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The Marshallian Macroeconomic Model

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Abstract

In this paper, background information on the origins and features of the Marshallian Macroeconomic Model (MMM) are presented. Sector models based on two alternative production functions are presented and compared. In addition, some empirical forecasting results for one of them are reviewed. Last, attention is focused on further development and implementation of the MMM.

1. Introduction

It is an honor and a pleasure to present my paper at this research conference honoring Professor Ryuzo Sato, a superb colleague and most productive scholar. His outstanding research analyzing production and technological change, Sato (1999a,b) has been appreciated worldwide. Indeed, these topics play a central role in almost all models of industrial sectors and economies, including the models to be discussed below.

On the origins of the MMM, in my experience it was a pleasure teaching undergraduate and graduate students the properties and uses of the Marshallian model of a competitive industry. On the other hand, teaching students macroeconomics was quite a different matter since there was no such comparable, operationally successful model available. See, e.g., Belgonia and Garfinkel (1992) for an excellent review of alternative macroeconomic models, including monetarist, neo-monetarist, Keynesian, post-Keynesian, and real business cycle models and Fair (1992) and Zellner (1992) who pointed out that not enough empirical testing of alternative models had been done and more is needed to produce macroeconomic models that explain the past, predict well and are useful in making policy.

To help achieve these objectives, the structural econometric, time series analysis (SEMTSA) approach, described and applied in Garcia-Ferrer, et al (1987), Palm (1976,1977,1983), Zellner and Palm (1974, 1975), Hong (1989), Min (1992), and Zellner (1979,1994, 1997) was used to generate an equation, a third order autoregression with lagged leading indicator variables (ARLI) for the annual rate of growth of real GDP that worked fairly well in point forecasting one year ahead and forecasting downturns and upturns in rates of growth with about 70% correct turning point forecasts in 211turning point episodes for 18 industrialized countries; see, e.g., Zellner and Min (1999) and references therein. While Hong (1989), Min (1992) and Zellner (1999) showed that this ARLI equation is mathematically implied by variants of a Hicksian IS-LM macro model,

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a generalized real business cycle model and an aggregate demand and supply model, respectively, it is not clear that these macroeconomic models are entirely adequate. In particular, they abstract from important phenomena involved in business cycles and growth, namely, industrial sectors with different cyclical properties, entry and exit of firms, sector linkages, etc. The challenge is to formulate a relatively simple model that accommodates such characteristics, works well in explanation and forecasting and is flexible enough to elaborate in various dimensions if necessary

One morning while shaving, the idea came to me to go back to my favorite Marshallian demand, supply AND entry model (see Veloce and Zellner (1985) for a formulation of such a model with an application to a Canadian industry) and use one for each sector of an economy. Note that many macroeconomic, general equilibrium and demand and supply models do not include an entry equation. Indeed, in some models, there is just a representative firm and one wonders what happens if this firm shuts down. Also, in many rational expectation models of competitive industries, see, e.g. Muth (1961) the number of firms is assumed constant. On aggregating supply functions over firms, the number of firms in operation, N, appears in the industry supply equation and thus an entry equation is needed, along with demand and supply functions, to close the model. Indeed, when the N variable is omitted from the supply function, very strange estimation results are obtained; see some examples in Veloce and Zellner (1985).

In the MMM model, we utilize industrial sectors, each with consumer demand, sector supply and entry equations and productive units that buy factors of production, labor, capital and money services and other inputs to production in national factor markets. With the addition of demand and supply models for factor markets and a foreign sector, we have a MMM model. Of course such a model can be elaborated in many ways. However, consistent with the SEMTSA approach, we start simply and complicate if necessary, in contrast to the "general to specific" approach used by many. Note that there are many general models, including VAR, MVARMA, nonlinear VARs, etc. and if the wrong one is chosen, users of the general to specific approach will be disappointed.

In what follows, in Sect. 2, the Veloce-Zellner sector model, based on use of Cobb-Douglas production functions, will be reviewed and extended to include technical change and factor augmentation effects. It will be shown how the model has been used in Zellner and Chen (2000) to forecast growth rates of sectors' real sales and of total real GDP of the U.S. economy. Then in Sect. 3, another production function, a "generalized production function," will be utilized and the model resulting from its use will be compared with that based on the Cobb-Douglas production function with neutral technical change and factor augmentation. Last, in Sect. 4 consideration is given to several properties of the models considered in this paper that can be relaxed or modified. Also, as is apparent, it will be pointed out that factor markets for labor, capital, money, and bonds can be added to the sector models to complete the MMM. Last, it is noted that the MMM can accommodate the birth of new and the death of old sectors.

2. A Competitive Marshallian Sector Model of an Economy

In this section, a slightly modified version of the competitive demand, supply and entry model put forward and estimated in Veloce and Zellner (1985) is presented. We assume a competitive industry with N=N(t) firms in operation at time t, each with a Cobb-Douglas production function, $q = AL^a K^b$, where $A = A(t) = A_N(t)\,A_L(t)^a A_K(t)^b$, the product of a neutral technological change factor and labor and capital augmentation factors that reflect changes in the quality of labor and capital inputs. Additional inputs, e.g. services of money, inventories and raw materials can be added without much difficulty. On assuming profit maximization with respect to inputs given factor prices,
w = w (t), the real wage rate and r = r (t), the real price of capital services and given real product price, p= p (t), the sector's real sales supply function is S = Npq = $NA^* p^{1/q} w^{-a/q} r^{-b/q}$, where $A^* = A^{1/q}$, and $0 < q = 1 - a - b < 1$. On logging both sides of the equation for S, real sales, and differentiating with respect to time, we obtain the industry real sales supply equation:

(1) $$\dot{S}/S = \dot{N}/N + \dot{A}^*/A^* + (1/q)\,\dot{p}/p - (a/q)\dot{w}/w - (b/q)\,\dot{r}/r$$ SUPPLY

where $\dot{x}/x = (1/x)dx/dt$. Further if we multiply both sides of the industry demand function by p, we obtain real sales = $S = pQ = Bp^{1-h} x_1^{h_1} x_2^{h_2} \ldots x_k^{h_k}$, where the x variables are demand shifters such as real income, real money balances, number of consumers, etc. On logging and differentiating this last equation with respect to time, the result is:

(2) $$\dot{S}/S = (1-h)\,\dot{p}/p + \sum_{i=1}^{k} h_i \dot{x}_i/x_i$$ DEMAND

Finally, the following entry equation will be utilized to complete the model for the three variables, price, p, real sales, S, and number of firms in operation, N:

(3) $$\dot{N}/N = g'(\Pi - F_e) = g(S - F)$$ ENTRY

where profits $\Pi = qS, g = g'q$, with $0 < g < 1$, and $F = F_e/q$, with F_e the equilibrium level of profits taking account of discounted entry costs.

In Veloce and Zellner (1985), data for the Canadian furniture industry were employed to estimate discrete versions of (1) and (3) taking factor prices and demand shift variables as exogenous variables. If it is assumed that all sectors sell in the final product market, similar analysis provides supply, demand and entry equations for each sector. Note that the above demand equations can be elaborated to take account of substitution and complementarity effects. Also, parameters' values will usually differ over sectors.

When equations (1)-(3) are solved for $\dot{S}/S$ by substituting from (3) in (1) and then eliminating the $\dot{p}/p$ variable, the result is

(4) $$\dot{S}/S = (a+g)[1 - S/(a+g)F]$$

where $a = gqF/(a+b-h)$ and $g =$ a linear function of rates of change of A*,w, r, and the x's, the demand shifters. If in (4), g = 0 or g = const., it is seen that the differential equation for S has a solution in the form of the well known logistic function. Further, if g = g(t), a given function of t, the equation is in the form of Bernoulli's differential equation; see Veloce and Zellner (1985,p.463) for its general solution. Note that g may change through time because of changes in the rate of growth of neutral technical change, factor augmentation and/or in the rates of change of exogenous variables affecting demand and supply, say real money balances, real income, the real wage rate, etc.

In the SEMSTA approach, mentioned above, it is considered important to test the forecasting performance of equations derived from theoretical models, such as that in (4). In Zellner and Chen (2000) we employed, among others, the following discrete approximation to (4) in analyzing real GDP of 11 sectors of the U.S. economy including agriculture, mining, construction, etc.

(5)

$$(1-L)\log S_t = a_o + a_1 S_{t-1} + a_2 S_{t-2} + a_3 S_{t-3} +$$
$$b_1(1-L)\log Y_t + b_2(1-L)\log M_{t-1} + b_3(1-L)\log w_t + b_4(1-L)\log SR_{t-1} + u_t$$

where L is the lag operator, S sector real GDP, Y aggregate real GDP, M real money balances, w the real wage rate, SR a real stock price index and u an error term. Lagged values of S were employed to reflect lags in the entry equation. Note that the rate of change of S is related to lagged levels of S, a "co-integration" effect that flows from the model.

With w and Y assumed exogenous relative to 11 individual sectors of the U.S. economy, equations in the form of (5), and variants of it, with sector specific parameters were fitted as a set of seemingly unrelated regressions, using U.S. annual data, 1949-1979, and then employed to produce one year ahead forecasts, with estimates updated year by year for the period, 1980-1997. Reduced form equations were employed to forecast the Y and w variables. While the models for certain sectors, namely agriculture, mining and construction did not perform very well given the great variability of these sectors' outputs, when the annual sector real GDP forecasts were added to provide a one year ahead forecast of total U.S. real GDP and its growth rate, it was found that such growth rate forecasts are better than those of benchmark models implemented with aggregate annual data including an AR(3) model for the rate of change of Y, the AR(3) model with added lagged leading indicator variables, and the same model with lagged levels of Y and a time trend variable. Thus, in this case it appears that it pays to disaggregate, mainly because sector specific variables can be employed along with many more observations than in an aggregate, one equation approach utilizing aggregate data. Similar results were obtained using equation (5), with Y and w assumed to be endogenous variables and various estimation techniques including OLS, 2SLS, Bayesian minimum expected loss (MELO) and shrinkage techniques; see Zellner and Chen (2000) for a detailed discussion of Bayesian and non-Bayesian estimation and prediction techniques. Predictive results using a real currency variable were somewhat better than those using a real M1 variable in sectors' demand equations. For the one year ahead forecasts of the annual growth rates of real U.S. GDP, 1980-1997, the mean absolute errors, MAEs ranged from 1.17 to 1.38 percentage points for the alternative approaches mentioned above applied to equation (5). For a benchmark AR(3) model that missed all the turning points, the MAE = 1.71, about 24 to 46 percent larger than the MAEs for the disaggregate forecasts. A MAE = 1.17 compares favorably with those reported in Zarnowitz (1986) for annual forecasts of the rate of growth of real U.S. GDP made by the Council of Economic Advisors and other forecasting organizations. With improvement of the models for certain sectors, e.g. the highly variable agricultural, mining and construction sectors, it may be that additional precision in forecasting performance, etc., can be realized. Last, joint estimation and forecasting using the three equations (3)-(5) for each sector, may produce even better results.

Thus in terms of aggregate forecasting, the disaggregate approach described above seems worthwhile. Further, it not only yields forecasts of aggregate output but, obviously, of its components sector by sector. In addition, the possibility of pooling data of various countries' sector models in estimation and prediction may result in further improvement in forecasting results. See, e.g. Garcia-Ferrer, et al. (1987), Quintana, Chopra and Putnam (1995), Zellner and Hong (1989), and Zellner (1994,1997) for empirical results showing that use of pooling or shrinkage techniques results in improved forecasts.

3. Sector Model Based on a Generalized Production Function

While the results above are useful, it is of interest to see how the sector models' form is affected by use of a "generalized production function," (GPF) rather than a Cobb-Douglas production function. In general a GPF is a monotonic function of a homogenous function, say a Cobb-Douglas function. That is, GPFs are in the class of homothetic production functions with pre-specified forms for the returns to scale function; see, e.g. Zellner and Revanker (1969), Greene (1993, pp. 324-328) and Zellner and Ryu (1998). GPFs, in contrast to the Cobb-Douglas function have associated long run average cost curves that are U or L shaped with a unique minimum. Further, the profit-maximizing labor's share is not a constant but varies with the level of output. A simple GPF that will be employed below is given by $\log q + a q = b_o + b_1 \log L + b_2 \log K + \log A$, with a and the b's strictly positive parameters and where A = A(t) denotes both general and factor augmenting technical change. This function has an elasticity of substitution = 1. In Zellner and Ryu (1998) many GPFs are presented and estimated, some with variable elasticity of substitution parameters, etc. Assuming competitive conditions and profit maximization on the part of firms in a sector yields the following expression for labor's share, $wL/pq = b_1/(1+a q)$. Thus labor's share is not constant but varies as q varies, say over the business cycle. Further, the supply function for an individual firm is given by:

(6) $$\ln Q/N = c_o + c_1 Q/N + c_2 \ln(1+a Q/N) + c_3 \ln p + c_4 \ln w + c_5 \ln r + c_6 \ln A$$

where Q/N = q, with N = number of identical firms in operation and the c's are parameters. A sector demand function is given by:

(7) $\ln p = a_o + a_1 \ln Q + a_2' x$

where a_2 is a vector of parameters and x is a vector of demand shifters including real income, money balances, etc. Last the entry equation is given by:

(8) $\dot{N} / N = d(p / pq) = da(q - q_m)/(1 + aq)$

where d is a positive adjustment coefficient and q_m is the output level associated with the minimum of the long-run average cost function associated with the GPF shown above. When q = Q/N = q_m, there is no change in the number of firms in operation, when q>q_m, there is entry and when q<q_m there is exit of firms.

After differentiating (6) and (7) with respect to t, we can solve the equations (6)-(8) for a differential equation for q with the exogenous variables as inputs, namely,

(9) $\dot{q} / q = [v_1(q - q_m) + (1 + aq)g]/[(1 + aq)^2 + v_2(1 + aq) + v_3]$

In (9), the v's are parameters, g denotes a linear combination of the rates of change of demand and supply shift variables, e.g. real income, real money, real wage rate, etc. and a is the parameter of the GPF given above. Note that the effect of a change in g on the growth rate is dependent on the level of output. If g = 0 or a constant, it is seen that the proportionate rate of growth of q = Q/N is given by the ratio of a linear function of q divided by a quadratic function. It is possible to solve the differential equation in (9) and variants of it and study their properties. Also solutions for the paths of p and N can be obtained, analytically or numerically.

As regards empirical work, if data are available on Q and N, then q = Q/N can be formed and used to fit discrete approximations to (9) that can be implemented sector by sector and tested in forecasting experiments. If these results are satisfactory, further work on the structural equations (6)-(8) can be undertaken. Given data on N, Q and p, discrete approximations to the structural equations in (6)-(8) can be estimated and checked for reasonableness and performance in prediction. On obtaining a joint predictive density for p, Q and N, it can be employed to compute the density of S = pQ, real sales for each sector. Forecasts of S so obtained can be compared to those provided by the Cobb-Douglas model, described above and actual outcomes. Simulation experiments can be performed to study dynamic properties of the two models and their responses to various policy changes, structural breaks, etc.

4. Concluding Remarks

In this paper, two sector models have been presented that are in the form of a Marshallian demand, supply and entry model for competitive sectors or industries that sell in a final product market. Of course such models can be modified in many different ways. For example, it is possible to take account of many additional elements, namely substitution effects in demand, additional factor inputs, monopolistic competition, expectations, inventories and inventory investment, birth and death of sectors, different forms for the entry equation, etc. Indeed, much work on these topics has appeared in the literature over the years. It is thought that integration of such research results in the MMM will provide improved explanatory and predictive performance. Further, by adding labor, capital and money service factor demands over the sectors and introducing supply functions for labor, capital and money services along with export and government sectors, a complete MMM is obtained. As mentioned above, new sectors can be introduced, e.g. using available models for new products and services, and thus make the model sensitive to Schumpeterian-like waves of innovation associated with new industries such as the computer industry, etc.

The starting point for this modeling work is not a very general model but relatively simple sector models that perform reasonably well in explanation and prediction. These tested components can then be improved step by step to accommodate the additional features mentioned above. In this way, in accordance with the SEMTSA approach, we shall have valuable components that work well empirically and when combined will probably result in a sensible, satisfactory MMM that has good explanatory and predictive properties and be useful to policy-makers.

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