A Broader View of Designing the Liver Allocation System

Mustafa Akan
Tepper School of Business, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, akan@cmu.edu

Oguzhan Alagoz
Department of Industrial and Systems Engineering, University of Wisconsin, Madison, Madison, Wisconsin 53706, alagoz@engr.wisc.edu

Baris Ata
Kellogg School of Management, Northwestern University, Evanston, Illinois 60208, b-ata@kellogg.northwestern.edu

Fatih Safa Erenay
Department of Management Sciences, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada, ferenay@connect.uwaterloo.ca

Adnan Said
Department of Medicine, University of Wisconsin, Madison, Madison, Wisconsin 53705, axs@medicine.wisc.edu

We consider the problem of designing an efficient system for allocating donated livers to patients waiting for transplantation. The trade-off between medical urgency and efficiency is at the heart of the liver allocation problem. We model the transplant waiting list as a multiclass fluid model of overloaded queues, which captures the disease evolution by allowing the patients to switch between classes, i.e., health levels. We consider the bicriteria objective of minimizing total number of patient deaths while waiting for transplantation (NPDWT) and maximizing total quality-adjusted life years (QALYs) through a weighted combination. On one hand, under the objective of minimizing NPDWT, the current policy of United Network for Organ Sharing (UNOS) emerges as the optimal policy, providing a theoretical justification for the current practice. On the other hand, under the metric of maximizing QALYs, the optimal policy is an intuitive dynamic index policy that ranks patients based on their marginal benefit from transplantation, i.e., the difference in benefit with versus without transplantation. Finally, we perform a detailed simulation study to compare the performances of our proposed policies and the current UNOS policy along the following metrics: total QALYs, NPDWT, number of patient deaths after transplantation, number of total patient deaths, and number of wasted livers. Numerical experiments show that our proposed policy for maximizing QALYs outperforms the current UNOS policy along all metrics except the NPDWT.

Subject classifications: organ transplantation; liver allocation system; healthcare operations; health policy design; fluid models; simulation; linear programming.

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1. Introduction

Organs donated for liver transplantation, the only viable therapy for patients with end-stage liver diseases (ESLD), are scarce resources. Because of the severe shortage of donated organs, ESLD patients are placed on a liver waiting list. Currently, there are approximately 17,000 patients on this list; 11,176 of them were added in 2008 alone. Although 6,745 cadaveric organs were donated in 2008, the median waiting time for transplantation was over 300 days (OPTN 2009). As a result of the insufficient organ supply, more than 20,000 patients died while waiting for a liver between 1999 and 2008. At the same time, each year approximately 6% of all donated livers are wasted and 12% of all patients who receive an organ die within the first year (OPTN 2009). These data underline the need to improve the allocation of available organs to patients.

It is estimated that tissue and organ transplantation in the United States was a $20.5 billion market in 2007. While Buchanan et al. (2009) report the base cost of a liver transplant to be approximately $450,000, a liver transplant for a patient with hepatitis C would cost over $1 million when the costs of surgery and medication are factored in (Naito 2005). These economic indicators suggest that any improvement on the efficient usage of donated organs would lead to significant savings in life expectancy and would help transplantation remain economically sustainable.

When an organ is harvested, United Network for Organ Sharing (UNOS) offers this organ to patients on the waiting
list based on their medical urgency, i.e., death risk while waiting. UNOS prioritizes ESLD patients in decreasing order of the Model of End-stage Liver Disease (MELD) score, which is an indicator of pre-transplant mortality (UNOS 2008). The MELD score has been criticized as a criterion for liver allocation because it ignores post-transplant outcomes (Schaubel et al. 2008). A medical urgency-based approach might sacrifice efficiency because patients with the greatest pre-transplant mortality risk might also have the least life expectancy and quality after the transplant. These competing factors highlight the crucial trade-off between medical urgency and efficiency.

The principle that patients in the greatest need deserve priority focuses on reducing the number of patient deaths while patients are waiting for transplantation (NPDWT). However, focusing primarily on medical urgency might fall short of capturing other important criteria, such as the impact of current decisions on the future well-being of patients. This shortcoming of the current allocation policy is recognized in the medical literature (Said et al. 2007). Indeed, recent investigations have emphasized the need to incorporate survival benefit and total life years gained, including both pre- and post-transplant survival into the allocation mechanism (Merion et al. 2005). The UNOS and Organ Procurement and Transplantation Network (OPTN) Ethics Committee also gives emphasis to the use of quality-adjusted life years (QALYs) in liver allocation (OPTN/UNOS 2009).

This paper studies the design of the liver allocation policy considering the trade-off between medical urgency and efficiency. We develop a dynamic fluid model to represent the liver allocation system and use optimal control theory to characterize the optimal policy. We formulate the objective function as a weighted combination of total NPDWT and total QALYs. Clearly, changing the weight parameter leads to different results. At one extreme, we consider the problem of minimizing the total NPDWT and show that the current UNOS policy is optimal under the medical urgency criterion; cf. Proposition 1. This result demonstrates the conditions under which the current policy would be optimal and provides a theoretical justification for the current UNOS policy. At the other extreme, the objective of maximizing QALYs gives rise to an intuitive dynamic index policy that prioritizes patients on the waiting list with respect to their marginal benefit from transplantation. More specifically, the policy compares the expected QALYs of each patient with versus without transplant and offers the organ to the patient for whom the difference is largest. The dynamic indices of these policies depend on the patients’ acceptance probabilities, benefit of receiving a transplant, and shadow prices capturing the future benefits of remaining on the waiting list.

Similarly, the optimal policy for the general case ranks patients according to their marginal benefit from transplantation, measured by a weighted average of QALYs and waiting list survival rate. We observe a common rationale across all optimal allocation policies obtained under different metrics: they calculate the difference in a patient’s expected benefit from receiving the allocated organ versus remaining on the waiting list. Multiplying this difference with acceptance probabilities, one obtains the optimal ranking for organ offers. Among the three determinants of patient priority, the transplant outcomes (e.g., QALYs after transplant) are available in the literature, and the acceptance probabilities are estimated using UNOS data. Hence, the optimal allocation rule boils down to estimating each patient’s expected benefit without a transplant. This is a challenging task because the benefit of a patient without transplant accounts for not only the expected QALYs she will earn while on the waitlist, but also the possibility of future organ offers and their benefits, which depend on the allocation policy used. This paper shows that the benefit without a transplant can be computed using shadow prices, which capture both the dynamics of the system and the impact of an allocation decision on future medical urgency and benefit.

We compare the performances of our proposed optimal policies and the current UNOS policy through an extensive simulation-based numerical analysis. We show that the policy we propose for maximizing QALYs (see §4) leads to significant improvements over the current UNOS policy. The proposed policy increases total QALYs while reducing number of wasted livers (NWL) and number of patient deaths after transplantation (NPDAT). In contrast, the current sickest-first policy of UNOS results in fewer patient deaths while patients are waiting for transplantation, as predicted by Proposition 1. Under the metric of total patient deaths (TPD), i.e., sum of NPDWT and NPDAT, we show that our proposed policy might perform better or worse compared to the current UNOS policy, depending on the length of the follow-up period over which post-transplant deaths are accounted for.

Several researchers investigate the problem of accepting/declining an organ offer for transplantation to optimize a particular patient’s welfare (Alagoz et al. 2004, 2007a, b; David and Yechiali 1985; Hornberger and Ahn 1997; Howard 2002; Sandikci et al. 2008). Several others use simulation models to evaluate the performance of various organ allocation policies (Shechter et al. 2005, Zenios et al. 1999); see Alagoz (2004) for a detailed review of this literature. Much of the research on the optimal allocation of organs (particularly kidneys) focuses on designing an optimal allocation system to maximize objectives such as mean expected QALYs, average one-year graft survival probability, and quality of the prospective matches (David and Yechiali 1995, Righter 1989). In a series of papers, Su and Zenios (2004, 2005, 2006) study the impact of patient choice on kidney allocation mechanisms. Another related paper is Zenios et al. (2000), which uses a fluid model to find the best kidney allocation policy with three criteria of maximizing efficiency (i.e., QALYs) and minimizing two measures of inequity. Zenios et al. (2000) explores the
efficiency-equity trade-off and uses various approximations to develop a heuristic dynamic index policy for the problem, which is effective and easy to implement.

This paper differs from Zenios et al. (2000) as it studies liver transplantation and analyzes the trade-off between medical urgency and efficiency. Focusing on the extreme points of the urgency-efficiency spectrum, we provide a theoretical justification for the optimality of the current UNOS policy for minimizing deaths in the waiting list while we lay the groundwork for other policies under alternative objectives. The dynamic health characteristics (i.e., laboratory values in the blood) of the patient at the time of transplantation significantly affect the outcome of the liver transplantation, whereas these are of secondary importance to the static health characteristics (i.e., antigen type) in determining the outcome of kidney transplantation (Roberts et al. 2004). As a result, we also incorporate the dynamic evolution of patient health in our model. Finally, unlike the previous research, we incorporated into our model the possibility of patients turning down the organ offers because Alagoz (2004) reports that 60% of all liver offers are declined by the ESLD patients.¹

Our paper also corroborates the recent work by the Scientific Registry for Transplant Recipients (SRTR), which proposes a policy of liver allocation taking post-transplant survival into account (Schaubel et al. 2009). The proposed policy of Schaubel et al. (2009) offers an available organ to the patient with the largest survival benefit, i.e., the difference between post-transplant and waitlist life expectancies. Therefore, our proposed policies and those of Schaubel et al. (2009) are both transplant benefit-based allocation schemes. However, our proposed policies and those of Schaubel et al. (2009) estimate marginal benefit of transplant and survival benefit of transplant differently: (i) Schaubel et al. (2009) calculate the waitlist life expectancy for each patient type based on observational data on the time until the earliest of death or transplant under the UNOS policy. On the other hand, our paper endogenously calculates patients’ expected benefit without transplant using a shadow price approach; (ii) Schaubel et al. (2009) consider factors other than the MELD score in estimating waiting list and post-transplant life expectancies. Despite these differences, the performance of our proposed policy compares well with that of Schaubel et al. (2009) (see §6).

We summarize our proposed policies and provide the intuition behind them in §4. Therefore, the readers interested in the policy implications of our results can skip the details in §§3 and 5 on the first reading because the proposed policies are overviewed in §4 in sufficient detail.

2. Current Liver Allocation System and Recent Trends

UNOS is responsible for managing the national organ allocation system. The new liver allocation procedure was approved in 2002 (Government Accounting Office 2003). Although the new policy is anticipated to “better identify urgent patients and reduce deaths among patients awaiting liver transplants” (UNOS 2008), there is evidence that some of the transplant community disagrees that the new allocation rules are satisfactory (Garber 2002, Trotter and Osgood 2004).

The national UNOS membership is divided into 11 geographic regions. Each region consists of several Organ Procurement Organizations (OPOs), nonprofit agencies responsible for coordinating the recovery and transportation of organs. UNOS maintains a patient waiting list to determine the transplant candidates among the patients. Available livers are allocated considering the liver/patient OPO and region, medical urgency, patient points, and patient waiting time. The medical urgency of the adult liver patients is represented by UNOS Status 1 and MELD scores, which indicate the status of the liver disease (Wiesner et al. 2001). A patient listed as Status 1 “has fulminating liver failure with a life expectancy without a liver transplant of less than 7 days” (UNOS 2008). The MELD score is a continuous function of total bilirubin, creatinine, and prothrombin time, whose formula is given in Appendix A. An electronic companion to this paper is available at http://dx.doi.org/10.1287/opre.1120.1064. This electronic companion contains the appendices. UNOS restricts the range of MELD scores to be between 6 and 40, where 6 and 40 correspond to the best and worst patient health, respectively (UNOS 2008). Patients on the waiting list are further stratified within Status 1 and each MELD score using patient “points” and waiting time, as explained in Appendix A.

In a nutshell, the current liver allocation system operates as follows: each liver available for transplant is first offered to Status 1 patients located within the harvesting OPO. If there are no suitable Status 1 matches within the harvesting OPO, the liver is then offered to Status 1 patients within the harvesting region. If a match has not yet been found at this point, then the liver is offered to all non-Status 1 patients with MELD scores greater than or equal to 15 in the harvesting OPO in descending order of MELD score. If this search for a suitable match fails, then the search is again broadened to the patients with MELD scores greater than or equal to 15 in the harvesting region, then the liver is offered to patients with MELD scores less than 15 in the harvesting OPO, and then to patients with MELD scores less than 15 in the harvesting region. Finally, if a match has not yet been found, then the liver is offered nationally to Status 1 patients followed by all other patients in descending order of MELD scores. The final decision to accept a liver “remains the prerogative of the transplant surgeon or physician responsible for the care of that patient” (IOM 1999). The surgeon or the physician have around one hour to make their decision, because the acceptable range for cold-ischemia time is 12 to 18 hours (SRTR
alcohol-induced liver disease, might improve temporarily at which their health status improves. Although it might of health status rate at which their health deteriorates. Let transplantation at time change over time, which corresponds to a change of class in dimension is indexed by cytomegalovirus, and dynamic patient characteristics representing patient characteristics (i.e., patient type) such as blood type, have been proposed to maximize survival benefit by looking at donor factors that impact transplant recipient outcomes differentially by MELD score (Schaubel et al. 2008).

To maximize the utility of organ allocation, a system based on survival benefit that balances both pre-transplant medical urgency and post-transplant survival is needed so that benefit to the patient population is maximized. Taking into account recent publications based on the SRTR, UNOS Board of Directors recognized the need for taking into account the total expected QALYs of the ESLD patients (OPTN/UNOS 2009), which are currently ignored (UNOS 2008).

3. The Fluid Model

Our model divides the ESLD population waiting for transplant into different classes along two dimensions: static patient characteristics (i.e., patient type) such as blood type, cytomegalovirus, and dynamic patient characteristics representing the health status such as MELD score. The former dimension is indexed by $i = 1, \ldots, I$, while the latter is indexed by $j = 1, \ldots, J$. Health status of a patient might change over time, which corresponds to a change of class in our model. The structure of classes in our model is depicted in Figure 1.

Patients of class $ij$ arrive to the system at rate $\lambda_{ij}(t)$ for $t \geq 0$, and the number of class $ij$ patients waiting for transplantation at time $t$ is denoted by $x_{ij}(t)$; there are $x_{ij}(0)$ patients in class $ij$ initially. We let $\alpha_j$ denote the rate at which patients of health status $j$ become patients of health status $j + 1$ for $j = 1, \ldots, J - 1$, that is, $\alpha_j$ is the rate at which their health deteriorates. Let $\beta_j$ denote the rate at which patients of health status $j$ become patients of health status $j - 1$ for $j = 2, \ldots, J$, that is, the rate at which their health status improves. Although it might seem counterintuitive, the MELD scores of ESLD patients, particularly the ones with chronic liver diseases such as alcohol-induced liver disease, might improve temporarily over time. We assume that $\alpha_j > \beta_j$ for $j = 2, \ldots, J - 1$ and $\alpha_j = \beta_j = 0$. The rate at which a patient of health status $j$ dies is denoted by $d_j$ for $j = 1, \ldots, J$. We assume that sicker patients are more likely to die, that is, $d_j > \cdots > d_1$, which is a reasonable assumption supported by various studies (Wiesner et al. 2001).

There are $K$ liver types, and type $k$ livers arrive at the system at rate $\mu_k(t)$ for $t \geq 0$. The liver type describes the liver quality defined by various factors such as donor age, donor blood type, donation after cardiac death, etc. UNOS must decide what fraction of the incoming livers of each type to allocate to patients of various classes waiting for transplantation dynamically over time. Equivalently, for each liver type $k = 1, \ldots, K$, UNOS chooses the rate $u_{ijk}(t)$ of organs to be allocated to class $ij$ patients dynamically over time for $t \geq 0$, $i = 1, \ldots, I$, and $j = 1, \ldots, J$.

Patients may reject the organs offered. Let $p_{ijk}$ denote the probability that a class $ij$ patient accepts an offered organ of type $k$. We define $p_{ijk}$ as a function of static patient characteristics such as blood type, patient MELD score, and liver type, because Howard (2002) and Alagoz (2004) report that the probability of organ acceptance depends highly on these factors. In our model, an organ can be offered to multiple patients. If the organ is offered to $n$ patients of class $ij$, then the probability that nobody accepts the organ (that is, the organ is wasted) is given by $(1 - p_{ijk})^n$. Then, letting $\pi_{ijk}^n$ denote the probability that the organ is transplanted when it is offered to $n$ patients of type $ij$, we have that $\pi_{ijk}^n = 1 - (1 - p_{ijk})^n$, where $n$ also represents the expected number of patients an organ is offered to. We denote the system state by $x(t) = (x_{i1}(t), \ldots, x_{ij}(t), \ldots, x_{iJ}(t))^t$ for $t \geq 0$, where $x_{ij}(t)$ represents the number of class $ij$ patients on the waiting list at time $t$. Similarly, letting $u(t) = (u_{11k}(t), \ldots, u_{ijk}(t), \ldots, u_{iJk}(t))^t$ for $t \geq 0$ and $k = 1, \ldots, K$, we denote our control by $u = [u^k(t) : k = 1, \ldots, K, t \geq 0]$. Then a feasible control $u$ must satisfy the following restrictions for $t \geq 0$ and $k = 1, \ldots, K$:

$$u^k(t) \geq 0, \quad e \cdot u^k(t) \leq \mu_k(t).$$

Figure 1. The class structure of the model.
where $e$ is an $IJ$-dimensional column vector of ones. The first part of Equation (1) imposes the nonnegativity constraint on the allocation rates, while the second part states that the total rate of livers assigned to each patient type should be less than or equal to the arrival rate of livers.

\[
\dot{x}(t) = \lambda(t) - \sum_{k=1}^{K} P^k u^k(t) - (d + \alpha - \beta)x(t), \quad t \geq 0. \tag{2}
\]

Given a feasible control $u$, the system state evolves according to Equation (2), where $P^k$ is an $IJ \times IJ$ dimensional diagonal matrix with entries $\pi_{ik}^j$: $i = 1, \ldots, I$ and $j = 1, \ldots, J$ for $k = 1, \ldots, K$. The arrival rate of the patients is modeled by the vector process $\lambda(t)$ whose $ij$th entry at time $t$ is $\lambda_{ij}(t)$. The square matrices $d$, $\alpha$, and $\beta$ represent the death, deterioration, and recovery of the patients' health status. We also require that the number of patients in each class is nonnegative, i.e.,

\[x(t) \geq 0 \quad \text{for } t \geq 0.\tag{3}\]

There are many important criteria to be taken into account while assigning the organs to patients in the waiting list. One of the essential objectives of the liver allocation system is to minimize the total number of patient deaths while waiting for transplantation, which is given by $\int_0^T d \cdot x(t) dt$. Another important consideration is to maximize the total QALYs of the patients, which requires us to take into account both post-transplant life expectancy of each patient who receives a transplant and how long each patient will wait for transplant. To capture the former, let $h_{ijk}$ denote the expected QALYs of a patient of class $ij$ transplanted with a liver of type $k$. The $h_{ijk}$ variables consider QALYs instead of life years because it is known that liver transplant recipients might experience a lower quality of life than people with no medical problems, because of the risk and discomfort associated with the use of antirejection immunosuppressive drugs (Trotter et al. 2002).

To capture the reward accrued by the patients while waiting for transplantation, let $q_{ij}$ denote the quality-of-life scores of class $ij$ patients waiting for transplantation. The QALYs accrue for the entire patient population on the waiting list at the instantaneous rate $q \cdot x(t)$ at time $t$, where $q = (q_{11}, \ldots, q_{1j}, \ldots, q_{i1}, \ldots, q_{ij}, \ldots)$. $q_{ij}$ depends on patient type because MELD scores provide a good proxy for the quality of life experienced by patients with ESLD. For instance, a patient with a MELD score over 30 has a higher risk of hospitalization, risk of complications related to ESLD such as bleeding and ascites, and consequently will experience a lower quality of life than a patient with a MELD score less than 10. Finally, let $\theta_{ij}$ denote the expected future QALYs of a patient of class $ij$ who is still waiting for transplantation at the end of the planning horizon, i.e., $x(T) \cdot \theta$ denotes the total terminal reward associated with the patients who are still on the waiting list at the end of the planning horizon, where $\theta = (\theta_{11}, \ldots, \theta_{1j}, \ldots, \theta_{i1}, \ldots, \theta_{ij})$. The total expected QALYs resulting from the organ allocation policy $u$ is given by $\sum_{k=1}^{K} \int_0^T h^k \cdot P^k u^k(t) dt + \int_0^T q \cdot x(t) dt + x(T) \cdot \theta$.

We consider the dual objective of maximizing the total QALYs and minimizing the total NPDWT through a weighted average of the two criteria. Let $\kappa$ and $1 - \kappa$ denote the weights of the objectives of maximizing the total QALYs and minimizing the total NPDWT, respectively. The choice of $\kappa$ is a policy question and will dictate the point at which the organ allocation system will operate along the efficient frontier of maximizing QALYs and minimizing patient deaths. The problem of UNOS can be stated as choosing an organ allocation policy $u$ to

\[
\text{maximize } \kappa \left( \sum_{k=1}^{K} \int_0^T h^k \cdot P^k u^k(t) dt + \int_0^T q \cdot x(t) dt, \right. \frac{\int_0^T d \cdot x(t) dt}{(1 - \kappa) \int_0^T d \cdot x(t) dt} \left. \right) \tag{P_\kappa}
\]

subject to (1)–(3).

When $\kappa = 1$, the objective of the organ allocation problem (P1) becomes one of solely maximizing QALYs, while for $\kappa = 0$, the objective of (P0) reduces to minimizing the NPDWT. Our modeling framework is quite flexible and can accommodate various other objective functions. The first alternative performance metric is the total number of wasted organs, minimization of which is equivalent to maximizing the total number of organs transplanted: Choose the organ allocation policy $u$ to

\[
\text{maximize } \sum_{k=1}^{K} \int_0^T e \cdot P^k u^k(t) dt, \quad \text{subject to (1)–(3)}.
\]

The second alternative is to minimize total patient deaths. Post-transplant deaths can be accounted for a given follow-up period, where one year has been a traditional time point to assess survival. Letting $\delta_{ijk}$ denote the death rate of class $ij$ patients at the end of the follow-up period when transplanted with a liver of type $k$, the problem is choosing the organ allocation policy $u$ to

\[
\text{minimize } \sum_{k=1}^{K} \int_0^T \delta^k \cdot P^k u^k(t) dt + \int_0^T d \cdot x(t) dt, \quad \text{subject to (1)–(3)}.
\]

The organ allocation problem (P_\kappa) constitutes an optimal control problem with state constraints and a linear objective function. We derive the optimal policies using optimal control theory (Seierstad and Sydsæter 1987), which characterizes the necessary and sufficient conditions for optimality.

4. Overview of the Proposed Policies

The basic principle of the proposed policies is simple: allocate the organ to the patient whose marginal benefit
from receiving it is the largest. The “benefit of transplantation” is best understood in the special case of maximizing total QALYs, i.e., \( \kappa = 1 \) in the formulation given in §3. There, the problem is to choose an organ allocation policy to maximize the total expected QALYs of all patients in the system. In that case, the marginal benefit a patient obtains from receiving an organ can be defined as the difference between the total expected QALYs with versus without transplantation. While models estimating the QALYs after the transplantation are available (Ghobrial et al. 2002, Roberts et al. 2004), there is no analytical framework to compute the QALYs without transplantation. Using a shadow pricing approach, we provide a proxy for the QALYs without transplantation. Intuitively, shadow prices \( y_{ij}(t) \) (derived in §5) associated with (\( P_1 \)) represent the total expected QALYs of a class \( ij \) patient awaiting transplantation at time \( t \).

Our proposed policy for the case of \( \kappa = 1 \) is referred to as the marginal benefit of transplant policy (MBTP-Q) for maximizing QALYs, because it considers not only the immediate benefits of transplantation but also possible future rewards. Hence, it truly captures the marginal benefit of transplantation for each possible allocation decision. For example, if a patient has a very low MELD score, she will have a low marginal benefit from the transplant due to a very high life expectancy without transplantation. Hence, MBTP-Q assigns a low priority to this patient. At the other extreme, if a patient has a high MELD score, this patient will also have lower priority in the allocation list because her/his post-transplant life expectancy is low. Therefore, MBTP-Q favors the patients with relatively high post-transplant life expectancy compared to their life expectancy without transplantation. MBTP-Q proceeds as follows.

### Marginal Benefit of Transplant Policy for Maximizing QALYs (MBTP-Q)

When a liver of type \( k \) arrives at time \( t \), the system manager ranks various patient classes \( ij \) (for \( i = 1, \ldots, I \) and \( j = 1, \ldots, J \)) with respect to \( (h_{ijk} - y_{ij}(t))\pi_{n_{ij}k}^{n} \), the expected difference in benefit with versus without transplantation. The dynamic index \( (h_{ijk} - y_{ij}(t))\pi_{n_{ij}k}^{n} \) consists of patients’ benefit \( h_{ijk} \) from transplantation, shadow price \( y_{ij}(t) \) capturing their future benefit without a transplant, and acceptance probability \( \pi_{n_{ij}k}^{n} \). The patients within the same class are ordered with respect to their waiting time in that class. The system manager offers the organ to the first \( n \) patients in class \( ij \) with the highest \( (h_{ijk} - y_{ij}(t))\pi_{n_{ij}k}^{n} \), where \( n \) denotes the number of parallel offers. In the case that there are fewer than \( n \) patients in that class, the system manager offers the organ to the patients in the lower ranked classes until the organ is offered to \( n \) patients. The organ is then transplanted to the patient with the highest index among those who accept the organ offer.

To see the logic behind MBTP-Q, recall that \( h_{ijk} \) represents total QALYs of a patient in class \( ij \) when she receives a type \( k \) liver, and \( y_{ij}(t) \) represents the total expected QALYs when she receives no organ and remains on the waiting list at time \( t \). Therefore, for a class \( ij \) patient, \( h_{ijk} - y_{ij}(t) \) captures the difference between future QALYs with transplantation versus without transplantation at time \( t \). Multiplying that with the probability of acceptance by some class \( ij \) patient yields the expected marginal total QALYs benefit \( (h_{ijk} - y_{ij}(t))\pi_{n_{ij}k}^{n} \) from transplantation. Theorem 1 justifies this policy analytically, cf. §5.

At the other end of the spectrum, the objective is to minimize only NPDWT, i.e., \( \kappa = 0 \). Like MBTP-Q, the optimal policy for minimizing NPDWT ranks patient classes based on the difference in their benefit with versus without transplantation. Unlike MBTP-Q, the “benefit” associated with transplantation or its absence is defined as the risk of dying on the waiting list. As shown by Proposition 1 in §5, to minimize NPDWT, under certain conditions, it is optimal to allocate organs on the basis of health status and prioritize the sickest patients. Therefore, Proposition 1 implies that if the emphasis is on preventing pre-transplant patient death, then the allocation policy can restrict itself to medical urgency, which corresponds to the current UNOS policy. Further details on how the “benefit” in this case translates into ranking patients according to their medical urgency are given in §5.

The optimal policy for the general case has a similar interpretation. Expected difference in benefits with versus without transplant is again the optimal index for organ allocation. The policy we propose, called the weighted marginal benefit policy (WMBP), cf. §5, is therefore similar to MBTP-Q. However, in this setting, the “benefit” for a patient is expressed as a weighted average of QALYs and pre-transplant mortality risk.

In summary, the optimal allocation rule offers an organ to the patient who is predicted to have the greatest difference between the benefit of receiving a transplant and the benefit of remaining on the waiting list. Therefore, optimal policy characterization boils down to accurately assessing patients’ benefit without a transplant. In the next section, we show that the shadow prices represent the benefit derived by not receiving an organ and can be used for making organ allocation decisions.

### 5. Formal Results

We use optimality conditions to characterize the optimal policy for the general case (\( P_\star \)). In doing so, we associate shadow prices to Equations (2) governing the dynamics of the number of patients in each class as they wait for transplantation. The following theorem establishes properties of shadow prices and characterizes the optimal allocation policy; its proof is given in Appendix B.

**Theorem 1.** A feasible organ allocation policy \( u \) with the corresponding state trajectory \( x \) is optimal for (\( P_\star \)) if and only if there exist shadow prices \( y \) such that

\[
y(t) \leq (d + \alpha - \beta)y(t) - \kappa q + (1 - \kappa)d
\]

for \( t \in [0, T] \), \( y(T) = \kappa \delta \), \hspace{1cm} (4)
and \( u \) satisfies the conditions (5) and (6) given below: For \( i, j, t \),
\[
y_{ij}(t) = -\kappa q_{ij} + (1 - \kappa)d_j + \left[y(t)(d + \alpha - \beta)\right]_t
\]
\[
\text{if } x_{ij}(t) > 0,
\]
\[
u^k(t) = \arg\max_{z \geq 0, \epsilon z \in \partial \mu_k(t)} \left[(k h^k - y(t)) \cdot P^k z\right]
\]
\[
\text{for } k = 1, \ldots, K.
\]

Conditions (5) and (6) of Theorem 1 should be satisfied by any optimal policy for \( (P_\kappa) \). Furthermore, a policy that satisfies (5) and (6) is globally optimal too. The shadow price \( y_{ij}(t) \) can be interpreted as the weighted average of total QALYs and death rate of class \( ij \) patients in the system if they are not transplanted an organ at time \( t \). We propose WMBP based on conditions (5) and (6) by utilizing the observation that given the shadow prices \( y \), the optimal allocation \( u^k(t) \) for \( t \in [0, T] \) and \( k = 1, \ldots, K \) is dictated as the solution to the maximization operation as in (6).

Weighted Marginal Benefit Policy (WMBP). When a liver of type \( k \) arrives, say at time \( t \), the system manager ranks various patient classes \( ij \) (for \( i = 1, \ldots, I \) and \( j = 1, \ldots, J \)) with respect to the effective reward \((k h_{jk} - y_{ij}(t)) \pi_{jk}^n \) for which \( k h_{jk} - y_{ij}(t) \) is highest. The patients in the same class are ordered according to their waiting time in that class. Given this ordering, the system manager offers the organ to the first \( n \) patients in class \( ij \) for which \((k h_{jk} - y_{ij}(t)) \pi_{jk}^n \) is highest. In the case that there are fewer than \( n \) patients in that class, the system manager offers the organ to the patients in the lower ranked classes as well until the organ is offered to \( n \) patients. The organ is transplanted to the patient with the highest rank, who accepts the offer.

Like MBTP-Q, PMPTD ranks patient classes based on the difference of their benefit with versus without transplantation. Unlike MBTP-Q, the “benefit” associated with transplantation or its absence is defined as the risk of dying on the waiting list, which follows from the objective of minimizing NPDWT. This definition of benefit ignores post-transplant outcomes. Moreover, remaining on the waiting list risks pre-transplant death, which is to be avoided. Therefore, the shadow price \( y_{ij}(t) \) can be interpreted as a cost rather than a benefit in absence of a transplant. Accordingly, the priority index is the cost with transplant, i.e., the risk of dying while waiting for transplantation (which is zero), minus the cost without transplant (i.e., \( y_{ij}(t) \)), multiplied by the acceptance probability \((\pi_{jk}^n)\). That is, \(-y_{ij}(t) \pi_{jk}^n\). Here, the cost (i.e., pre-transplant death risk) incurred when a patient continues on the waiting list is multiplied by her acceptance probability to obtain the index for optimal prioritization. PMPTD is based on this logic and assigns livers to patients whose removal from the waiting list avoids the highest risk of a pre-transplant death. Under certain conditions, the patients in the worst medical condition (i.e., having the highest MELD score) will be those facing the biggest risk of dying while waiting for transplantation. In this case, the shadow prices will be ordered in MELD score and the optimal priority scheme coincides with the sickest-first rule of UNOS. The following proposition formalizes this intuition and proves that under certain conditions UNOS policy is optimal for minimizing NPDWT.

**Proposition 1.** Suppose that \( p_{ijk} = p_{ijk} = p_{ijk} \) for all \( i, j, k \). If \( p_{j,k} \geq p_{j-1,k} \geq \cdots \geq p_{1,k} \) for all \( k = 1, \ldots, K \), then an optimal solution to \( (P_\kappa) \) allocates organs to patients in the order of their health status, giving the priority to the sickest patients provided that \( x_{ij}(t) > 0 \) for all \( i, j \) and \( t \in [0, T] \) and \( d_{j+1} - d_j - \beta_j > 0 \) for all \( j = 1, \ldots, J - 1 \).

Proposition 1 provides a justification for why the current UNOS policy gives precedence to the sickest patients. We make three main assumptions in this proposition. First, we assume that the probability of acceptance is nondecreasing in MELD score. Second, we assume that acceptance probabilities are the same for each static type. These two assumptions are in line with the current UNOS practice because UNOS policy is blind to the differences between the acceptance probabilities of the patients in allocating livers. The third assumption implies that the reduction in the probability of death as the patient gets sicker is strictly less than the probability of getting better. This assumption holds for patients with ESLD, such as acute liver disease, because these diseases cause fast deterioration of patients’ health. However, it might not be true for other diseases. Note that these assumptions are sufficient but not necessary,
and we will demonstrate through numerical analysis based on clinical data, cf. §6, that the prediction of Proposition 1 is still valid under more general conditions. The proof of Proposition 1 is provided in Appendix C.

In reality, the acceptance probabilities might differ from one static type to the other. If we relax the assumption that \( p_{ijk} = p_{r,ijk} = p_{jk} \) for all \( i, i', j, k \), one can still show that the shadow prices satisfy \( y_{ij}(t) = y_{ij}(t) = y_{ij}(t) \) for all \( i, i', j \) and \( y_{ij}(t) < y_{ij}(t) \) if \( j > j' \) for \( t \in [0, T] \). In that case, the optimal policy is to assign priorities based on the dynamic index \( y_{ij}(t) \pi_{ijk}^{n} \), where the patient class with the lowest index has the highest rank. Note that because \( y_{ij}(t) < y_{ij}(t) \) if \( j > j' \), implementation of the dynamic index \( y_{ij}(t) \pi_{ijk}^{n} \) ignoring the acceptance probabilities \( p_{ijk}^{n} \) would correspond to the current UNOS policy of allocating organs based solely on medical urgency. Next, we consider the other end of the medical urgency-efficiency spectrum.

**Maximizing Total QALYs.** When the objective is only to maximize the total QALYs, i.e., \( \kappa = 1 \), the problem of UNOS is to choose an organ allocation policy \( u \) to

\[
\begin{align*}
\text{maximize} & \quad \sum_{k=1}^{K} \int_{0}^{T} h^{k} \cdot P^{k} u^{r}(t) \, dt \\
& \quad + \int_{0}^{T} q \cdot x(t) \, dt + x(T) \cdot \theta, \\
\text{subject to} & \quad (1)-(3).
\end{align*}
\]

Theorem 1 for \( \kappa = 1 \) provides the motivation for MBTP-Q, which prioritizes patient classes \( ij \) (for \( i = 1, \ldots, I \) and \( j = 1, \ldots, J \)) based on their marginal benefit in QALYs due to transplantation \( (h_{ij} - y_{ij}(t)) \pi_{ijk}^{n} \). Given this ranking, the system manager offers the organ to the top \( n \) patients.

The next proposition advances an intuitive partitioning policy under additional assumptions. First, this proposition assumes that the terminal rewards and the utility scores are the same across static patient health characteristics implying that patients within the same MELD scores have the same utility. For instance, consider two patients with identical MELD scores: one has cytomegalovirus and the other one has no cytomegalovirus. The existence of cytomegalovirus (like other static patient characteristics) affects only post-transplant outcome; it does not affect the quality of life or pre-transplant life expectancy. Patients’ disutility of being at a particular MELD score is the same. In fact, UNOS also does not primarily consider any differences among various static patient types in determining its priority rule, i.e., UNOS ranks patients according to their MELD scores and then uses only blood type compatibility to rank patients with identical MELD scores. Proposition 2 also reasonably assumes that for a given liver type, there exists an ordering of static patient types according to post-transplant QALYs and acceptance probabilities. Note that the ordering assumption on acceptance probabilities across static patient types provides a more general condition than identical acceptance probabilities across static patient types, an assumption currently used by UNOS as stated before. Under identical acceptance probabilities, the ordering in post-transplant QALYs of patients for a given liver type can be obtained using any post-transplant survival model (Roberts et al. 2004). Although the ordering might be different for each liver type, the proposition provides the ordering in static patient types without loss of generality because the static patient types can be re-labeled according to the post-transplant QALYs.

**Proposition 2.** Suppose that the terminal rewards and the quality-of-life scores of patients waiting for transplantation are the same across static types, that is, if \( \vartheta_{ij} = \vartheta_{i'j} \) and \( q_{ij} = q_{ij} \) for all \( i, i', j \). Suppose also that for each liver type \( k \), static patient types are ordered with respect to acceptance probabilities and expected QALYs after transplantation. That is, for each liver type \( k \), there exists a permutation \( r_{k}(\cdot) \) of \{1, \ldots, I\} such that

\[
\begin{align*}
h_{r_{k}(1)jk} \geq h_{r_{k}(2)jk} \geq \cdots \geq h_{r_{k}(I)jk}, \quad \text{and} \\
p_{r_{k}(1)jk} \geq p_{r_{k}(2)jk} \geq \cdots \geq p_{r_{k}(I)jk}, \quad j = 1, \ldots, J.
\end{align*}
\]

Then, livers of type \( k \) are only assigned to static patient type \( r_{k}(1) = \arg \max \{h_{ijk}\} \) provided that \( x_{ij}(t) > 0 \) for all \( i, j \) and \( t \in [0, T] \).

Proposition 2 shows that there exists a partition \( K_{1}, \ldots, K_{t} \) of liver types \( \{1, \ldots, K\} \) such that liver types in \( K_{i} \) are only assigned to patients of static type \( i \). Thus, organs can be allocated in a hierarchical way: For a liver of type \( k \), only consider the static types \( i \) such that \( k \in K_{i} \). Then, allocate a type \( k \) liver among the static types \( \{i: k \in K_{i}\} \) depending on their health status.

### 6. Computational Results

We simulate the national liver allocation system between 2002–2015 and compare the performances of the proposed policies to the new UNOS policy in NPDWT, total QALYs (TQALYs), NWL, NPDAT within one year after the transplant, and TPD criteria. In addition, the simulation model, described in Appendix D in the electronic companion, is used for the validation of some of the structural results under more general assumptions. We use data from UNOS for the period 2002–2006, whereas a projection of data from 2002–2006 is used for simulating the period 2007–2015. We cross-validated the projection using 2007 UNOS data.

The proposed liver allocation policies are based on shadow prices for the fluid model. We discretize the fluid model on a daily basis, i.e., the decision horizon \([0, T]\) is replaced with \([0, 1, 2, 3, \ldots, T]\). We then solve the discretized model with linear program (LP) to derive the shadow prices. We solve an LP for each region and use 10 replications to calculate the system performance. The variation within the replications is very small (i.e., the standard
deviation is less than 5% of the mean for most cases); therefore, we report only the mean values throughout for brevity.

Because the input parameters of the LPs would change over time, we use a rolling horizon strategy to evaluate our policies and compare our results to the current UNOS policy. Our rolling horizon strategy solves the discretized model periodically for a fixed duration and interacts with the simulation model. For example, if the rolling horizon solution frequency (rf) is 1 year and rolling horizon length (rh) is 10 years, the first LP is solved at the beginning of 2002 for years 2002–2011. Then, using the shadow prices of the solution for 2002, the simulation model is run for 1 year and the system state for 2003 is obtained. The process is repeated until the end of the decision horizon. If the simulation is run between 2002 and 2015, then a total of 14 LPs are solved for each replication.

Recall that we consider the objective of maximizing the total QALYs and minimizing NPDWT through a weighted average of the two criteria. The choice of the parameter $\kappa$ dictates the balance between the two objectives. Figure 2 demonstrates the performance of WMBP relative to the current UNOS policy as the weight parameter $\kappa$ varies when $rh = 2$ years and $rf = 1$ year. As $\kappa$ reduces, the improvement in QALYs over UNOS policy diminishes and the performance in terms of NPDWT improves because a reduction in $\kappa$ shifts the emphasis toward minimizing deaths criterion rather than maximizing QALYs. Indeed, as $\kappa$ approaches zero, WMBP converges to PMPTD, which matches the performance of the UNOS policy.

Table 1 assesses the performance of PMPTD relative to the current UNOS policy under various rh and rf settings between 2002 and 2015. As presented in Table 1, MBTP-Q outperforms the current UNOS policy in TQALY, NWL, and NPDAT criteria, whereas the UNOS policy outperforms MBTP-Q in the NPDWT criterion. Our proposed MBTP-Q results in a 3.98% to 4.9% improvement in total expected QALYs. The relatively worse performance of MBTP-Q in the NPDWT criterion compared to UNOS policy is reflective of Proposition 1, which proves that current UNOS policy is optimal under the objective of minimizing NPDWT. MBTP-Q outperforms UNOS policy in the NWL criterion because MBTP-Q accounts for the acceptance probabilities of the patients, while UNOS does not consider them in prioritizing the patients. We also observe that as the rolling horizon and frequency increase the performance of our policies do not change significantly, which might indicate that our model is robust to parameter changes. We use $rh = 5$ years and $rf = 1$ year unless stated otherwise.

Note that it is not clear how to estimate the terminal rewards. Ideally, one would simulate the optimal policy to
Performance of PMPTD for years 2002–2015 for various rh and rf.

<table>
<thead>
<tr>
<th>PMPTD (α = 0)</th>
<th>% Improvement over current UNOS policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TQALY</td>
</tr>
<tr>
<td>rh = 2 yrs, rf = 3 months</td>
<td>0.66</td>
</tr>
<tr>
<td>rh = 2 yrs, rf = 1 yr</td>
<td>0.76</td>
</tr>
<tr>
<td>rh = 5 yrs, rf = 1 yr</td>
<td>0.82</td>
</tr>
<tr>
<td>rh = 10 yrs, rf = 1 yr</td>
<td>1.10</td>
</tr>
<tr>
<td>rh = 2 yrs, rf = 2 yrs</td>
<td>0.58</td>
</tr>
<tr>
<td>rh = 5 yrs, rf = 2 yrs</td>
<td>0.89</td>
</tr>
<tr>
<td>rh = 10 yrs, rf = 2 yrs</td>
<td>0.95</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>MBTP-Q (α = 1)</th>
<th>% Improvement over current UNOS policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TQALY</td>
</tr>
<tr>
<td>rh = 2 yrs, rf = 3 months</td>
<td>4.67</td>
</tr>
<tr>
<td>rh = 2 yrs, rf = 1 yr</td>
<td>4.90</td>
</tr>
<tr>
<td>rh = 5 yrs, rf = 1 yr</td>
<td>4.37</td>
</tr>
<tr>
<td>rh = 10 yrs, rf = 1 yr</td>
<td>3.98</td>
</tr>
<tr>
<td>rh = 2 yrs, rf = 2 yrs</td>
<td>4.65</td>
</tr>
<tr>
<td>rh = 5 yrs, rf = 2 yrs</td>
<td>4.41</td>
</tr>
<tr>
<td>rh = 10 yrs, rf = 2 yrs</td>
<td>4.07</td>
</tr>
</tbody>
</table>

The numerical experiments described above do not discount future QALYs, whereas there is a debate on whether to discount the QALYs or not (Gold et al. 1996). Furthermore, although the above numerical experiments use QALYs as the performance criterion, several studies suggest that MELD itself may not be a good predictor of the QALYs on the waiting list (Jacob et al. 2005a). Therefore, it is important to evaluate the performance of our model using life years (LYs) instead of QALYs. For this purpose, we perform a sensitivity analysis on the discount factor and quality-of-life scores. Table 3 presents the results of our sensitivity analysis, where the top and bottom panels display the cases when QALYs and LYs are considered as the reward criterion, respectively. Although the improvements obtained by our proposed policy slightly decrease when LYs are used, the trends are generally preserved, and our policy still outperforms the current UNOS policy.

In Table 3, TPD metric is obtained by adding NPDWT and NPDAT, which counts patients who die within a given follow-up period after their transplant surgery. We take into account the post-transplant deaths occurring over 1, 5, and 10 years after the transplant to calculate TPD metric, cf. TPD(1), TPD(5), and TPD(10), respectively. Table 3 shows that MBTP-Q always reduces NPDAT and increases NPDWT over the UNOS policy. If MBTP-Q is implemented, whether the improvement in NPDAT or the deterioration in the NPDWT outweighs depends on the following conditions: If we accept patient deaths that occur within a short timeframe after transplantation, MBTP-Q cannot outperform the UNOS policy in TPD(1) because reduction it achieves for NPDAT is smaller than the increase in NPDWT. With a very long follow-up period, both MBTP-Q and UNOS policy will result in the same TPD because all organ recipients die eventually. Only under moderate time intervals (e.g., 5–10 years), the reduction in NPDAT under MBTP-Q may overweight the increase in NPDWT and MBTP-Q may outperform the UNOS policy under the TPD metric.

7. Concluding Remarks

We develop a dynamic fluid model of the liver allocation system that captures the trade-off between the medical
urgency and the impact of current decisions on the future well-being of the patients on the waiting list, while accounting for disease evolution and patient preferences with the objective to maximize total QALYs and minimize total NPDWT. We analytically solve the model and propose a policy that assigns organs based on the expected marginal benefit (in terms of the weighted average of total QALYs and patient deaths) of transplantation. At one end of the medical urgency-efficiency spectrum, we consider the objective of minimizing total NPDWT. In that case, the current UNOS policy of assigning the organs based solely on medical urgency arises as the optimal policy. This result provides a theoretical justification for the existing UNOS policy. At the other extreme, we solve the dynamic fluid model with the objective of maximizing QALYs. Our proposed policy can be interpreted as follows: when an organ is donated, rank patients based on their marginal benefit when transplanted with that organ, i.e., the difference between their expected QALYs with versus without transplant, and offer that organ to these patients sequentially.

We show that MBTP-Q has the potential to improve the current system along several performance measures. For instance, the alternative policy generates approximately 5% more QALYs and 10% fewer number of wasted livers than the current UNOS policy. Even though MBTP-Q performs relatively worse in the NPDWT criterion, it achieves a significant improvement in one-year post-transplant survival rates. The comparison along the total deaths criterion essentially depends on the length of the follow-up period over which post-transplant deaths are accounted for.

There are several policy implications of our models. The current UNOS policy uses medical urgency as the primary criterion for allocating organs to ESLD patients, which we show is the optimal policy under the objective of minimizing total number of deaths in the waiting list. On the other hand, a prioritization scheme depending on both the medical urgency and the potential future savings of transplanting with a particular organ is optimal when the objective includes maximizing total QALYs. UNOS uses MELD scores for determining the medical urgency of patients, which might still be used for determining the medical urgency of patients in a new liver allocation system, or might be extended with more dynamic clinical characteristics that indicate the status of the liver disease, such as encephalopathy, blood albumin level, history of variceal bleeding, spontaneous bacterial peritonitis, and ascites (Roberts et al. 2004). UNOS uses only blood type compatibility as a secondary factor in determining the priority, which is an important factor indicating post-transplant success. However, blood type compatibility is only one of the many clinical factors that affect the outcome of the transplantation (Roberts et al. 2004). UNOS may consider the use of a more comprehensive model to represent the post-transplant outcomes.

There exist models estimating the expected transplant outcomes for given patient and organ characteristics (Roberts et al. 2004, Olthoff et al. 2005). Although we use the model by Roberts et al. (2004), any post-transplant survival model can be used to implement our proposed policy. As discussed before, our policy’s suggestion to allocate the donated livers based on the marginal benefit of transplant and to include factors other than the MELD score is also supported by the findings of Schaubel et al. (2009). Schaubel et al. (2009) show that the prioritization of the patients based on the difference between post-transplant and waitlist life expectancy, which is similar in spirit to MBTP-Q, improves the performance of liver allocation system significantly.

An important issue in any public policy design problem such as the organ transplant problem is equity among various patient groups. That is, the organ allocation system should not put any part of the society in a disadvantageous position for having access to available organs. Some of the optimization studies in the organ transplant area (Su and Zenios 2006, Zenios et al. 2000) explicitly address the equity issue. This paper did not explicitly consider the equity issue, and one might argue that our proposed policy disfavors some classes of patients over others, which might violate the equity among various patient types. For instance, the post-transplant survival model of Roberts et al. (2004) suggests that the post-transplant life expectancy of a young patient is significantly higher than that of an identical older patient when transplanted with the same organ, which might reduce the access of older patients to organs. Note that our analytical model is able to explicitly model the equity issue by simply adding a new term

### Table 3. Sensitivity analysis on quality-of-life scores and discount factors with \( \kappa = 1 \), \( rh = 5 \), \( rf = 1 \).

<table>
<thead>
<tr>
<th>Quality-of-life score</th>
<th>Annual discount factor</th>
<th>% Improvement over current UNOS policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TQALY</td>
<td>NWL</td>
</tr>
<tr>
<td>QALYs</td>
<td>1</td>
<td>4.37</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>3.77</td>
</tr>
<tr>
<td></td>
<td>0.97</td>
<td>3.26</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>2.93</td>
</tr>
<tr>
<td>LYs</td>
<td>1</td>
<td>3.88</td>
</tr>
<tr>
<td></td>
<td>0.99</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td>0.97</td>
<td>2.63</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>2.54</td>
</tr>
</tbody>
</table>
to the objective function or an additional constraint. On the other hand, instead of explicitly modeling the equity issue in this paper, we suggest the following to implement our proposed policy: Calculate the post-transplant and pre-transplant life expectancy for each patient class by removing factors that might violate the equity among patients, such as age, gender, and race, while keeping clinical factors that do not cause an inequity, such as blood type compatibility with the donor, cytomegalovirus, encephalopathy, and/or disease type. The determination of which factors should be included in such an alternative prioritization scheme requires a comprehensive multidisciplinary study by experts in public policy, ethical studies, and transplantation. Such a study is beyond the scope of this research.

Our proposed policies consider the acceptance probability of ESLD patients in allocating a liver; however, implementing such a policy might not be practical because of potential ethical issues. On the other hand, the transplant community has long debated whether to penalize transplant centers who decline organs more frequently than others (Howard 2002). Indeed, the European transplant system implemented a variant of this penalizing policy where the number of offers per organ declined from 3.5 to 2.3 within 18 months (Jost et al. 1993). To the best of our knowledge, designing a mechanism for implementing a policy that penalizes transplant centers for declining more organ offers is an open research question. Our proposed policy provides an analytical model for considering organ refusal rates in allocating organs to patients and hence may help UNOS to implement such a policy.

Admittedly, our study has several limitations and does not address all of the policy questions regarding a liver allocation system such as geography. The efficient region design has been studied by Stahl et al. (2005) and Kong et al. (2010) for the liver allocation system. Moreover, we assume that the organ acceptance probabilities are not affected by the organ allocation policy, which might be a strong assumption. Analyzing the effects of changing the allocation policy on patient preferences in accepting/failing liver offers requires a dynamic mechanism design approach, which is left for future research. Another important issue related to the liver allocation system is based on the value of “hope,” an issue studied extensively previously by Howard (2001). The society might prefer an allocation system where all ESLD patients have a nontrivial probability of receiving an organ, which is not explicitly addressed in our paper. On the other hand, there is no foreseeable difference between our proposed policy and the current UNOS policy in terms of the hope of the patients.

Furthermore, there is evidence that the severity of the liver disease represented by MELD in our model is a poor predictor of post-transplant survival, whereas a patient’s perceived health status is a better predictor (Jacob et al. 2004 and 2005a, b), which is ignored by the post-transplant survival model of our numerical study. As Jacob et al. (2005a) note, the inclusion of a patient’s functional status or more general measures of a patient’s pre-transplant health status might improve the prediction of post-transplant survival. Note that our model can account for the functional status by incorporating it into the dynamic patient health states.

Electronic Companion

An electronic companion to this paper is available as part of the online version at http://dx.doi.org/10.1287/opre.1120.1064.

Endnotes

1. Note that in some cases, livers are declined due to unavailability of the surgeon and/or the patient.

2. More precisely, $d$ is a diagonal matrix such that $i$th diagonal entry is $d_i$. In matrix $\beta$, for all $i = 1, \ldots, I$, in row $i1$, only the 12th entry is nonnegative and is equal to $\beta_{2i}$, in row $ij$ for $j = 2, \ldots, J - 1$, $i$th column is equal to $-\beta_j$, and $(ij + 1)$st column is equal to $\beta_{j+1}$, and all other entries are zero. In row $ij$, only $ij$th column is equal to $-\beta_j$ and the rest are zero. Similarly, in matrix $\alpha$, for all $i = 1, \ldots, I$, in row $i1$, only the 1st entry is nonnegative and is equal to $\alpha_i$, in row $ij$ for $j = 2, \ldots, J - 1$, $ij$th column is equal to $\alpha_j$ and $(ij - 1)$st column is equal to $-\alpha_{j-1}$ and all other entries are zero. Finally, in row $ij$, only $(ij - 1)$st column is equal to $-\alpha_{j-1}$ and the rest are zero. 3. We normalize $k$ so as to ensure that QALYs and number of deaths are of the same order.

Acknowledgments

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References


in medical decision making and health operations management problems, including those related to cancer screening and liver transplantation.

Adnan Said is an associate professor in the Division of Gastroenterology and Hepatology, Department of Medicine, at the University of Wisconsin School of Medicine and Public Health. He also serves as the chief of gastroenterology and hepatology at the William S. Middleton Veterans Affairs Medical Center. His research interests lie in studying outcomes with chronic liver disease, including post-transplant outcomes. He also has a research interest in obesity-related liver disease linked to the metabolic syndrome (fatty liver disease) and has published in these areas.