THE USE OF A MARSHALLIAN MACROECONOMIC MODEL FOR POLICY EVALUATION: CASE OF SOUTH AFRICA

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Abstract

Using a disaggregated Marshallian Macroeconomic Model (MMM-DA), this paper investigates how the adoption of a set of 'Freedom Reforms' may affect the economic growth rate of South Africa. This study makes use of transfer functions as a way to overcome data shortfalls that often arise in the building of disaggregated models. Our findings suggest that the institution of the proposed policy reforms would yield an aggregate annual RGVA (Real Gross Value Added) growth rate between 8.0 percent and 9.8 percent depending on which variant of the reform policies is implemented.

Keywords:Evaluation of South African economic reforms; Macroeconomic policy analysis; Marshallian Macroeconometric Model; Dissagregation.

JEL Code: E27

I. INTRODUCTION

The role played by adopting certain economic policy reforms, which we refer to as "freedom reforms," in inducing changes in economic growth has been widely discussed in the literature. Countries such as Great Britain, India, China, Estonia, Georgia and others have experienced substantial increases in their growth rates after instituting various "freedom reforms." In Great Britain, e.g., reforms instituted by Mrs. Thatcher that involved freeing up product and labor markets from undue restrictions as well as tax and monetary reforms led to considerable improvement in economic growth. Post-apartheid South Africa has seen a succession of interesting reforms but still the growth rate remains low (below five percent) and unemployment extremely high (23.1 percent)[1]. E.g. the RDP (Reconstruction and Development Program) was implemented with the primary focus of redressing social imbalances that were introduced and supported throughout pre-1994 South Africa. The RDP contained socio-economic goals that ranged from delivery of primary services to more advanced legislation and policies. Also, in order to secure more openness, more privatization and more foreign direct investment, the South African government implemented the Growth, Employment and Redistribution (GEAR) strategy in 1996. As one of its objectives, the GEAR policy aimed to establish a real GDP growth rate of at least six percent by year 2000. In addition it also aimed to reduce the unemployment rate by creating 400,000 new jobs each year on average. The RDP and GEAR have made valuable contributions to South African development since 1994. However, these policies have not achieved several very important policy objectives. South Africa still faces a very high unemployment rate, as we mentioned earlier, coupled with: (1) rising inflation (slightly above 13 percent), (2) high interest rates (15.5 percent prime rate), (3) a general electricity crisis, (4) declining business confidence due to political uncertainty preceding elections, etc. Also, some critics have pointed to the

[1] This and other data cited below have been obtained from Statistics South Africa, Q2 2008.

inability of these policies to deal effectively with problems associated with income inequality, poverty, promotion of employment growth through educational programs, etc. The need for additional constructive policies and reforms seems obvious. And indeed recently (2006), the South African government has embarked on a new strategy: the Accelerated and Shared Growth Initiative (ASGISA-SA). This strategy aims at promoting faster growth while reducing income inequality.

The South African economy has several problems that none of the previous reforms fully addressed. Labor unions are overwhelmingly powerful and exert a negative influence on workers' freedom to seek employment. Also, a high proportion of the labor force suffers from a lack of education, skills and good health that are needed to obtain employment. Further, many wishing to set up new firms find it difficult to do so under current regulations. In order to establish a new paradigm for the current South African economy, this paper suggests a set of policy changes similar to those that Mrs. Thatcher implemented successfully in the United Kingdom during her tenure as British Prime Minister. While controversial, her reform policies that involved an emphasis on free enterprise and competition produced remarkable growth in the British economy. Promoting free enterprise and competition involved adopting policies that increased the ability of firms to enter industries freely, e.g. by substantially lowering the cost of entry. In order to assess the possible effects of lowering the cost of firm entry, we shall use our MMM-DA that includes a cost of firm entry for each of the industrial sectors of the South African economy. In principle, the entry price should be a combination of all the financial requirements and the normal operating and production costs that a firm bears in order to enter a particular economic sector. This includes accessibility to loans since it constitutes an instrumental element for firms' entry into a specific sector. However, our model makes use of "other taxes on production" as a proxy for this entry cost since no other data measuring this effect are currently available. As the estimation results show, there is a significant and negative relationship between the proxy for the entry price and the growth of an industrial sector's real sales. Also, with higher entry prices, the model predicts a higher rate of general inflation. Last, the model predicts that lowering firm entry cost, ceteris paribus, generally leads to increases in output growth rates.

The Thatcher reforms also included a series of socio-economic reforms that raised the quality of education by giving British parents the freedom to choose schools for their children rather than have them routinely assigned to schools. Apparently, this reform measure, that resembles somewhat Friedman's suggested voucher system for education, produced increases in the quality of education in Britain. Also, her reforms involved the privatization of the National Health Service that led to improved health services, healthier workers and increased labor productivity. In this regard, our model embodies a measure of labor effectiveness that represents the role played by social ingredients (health and education) affecting economic sectors' growth. Since different economic sectors of an economy, say agriculture, manufacturing, services, etc. may respond differently to reform policies, it is fortunate that our Marshallian macroeconometric model (MMM-DA) that we use to analyze the effects of reforms is a disaggregated model featuring individual sectors of economies that renders it useful for policy analysis and guidance.

In addition, the Thatcher reforms involved (1) reduction of trade unions' influence, rendering the labor markets much less rigid, (2) improved management of monetary policy and the money supply, and (3) a tax cut for high income groups and later the institution of a traditional "poll tax." While much has been and could be said about these reforms, in this paper we shall just feed certain combinations of these reform measures into our MMM-DA and predict the resulting effects on important variables of the model.

What is unique about our study is our use of a MMM-DA to appraise effects of these reforms. While there are many aggregate models, available, e.g. Keynesian, neo-Keynesian, Monetarist, neo-Monetarist, Real Business Cycle, generalized Real Business Cycle, Dynamic Stochastic General Equilibrium, and others, these aggregate models do not forecast very well and do not provide an adequate framework for the evaluation of our sector specific policies that we wish to evaluate. We have to take account of the very different effects that health, education, entry cost changes and other policies have on different sectors of the economy, e.g. agriculture, services, mining, etc, in order to get good predictions of outcomes. As will be shown, and emphasized strongly for many years by Stone, Orcutt, Friedman, Modigliani, and many others, disaggregating wisely and using good disaggregated data can lead to much improved explanations and predictions; for some recent empirical examples and references, see Zellner et al. (2000, 2001 and 2005). Most fortunately, our evaluation of the policy measures that we consider leads to rather optimistic results that we hope can be realized by actual implementation of the "freedom reforms" that we consider.

An overview of the paper is as follows. In Section II, we provide a description of: (1) our model (MMM-DA) and variants of it; (2) our estimation techniques; and (3) our data. In Section III, we discuss the fit and the predictive performance of the MMM-DA as compared to those of a benchmark autoregressive leading indicator (ARLI) model. Section IV is devoted to an evaluation of the Freedom Reforms' effects on the growth rate of the South African economy using our MMM-DA. Finally, in our concluding section, we summarize our results and indicate the direction of future research.

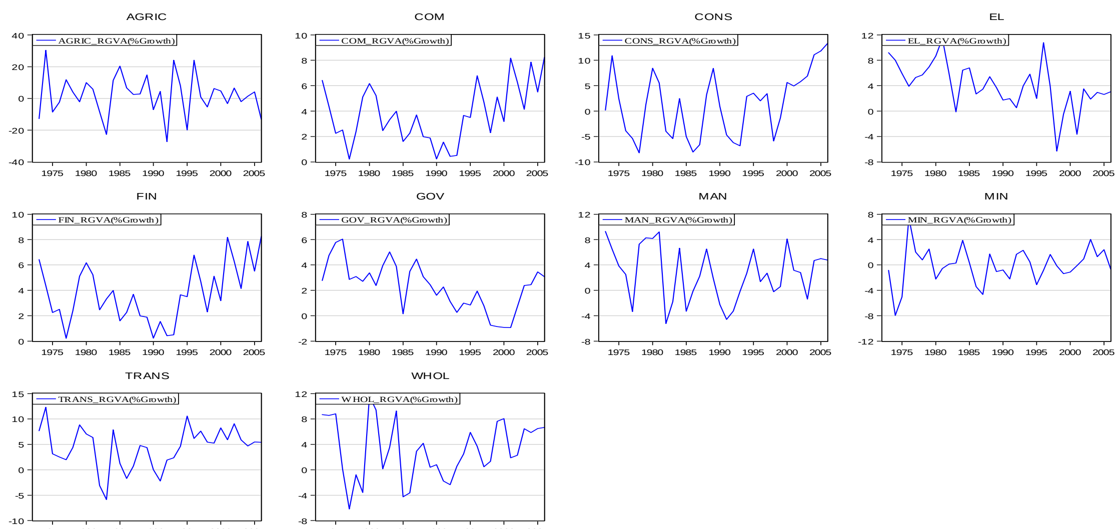
II. MODEL SPECIFICATION, ESTIMATION TECHNIQUES AND DATA

1. Model specification, Aggregation and Disaggregation

1.1 Aggregation and Disaggregation

In this paper we make use of a model disaggregated by economic sectors[2] of the South African economy. Good disaggregated models and data can lead to a better understanding of sectors' very different behavior (see Fig. I for plots of the sectoral rates of real gross value added (RGVA) growth). From Fig. I, we see that the sectors' growth rates present disparate behavior to such an extent that using aggregate data entails loss of much useful information. Moreover, aggregate models are unable to analyze how policy changes affect specific sectors. Also, use of aggregate data can lead to inaccurate policy recommendations. Last, use of aggregate data and relations can lead to a loss in forecast accuracy as shown, e.g., in de Alba and Zellner (1991), Zellner and Tobias (2000), and Zellner and Guillermo (2005).

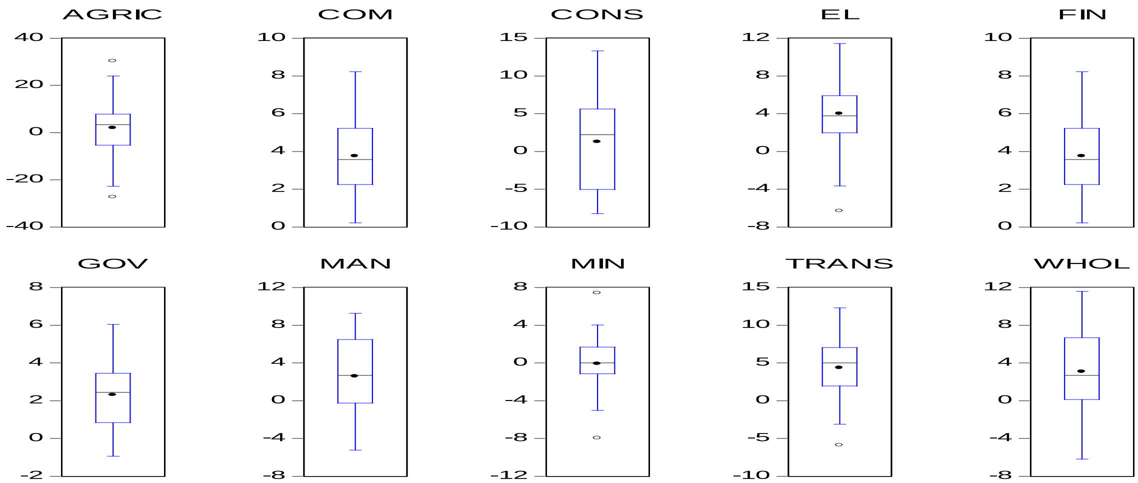
FIGURE I: RGVA SECTORAL ANNUAL GROWTH (%): DATA PLOTS, 1972 - 2006



Notes: The growth rates of real gross value added (RGVA) of the 10 South African economic sectors are annual series constant 2000 prices and seasonally adjusted obtained from the SARB (South African Reserve Bank) database, home page http://www.reservebank.co.za/.

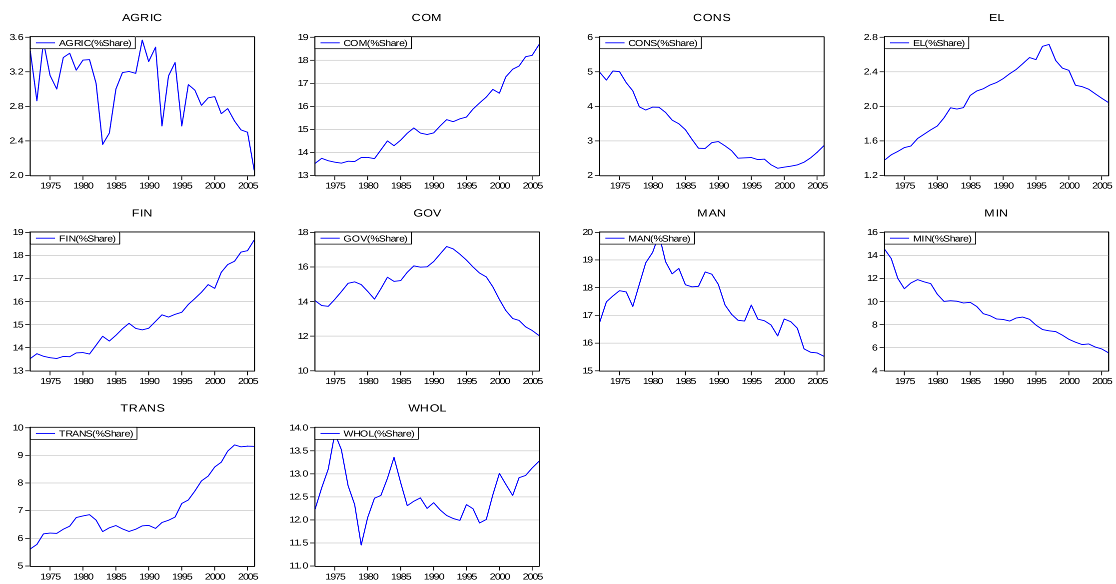
[2] The economic sectors considered are: (1) Agriculture (AGRIC); (2) Mining (MIN); (3) Manufacturing (MAN); (4) Financial services (FIN); (5) Wholesale (WHOL); (6) Transport and Communication (TRANS); (7) Construction and Building (CONS); (8) Government (GOV); (9) Community services (COM); and (10) Electricity (EL).

FIGURE II: RGVA ANNUAL GROWTH (%) RATES OF 10 SOUTH AFRICAN ECONOMIC SECTORS[3]: BOXPLOTS, 1972 - 2006



Notes: The growth rates of RGVA by sector are obtained from exactly the same series as those used in Fig. I.

FIGURE III: SECTORS' ANNUAL SHARES, 1972 - 2006



Notes: The sectors' annual shares represent each sector's contribution to the country's RGVA using SARB, data home page http://www.reservebank.co.za/.

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[3] We make use of well-known boxplots to provide information about the distributions of annual sector growth rates. Our boxplots include the following elements: (1) the mean (point in bold); (2) a median (middle line in the box); (3) the length of the box is the interquratile range of the growth rates; (4) the outliers (extreme limits); and (5) the whiskers (vertical lines joining the outliers and the box).

While some sectors have relatively stable shares of aggregate RGVA, we see that other sectors’s shares exhibit upward movements, namely Financial Services (Fin), Transport & Communication (Trans), and Community Services (Com). Electricity (El) has followed an upward trend until the early 1990s and then dropped. The largest decline is observed for Mining. Also, there is a slight fall in Agriculture's share in recent years. The drastic changes that occurred in the political history of South Africa together with shocks from the global economy have largely influenced this mapping. For example, Financial Services as well as Transport & Communication have experienced larger increases in their shares after the abolishment of the apartheid regime.

Development of the Marshallian Model

Our current MMM-DA involves disaggregating the South African economy into the 10 industrial sectors that we referred to earlier. For each sector, we, along with Alfred Marshall and others have introduced a product market involving demand and supply equations derived from assumed optimizing behavior of firms and consumers. On aggregating over firms, we obtain the industry supply equation that depends on the number of firms in operation, a variable that does not appear in many macroeconomic models. To determine the number of firms in operation, we introduce a firm entry-exit equation in each sector such that when positive profits exist in the industrial sector, firms enter to compete away the profits and to help the sector return to a new equilibrium, as described in many price theory texts. Further, the firms in our industrial sectors demand labor, capital and money services in markets for these factors of production. Also, consumers demand outputs and money services and supply labor and savings and the government supplies money to the money market. Also, the government taxes and produces a range of goods and services in the model that are demanded by firms and consumers. See Zellner and Israilevich (2005) for one sector, two sector and n-sector versions of our MMM and their properties along with some results of forecasting experiments with the model (Zellner et al., 2001). Further, in their paper Zellner and Israilevich (2005) ascertained that an MMM in its discrete version is in the form of a chaotic model that has various types of oscillatory behavior. Using their two-sector version of the model, they have established that the it can provide a wide variety of possible solutions, including output rates of growth with "bubbles and busts" behavior.

The complete model (see appendix) includes five major components. First, it includes the sectors' supply equations derived by aggregating individual firms' supply functions[4]. Second, the MMM-DA includes the sectors' product demand equations derived by aggregating individual consumers' demand functions that include traditional demand shifters such as real disposable income, real money balances, etc. Third, sectors' firm entry/exit incorporate the link between firms' entry-exit behavior and the gap between actual and equilibrium profits described in many price theory texts. In this regard, Veloce and Zellner (1985) have discussed the effects of a failure to take account of entry and exit behavior on the analysis of industries' behavior using data for a Canadian manufacturing industry. And fourth, the MMM-DA incorporates firms' factor demand functions for labor, capital and money services and supply functions for these factors in a set of factor markets. And last, the fifth major component is the government sector that produces goods and services, demands factors of production in the factor markets, supplies money to the money market, taxes producers and consumers, and provides regulatory policies.

Each industrial sector has a number of firms operating at time *t*, each with a Cobb-Douglas production function $Q_i = A_i (z_i L_i)^{\alpha_i} K_i^{\beta_i}$, where A_i includes (1) A_N, a neutral technological change factor, (2) A_K, a capital augmentation factor, and (3) A_L, the remaining labor technological augmentation factor that is not explained by z_i (level of

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human capital in per capita terms)[5]. Therefore, $z_i L_i$ constitutes a sector's level of effective labor and K_i represents a firm's level of capital services. Also, as suggested in many studies, e.g. Zellner and Israilevich (2005), money services can be introduced as an additional input in the production process: $Q_i = A_i (z_i L_i)^{\alpha_i} K_i^{\beta_i} M_i^{\gamma_i}$.

1.4 Transfer Function Equations

As discussed in the literature (see, e.g. Zellner and Palm (1974, 2004)), dynamic simultaneous equations models, such as our MMM-DA, have a variety of associated algebraic representations, including reduced form equations, restricted reduced form equations, final equations, transfer function equations, etc. For our purposes, the transfer function representation of our MMM-DA is very useful given that we do not have data on all of our variables. Each transfer equation links current and lagged values of an endogenous variable, e.g., each sector's output growth rate, to its own lagged values and to current and lagged values of the exogenous variables. Thus, for the sector output growth rate variables, we have a set of ten transfer equations that can be estimated and used in forecasting and policy analysis without the need for data on other endogenous variables, e.g. prices and numbers of firms in operation that we do not have.

Shown below are the transfer functions, derived from the complete model in Appendix A. 4, for the rates of change of real sectoral sales of the i'th sector with i = 1,2,…, 10, where $\lambda(L)$ and $\gamma(L)$ are lag operators. Also, X_t is a set of exogenous variables, S the RGVA, W the wage rate, r the interest rate, A the technological factor productivity, Γ the entry cost which represents a non-linear combination of all costs incurred in starting a new firm, Y disposable income, IY world income using the United States income as proxy since the US is

[5] There is an extensive literature that exists on the relationship between the labor augmentation factor and health, education or other social components. Ngoie et al. (2008) constitutes our closest reference using South African data. In that paper, the authors have estimated the parameters that link education and health to labor augmentation using sectoral data.

one of the largest export destinations for South African products, *D* the number of demanders of sectors' outputs, *SP* the stock price index, *M* the money supply *M2*, and ε, μ and ν are the error terms. With the exception of r, each of the small letters represents the rate of change of the corresponding capital letters, e.g. $\ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right) = s_{i,t}$. In addition, we have $\ln\left(\frac{\Gamma_{i,t}}{\Gamma_{i,t-1}}\right) = \Omega_{i,t}$ for i = 1,2,.., 10.

$$[\lambda(L) - \gamma(L)]s_{i,t} = -\gamma(L)[\delta_{0,i} - \delta_{1,i}S_{i,t-1} - \kappa_{1,i}w_{i,t} - \kappa_{2,i}r_t - \kappa_{3,i}a_{i,t} - \kappa_{4,i}z_{i,t} - \kappa_{5,i}\Omega_{i,t} - \kappa_{6,i}X_t$$

$$-\varepsilon_{t,i} - \nu_{t,i}] + \lambda(L)[\Delta_{1,i}y_t + \Delta_{2,i}iy_t + \Delta_{3,i}d_t + \mu_{Ti,t}] \tag{1}$$

As regards the error term properties, we note that when white noise error terms are introduced in the structural equations, the error terms in the transfer functions will be auto-correlated. If the structural equations' error terms are auto-correlated, then the transfer functions' error terms could have a variety of possible properties, e.g. MA (1), and perhaps white noise in certain cases. Since data on all the structural equations' variables are not available, it is not possible to estimate the structural equations and determine the properties of the structural error terms. Thus, we decided to fit the transfer functions using a GLS criterion and to check whether the error terms are auto-correlated. We find that they are not according to estimates of the autocorrelation functions for each sector.

Due to unavailability of disaggregated data on sector prices and numbers of firms in each sector, we have only estimated the ten sectoral RGVA transfer function equations shown above in (1).

2. The Iterative Seemingly Unrelated Regression (ISUR) Transfer Function Estimation Technique

In order to estimate the set of ten transfer functions in (1), associated with our MMM-DA model, the ISUR technique has been utilized. The ISUR method provides estimates using a GLS (Generalized Least Square) approach. (See Zellner (1962), and Judge et al (1985) for discussions of iterative SUR GLS estimation of a set of regression equations. Also see Rossi et al (2005) and Zellner and Ando (2008) for Bayesian estimation techniques for the SUR model. Note that, as is well known, the use of ISUR takes account of differing variances of error terms in different equations as well as correlations of error terms in different equations and does not involve assuming that the zero mean error terms are iid, normally distributed. Further, it yields consistent and asymptotically efficient estimators.

In future work, it will be interesting to compute Bayesian estimates and predictions for our transfer function system and to compare results to those produced by the ISUR procedure. In this connection, we note that with a flat prior and a normal likelihood function, the ISUR estimate is equal to the modal value of the posterior distribution. And the modal value is an optimal estimate relative to a zero-one loss function, as is well known. Further, in large samples, the posterior distribution will assume a normal shape with the posterior mean equal to the maximum likelihood estimate (MLE), as shown by Jeffreys (1961) and others that will also be equal to the ISUR estimate. Thus in large samples, the MLE will be equal to the mean and to the modal value of the posterior and our ISUR estimates have a number of alternative justifications. In small to medium sized samples, the ISUR estimate will be equal to the modal value of the posterior density based on a uniform prior, as noted above and will thus be optimal relative to a zero one loss function, that is zero loss if the estimate is close to the true value and unit loss if it is not. Similar considerations relate to properties of ISUR predictors and Bayesian predictors.

3. Data

The data used in this paper for implementing our ten equation transfer function model were collected from 1972 on a yearly basis. Ten economic sectors were considered that account for the overall national sales output[6]. The main sources of data collection are all official in addition to some from data bases mentioned below. The main data sources used in this paper are (1) the SARB (South African Reserve Bank) database, (2) the International Financial Statistics (IFS) database, and (3) the World Bank Indicators (WBI). Data on local leading indicators such as Stock (all shares) Prices (SP)[7] and Money Supply (M2)[8] were obtained from the IFS database while data on world leading indicators (IY, the US Gross National Income) originate from the WBI[9]. Other types of national data such as (1) sales supply (S), (2) disposable income (Y_d), (3) interest rates (r), (4) wage rates (W), (5) number of households (D), (6) labour effectiveness (z), and (7) firms' entry cost (Γ) were collected from the SARB database (http://www.reservebank.co.za/).

III. PREDICTIVE PERFORMANCE: MMM-DA versus AR(3)LI

The analysis reported below are based on the use of transfer functions for sector output growth rates derived from our MMM-DA. As mentioned earlier, our estimation results have been obtained using ISUR estimation of the transfer functions in (1) for 10 sectors of the South African economy. As shown by the following results, not only do the sector transfer functions fit the South African data very well (see Fig. IV) but they also provide reasonable one year ahead forecasts (see Fig. V).

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[6] The sectors considered are the following: (1) Manufacturing; (2) Agriculture, Fishing and Forestry; (3) Construction and Buildings; (4) Mining; (5) Government; (6) Community services; (7) Transport and Telecommunication; (8) Financial services; (9) Wholesales, Retail, Catering and Accommodation; and (10) Electricity, Gas and Water.

[7] SP is published in the International Financial Statistics (IFS) under the code IFS; 19962 MB.ZF.

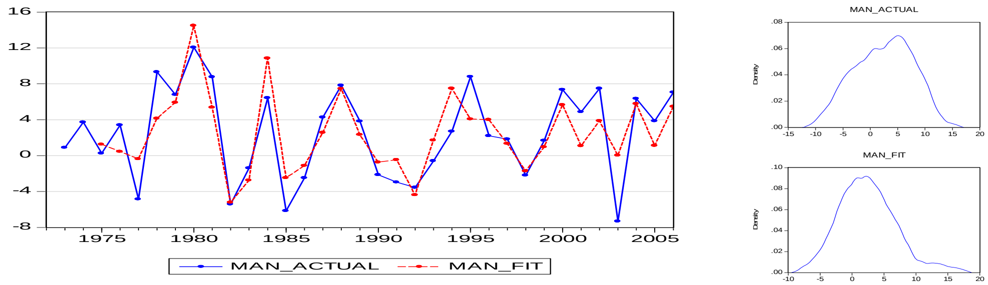
[8] The series for M2 is published in the IFS under the code IFS; 19959 MB.ZF.

[9] The series for IY is published under the code WDI; A111, NYGNPMKTPCD.

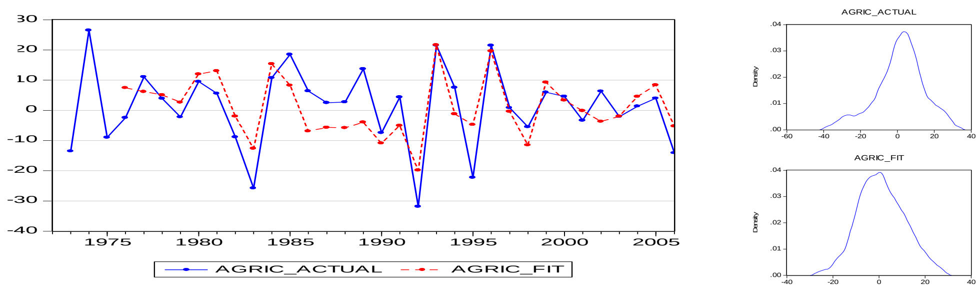
FIGURE IV: ACTUAL SERIES VERSUS FITTED VALUES[10], 1972 - 2006

Data and Fitted Values **Kernel Density**[11]

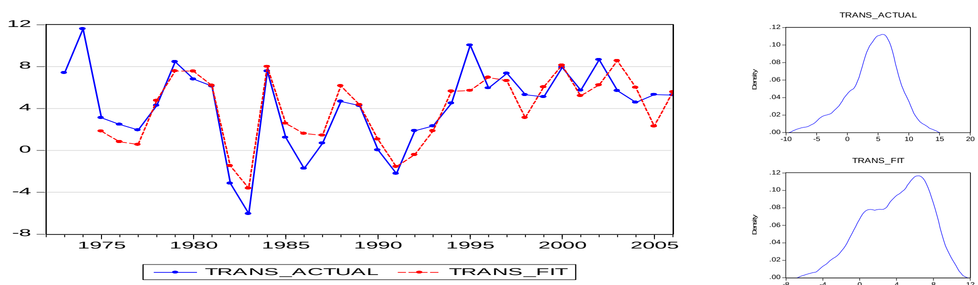
RGVA Growth (%)



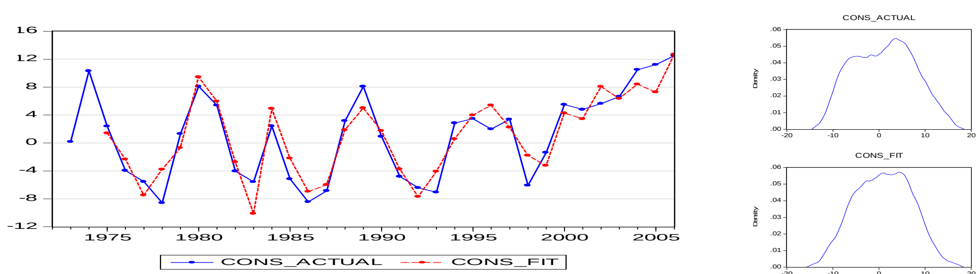
RGVA Growth (%)



RGVA Growth (%)



RGVA Growth (%)



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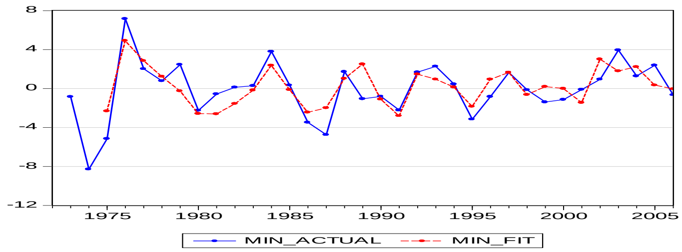
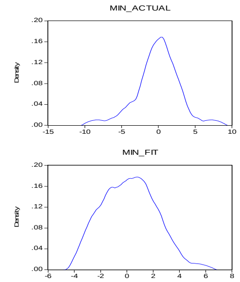
[10] See Fig. 1 for acronyms.

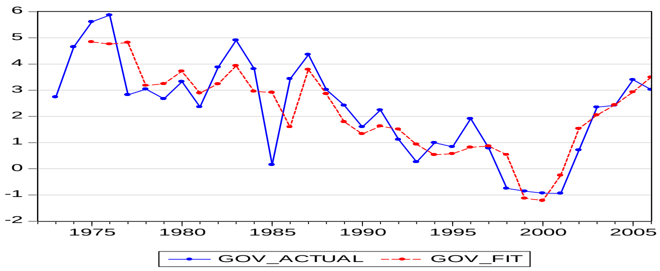
[11] The Kernel density constitutes a refined version of the histogram of the growth rate of RGVA computed using an advanced algorithm, the Fast Fourier Transform.

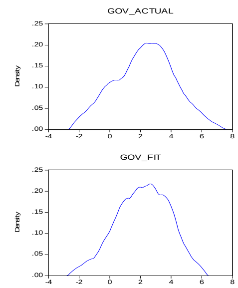
RGVA Growth (%)



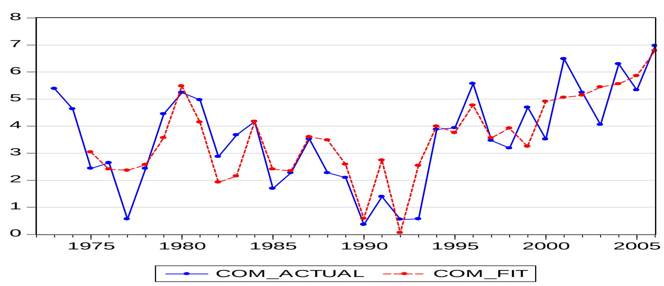


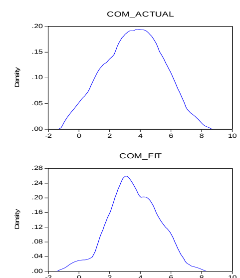
RGVA Growth (%)



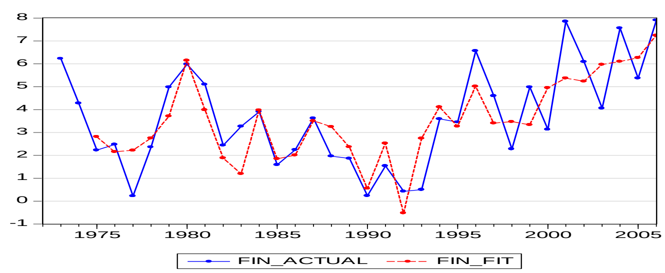


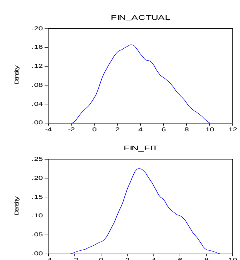
RGVA Growth (%)



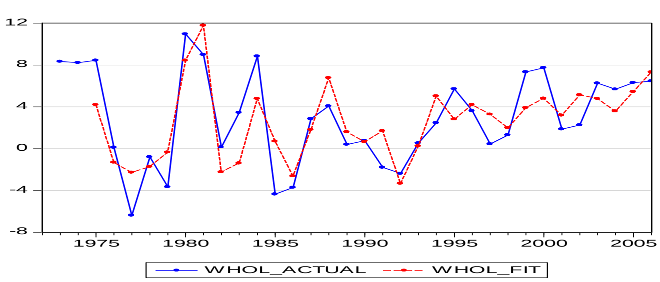


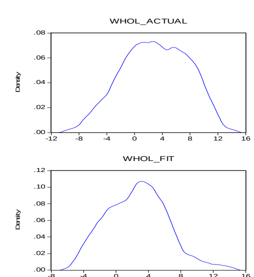
RGVA Growth (%)



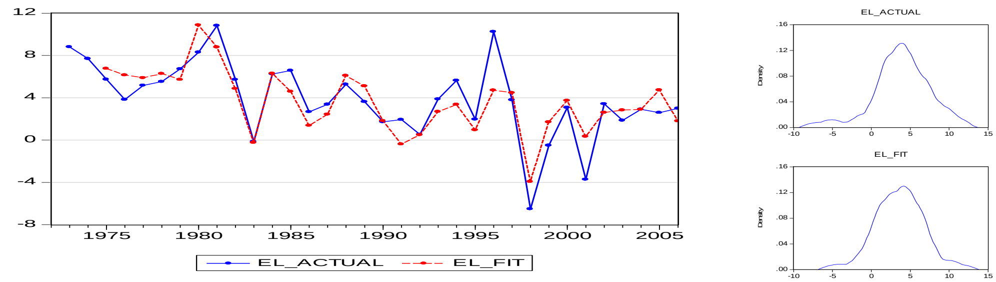


RGVA Growth (%)





RGVA Growth (%)



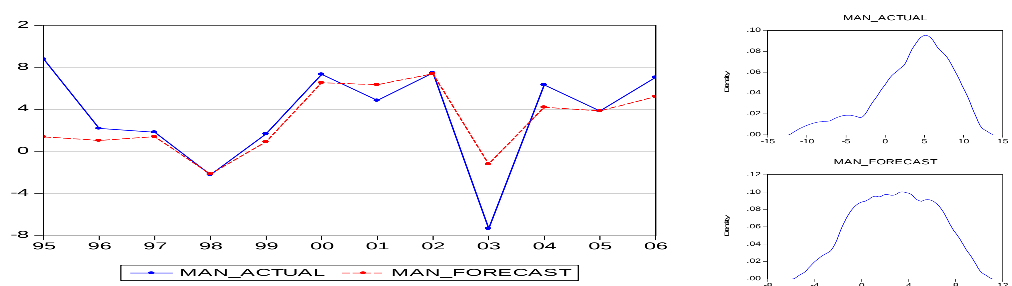
Note: Actual series represent sectors' RGVA annual growth rates obtained from the SARB database. The fits are obtained from estimating our transfer equations using ISUR.

With a few exceptions, mainly caused by uncontrolled structural breaks, our disaggregated model fits remarkably well each of the 10 sectors of the South African economy. These results are encouraging especially when we note that our equations are not highly over-parameterized. .

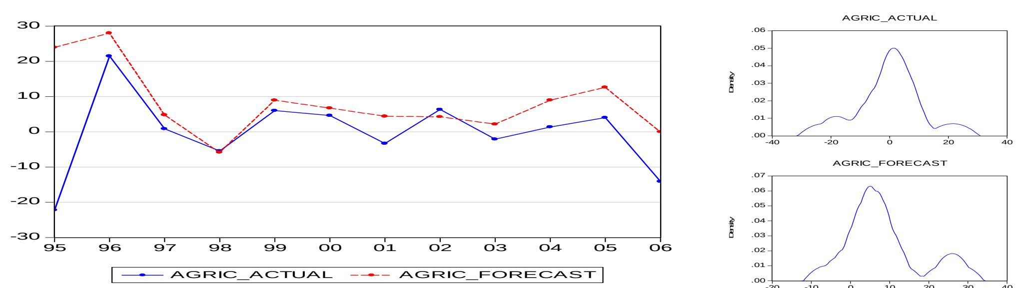
FIGURE V: ACTUAL VERSUS ONE YEAR AHEAD PREDICTIONS[12], 1972 – 2006

Data and Predicted Values **Kernel Density**

RGVA Growth (%)

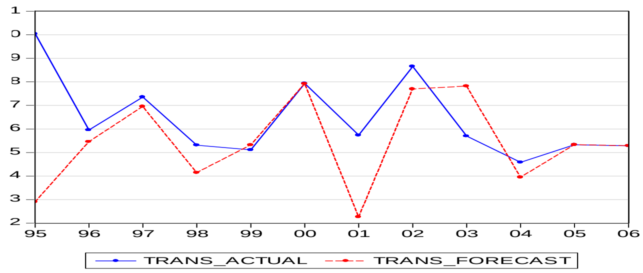


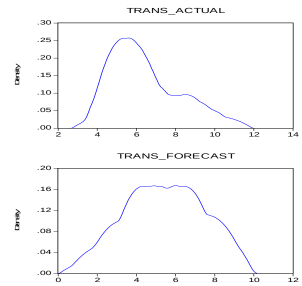
RGVA Growth (%)



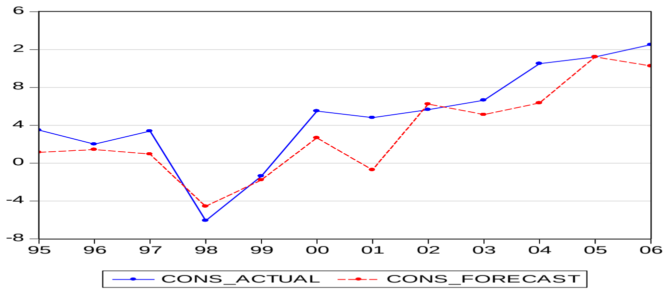
[12] See Fig. I for acronyms.

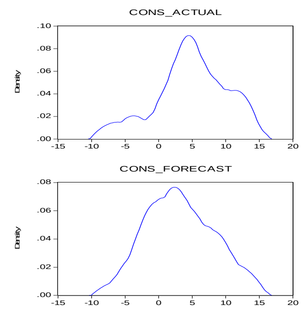
RGVA Growth (%)



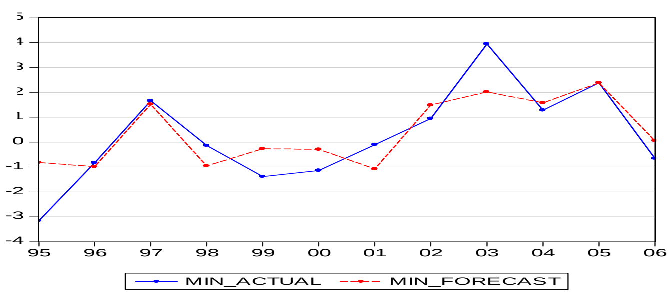


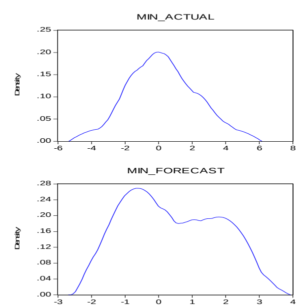
RGVA Growth (%)



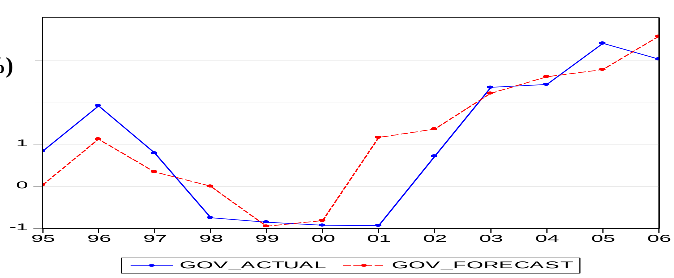


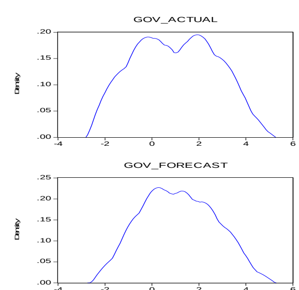
RGVA Growth (%)



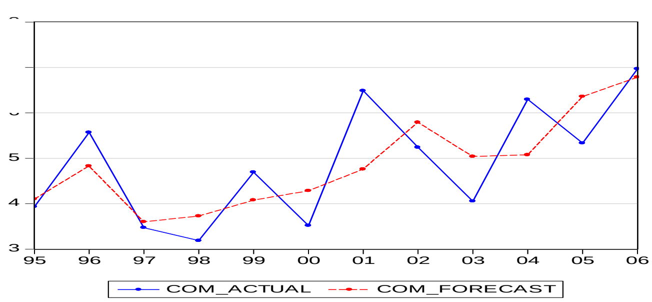


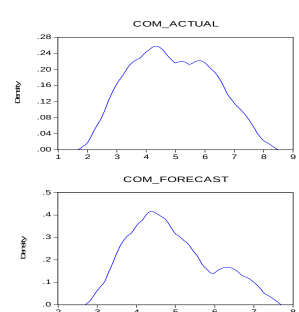
RGVA Growth (%)

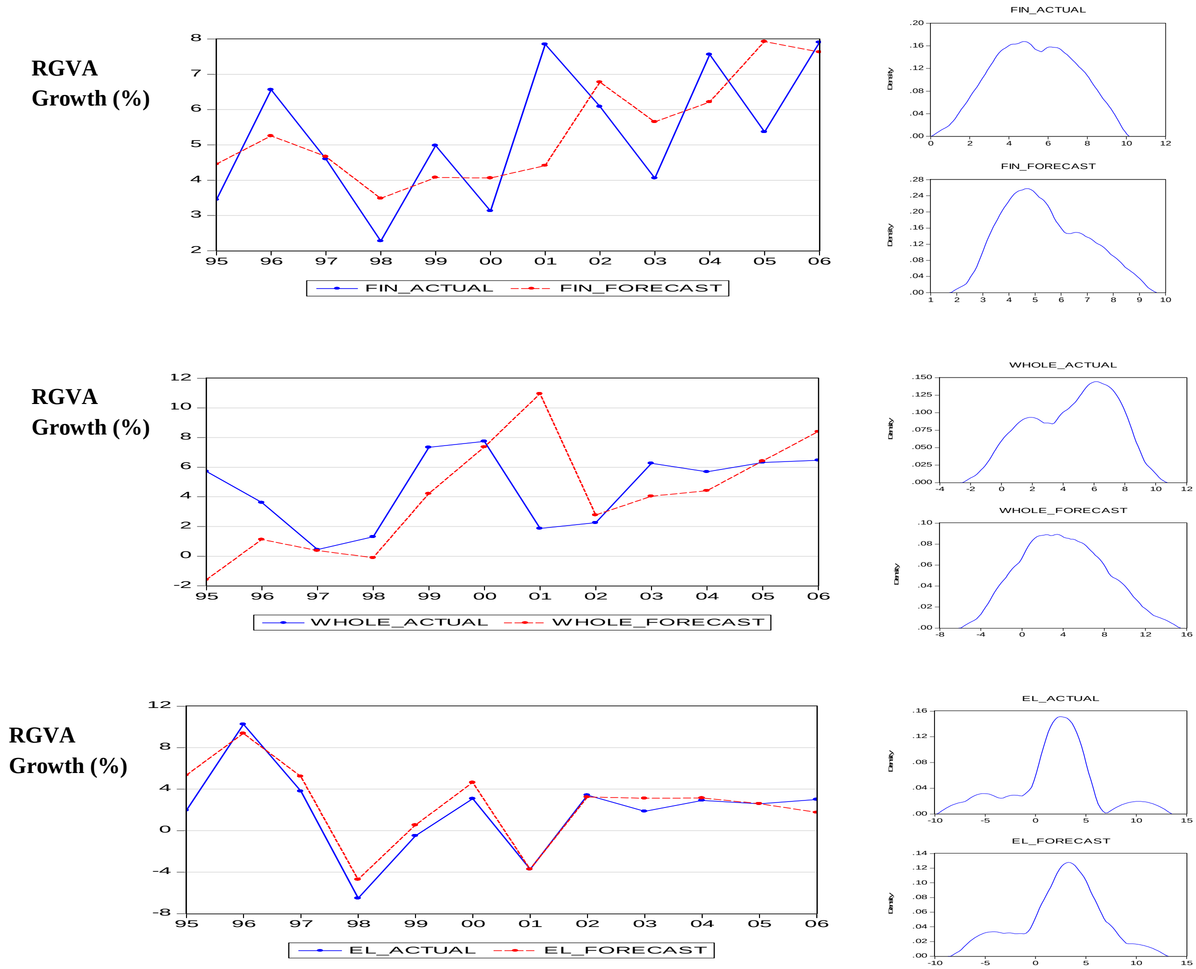




RGVA Growth (%)







Notes: Results (Forecasts) are obtained by conducting one year ahead forecast (prediction) on individual sectors' RGVA growth rates. The exogenous variables in the prediction period are assumed to have known values equal to their observed values.

In general, Fig. V shows the predictive performance of our MMM-DA for 12 point forecasts [1995 – 2006] with some sectors such as Electricity, Manufacturing, Agriculture, and Mining presenting outstanding predictions of turning points. Weaker prediction performance of some sectors probably indicates that these sectors' equations need to be improved, perhaps by introducing additional explanatory variables and/or changing the lag structures. Also, the year 1995 was hard to predict especially for agriculture. This was mainly due to the major political outbreak characterizing the shift from the apartheid regime to the democratic South Africa.

As mentioned earlier, the benchmark model used in this paper is an Autoregressive Leading Indicator Model of order 3, denoted by AR(3)LI, that is given as follows:

$$s_{i,t} = \theta_0 + \theta_1 s_{i,t-1} + \theta_2 s_{i,t-2} + \theta_3 s_{i,t-3} + \theta_4 \ln\left(\frac{SP_{Q_{(t-3)}}}{SP_{Q_{(t-4)}}}\right) + \theta_5 \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right) + \varepsilon_{St} \qquad (2)$$

Autoregressive (AR) models in general have been extensively used in the forecasting literature. As opposed to autoregressive models of order 1, AR (1), the AR(3)LI model allows for both real and complex roots. Also, the use of rates of change of real stock prices (SP) and of real money (M) as leading indicators in the AR (3) models has produced substantial improvement in the predictive ability of this class of models (see Zellner et al, 1999). Therefore, choosing the forecasting performance of an AR(3)LI as a benchmark for comparison with the forecasting performance of our MMM-DA provides a rather good test of the latter's predictive ability.

TABLE I- RMSE AND MAE (%) OF ONE YEAR AHEAD PREDICTIONS, 1997 - 2006

	AR(3)LI		MMM-DA	
	MAE	**RMSE**	**MAE**	**RMSE**
Community	1.83	1.22	0.65	0.82
Electricity	2.44	3.29	0.73	1.00
Finance services	3.70	1.80	1.08	1.48
Wholesale	2.41	2.57	1.68	2.96
Transport & Communication	1.66	1.89	0.75	1.27
Mining	1.42	1.57	0.61	0.83
Manufacturing	3.53	4.07	1.14	2.02
Government	0.92	0.99	0.47	0.73
Construction	4.22	4.93	1.77	2.44
Agriculture	2.86	4.90	4.48	6.06

Notes: Results are obtained from computing sectors' predictions Root Mean Square Errors (RMSEs) and Mean Average Errors (MAEs).

In Table I, we compare the predictive ability of our MMM-DA and of the benchmark AR(3)LI model. At this stage, the results presented in Table I represent our prediction experiments with known exogenous variables since the forecasting of some variables such the ‘world income’ would require additional modeling. However, after running the model with predicted exogenous variables we conclude that predicting using predicted exogenous variables induces larger MAEs or RMSEs since the errors in predicting the exogenous variables will inflate the MAEs or the RMSEs of the MMM-DA predictions.

From the information in Table I and in Fig. V, it is evident that MMM-DA predicts reasonably well and much better than the benchmark AR(3)LI model. However, it is important to note that the MAEs and RMSEs results for the agriculture sector are quite large. When we look at the actual series we find that the growth rate of agriculture jumped from negative 20 percent to positive 20 percent in the period 1995 to 1996 after facing a major decline in 1994. It was due to a major structural break linked to the end of apartheid. Land in the country was owned entirely by white farmers who had growing concerns about their future after the 1993 national elections. There was heavy pressure for instituting land redistributions starting at that time and farmers had the fear that it could all turn into chaos as happened in other African states and that affected tremendously the sector’s output growth rates and our ability to forecast them.

Additionally, the MMM-DA’s predictive ability is well demonstrated on observing the number of turning points that are well forecasted across different sectors (see Fig V). In general, the model seems to do well in forecasting turning points correctly in a number of cases. However, the performance in certain sectors is not entirely satisfactory. This may be explained by the fact that the model specification is more appropriate for some sectors as opposed to others and/or that some sectors have higher data quality. Improvement may be made in this regard by allowing different model specifications for different sectors in future work.

IV. IMPLEMENTING AND EVALUATING THE FREEDOM REFORMS

As mentioned earlier, this paper focuses on three types of freedom reforms: (1) freeing firms' entry by lowering their cost of entry, (2) a tax-cut on the incomes of workers and employers and (3) an improvement of labor effectiveness by increased qualitative public investment in education and health. Due to data constraints, the paper makes use of 'other cost of production' as a proxy for entry cost. The tax-cut is simply captured through an overall increase in national disposable income. We do not suggest any specific type of tax-cut. We rather allow the policy- makers to choose any combination that helps increase national disposable income. As regards the labor effectiveness (z), the details of its linkages with health and education are well described in a related study that was conducted using South African data see Ngoie et al, (2007). Therefore, when we introduce an increase in z, it is the result of an increase in qualitative investment on health and education programs (using the appropriate parameters) that is translated into a more effective labor force. Reforms that result in increased labor effectiveness of course must be well-designed and well-implemented. In this paper, we simply utilize the outcome of such reforms without necessarily providing the design and implementation techniques that produced them. Generally, freeing up markets induces more competitiveness in different sectors and therefore we may observe an increase not only in the number of firms in existence but also more firms seeking to make their employees more productive. Also, having more competitive firms in the sectors helps to increase tax revenues. With more money available, the government can invest more in good health and education programs.

The three sets of freedom reforms that we have implemented using this model are interlinked. Freeing up the market by introducing lower entry costs helps to increase the number of firms operating in the sectors. Having more firms translates into more employees and therefore higher disposable income. Also, having firms become more competitive provides incentives for them to provide appropriate training for their employees that make them more cost effective.

TABLE II - COUNTRY'S AGGREGATE GROWTH RATE (RGVA GROWTH) RESULTING FROM THE IMPLEMENTED REFORMS

REFORM TYPES	SECTORS' ALLOCATION (scenarios)	SIZE OF THE REFORM	DURATION OF THE REFORM (Shock of the specific variables)	
			ONE[13] YEAR (2006)	FIVE YEARS[14] (2002-2006)
Tax cut only	**Equal**	1 PERCENT	5.3 % (0.98)	7.9 % (1.01)
		5 PERCENT	8.0 % (1.00)	-
Tax cut & Labor Effectiveness (increase)	**Equal**	1 PERCENT	5.5 % (0.99)	9.6 % (1.02)
		5 PERCENT	8.8 % (1.01)	
Tax cut & Labor Effectiveness (increase)	**Optimal**	1 PERCENT	5.7 % (0.99)	9.8 % (1.02)
		5 PERCENT	9.8 % (1.01)	-

Note: The values in parentheses represent the standard errors of regressions corresponding to each scenario[15].

As shown in Table II, when all the reforms are implemented simultaneously with a 1 percent shock (level at which the concerned variable is increased) on each of the variables (Γ, Y_d, z), namely, the entry cost, real disposable income and labor effectiveness, our model predicts that the country's growth of real GVA will gain 0.8 percent in a year. That produces a growth rate of real GVA of 5.5 percent compared to 4.7 percent initially recorded for the year 2006. Supposing that the reforms are much stronger, e.g. a 5 percent increase in the

[13] Immediate shock of 1 percent in a year.
[14] Shock of 1 percent sustained along 5 years.
[15] The standard error of regression (SE) constitutes a summarized measure of the estimated variance of the equation's residual. In this case, we first obtained the SEs for each sector and then compute the aggregate SE using a median growth equation. The median growth equation is obtained by converting the sectors' growth equations into levels and multiplying each level by the corresponding weight. After obtaining the median growth rate we then compute the standard error of regression.

growth rate of the same variables, a gain of 4.1 percent in real GVA is produced. That will increase the country's growth rate to 8.8 percent. The 5 percent policy shock can be considered as an extremely large program in implementing the reforms while the 1 percent shock represents a more modest program. Also, since the implementation of education and health reforms as a way to raise labor efficiency produces long term effects, we have decided to reassess the growth outcomes by (1) allowing for the reforms to raise z gradually (along 5 years) rather than instantaneously and (2) without the health and education reforms.

Assuming a gradual increase in the growth of human capital as a result of rigorous reforms in health and education, meaning 5 years before z reaches a 1 percent increase beside the 1 percent increase in the growth of disposable income and 1 percent decrease in the growth of entry cost, the resulting RGVA growth will amount to 9.6 percent.

Furthermore, for a 1 percent increase in the growth rate of disposable income (Y_d) concomitant to a 1 percent decrease in the growth rate of entry cost (Γ) with change in z, the country's RGVA growth rate rises by 0.6 percent from 4.7 percent to 5.3 percent. Assuming a 5 percent shock, the resulting RGVA growth is 8.0 percent. From such findings, we may conclude that reforms on education and health do not have instantaneous effects and therefore require more time before producing substantial effects. Also, the use of a disaggregated model has permitted us to determine how different sectors react to the reforms.

In Table II, we report results of what we call a more 'optimal' implementation of these three types of reforms. The reforms are calibrated according to the sectors' level of responsiveness. For example, instead of reducing entry cost in sectors that are naturally regulated monopolies, this money can be used to reduce entry costs of other more open sectors. As we mentioned earlier, sectors may have different types of reactions when it comes to improving labor effectiveness. Capital intensive sectors may react differently than labor intensive sectors. It is therefore relevant to reallocate funds by shifting them from sectors where they are less productive to sectors with higher returns. Indeed, given a social welfare function of the type used by Tinbergen and others, it may be possible to determine an optimal allocation of a given amount of funds for different reform programs in different sectors.

In another calculation, we have reallocated our funds for financing reforms such that sectors having recorded larger growth gains from increased labor effectiveness as compared to lower firms' entry cost will receive more funds to promote labor effectiveness and vice-versa. As a result, we have noticed that such a reallocation of funds for reforms provides a much larger RGVA growth gain, 5.2 percent instead of 4.1 percent. That will raise the predicted annual growth rate of RGVA to (1) 5.7 percent for a 1 percent shock, and (2) 9.8 percent for a 5 percent shock.

CONCLUSION AND STUDY LIMITATIONS

The present research considered the impact of freedom reforms on the performance of the South African economy. In this regard, the paper made use of a disaggregated Marshallian macro-econometric model that was shown to fit the data and to predict well, to evaluate the economic effects of the reforms. As regards freedom reforms, similar to Thatcher-like reforms, the results in this paper indicate that such reforms are likely to produce a remarkable improvement in the South African growth rate. When carefully implemented, institution of all the three sets of freedom reforms is predicted to raise the South African annual real GVA growth rate to 8.8 percent with a uniform allocation of the reforms over the 10 economic sectors and to 9.8 percent using a more reasonable allocation of reforms over sectors. It is interesting to note that institution of only the tax reform is predicted to raise the country's RGVA growth rate to 8.0 percent. These results are indeed encouraging and in the future will be studied further with other variants of our MMM-DA with data not only for S. Africa but also for other countries.

In addition, the current research provides evidence that a disaggregated Marshallian Macroeconometric model is a useful tool for understanding and predicting a country's overall economic behavior and the behavior of important industrial sectors. In the present study, lack of data on important sector variables, led to the use of the model's implied transfer function equations for the sectors' real sales growth rates. With additional data on sector prices, number of firms in operation, etc., the full MMM-DA model can be estimated and used to explain and predict a wider range of variables, probably with added precision given that use of more data involves an increase in information available for estimation and prediction purposes. Also disaggregation helps to avoid aggregation biases emphasized by Theil (1978) and many others; see, e.g., de Alba and Zellner (1991), Zellner and Tobias (2000), Zellner and Chen (2001), Zellner and Israilevich (2005) and Kim (2006) for some effects of disaggregation on the quality of models' forecasts. Moreover, in the present paper, the Marshallian modeling process has been broadened by (1) further analysis and implementation of entry costs, (2) more explicit allowance for a human capital component in the production process (labor effectiveness), and (3) addition of a foreign sector.

It is anticipated that further use and development of the MMM-DA will yield additional explanatory, predictive and policy-making results that will be useful to many.

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APPENDIX A: MODEL SPECIFICATION

Assuming that all firms are optimizers, with fixed entry costs, the sectors' real rates (sectors' nominal rates deflated by a general price index) are given as follows:

SALES SUPPLY EQUATION (RGVA):

$$S_{Sit} = A_{it}^{\frac{1}{1-\alpha_i-\beta_i}} .\alpha_i^{\frac{\alpha_i}{1-\alpha_i-\beta_i}} .\beta_i^{\frac{\beta_i}{1-\alpha_i-\beta_i}} .N_{it} .w_{it}^{\frac{-\alpha_i}{1-\alpha_i-\beta_i}} .r_{it}^{\frac{-\beta_i}{1-\alpha_i-\beta_i}} .P_{it}^{1+\alpha_i\phi_{Li}+\beta_i} .\left(P_{Q_{it}}^{e}\right)^{-\alpha_i\phi_{Li}-\beta_i\phi_{Ki}+\frac{\alpha_i+\beta_i}{1-\alpha_i-\beta_i}} .z_{it}^{-\alpha} \tag{A.1}$$

SALES DEMAND EQUATION:

$$S_{Dit} = P_{it} . \left[C_{Si} (P_{Q_{it}}^{e})^{\lambda_{1i}} .(Y_{dt})^{\lambda_{2i}} .(D_{ti})^{\lambda_{3i}} \prod_{j=1}^{m} X_{jt}^{\chi_{ji}} . \left(\frac{P_{Q_{it}}^{e}}{P_{Q_{it}}} \right)^{\Delta_i} \right] \tag{A.2}$$

ENTRY/EXIT EQUATION:

$$\frac{\dot{N}_{it}}{N_{it}} = C_{Ei}^{'} (\pi_{it}^{a} - \overline{\pi}_{it}) \tag{A.3}$$

In (A.3), the market equilibrium profit within a given sector is represented by $\overline{\pi}_{it}$. Assuming that a firm's actual profit π_i^a constitutes a proportion ℓ of its sales supply S_{Si} and $\pi_{it}^{a} = \ell S_{Sit}$, we can transform (A.3) as follows:

$$\frac{\dot{N}_{it}}{N_{it}} = C_{Ei} (S_{Sit} - \pi_{it}^{e}) \tag{A.4}$$

To this regard, we assume that (1) $\pi_i^e = \frac{\overline{\pi}_i}{\ell}$, (2) $C_{Ei} = a\Gamma_i^{\kappa} = C_{Ei}^{'}\ell$ and (3) Γ is the firms' entry cost per sector that exert a negative impact on firms' entry.

Additionally, we have considered two output prices, the expected price (P_Q^e) and the current price (P_Q). At the beginning of period *t*, firms base all their production activities on the expected price. However, should the actual price be set, firms follow an adjustment process that is captured through the parameter ϕ in our optimizing equations.

As we have done for all the equations below, the above demand and supply equations can be expressed in growth terms (discrete time denoting variables' rates of change) by logging both sides and differentiating with respect to time. The new equations are as follows:

$$\frac{\dot{S}_{Sit}}{S_{Sit}} = \theta_{1i}\frac{\dot{A}_{it}}{A_{it}} + \frac{\dot{N}_t}{N_t} + \theta_{2i}\frac{\dot{P}_{it}}{P_{it}} + \theta_{3i}\frac{\dot{w}_{it}}{w_{it}} + \theta_{4i}\frac{\dot{r}_{it}}{r_{it}} + \sum_{l=1}^{T}\sigma_{li}\frac{\dot{P}_{lit}}{P_{lit}} + \theta_{5i}\frac{\dot{z}_{it}}{z_{it}} \tag{A.5}$$

$$\frac{\dot{S}_{Dit}}{S_{Dit}} = (1-\Delta_i)\frac{\dot{P}_{Q_{it}}}{P_{Q_{it}}} + (\lambda_{1i}+\Delta_i)\frac{\dot{P}^e_{Q_{it}}}{P_{Q_{it}}} + \lambda_{2i}\frac{\dot{Y}_{dit}}{Y_{dit}} + \lambda_{3i}\frac{\dot{D}_{it}}{D_{it}} + \chi_{j1i}\frac{\dot{IY}_{it}}{IY_{it}} \tag{A.6}$$

FACTOR MARKETS

1. Labor

1.1. Labor Supply Equation

We assume that the sectoral labor supply function is given by:

$$zL_{it} = C_{Li}\left(\frac{w_i}{P_{Q_i}}\right)^{\psi_1}\left(\frac{Y_{it}}{P_{Qi}}\right)^{\psi_2}\left(\frac{P_{Qit}}{P^e_{Qit}}\right)^{\psi_3}\left(D_{Lit}\right)^{\psi_4}\left(\prod_{j=1}^{l} v_j^{\varphi_j}\right) \tag{A.7}$$

where D_L is the total number of labor providers (mainly households) within the sector and the v variables are labor supply shifters.

$$\frac{\dot{zL}_{it}}{zL_{it}} = \psi_1\left(\frac{\dot{w}_{it}}{w_{it}} - \frac{\dot{P}_{Qit}}{P_{Qit}}\right) + \psi_2\left(\frac{\dot{Y}_{it}}{Y_{it}} - \frac{\dot{P}_{Qit}}{P_{Qit}}\right) + \psi_3\left(\frac{\dot{P}_{Qit}}{P_{Qit}} - \frac{\dot{P}^e_{Qit}}{P^e_{Qit}}\right) + \psi_4\left(\frac{\dot{D}_{Lit}}{D_{Lit}}\right) + \sum_{j=1}^{l}\varphi_j\left(\frac{\dot{v}_{jt}}{v_{jt}}\right)$$

(A.8)

1.2. Labor Demand Equation (Efficient Labor)

The demand for efficient labor, derived from profit maximization on the part of firms is given by:

$$zL_{it} = \alpha.\left(\frac{S_{Sit}}{w_{it}}\right).\left(\frac{P^e_{Qit}}{P_{Qit}}\right)^{1+\beta_i\phi_{Ki}+(\alpha_i-1)\phi_{Li}} \quad \text{(A.9)}$$

$$\frac{\dot{zL}_{it}}{zL_{it}} = \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{w}_{it}}{w_{it}} - (1+\beta_i\phi_{Ki} + (\alpha_i - 1)\phi_{Li})\frac{\dot{P}_{Qit}}{P_{Qit}} + (1+\beta_i\phi_{Ki} + (\alpha_i - 1)\phi_{Li})\frac{\dot{P}^e_{Qit}}{P^e_{Qit}} \quad \text{(A.10)}$$

$$\frac{\dot{zL}_{it}}{zL_{it}} = \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{w}_{it}}{w_{it}} + (1+\beta_i\phi_{Ki} + (\alpha_i - 1)\phi_{Li})\left[\frac{\dot{P}^e_{Qit}}{P^e_{Qit}} - \frac{\dot{P}_{Qit}}{P_{Qit}}\right] \quad \text{(A.11)}$$

2. Capital

As for labor, capital equations are obtained from firms' profit maximization.

2.1. Capital Supply Equation

$$K_{it} = C_{Ki}(r_t)^{\gamma_1}\left(\frac{Y}{P_{Qit}}\right)^{\gamma_2}\left(\frac{P_{Qit}}{P^e_{Qit}}\right)^{\gamma_3}(D_{Kit})^{\gamma_4}\left(\prod_{j=1}^{n} u_j^{\delta_j}\right) \quad \text{(A.12)}$$

$$\frac{\dot{K}_{it}}{K_{it}} = \gamma_1\left(\frac{\dot{r}_t}{r_t}\right) + \gamma_2\left(\frac{\dot{Y}_t}{\dot{Y}_t} - \frac{\dot{P}_{Qit}}{\dot{P}_{Qit}}\right) + \gamma_3\left(\frac{\dot{P}_{Qit}}{P_{Qit}} - \frac{\dot{P}^e_{Qit}}{P^e_{Qit}}\right) + \gamma_4\left(\frac{\dot{D}_{Kit}}{D_{Kit}}\right) + \sum_{j=1}^{n}\delta_j\left(\frac{\dot{u}_{jt}}{u_{jt}}\right)$$

(A.13)

Where:

- D_K represents the total number of capital providers that includes (1) Government, (2) Domestic providers, and (3) Foreign providers;
- u represents the capital supply shifters; and
- r represents the real interest rate.

2.2. Capital Demand Equation

$$K_{it} = \beta_i . \left(\frac{S_{Sit}}{r_t} \right) . \left(\frac{P^e_{Qit}}{P_{Qit}} \right)^{1+\alpha_i \phi_{Li} + (\beta_i - 1)\phi_{Ki}} \quad \text{(A.14)}$$

$$\frac{\dot{K}_{it}}{K_{it}} = \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{r}_t}{r_t} - [1 + \alpha_i \phi_{Li} + (\beta_i - 1)\phi_{Ki}] \left(\frac{\dot{P}_{Qit}}{P_{Qit}} \right) + [1 + \alpha_i \phi_{Li} + (\beta_i - 1)\phi_{Ki}] \left(\frac{\dot{P}^e_{Qit}}{P^e_{Qit}} \right)$$

(A.15)

$$\frac{\dot{K}_{it}}{K_{it}} = \frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{r}_t}{r_t} + [1 + \alpha_i \phi_{Li} + (\beta_i - 1)\phi_{Ki}] \left(\frac{\dot{P}^e_{Qit}}{P^e_{Qit}} - \frac{\dot{P}_{Qit}}{P_{Qit}} \right) \quad \text{(A.16)}$$

3. The Money Market

Real money balances as a factor of production is demanded by firms and the government while households also require services of real money balances.

3.1. Money Supply Equation

$$M_{Sit} = C_{M_{Si}} . P^{\pi_1}_{Qit} . r_t^{\pi_2} \quad \text{(A.17)}$$

$$\frac{\dot{M}_{Sit}}{M_{Sit}} = \pi_1 \left(\frac{\dot{P}_{Qit}}{P_{Qit}} \right) + \pi_2 \left(\frac{\dot{r}_t}{r_t} \right) \quad \text{(A.18)}$$

3.2. Money Demand Equation

$$M^d_{it} = C_{M^d_{it}} . (D_{it})^{\nabla_1} . (N_{it})^{\nabla_2} . \left(\frac{r_t}{P^e_{Qit}} \right)^{\nabla_3} . \left(\frac{S_{Sit}}{P^e_{Qit}} \right)^{\nabla_4} . \left(\frac{P_{Qit}}{P^e_{Qit}} \right)^{\nabla_5} \quad \text{(A.19)}$$

$$\frac{\dot{M}^d_{it}}{M^d_{it}} = \nabla_1 \left(\frac{\dot{D}_{it}}{D_{it}} \right) + \nabla_2 \left(\frac{\dot{N}_{it}}{N_{it}} \right) + \nabla_3 \left(\frac{\dot{r}_t}{r_t} - \frac{\dot{P}^e_{Qit}}{P^e_{Qit}} \right) + \nabla_4 \left(\frac{\dot{S}_{Sit}}{S_{Sit}} - \frac{\dot{P}^e_{Qit}}{P^e_{Qit}} \right) + \nabla_5 \left(\frac{\dot{P}_{Qit}}{P_{Qit}} - \frac{\dot{P}^e_{Qit}}{P^e_{Qit}} \right)$$

(A.20)

4. TRANSFER FUNCTIONS

As shown in Zellner and Palm (2004), transfer functions can be derived mathematically from dynamic linear structural equation models. In his seminal work, Quenouille (1957) has indicated a specific way to represent linear multiple time series processes as follows (see Zellner and Palm, 2004):

$$\underset{mxm}{A(L)}\,\underset{mx1}{y_t} = \underset{mxm}{R(L)}\,\underset{mx1}{\varepsilon_t} \quad , \tag{A.21}$$

where (1) $y_t^{'} = (y_{1t}, y_{2t}, ..., y_{mt})$ is the vector of random variables, and (2) $\varepsilon_t^{'} = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{mt})$ is the random errors' vector. $A(L)$ and $R(L)$ are full rank matrix containing lagged polynomial elements. We can solve for y_t as follows:

$$y_t = \left[A^*(L)/|A(L)|\right]R(L)\varepsilon_t, \tag{A. 22}$$

where $A^*(L)$ is the adjoint matrix and $|A(L)|$ is the determinant, both obtained from $A(L)$. As y_t is expressed as an infinite moving average process, we can derive a system of finite order ARMA (Autoregressive Moving Average) functions as follows:

$$|A(L)|y_t = A^*(L)R(L)\varepsilon_t \tag{A. 23}$$

Our specific product market model for the i'th sector, expressed in matrix lag operator form is:

$$\begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \delta_{0,i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \delta_{1,i} \end{bmatrix} S_{i,t-1} + \begin{bmatrix} \kappa_{1,i} \\ 0 \\ 0 \end{bmatrix} w_{i,t} + \begin{bmatrix} \kappa_{2,i} \\ 0 \\ 0 \end{bmatrix} r_t + \begin{bmatrix} \kappa_{3,i} \\ 0 \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} \kappa_{4,i} \\ 0 \\ 0 \end{bmatrix} z_{i,t}$$

$$+ \begin{bmatrix} \kappa_{5,i} \\ 0 \\ 0 \end{bmatrix} \Omega_{i,t} + \begin{bmatrix} \kappa_{6,i} \\ 0 \\ 0 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} 0 \\ \Delta_{2,i} \\ 0 \end{bmatrix} iy_t + \begin{bmatrix} 0 \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t + \begin{bmatrix} \varepsilon_{Ti,t} \\ \mu_{Ti,t} \\ \nu_{Ti,t} \end{bmatrix} .$$

(A.24)

In order to obtain the transfer equations, multiply both sides of equation A.24 by the adjoint matrix A^* ($A^* = \det A . A^{-1}$), with:

$$A = \begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad A^{-1} = \frac{1}{\det A}\begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L)-\gamma(L) \end{bmatrix}$$

Therefore: $A^* = \begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L)-\gamma(L) \end{bmatrix}$

After multiplying both sides of equation A.24 by A^*:

$$\left[\lambda(L)-\gamma(L)\right].\begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} -\gamma(L)\delta_{0,i} \\ -\delta_{0,i} \\ \delta_{0,i}\left[\lambda(L)-\gamma(L)\right] \end{bmatrix} + \begin{bmatrix} -\gamma(L)\delta_{1,i} \\ -\delta_{1,i} \\ \delta_{1,i}\left[\lambda(L)-\gamma(L)\right] \end{bmatrix} S_{i,t-1} + \begin{bmatrix} -\gamma(L)\kappa_{1,i} \\ -\kappa_{1,i} \\ 0 \end{bmatrix} w_{i,t}$$

$$+\begin{bmatrix} -\gamma(L)\kappa_{2,i} \\ -\kappa_{2,i} \\ 0 \end{bmatrix} r_t + \begin{bmatrix} -\gamma(L)\kappa_{3,i} \\ -\kappa_{3,i} \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{4,i} \\ -\kappa_{4,i} \\ 0 \end{bmatrix} z_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{5,i} \\ -\kappa_{5,i} \\ 0 \end{bmatrix} \Omega_{i,t}$$

$$+\begin{bmatrix} -\gamma(L)\kappa_{6,i} \\ -\kappa_{6,i} \\ 0 \end{bmatrix} X_t + \begin{bmatrix} \lambda(L)\Delta_{1,i} \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} \lambda(L)\Delta_{2,i} \\ \Delta_{2,i} \\ 0 \end{bmatrix} iy_t + \begin{bmatrix} \lambda(L)\Delta_{3,i} \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t +$$

$$+\begin{bmatrix} -\gamma(L)\varepsilon_{Ti,t} + \lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t} \\ -\varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t} \\ [\lambda(L)-\gamma(L)]v_{Ti,t} \end{bmatrix}.$$

(A.25)

Equation A.25 can be transformed into a system of linear equations for both price and sales supply:

$$\left[\lambda(L)-\gamma(L)\right].s_{i,t} = -\gamma(L)\delta_{0,i} - \gamma(L)\delta_{1,i}S_{i,t-1} - \gamma(L)\kappa_{1,i}w_{i,t} - \gamma(L)\kappa_{2,i}r_t - \gamma(L)\kappa_{3,i}a_{i,t} - \gamma(L)\kappa_{4,i}z_{i,t}$$

$$-\gamma(L)\kappa_{5,i}\Omega_{i,t} - \gamma(L)\kappa_{6,i}X_t + \lambda(L)\Delta_{1,i}y_t + \lambda(L)\Delta_{2,i}iy_t + \lambda(L)\Delta_{3,i}d_t - \gamma(L)\varepsilon_{Ti,t}$$
$$+\lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t}$$

(A.26a)

$$s_{i,t} = \frac{1}{\lambda(L)-\gamma(L)}[-\gamma(L)\delta_{0,i} - \gamma(L)\delta_{1,i}S_{i,t-1} - \gamma(L)\kappa_{1,i}w_{i,t} - \gamma(L)\kappa_{2,i}r_t - \gamma(L)\kappa_{3,i}a_{i,t} - \gamma(L)\kappa_{4,i}z_{i,t}$$
$$-\gamma(L)\kappa_{5,i}\Omega_{i,t} - \gamma(L)\kappa_{6,i}X_t + \lambda(L)\Delta_{1,i}y_t + \lambda(L)\Delta_{2,i}iy_t + \lambda(L)\Delta_{3,i}d_t - \gamma(L)\varepsilon_{Ti,t}$$
$$+\lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t}]$$

(A.26b)

$$[\lambda(L)-\gamma(L)]P_{i,t} = -\delta_{0,i} - \delta_{1,i}S_{i,t-1} - \kappa_{1,i}w_{i,t} - \kappa_{2,i}r_t - \kappa_{3,i}a_{i,t} - \kappa_{4,i}z_{i,t} - \kappa_{5,i}\Omega_{i,t} - \kappa_{6,i}X_t + \Delta_{1,i}y_t$$
$$+\Delta_{2,i}iy_t + \Delta_{3,i}d_t - \varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t}$$

(A.27)

where:

- $\lambda(L)$ and $\gamma(L)$: lag operators
- X : set of other exogenous variables obtained from the ARLI (3) model: *SP* (Stock Prices) and M (Money Supply: M2);
- $\ln\left(\frac{S_{i,t}}{S_{i,t-1}}\right) = s_{i,t}$; $\ln\left(\frac{N_{i,t}}{N_{i,t-1}}\right) = n_{i,t}$; $\ln\left(\frac{w_{i,t}}{w_{i,t-1}}\right) = n_{i,t}$; $\ln\left(\frac{r_{i,t}}{r_{i,t-1}}\right) = n_{i,t}$;

$\ln\left(\frac{A_{i,t}}{A_{i,t-1}}\right) = n_{i,t}$; $\ln\left(\frac{z_{i,t}}{z_{i,t-1}}\right) = z_{i,t}$; $\ln\left(\frac{\Gamma_{i,t}}{\Gamma_{i,t-1}}\right) = \Omega_{i,t}$.