A Note on Aggregation, Disaggregation and Forecasting Performance

Arnold Zellner and Justin Tobias *

June 1,1999

In this note we report the results of an experiment to determine the effects of aggregation and disaggregation in forecasting the median growth rate of eighteen industrialized countries' annual output (GDP) growth rates; see Figure 1 for a plot of our data and Table 5 for the names of the countries in our sample. In one approach, following Zellner and Hong (1989), we model the aggregative annual median growth rate, w_t, as an autoregression of order three with lagged leading indicator input variables, denoted by AR(3)LI, as follows:

$$w_t = \alpha_0 + \beta_1 w_{t-1} + \beta_2 w_{t-2} + \beta_3 w_{t-3} + \beta_4 MGM_{t-1} + \beta_5 MSR_{t-1} + \epsilon_t, \tag{1}$$

where MGM_t is the median annual growth rate of real money in year t, MSR_t denotes the median annual growth rate in real stock prices in year t and ϵ_t is a zero mean, non-autocorrelated, constant variance error term. Given data on 18 industrialized countries' annual output growth rates, it is possible to compute annual median growth rates, and use them and data on the other input variables appearing in (1) to obtain point and turning point forecasts for future median annual growth rates of the 18 countries. The results of such calculations will be reported below after describing alternative approaches to forecasting the median growth rate using disaggregated data and disaggregated forecasting models.

*Research financed in part by the National Science Foundation and by income from the H.G.B. Alexander Endowment Fund, Graduate School of Business, U. of Chicago.

As an alternative to (1), we can employ the disaggregated ARLI relationships,

$$y_{it} = \gamma_i + \delta_{1i} y_{it-1} + \delta_{2i} y_{it-2} + \delta_{3i} y_{it-3} + \delta_{4i} GM_{it-1} + \delta_{5i} SR_{it-1} + \delta_{6i} SR_{it-2} + \delta_{7i} MSR_{t-1} + u_{it} \quad (2)$$

where the subscripts i and t denote the value of a variable for the i^{th} country in the t^{th} year, u_{it} is an error term, and y, GM, SR, and MSR denote the annual growth rates of real GDP, real money, real stock prices and the median growth rate of real stock prices, respectively. See Garcia-Ferrer *et al.* (1987) and Zellner and Hong (1989) for discussions and uses of (2) in forecasting. In (2) we allow the regression coefficients to vary in value across countries. We also consider two variants of (2) which involve restrictions on the regression coefficients. In the first model, the coefficients are restricted to be the same across countries; that is, $\gamma_i = \gamma, \ \ \delta_{ji} = \delta_j, \ \ j = 1, 2, \cdots 7$. In the second model, we assume that the coefficients associated with the leading indicators are the same across countries while the AR and intercept coefficients vary across countries; that is, $\delta_{ji} = \delta_j \ \ j = 4, 5, 6, 7$.[1] Then, given forecasts of 18 countries' annual output growth rates from (2) and its variants, it is clearly possible to compute the medians of the 18 forecasts year-by-year and use them as forecasts of the median output growth rates and compare them to the forecasts obtained by use of equation (1), as will be done below.[2]

Further, equation (2) can be expanded to include the current median output growth variable to obtain the following equation:

$$\begin{aligned} y_{it} \;=\; & \phi_i w_t + \gamma_i + \delta_{1i} y_{it-1} + \delta_{2i} y_{it-2} + \delta_{3i} y_{it-3} + \delta_{4i} GM_{it-1} + \\ & \delta_{5i} SR_{it-1} + \delta_{6i} SR_{it-2} + \delta_{7i} MSR_{t-1} + u_{it}. \end{aligned} \quad (3)$$

As Zellner and Hong (1989) do, we use equation (1) to obtain forecasts of the w_t's and use them and equation (3) to forecast the output growth rates of the 18 countries year-by-year. As described above, we analyze two additional variants of (3). In the first case all coefficients are restricted to

[1]We thank a referee for suggesting these alternative models.

[2]For the model in (2), we also estimated a random effects specification: $y_{it} = X_{it}\theta + \alpha_i + u_{it}$, where $u_{it} = \alpha_i + \epsilon_{it}$, and X_{it} denotes all the input variables in (2), including lagged y's, and θ denotes the associated regression parameters. Unlike the model in (2), this specification does not assume that the error terms for the same country are uncorrelated over time. We made the standard assumptions that the α's are uncorrelated across countries with common variance σ^2_α, and $E(\epsilon^2_{it}) = \sigma^2_\epsilon$, $E(\epsilon_{it}\epsilon_{i't'}) = 0 \ \forall \ i \neq i', t \neq t'$. We found that predictive RMSE's and MAE's are often lower using least-squares forecasts from (2) than results obtained using the above random effects specification. For example, updating the one-year ahead forecasts through the hold-out period 1985-1995, least-squares on (2) gives RMSE and MAE of 1.55 and 1.37, while forecasts from the random effects model above gives 1.68 and 1.44 for RMSE and MAE, respectively.

be equal across countries (*i.e.* $\phi_i = \phi, \gamma_i = \gamma, \delta_{ji} = \delta_j,\ j = 1, 2, \cdots 7$), and in the second case only the coefficients of the current median growth rate and the leading indicator variables are restricted to be equal across countries (*i.e.* $\phi_i = \phi,\ \ \delta_{ji} = \delta_j\ \ j = 4, 5, 6, 7.$) . Thus, we analyze the performance of 7 different models: the aggregate specification in (1), and 3 models for each of the specifications in equations (2) and (3).

In the first experiment, we use annual data, 1954-73 to fit our three models (see Appendix for estimation results) and then employ them to forecast the median of the annual output growth rates year by year for the period 1974-84, updating our parameter estimates as we move through the forecast period. This time period was chosen to be similar to the forecasting period used in previous work by Zellner and Hong (1989). For this time period, we present results only for the aggregate model in (1) and the models in (2) and (3) with all coefficient vectors restricted to be the same across countries. Forecasting results are shown in Table 1, Part A. It is seen that use of the disaggregated equations in (3) which include the aggregate variable w_t , perform the best with a RMSE and MAE of prediction of 1.22 and 1.08, respectively. Second best is the performance of the aggregate relation in (1), with RMSE = 1.54 and MAE = 1.44. Last in performance are the disaggregated relations in (2) that do not include the variable w_t with RMSE = 1.78 and MAE = 1.51. The empirical results for equation (1) using revised data are very similar to those reported in Zellner and Hong (1989) using unrevised data and which are better than those using just an AR(3) for w_t without leading indicator variables.

As a second experiment, we employed annual data, 1954-79 to fit our models and forecasted the median growth rate of the 18 countries year-by-year for the period 1980-95 with results shown in Table 1B. For this experiment, we obtain results using the 7 models based on equations (1), (2) and (3) described above. Again it is the case that use of the disaggregated equations in (3) with coefficients pooled over countries including the aggregate variable w_t, performed best with RMSE = 1.40 and MAE= 1.21. The results also indicate that RMSE's and MAE's increase as we add extra parameters to the models and allow some or all of the regression coefficients to vary across countries. Further, RMSE's and MAE's using the model in (3) that includes the forecasted current median growth rate are smaller than the RMSE's and MSE's associated with the corresponding models based on (2) which do not include the current median growth rate. These results also show improvement over some naive forecasting rules. For example, forecasting 0 percent as the median growth rate in each year yields RMSE = 2.70 and MAE = 2.28, respectively, while a 3 percent forecasting rule yields RMSE = 1.81 and MAE = 1.49.

Using the same data, we also performed calculations to determine which of the three models performed best in forecasting turning points in the median growth rate of the 18 countries over the period, 1980-1995. As in previous work, we define a downturn (DT) in period $T+1$ as occurring if the following median output growth rate sequence occurs:

$$w_{T-2}, w_{T-1} < w_T > w_{T+1}.$$

Also, by definition, no downturn (NDT) occurs when the following sequence is observed:

$$w_{T-2}, w_{T-1} < w_T \leq w_{T+1}.$$

Similarly, an upturn (UT) is said to occur in period $T+1$ if the following sequence of observations occurs:

$$w_{T-2}, w_{T-1} > w_T < w_{T+1}.$$

No upturn (NUT) occurs in period T+1 if the following sequence is observed:

$$w_{T-2}, w_{T-1} > w_T \geq w_{T+1}.$$

Given that we have a predictive density for w_{T+1}, we can easily compute the probability of DT and or NDT and use these probabilities along with a 2×2 loss structure to determine the forecast that minimizes expected loss. If the 2×2 loss structure is symmetric, a DT is the optimal forecast given that the probability of a DT is greater than 1/2.[3] If the probability of a DT is less than 1/2, then the optimal forecast is NDT. Similar considerations relate to forecasting UTs and NUTs. See LeSage (1996), Zellner, Hong and Min (1991) and Zellner, Tobias and Ryu (1997) for further discussion and applications of this turning point forecasting methodology.

In Table 2 are the number of DT, NDT, UT and NUT events that actually occurred in our sample and the number of correct forecasts using the procedure described above with each of our three

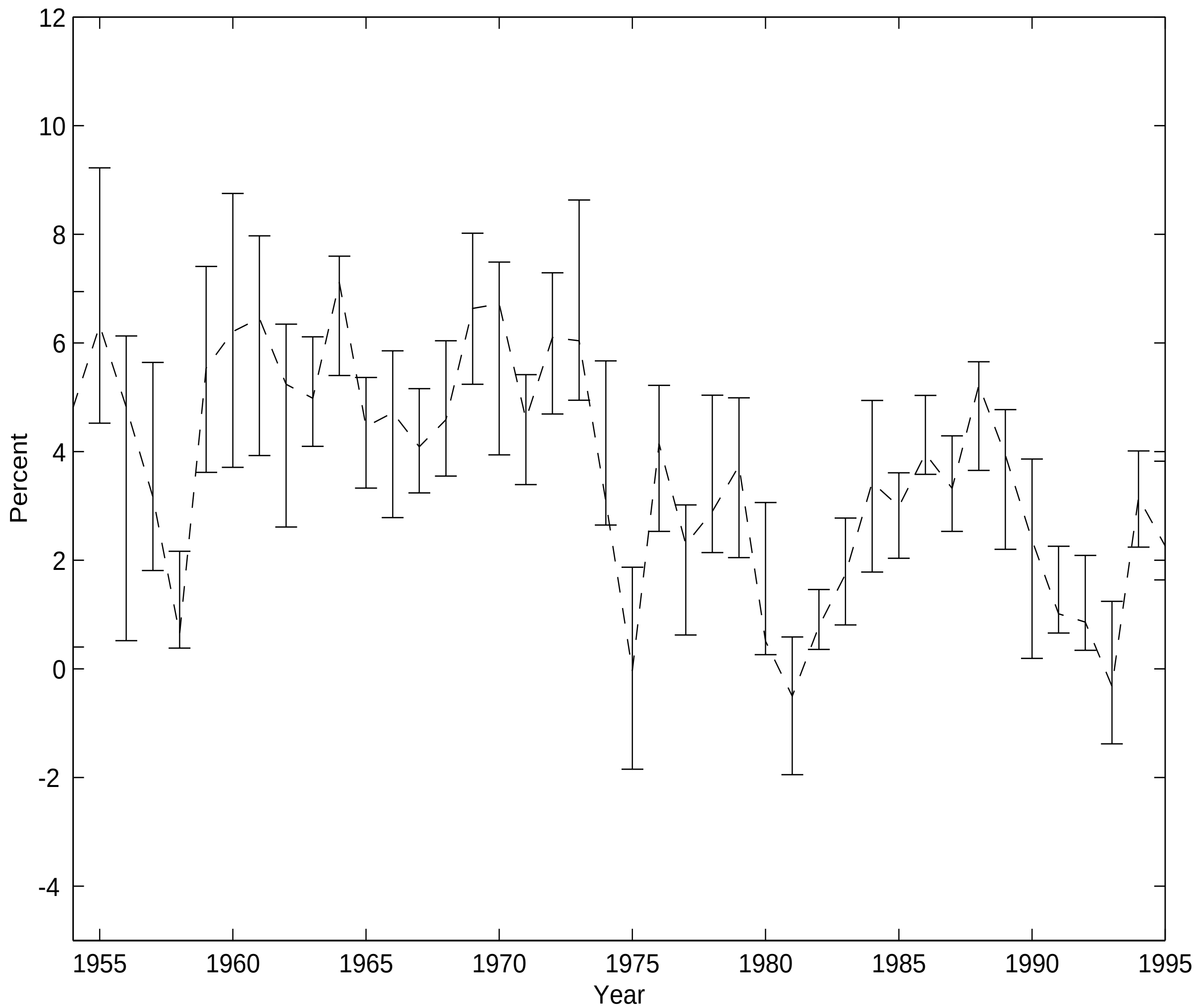
[3] We use the forecasted median from the alternative models as an approximation to the posterior predictive mean of w_{T+1}, and assume the predictive density is symmetric about the forecasted median.

models, shown in equations (1), (2) and (3). It is seen that use of the disaggregated AR(3)LI relations including the aggregate variable w_t in equation (3) produced the best results, namely 4 of 4 correct DT forecasts, 2 of 2 correct NDT forecasts, 2 of 2 correct UT forecasts and 1 of 4 correct NUT forecasts. Thus, the best model in terms of RMSE and MAE is also the best model in terms of turning point forecasting. As regards the poor NUT forecasts, in Zellner,Tobias and Ryu (1998), it was found that use of "trend" add factors that represent inertia effects produced improved NUT forecasts for individual countries' annual output growth rates. From Table 5, we see that adding the forecasted median to the individual countries' equations did not always improve the forecasting results. From the results reported in Table 1, we found that adding the forecasted median and restricting coefficients to be the same over countries lowered RMSE's and MAE's. It is important to note that this restricted version of (3) produces the lowest RMSE and MAE, and correctly forecasts the most turning points.

In summary, our forecasting experiments provide some evidence that improved forecasting results can be obtained by disaggregation given that an aggregate variable, w_t, appears in the disaggregated relations, as shown in (3). With disaggregation, there are more observations to estimate parameters and given that the disaggregated relations are reasonably specified, it is possible to obtain improved forecasts of an aggregate variable, here w_t, the median growth rate, a result that is in accord with some views expressed in the literature; see, *e.g.* Espassa (1994) and Palm and Zellner (1992).

Figure 1: Medians and Interquartile Ranges for Growth Rates of Real Output (A), Real Money (B) and Real Stock Prices (C): for 18 Industrialized Countries: 1954-1995.[4]

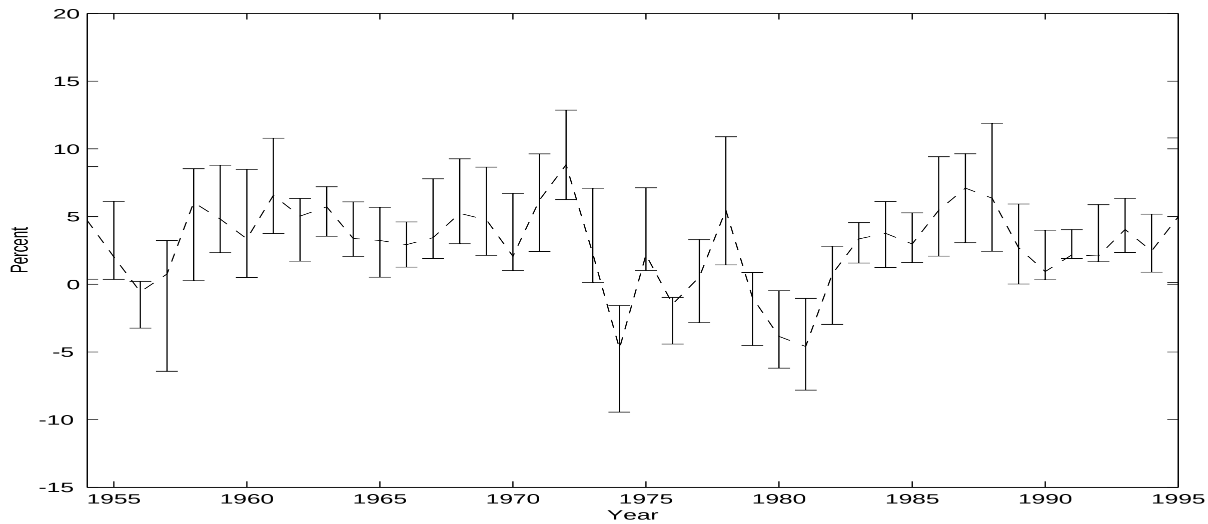
(A) Growth Rates of Real Output



[4]The dashed line connects the annual median growth rates (the w_t's) and the vertical lines give the interquartile ranges. See Table 5 for a list of the 18 countries included in our sample.

Figure 1 Continued

(B) Growth Rates of Real Money



(C) Growth Rates of Real Stock Prices

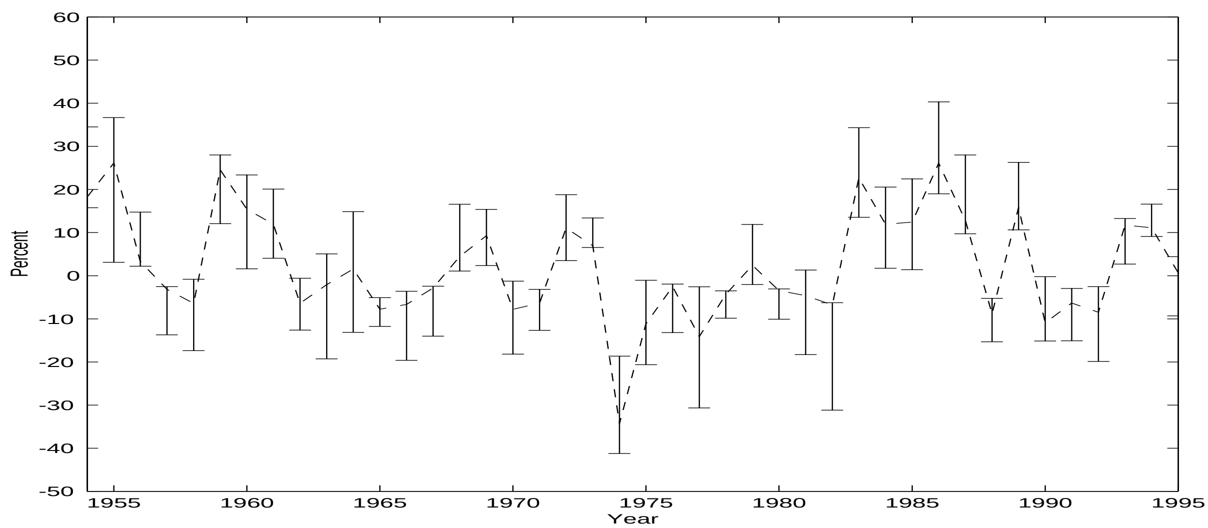


Table 1

RMSE's and MAE's of One-Year Ahead Aggregate and Disaggregate Forecasts of the Median of 18 Countries' Annual Real GDP Growth Rates [5]

A: 1974-1984

	Disaggregated AR(3)LI with $\hat{w}_t$ [eqn. (3)]	Aggregate AR(3)LI for w_t [eqn. (1)]	Disaggregated AR(3)LI [eqn. (2)]
RMSE	1.22	1.54	1.78
MAE	1.08	1.44	1.51

B: 1980-1995

	Aggregate AR(3)LI for w_t [eqn. (1)]		
RMSE	1.63		
MAE	1.46		
	Disaggregated AR(3)LI with $\gamma_i = \gamma, \delta_{ji} = \delta_j$ [eqn. (2)]	Disaggregated AR(3)LI with $\delta_{ji} = \delta_j, \quad j = 4, 5, 6, 7$ [eqn. (2)]	Disaggregated AR(3)LI [eqn. (2)]
RMSE	1.70	1.85	1.95
MAE	1.42	1.56	1.61
	Disaggregated AR(3)LI with $\hat{w}_t$ $\phi_i = \phi, \ \gamma_i = \gamma, \delta_{ji} = \delta_j$ [eqn. (3)]	Disaggregated AR(3)LI with $\hat{w}_t$ $\phi_i = \phi, \ \delta_{ji} = \delta_j, \quad j = 4, 5, 6, 7$ [eqn. (3)]	Disaggregated AR(3)LI with $\hat{w}_t$ [eqn. (3)]
RMSE	1.40	1.48	1.64
MAE	1.21	1.28	1.44

[5] The data are taken from the IMF computerized database at the University of Chicago. We use data for the following 18 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Spain, Sweden, Switzerland, UK and the U.S. Observations are available from 1954-1995 (with some country's data dating back to 1948) for most countries, but begin in 1971 for Germany. Omitting Germany from the analysis produced similar results. We define $RMSE = \sqrt{(\sum_{t=1}^{T} (\hat{w}_t - w_t)^2)/T}$, and similarly, $MAE = (\sum_{t=1}^{T} \mid \hat{w}_t - w_t \mid)/T$.

Table 2

Results of Forecasting Turning Points in the Median Output Growth Rate of 18 Countries: 1980-1995. Number of Correct Forecasts for Alternative Models

	DT	NDT	UT	NUT
	Observed Outcomes			
	4	2	2	4
Model	Number Correctly Forecasted			
Aggregate AR(3)LI for w_t. [eqn. (1)]	4	2	2	0
Disaggregated AR(3)LI [eqn. (2)] with	2	2	2	0
$\gamma_i = \gamma, \delta_{ji} = \delta_j \;\; \forall j$	—	—	—	—
Disaggregated AR(3)LI [eqn. (2)] with	3	2	2	0
$\delta_{ji} = \delta_j, \;\; j = 4, 5, 6, 7$	—	—	—	—
Disaggregated AR(3)LI [eqn. (2)]	2	2	2	1
Disaggregated AR(3)LI [eqn. (3)] with $\hat{w}_t$	4	2	2	1
$\phi_i = \phi, \; \gamma_i = \gamma, \delta_{ji} = \delta_j \;\; \forall j$	—	—	—	—
Disaggregated AR(3)LI [eqn. (3)] with $\hat{w}_t$	1	2	2	1
$\phi_i = \phi, \; \delta_{ji} = \delta_j, \;\; j = 4, 5, 6, 7$	—	—	—	—
Disaggregated AR(3)LI [eqn. (3)] with $\hat{w}_t$	1	2	2	1

APPENDIX
ESTIMATION RESULTS

The output, money, and stock price variables used in this paper are first converted to real quantities by dividing each variable by a country-specific price index. The variables are then logged, first-differenced, and multiplied by 100 to convert to growth rates. Estimation results for the AR(3)LI model in equation (1) are presented in Table 3, and coefficient posterior means and standard deviations for the models in (2) and (3) with coefficients restricted to be equal across countries are presented in Table 4. On computing the roots of the AR(3) process for the countries' output growth rates from equation(2),[6] we obtain one real root equal to .6646, and two complex conjugate roots, $.1488 \pm .4676i$, associated with a damped oscillatory component with estimated amplitude $= .491$ and estimated period $= 4.98$. See Geweke (1986) and Hong (1989) for Bayesian procedures for making posterior inferences about the properties of roots of an AR(3) process. Using earlier data for 18 countries included in our sample, Hong found that there is a high posterior probability that: (a) there are two complex roots and one real root, (b) the real root has amplitude less than one, and (c) the complex roots are associated with a damped oscillatory component with a period of about 4 to 6 years. We arrive at similar conclusions using updated and expanded data.

The addition of the "world return" variable, MSR_t has been shown in past work to reduce contemporaneous correlation among the error terms. See also Zellner Hong and Gulati (1990) and Min and Zellner (1993) for discussions of time-varying parameter models and shrinkage techniques for obtaining point and turning point forecasts, and the uses of posterior odds for comparing, choosing between and/or combining alternative forecasting models.

[6]Estimation results are obtained by inlcuding the leading indicators in equation (2). The roots, amplitude, and period are then computed from the AR(3) relationship using the posterior mean as a point estimate of the regression parameters. By drawing from the multivariate Student-t distribution, (the posterior distribution for the regression coefficients), we can also compute the probability that the AR(3) will have one real root associated with a cycle and two complex conjugate roots associated with a trend. Posterior distributions of the amplitude and period can also be obtained. See Hong (1988) for discussions.

Table 3

Diffuse Prior Posterior Means and Standard Deviations for Coefficients of AR(3)LI Model for the Annual Median Output Growth Rate (Eqn. (1)). $R^2 = .68$[7]

	Constant	w_{-1}	w_{-2}	w_{-3}	MGM_{-1}	MSR_{-1}
Coefficient Mean	1.05	.284	.088	-.008	.0132	.423
Posterior Std. Dev.	(.526)	(.144)	(.152)	(.143)	(.0220)	(.082)

Table 4

Diffuse Prior Posterior Means and Standard Deviations for Coefficients of Equations (2) and (3) with Coefficients Restricted to be the Same Across Countries

Variable	Disaggregated AR(3)LI with $\hat{w}_t$ eqn. (3)		Disaggregated AR(3)LI eqn. (2)	
	Coeff. Mean	Std. Dev.	Coeff. Mean	Std. Dev.
$\hat{w}_t$	.848	(.055)	—	—
Constant	-.341	(.213)	1.54	(.203)
y_{-1}	.222	(.035)	.367	(.039)
y_{-2}	-.099	(.036)	-.043	(.042)
y_{-3}	.086	(.031)	.160	(.036)
SR_{-1}	.032	(.007)	.025	(.008)
SR_{-2}	-.008	(.006)	-.021	(.006)
GM_{-1}	.045	(.012)	.076	(.014)
MSR_{-1}	-.042	(.011)	.017	(.012)
R^2	.47		.29	

[7] With use of diffuse priors and an IID normal likelihood function, coefficients' posterior means are equal to least squares estimates. Posterior standard deviations are equal to usual least squares asymptotic standard errors times $\sqrt{v_i/(v_i-2)}$ where $v_i = n_i - k_i$. Estimation results are computed using the full sample, 1954-1995. R^2 is an approximate mean of the posterior density of the population R^2 parameter.

Table 5[8]
Diffuse Prior Posterior Means and Standard Deviations
Allowing Regression Coefficients to Vary Across Countries.

Country	$\hat{\phi}_i$ $\hat{w}_t$	γ_i Const	δ_{1i} y_{-1}	δ_{2i} y_{-2}	δ_{3i} y_{-3}	δ_{4i} GM_{-1}	δ_{5i} SR_{-1}	δ_{6i} SR_{-2}	δ_{7i} MSR_{-1}	R^2
Australia	.817	.172	.134	-.035	.073	.046	.021	-.003	.020	.61
	(.192)	(.945)	(.139)	(.148)	(.134)	(.067)	(.020)	(.018)	(.033)	
Austria	.876	.525	-.139	-.083	.175	.145	.025	-.012	-.066	.44
	(.305)	(1.19)	(.177)	(.174)	(.168)	(.081)	(.039)	(.029)	(.057)	
Belgium	.924	-1.23	.111	.109	.069	.030	.003	-.008	.020	.87
	(.113)	(.423)	(.102)	(.102)	(.098)	(.048)	(.025)	(.016)	(.030)	
Canada	.800	-.128	.225	.029	-.066	.050	.057	-.034	-.065	.55
	(.277)	(.918)	(.192)	(.202)	(.173)	(.048)	(.045)	(.038)	(.055)	
Denmark	1.38	-.623	.042	-.479	-.056	.066	-.017	.039	-.074	.53
	(.311)	(.857)	(.162)	(.172)	(.144)	(.078)	(.027)	(.025)	(.039)	
Finland	1.24	-.881	.252	-.400	.110	-.001	.060	-.010	-.022	.63
	(.315)	(1.16)	(.164)	(.146)	(.117)	(.032)	(.034)	(.031)	(.060)	
France	.863	-.141	.154	-.179	.152	.075	-.041	.028	.019	.70
	(.202)	(.652)	(.130)	(.145)	(.130)	(.061)	(.028)	(.017)	(.047)	
Germany	.416	-.136	.281	-.175	.099	.263	.078	.017	-.051	.75
	(.417)	(1.37)	(.194)	(.201)	(.201)	(.125)	(.059)	(.048)	(.075)	
Ireland	.426	3.06	.001	-.021	-.082	.034	.073	-.038	-.175	.33
	(.304)	(1.67)	(.187)	(.187)	(.163)	(.106)	(.052)	(.031)	(.062)	
Italy	.981	.621	-.011	-.018	.027	.087	.028	-.014	.012	.66
	(.244)	(.956)	(.133)	(.139)	(.130)	(.042)	(.021)	(.016)	(.047)	
Japan	.782	-.557	.378	.190	.023	-.026	.117	-.082	-.114	.70
	(.401)	(1.06)	(.174)	(.200)	(.166)	(.120)	(.052)	(.039)	(.065)	
Netherlands	1.39	-1.07	.110	-.099	.019	-.088	.015	-.008	-.036	.73
	(.217)	(.668)	(.119)	(.122)	(.118)	(.081)	(.027)	(.020)	(.039)	
Norway	.367	2.46	.278	-.398	.017	.010	-.002	-.030	-.002	.45
	(.215)	(1.33)	(.169)	(.153)	(.167)	(.057)	(.028)	(.020)	(.049)	
Spain	1.08	-.167	.210	-.060	-.108	.114	.032	-.002	-.040	.66
	(.269)	(1.03)	(.174)	(.173)	(.155)	(.118)	(.034)	(.028)	(.061)	
Sweden	.983	-.617	.079	-.178	-.080	.124	.032	.024	-.084	.61
	(.202)	(.673)	(.176)	(.166)	(.173)	(.099)	(.022)	(.021)	(.038)	
Switzerland	.914	-1.09	.407	-.089	-.052	-.008	.032	.008	-.016	.74
	(.245)	(.756)	(.169)	(.196)	(.161)	(.065)	(.035)	(.025)	(.045)	
UK	.701	.962	-.137	-.178	-.248	.060	.044	.020	-.098	.55
	(.234)	(.755)	(.230)	(.200)	(.184)	(.035)	(.035)	(.026)	(.049)	
USA	.795	1.27	-.305	-.053	-.168	.213	.140	.002	-.145	.63
	(.184)	(.748)	(.137)	(.118)	(.095)	(.075)	(.038)	(.030)	(.039)	

[8]The coefficient posterior means presented above are numerically equivalent to least squares estimates. The posterior standard deviations equal least squares asymptotic standard errors times $\sqrt{v_i/(v_i-2)}$ where $v_i = n_i - k$. Estimation results are computed using the full sample, 1954-1995.

References

[1] Espassa, A., “Comment on ‘Time Series Analysis, Forecasting and Econometric Modeling: The Structural Econometric Modeling, Time Series Analysis (SEMSTA) Approach’,” *Journal of Forecasting* 13 (1994).

[2] Geweke, J., “Exact Inference in the Inequality Constrained Normal Linear Regression Model,” *Journal of Applied Econometrics* 1:127-141 (1986).

[3] Garcia-Ferrer, A., R.A. Highfield, F. Palm and A. Zellner, “Macro-Economic Forecasting Using Pooled International Data,” *Journal of Business and Economic Statistics* 5, 53–67 (1987).

[4] Lesage, J. P., “Forecasting Turning Points in Metropolitan Employment Growth Rates Using Bayesian Exponentially Weighted Regression, Time-Varying Parameter and Pooling Techniques” *Journal of Regional Science* 30(1990), 533–548.

[5] Min, C-K. and A. Zellner, “Bayesian and Non-Bayesian Methods for Combining Models and Forecasts with Applications to Forecasting International Growth Rates,” *Journal of Econometrics* 56 (1993), 89–119, reprinted in Zellner (1997).

[6] Palm, F. C. and A. Zellner, “To Combine or Not to Combine? Issues of Combining Forecasts,” *Journal of Forecasting* 11 (1992), 687–701, reprinted in Zellner (1997).

[7] Zellner, A., *Bayesian Analysis in Econometrics and Statistics: The Zellner View and Papers,* Edward Elgar Publ. Ltd., U.K. and U.S., (1997).

[8] Zellner, A., “Time Series Analysis, Forecasting and Econometric Modeling: The Structural Econometric Modeling, Time Series Analysis (SEMSTA) Approach,” *Journal of Forecasting* 13 (1994), 215-233.

[9] Zellner, A. and C. Hong, “Forecasting International Growth Rates Using Bayesian Shrinkage and Other Procedures,” *Journal of Econometrics* 40 (1989), 183–202, reprinted in Zellner (1997).

[10] Zellner, A., C. Hong, and G.M. Gulati, “Turning Points in Economic Time Series, Loss Structures, and Bayesian Forecasting,” in S. Geisser, J. Hodges, S.J. Press, and A. Zellner, eds., *Bayesian and Likelihood Methods in Statistics and Econometrics: Essays in Honor of George A. Barnard* Amsterdam: North-Holland (1990) 371–393.

[11] Zellner, A., C. Hong and C-k Min, “Forecasting Turning Points in International Output Growth Rates Using Bayesian Exponentially Weighted Autoregression, Time- Varying Parmaeter, and Pooling Techniques,” *Journal of Econometrics* 49 (1991), 275–304, reprinted in Zellner (1997).

[12] Zellner, A., J. Tobias and H. Ryu, “Bayesian Method of Moments Analysis of Time Series Models with an Application to Forecasting Turning Points in Output Growth Rates,” H.G.B. Alexander Research Foundation, U. of Chicago, (1998).