980 is 3.57 percentage points. For the Durables sector, the 1980 predictive mean is 6.31 percentage points with a standard deviation of 8.06. The actual 1980 growth rate for this sector is 7.33. Both predictive densities appear to be slightly skewed to the left and rather spread out. As is well known, such densities can be employed in making probability statements regarding possible outcomes, for example a downturn in the growth rate and in implementing a decision theoretic approach for making optimal turning point forecasts. Also, these predictive densities and predictive densities for other models can be used to form Bayes' factors for model comparison and/or model combining. That these predictive densities can be computed relatively easily using the “direct” Monte Carlo approach described in Section 3 is fortunate.

Last in Table 4, we present some MAEs of forecast for various types of forecasts of one year ahead growth rates of real GDP for the U.S. compiled by Zarnowitz (1986). For several different periods and forecasting units, the average of their MAEs associated with annual forecasts of the growth rate of real GNP in 1972 dollars are given in Table 4 below.

Table 4
MAEs for Annual Forecasts of Growth Rates of
Real GNP Made by Various Forecasters*

Period	MAEs	Average
1953-67	1.3(e), 1.0(d)	1.2
1962-76	1.1(a), 1.4(d)	1.2
1969-76	1.2(a), 1.0(b), 1.6(d), 0.9(c)	1.2
1977-84	1.2(a), 1.0(b), 1.0(d), 1.0(c)	1.0

*Source: Zarnowitz (1986), Table 1, p. 23. The forecasts are those of (a) Council of Economic Advisers, (b) ASA&NBER Surveys, (c) Wharton Newsletter, U. of Pennsylvania, (d) U. of Michigan and (e) an average of forecasts from the following sources: Fortune Magazine, Harris Bank, IBM, NICB, Nat. Securities and Research Corp., U. of Missouri, Prudential Insurance Co. and U. of California at Los Angeles.

Many of the MAEs in Table 4 are of magnitude comparable to those associated with the MMM(DA) annual one year ahead, reproducible forecasts for the period 1980-1997 shown in Table 2. Some forecasters use informal judgment along with models and data to produce forecasts. Adding “outside” information through the use of informative prior distributions may possibly improve the precision of MMM(DA) forecasts. See Zellner, Tobias and Ryu (1999) for some examples of the use of judgmental information in forecasting turning points in output growth rates. Also, averaging forecasts from different sources may improve forecast precision as many have pointed out. Note that the MAEs labeled (b) and (d) in Table 4 are such averages. On the other hand, on-line forecasters have problems associated with the use of preliminary estimates of economic variables that we do not have in our forecasting experiments using revised data throughout. The results of some on-line forecasting experiments would be of great value in assessing the importance of the “preliminary data” problem.

5. Summary and Conclusions

In the present research, we found that several disaggregated MMM forecasting equations performed the best in our forecasting experiments. Given the theoretical appeal of the

Marshallian sector models, it is indeed satisfying that they yielded forecasts of the growth rates of aggregate real GDP, 1980-1997, that were quite a bit better than those yielded by several aggregate benchmark models and competitive with other forecasting models and techniques. Shrinkage techniques and use of currency as the money variable in our models led to improved forecasts. However, the performance of various of our sector models, particularly those for the agricultural and mining sectors has to be improved. In addition, factor market models for labor, capital, money and bonds as well as other equations are in the process of being formulated to complete our MMM.

Bayesian and certain non-Bayesian point forecasts performed about equally well for our disaggregated MMM models. However, the Bayesian approach provides exact finite sample posterior densities for parameters and predictive densities. The latter are very useful for forecasting turning points and making probabilistic statements about various future outcomes. The "direct" Monte Carlo numerical procedure for computing finite sample Bayesian posterior densities for parameters and predictive densities for simultaneous equations models appears convenient and useful.

We recognize that writing a single structural equation in restricted reduced form and allowing for error term effects in the equation, along with other unrestricted reduced form equations, yields a non-linear SUR system with an important restriction on the error term covariance matrix. This form for the system is very useful in terms of understanding it, analyzing it and computing posterior and predictive densities.

While much can be said about the topic of data improvement, here we shall just remark that better data on (1) the numbers of firms and plants in operation within sectors, (2) sectoral stock price and wage rate indices, (3) weather variables, and (4) quality corrected output price data would be very useful and may lead to improved forecasts. Futher, having monthly or quarterly data for individual sectors would be useful in dealing with temporal aggregation problems. However, seasonality must be treated carefully. Mechanical seasonal adjustment procedures may not be the best alternative. Improvement of preliminary estimates of variables is another important issue in "on-line" forecasting. Preliminary estimates of variables that are contaminated with large errors can obviously lead to poor forecasts.

Last, with better data for sectors of a number of economies and reasonably formulated MMMs, past work on use of Bayesian shrinkage forecasting and combining techniques can be extended in an effort to produce improved point and turning point forecasts for many countries.

Fig. 1a

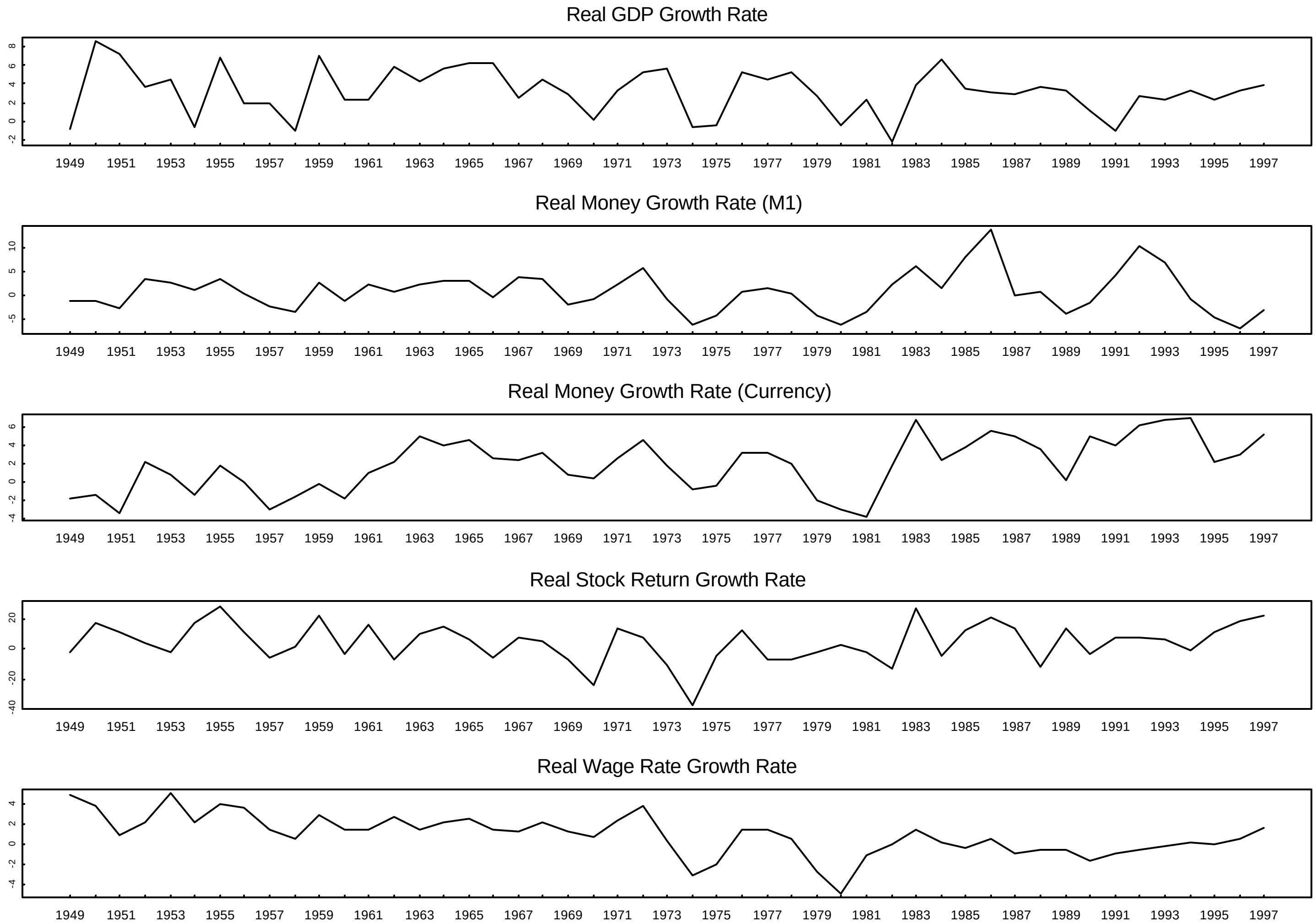
Real GDP Growth Rate
-2 0 2 4 6 8
1949 1951 1953 1955 1957 1959 1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997
Real Money Growth Rate (M1)
-5 0 5 10
1949 1951 1953 1955 1957 1959 1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997
Real Money Growth Rate (Currency)
-4 -2 0 2 4 6
1949 1951 1953 1955 1957 1959 1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997
Real Stock Return Growth Rate
-40 -20 0 20
1949 1951 1953 1955 1957 1959 1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997
Real Wage Rate Growth Rate
-4 -2 0 2 4
1949 1951 1953 1955 1957 1959 1961 1963 1965 1967 1969 1971 1973 1975 1977 1979 1981 1983 1985 1987 1989 1991 1993 1995 1997

Fig. 1b

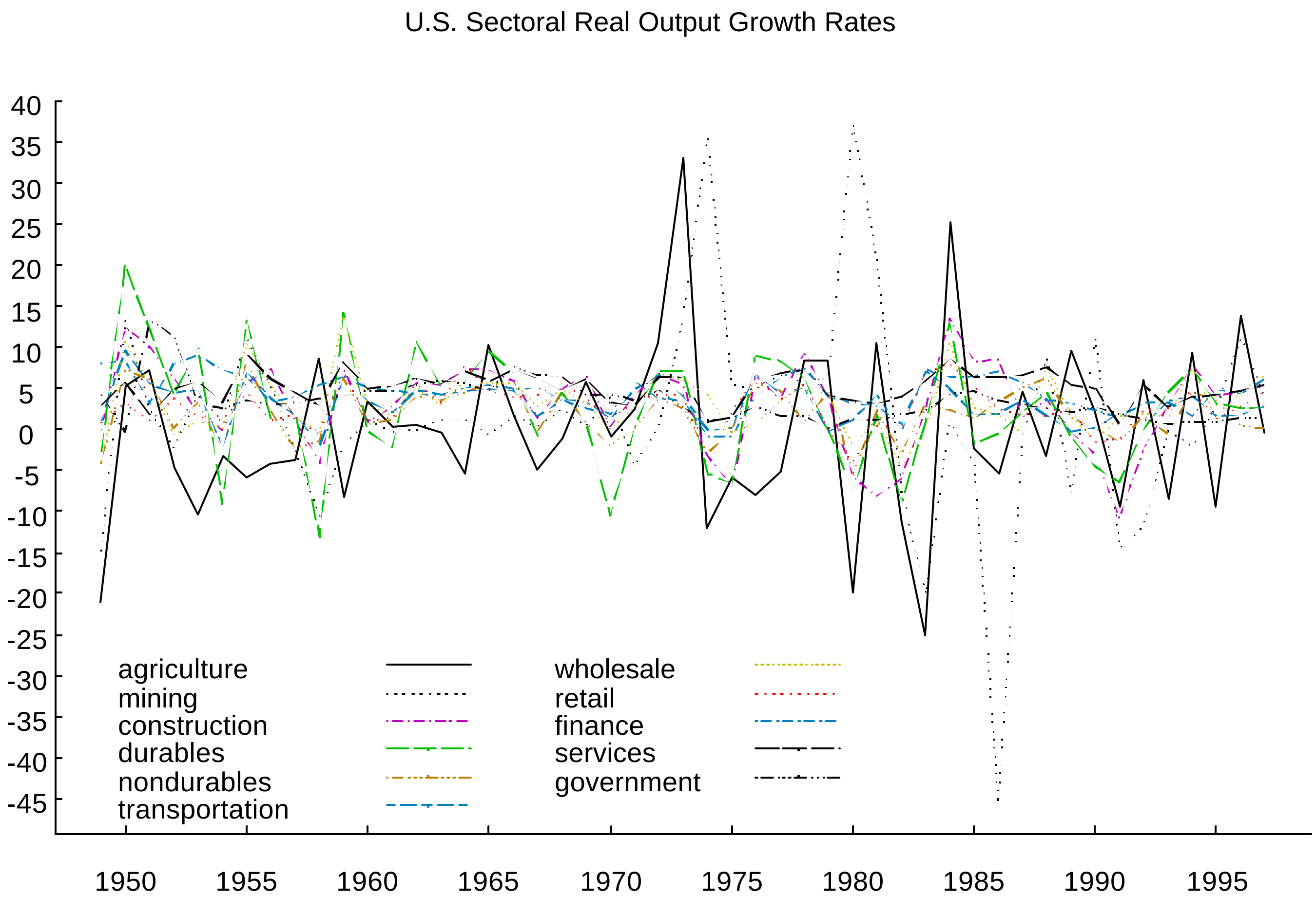
U.S. Sectoral Real Output Growth Rates
40
35
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
1950
1955
1960
1965
1970
1975
1980
1985
1990
1995
agriculture
mining
construction
durables
nondurables
transportation
wholesale
retail
finance
services
government

Fig. 1c

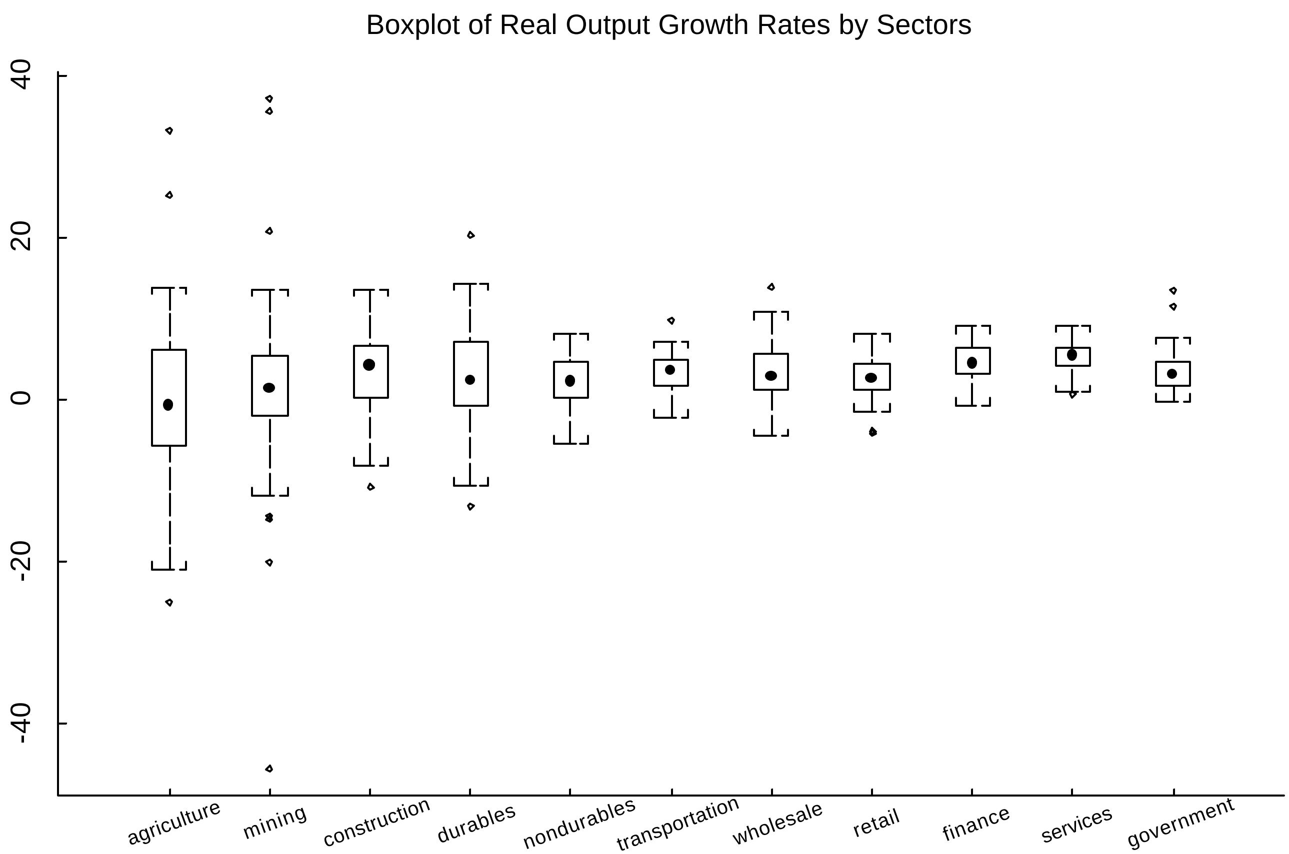
Boxplot of Real Output Growth Rates by Sectors
40
20
0
-20
-40
agriculture
mining
construction
durables
nondurables
transportation
wholesale
retail
finance
services
government

Fig. 2a

Aggregate GDP Growth Rate Forecasts Using the Currency Variable

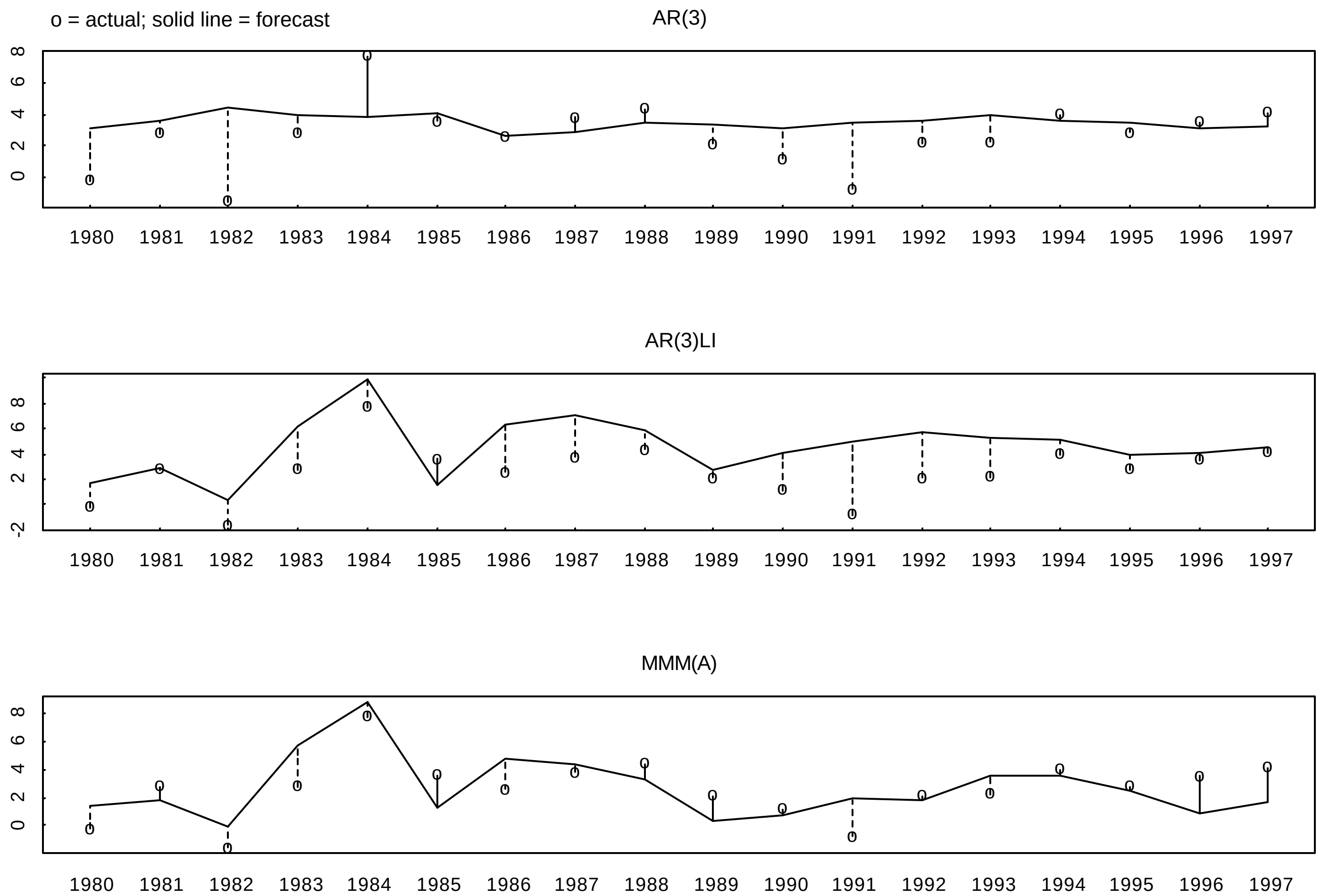

Fig. 2b
Disaggregated GDP Growth Rate Forecasts Using the Currency Variable (Complete Shrinkage)

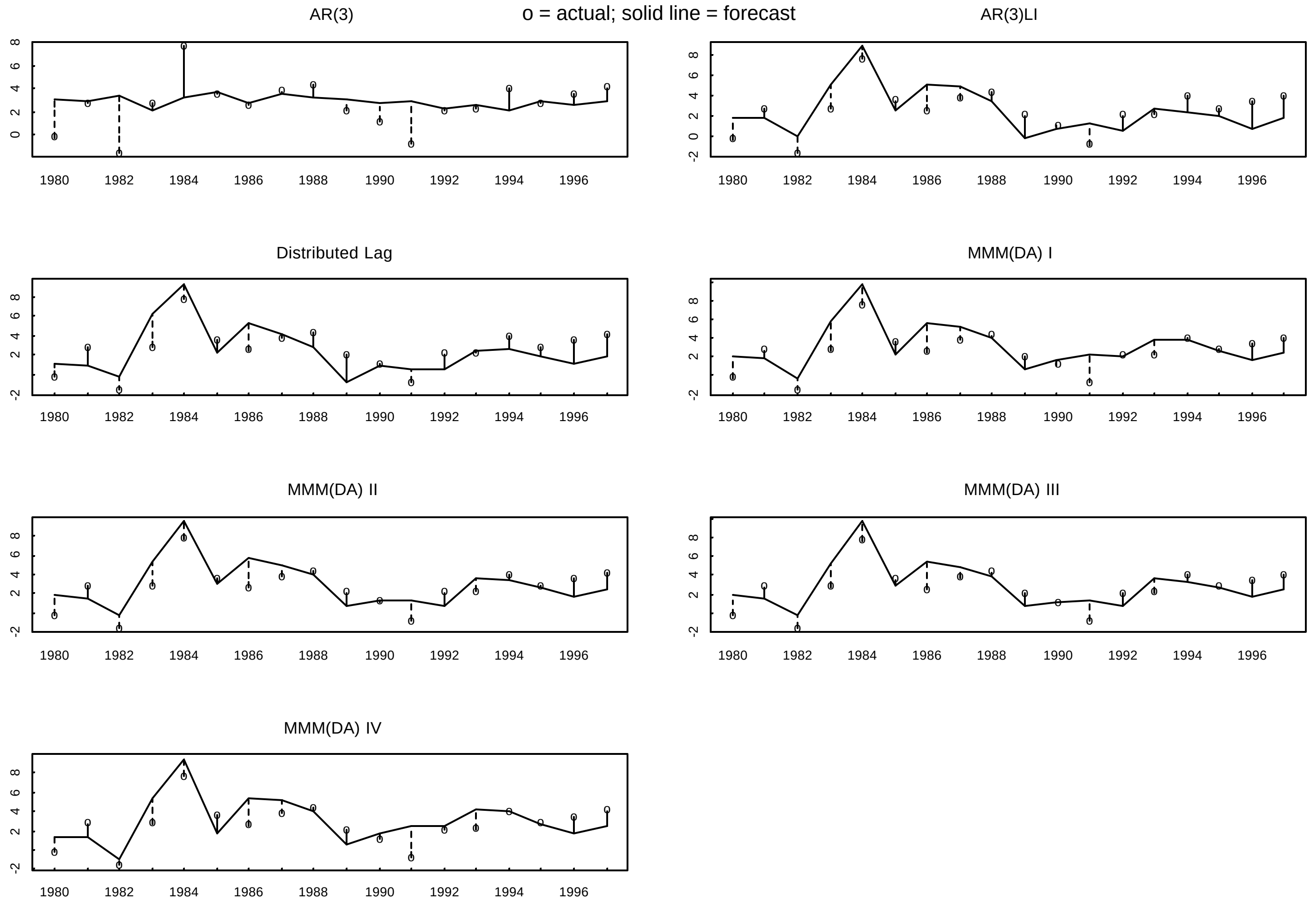

Fig. 3a

Aggregate GDP Growth Rate Forecasts Using the M1 Variable

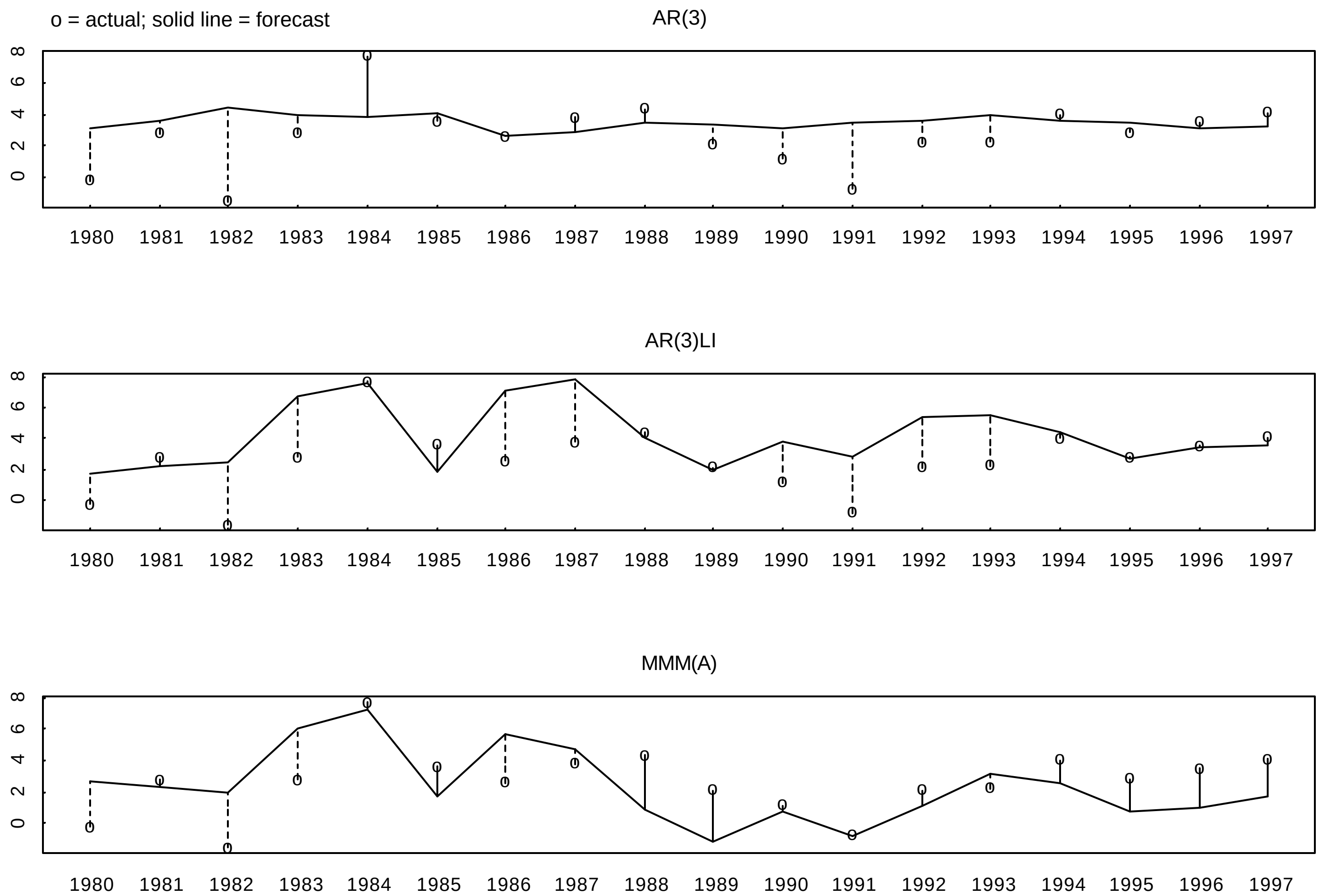

Fig. 3b
Disaggregated GDP Growth Rate Forecasts Using the M1 Variable (Complete Shrinkage)

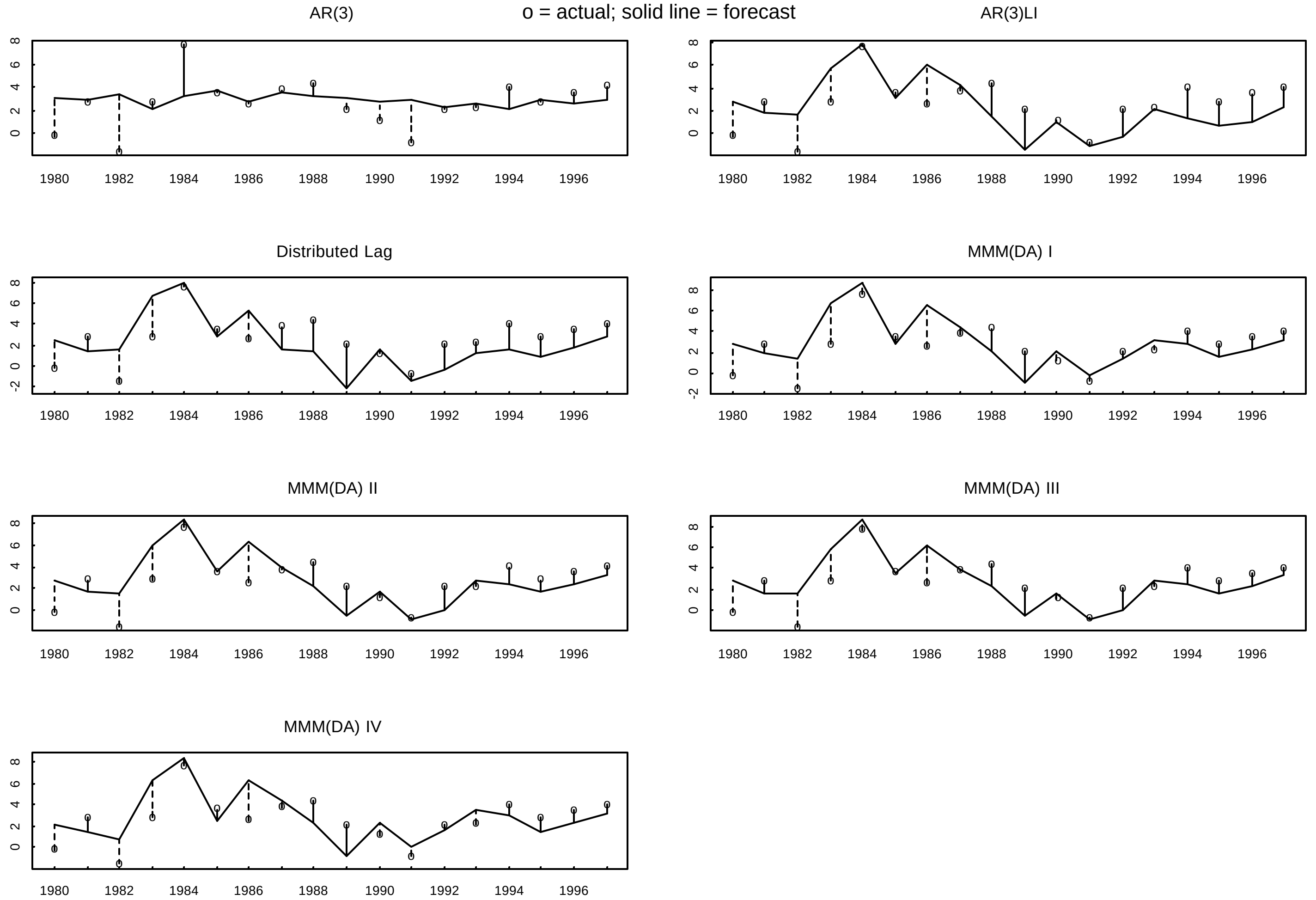

Fig. 4

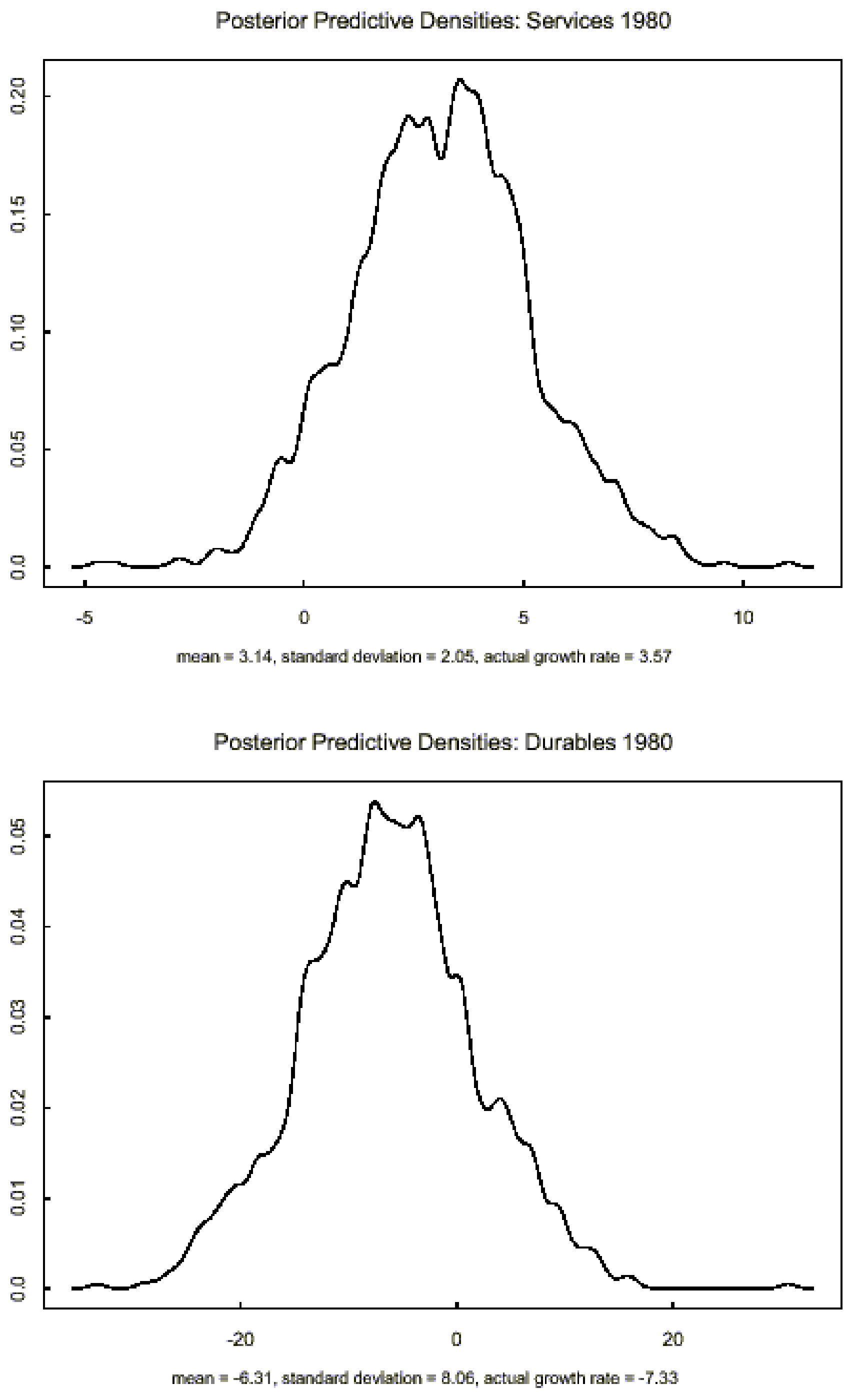
Posterior Predictive Densities: Services 1980
0.0
0.05
0.10
0.15
0.20
-5
0
5
10
mean = 3.14, standard deviation = 2.05, actual growth rate = 3.57
Posterior Predictive Densities: Durables 1980
0.0
0.01
0.02
0.03
0.04
0.05
-20
0
20
mean = -6.31, standard deviation = 8.06, actual growth rate = -7.33

Table 1

Forecasting Equations

Reduced Form Equations:

Real US GDP:

AR(3)(A): $(1\text{-}L)\log Y_t = \alpha_0 + \alpha_1(1\text{-}L)\log Y_{t-1} + \alpha_2(1\text{-}L)\log Y_{t-2} + \alpha_3(1\text{-}L)\log Y_{t-3} + u_t$

AR(3)LI(A): $(1\text{-}L)\log Y_t = \alpha_0 + \alpha_1(1\text{-}L)\log Y_{t-1} + \alpha_2(1\text{-}L)\log Y_{t-2} + \alpha_3(1\text{-}L)\log Y_{t-3} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + u_t$

MMM(A): $(1\text{-}L)\log Y_t = \alpha_0 + \alpha_1(1\text{-}L)\log Y_{t-1} + \alpha_2(1\text{-}L)\log Y_{t-2} + \alpha_3(1\text{-}L)\log Y_{t-3} + \alpha_4 Y_{t-1} + \alpha_5 Y_{t-2} + \alpha_6 t + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + u_t$

Real Wage:

AR(3) (A): $(1\text{-}L)\log W_t = \alpha_0 + \alpha_1(1\text{-}L)\log W_{t-1} + \alpha_2(1\text{-}L)\log W_{t-2} + \alpha_3(1\text{-}L)\log W_{t-3} + u_t$

AR(3)LI(A): $(1\text{-}L)\log W_t = \alpha_0 + \alpha_1(1\text{-}L)\log W_{t-1} + \alpha_2(1\text{-}L)\log W_{t-2} + \alpha_3(1\text{-}L)\log W_{t-3} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + u_t$

MMM(A): $(1\text{-}L)\log W_t = \alpha_0 + \alpha_1(1\text{-}L)\log W_{t-1} + \alpha_2(1\text{-}L)\log W_{t-2} + \alpha_3(1\text{-}L)\log W_{t-3} + \gamma_1 W_{t-1} + \gamma_2 W_{t-2} + \gamma_3 t + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + u_t$

Sectoral Forcast Equations:

AR(3)(DA): $(1\text{-}L)\log S_t = \alpha_0 + \alpha_1(1\text{-}L)\log S_{t-1} + \alpha_2(1\text{-}L)\log S_{t-2} + \alpha_3(1\text{-}L)\log S_{t-3} + u_t$

AR(3)LI(DA): $(1\text{-}L)\log S_t = \alpha_0 + \alpha_1(1\text{-}L)\log S_{t-1} + \alpha_2(1\text{-}L)\log S_{t-2} + \alpha_3(1\text{-}L)\log S_{t-3} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + \beta_3(1\text{-}L)\log W_t + \beta_4(1\text{-}L)\log Y_t + u_t$

Distrib.Lag(DA): $(1\text{-}L)\log S_t = \alpha_0 + \alpha_1(1\text{-}L)\log S_{t-1} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + \beta_3(1\text{-}L)\log W_t + \beta_4(1\text{-}L)\log Y_t + \beta_5(1\text{-}L)\log W_{t-1} + \beta_6(1\text{-}L)\log Y_{t-1} + u_t$

MMM(DA)I: $(1\text{-}L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + \beta_3(1\text{-}L)\log W_t + \beta_4(1\text{-}L)\log Y_t + u_t$

MMM(DA)II: $(1\text{-}L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + \alpha_2 S_{t-2} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + \beta_3(1\text{-}L)\log W_t + \beta_4(1\text{-}L)\log Y_t + u_t$

MMM(DA)III: $(1\text{-}L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + \alpha_2 S_{t-2} + \alpha_3 S_{t-3} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + \beta_3(1\text{-}L)\log W_t + \beta_4(1\text{-}L)\log Y_t + u_t$

MMM(DA)IV: $(1\text{-}L)\log S_t = \alpha_0 + \alpha_1 S_{t-1} + + \alpha_2 S^2_{t-1} + \beta_1(1\text{-}L)\log SR_{t-1} + \beta_2(1\text{-}L)\log m_{t-1} + \beta_3(1\text{-}L)\log W_t + \beta_4(1\text{-}L)\log Y_t + u_t$

Table 2
Forecast RMSEs and MAEs for Aggregate and Disaggregated Models Using Currency as the Money Variable (percentage points)

Aggregate Forecasts: 1952-1979 Þ 1980-1997

Real Income Y_t (Real GDP)

	AR(3)(A)	AR(3)LI(A)	MMM(A)
RMSE	2.32	2.61	1.72
MAE	1.71	2.19	1.48

Real Wage Rate W_t

	AR(3)(A)	AR(3)LI(A)	MMM(A)
RMSE	1.43	1.71	1.49
MAE	0.98	1.10	1.11

Disaggregated Forecasts: 1952-1979 Þ 1980-1997

** Using MMM(A) reduced form equations to forecast real income and real wage rate growth*

OLS

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.26	1.62	1.61	1.61	1.52	1.47	1.80
MAE	1.65	1.32	1.35	1.31	1.28	1.25	1.47

Extended MELO

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.26	1.58	1.62	1.55	1.55	1.50	1.80
MAE	1.65	1.23	1.34	1.26	1.31	1.26	1.46

2SLS

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV

RMSE	2.26	1.60	1.63	1.59	1.49	1.48	1.78
MAE	1.65	1.31	1.38	1.29	1.25	1.24	1.45

Table 2 (cont.)

SUR

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.21	1.70	1.66	1.68	1.61	1.40	1.92
MAE	1.52	1.41	1.36	1.39	1.38	1.17	1.60

Complete Shrinkage

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.11	1.73	1.82	1.76	1.57	1.59	1.70
MAE	1.45	1.57	1.60	1.46	1.37	1.38	1.43

g-Shrinkage

$\gamma = 0$ (same as OLS)

$\gamma = 0.25$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.21	1.62	1.61	1.61	1.49	1.46	1.74
MAE	1.59	1.36	1.38	1.34	1.26	1.25	1.41

$\gamma = 0.5$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.18	1.62	1.63	1.62	1.49	1.46	1.71
MAE	1.56	1.39	1.42	1.36	1.27	1.27	1.38

$\gamma = 1$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.15	1.64	1.66	1.64	1.49	1.48	1.69
MAE	1.52	1.44	1.46	1.38	1.29	1.29	1.39

$\gamma = 2$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV

RMSE	2.13	1.66	1.70	1.67	1.51	1.50	1.68
MAE	1.49	1.48	1.51	1.41	1.32	1.32	1.40

Table 2 (cont.)

$\gamma = 5$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.11	1.69	1.75	1.71	1.53	1.54	1.68
MAE	1.47	1.52	1.56	1.44	1.34	1.35	1.41

$\gamma = 10^6$ (same as complete shrinkage)

η-Shrinkage

$\eta = 0$ (same as OLS)

$\eta = 0.25$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.26	1.66	1.62	1.59	1.49	1.44	1.77
MAE	1.63	1.36	1.40	1.34	1.25	1.24	1.48

$\eta = 0.5$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.28	1.73	1.70	1.60	1.48	1.42	1.76
MAE	1.63	1.43	1.50	1.37	1.23	1.23	1.52

$\eta = 0.75$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.33	1.82	1.82	1.62	1.48	1.42	1.79
MAE	1.68	1.55	1.64	1.41	1.21	1.23	1.57

$\eta = 1$

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.40	1.93	1.98	1.65	1.49	1.44	1.83

MAE	1.76	1.67	1.79	1.44	1.22	1.25	1.62

Table 3
Forecast RMSEs and MAEs for Aggregate and Disaggregated Models Using M1 as the Money Variable
(percentage points)

Aggregate Forecasts: 1952-1979 Þ 1980-1997

Real Income Y_t (Real GDP)

	AR(3)(A)	AR(3)LI(A)	MMM(A)
RMSE	2.32	2.57	2.23
MAE	1.71	1.98	1.90

Real Wage Rate W_t

	AR(3)(A)	AR(3)LI(A)	MMM(A)
RMSE	1.43	1.73	1.66
MAE	0.98	1.07	1.29

Disaggregated Forecasts: 1952-1979 Þ 1980-1997

** Using MMM(A) reduced form equation to forecast real income and real wage rate growth*

OLS

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.26	2.03	2.01	2.04	1.96	1.89	2.17
MAE	1.65	1.77	1.74	1.78	1.76	1.67	1.88

Extended MELO

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.26	1.97	2.07	1.95	2.00	1.93	2.14
MAE	1.65	1.74	1.81	1.73	1.83	1.76	1.89

2SLS

				MMM(DA)			
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV

RMSE	2.26	2.06	2.06	1.99	1.91	1.89	2.12
MAE	1.65	1.78	1.76	1.75	1.73	1.69	1.85

Table 3 (cont.)

SUR

					MMM(DA)		
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.21	2.21	2.07	2.14	2.01	2.00	2.30
MAE	1.52	1.87	1.74	1.87	1.79	1.75	1.96

Complete Shrinkage

					MMM(DA)		
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.11	2.21	2.34	2.05	1.94	1.93	1.89
MAE	1.45	1.81	2.02	1.70	1.55	1.56	1.63

g-Shrinkage

γ = 0 (same as OLS)

γ = 0.25

					MMM(DA)		
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.21	2.04	2.02	2.01	1.91	1.85	2.06
MAE	1.59	1.75	1.67	1.74	1.68	1.60	1.78

γ = 0.5

					MMM(DA)		
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.18	2.06	2.04	1.99	1.89	1.84	2.01
MAE	1.56	1.73	1.65	1.71	1.62	1.55	1.71

γ = 1

					MMM(DA)		
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV
RMSE	2.15	2.08	2.09	1.99	1.88	1.84	1.95
MAE	1.52	1.74	1.71	1.69	1.56	1.52	1.66

γ = 2

					MMM(DA)		
	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	II	III	IV

RMSE	2.13	2.12	2.16	2.00	1.88	1.85	1.91
MAE	1.49	1.75	1.80	1.69	1.53	1.53	1.65

Table 3 (cont.)

$\gamma = 5$

	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	MMM(DA) II	III	IV
RMSE	2.11	2.16	2.24	2.02	1.90	1.89	1.89
MAE	1.47	1.76	1.91	1.69	1.53	1.54	1.64

$\gamma = 10^6$ (same as complete shrinkage)

η-Shrinkage

$\eta = 0$ (same as OLS)

$\eta = 0.25$

	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	MMM(DA) II	III	IV
RMSE	2.26	2.04	2.03	2.00	1.93	1.85	2.13
MAE	1.63	1.73	1.70	1.72	1.70	1.60	1.82

$\eta = 0.5$

	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	MMM(DA) II	III	IV
RMSE	2.28	2.07	2.15	1.99	1.92	1.84	2.13
MAE	1.63	1.71	1.82	1.66	1.63	1.52	1.79

$\eta = 0.75$

	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	MMM(DA) II	III	IV
RMSE	2.33	2.14	2.33	2.00	1.92	1.86	2.16
MAE	1.68	1.74	2.00	1.64	1.61	1.53	1.78

$\eta = 1$

	AR(3)(DA)	AR(3)LI(DA)	Distrib.Lag(DA)	I	MMM(DA) II	III	IV
RMSE	2.40	2.23	2.57	2.03	1.95	1.89	2.22
MAE	1.76	1.82	2.19	1.70	1.65	1.61	1.86

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