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Author(s): Canice J. Prendergast
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A Theory of Responsibility in Organizations

Canice J. Prendergast, *University of Chicago*

This article considers the implications of allowing a manager discretion over task assignment. If employees earn rents from carrying out tasks, and the manager cannot "sell" the jobs to her subordinates, she has an incentive to take on more tasks than is optimal and delegate too few to a subordinate. I show that although firms can alleviate this incentive by offering output-contingent contracts, even with the optimal contract, (i) the manager carries out too many tasks, (ii) she exerts too much effort on her own tasks, and (iii) her subordinate exerts too little effort on his tasks.

Managers typically hold considerable discretion over the organization of production within their departments. In many cases, the defining characteristic of a manager is that she must make decisions to cater for unforeseen circumstances so that discretion is required by definition. In this article, I consider whether a manager is likely to use that discretion in an efficient fashion. In particular, are managers willing to delegate tasks to subordinates if the subordinates are more capable of carrying out these tasks? The principal result of the article is that a manager who chooses how production is organized within a department is likely to hoard responsibility, in the sense that she carries out more tasks than is optimal herself and refuses to delegate tasks to subordinates, even when firms can write output-contingent contracts which can constrain such behavior.

There is a large literature on the optimal assignment of responsibility within organizations. For example, see Calvo and Wellisz (1978), Rosen (1982), and Geanakoplos and Milgrom (1991). This literature has been

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primarily concerned with technological issues, such as whether responsibility (or span of managerial control) is complementary with managerial ability. This work has been used to explain, for example, why wages rise rapidly as workers ascend an organization's hierarchy. In contrast, managerial theories of the firm argue that management often uses its discretion in inefficient ways. For example, Marris (1964) argues that managers may be growth maximizers, while Shleifer and Vishny (1989) contend that they may pursue strategies which make themselves indispensable to the firm.

This article combines the themes of allocation of authority and rent seeking by assuming that employees derive returns from carrying out tasks through on-the-job learning. In particular, it is assumed that by carrying out tasks, agents obtain skills that increase future productivity. For example, by operating a machine, a worker is assumed to develop an understanding of when it is likely to break down. Similarly, holding a client's account may provide information on client needs. It is further assumed that these skills translate into higher wages, as in Becker (1964). An immediate implication of this is that unless the manager can "sell" the jobs to subordinates, there is an incentive for the manager to hoard responsibility.

If managers collect skills from carrying out tasks, it is not surprising that they have an incentive to overextend themselves if they are not responsible for the output that is produced as a result. However, the principal result of the article is that even when a firm can potentially constrain the manager's inefficiencies by writing output-contingent contracts, the manager still continues to carry out more tasks than is efficient in the optimal contract.

In the model constructed below, a manager chooses whether she or a subordinate (the worker) carries out each of a set of tasks. Output in any task is assumed to depend on (i) the effort exerted by the person assigned to the task, and (ii) how many other tasks the agent carries out (with output in any task falling as the number of other tasks carried out increases). Skills are collected by carrying out tasks which translate into higher future wages. A key assumption is that the employees are liquidity constrained; this implies that the manager cannot extract future rents from her subordinate by reducing his initial wage. It is further assumed that outputs can be contracted on, though the task assignment itself is too nebulous to allow contracting. Thus, the emphasis of the article is on arm's-length contracting with the manager, who is evaluated only on output. The manager provides incentives to the worker by offering him output-based contracts for those tasks he does. The principal results of the article are that given this environment, the manager (i) carries out more tasks than is optimal and (ii) offers the most routine tasks to her subordinate. Furthermore, (iii) in equilibrium she works too hard on her own tasks to cover for hoarding responsibility, whereas (iv) her subordinate exerts too little effort on his tasks.

The intuition for the results is as follows. By taking on too many tasks, the manager's productivity falls because she is spreading herself too thin. But her effort can increase to compensate for this. Therefore, when choosing how to allocate tasks, the manager realizes that for any task, she faces an isoquant over task assignments and effort to attain any given level of output. Then the firm's objective is to choose output-contingent contracts that maximize surplus subject to the manager's incentives, which are determined by the manager's preferred point on the isoquant described above. It turns out that if the firm induces the efficient level of outputs in all jobs, then the manager assigns herself too many tasks but works too hard to cover for this. The worker is induced to exert too little effort.

But the firm can induce the manager to produce any level of output through its choice of contracts so in principle it could induce efficient task assignment. In the model, the firm can write forcing contracts that induce the worker to produce some "required" level of output, so that the firm can effectively choose an output level for each task. When the firm chooses optimal output levels, it trades off the efficiency-enhancing effect of better job allocation with distortions from excessive or deficient effort on those tasks. More specifically, by increasing output on the manager's tasks beyond the efficient level, the manager assigns tasks more efficiently but exerts more effort. Since the manager is already exerting too much effort when the efficient level of output is required, this involves further distortions on the effort dimension. Trading off the distortions on these two dimensions implies that the optimal task assignment involves too many tasks carried out by the manager and too much effort exerted by her.

Now consider the effect of increasing required output on one of the worker's tasks beyond the "efficient" level. This *increases* the incentive for the manager to take on more tasks herself, in order that the worker carries out his tasks better (as he has fewer to concentrate on). To mitigate the manager's incentive to hoard responsibility, the firm chooses "low" rather than "high" output in those jobs known to be carried out by the worker. In equilibrium, this implies that the worker is required to exert less effort than the efficient level. Hence the equilibrium involves not only that the manager carries out too many tasks but also that she works harder than her subordinate, ceteris paribus.

I. The Model

There are two employees, a manager and a worker, who are employed in a firm for a single period. There is a continuum of tasks, defined over the interval [0, 1], where each task must be assigned to one of the two agents.[1] Task assignments are made at the beginning of the period by the manager.

[1] Assuming a continuum of tasks is useful only because it allows me to treat the number of tasks as a real number in the analysis below.

Output in task i depends on two factors: (i) the effort exerted by the agent assigned to task i, e^i, and (ii) task assignments. More specifically, let α be the fraction of tasks carried out by the manager, with $1 - \alpha$ being carried out by the worker. Then output in task i is assumed to be given by

$$y^i = \begin{cases} e^i - \mu_m(\alpha) & \text{if the manager carries out the task,} \\ e^i - \mu_w(1-\alpha) & \text{if the worker carries out the task.} \end{cases} \tag{1}$$

It is assumed that the μ_i functions are increasing and convex, that is, $\mu_i' > 0$, $\mu_i'' > 0$, $\mu_i'(0) = \infty$, and $\mu_i'(1) = -\infty$. The cost of effort in task i is assumed to be $C^i(e^i)$, where $C^i(0) = 0$, $C^{i\prime} > 0$, and $C^{i\prime\prime} > 0$. Note that the cost of exerting effort is independent of (i) other tasks carried out and (ii) the identity of the agent carrying out the task. This implies that the inefficiencies that arise here are caused by the manager carrying out the wrong proportion of tasks, rather than the manager simply doing the *wrong* tasks.

The economic interpretation of this technology is that output depends on (i) effort exerted by the agent assigned to that task, and (ii) how many other tasks that agent also has to carry out. As the number of other tasks that the agent carries out increases, the agent can devote less attention to any given task, and so output falls. Therefore there are costs to any single agent carrying out too many tasks.

Skill Collection

Skills are assumed to be collected through on-the-job learning and have value to the agent as his or her reservation wage increases in an (unmodeled) future period. In particular, the agent who carries out task i is assumed to collect skills which increase his or her future utility by $\nu^i > 0$. Without loss of generality, let jobs be ordered such that $\nu^i \le \nu^j$ for $i > j$ so that job 0 has the highest rents and task 1 has the fewest.

ASSUMPTION 1. Task assignments are too nebulous to be included in a labor contract. Contracts can be based only on output in any given task.

Preferences

It is assumed that the manager and the worker are risk neutral over all wages above their reservation wages. Utility from carrying out task i is

$$U^i = w^i - C^i(e^i) + \nu^i, \tag{2}$$

where w^i is the transfer to the agent for job i. The total utility for an agent is merely U^i integrated over all tasks carried out by the agent. The manager is assumed to have a reservation wage of m, while the worker has a reservation wage of w.

The purpose of the article is to illustrate the impact of rent seeking on task allocation so that I assume liquidity constraints that imply an inability to extract these rents when the manager assigns tasks.[2]

ASSUMPTION 2. The manager and the worker must be paid at least their reservation wages.

The symmetric way in which the liquidity constraints bind is discussed in more detail below.

Contracting

It is assumed that the firm offers the manager a contract offering a transfer $t^i(y^i)$ if output in task i is y^i. The contract is offered to the manager for all tasks. The manager then contracts the worker to carry out some fraction $1 - \alpha$ of the tasks where he is paid $\tau^i(y^i)$ for output y^i.[3] Contracts to the worker and the manager must be incentive compatible and individually rational.

Strategies

To summarize, the strategies of the three actors are as follows.

The firm.—It chooses $t^i(y^i)$, the contracts offered to the manager for all i.

The manager.—She chooses (i) α, task assignments, (ii) e^i, her own efforts on those tasks she allocates herself, and (iii) $\tau^i(y^i)$, contracts offered to the worker for his tasks.

The worker.—He chooses e^i, his effort on those tasks assigned to him by the manager.

The Use of Forcing Contracts

The firm offers contracts $t^i(y^i)$ to the manager for all jobs. The manager then subcontracts with the worker, offering $\tau^i(y^i)$ for all jobs that the worker carries out. The form of the contract offered by the manager and the firm is simple to construct: forcing contracts can be designed that induce effort

[2] Assuming liquidity constraints rather than more generally assuming risk aversion implies that the firm can extract none of the rents "up front" from the workers. When the worker and manager have risk-averse utility functions, this is no longer true, and the contract designed by the firm may no longer maximize surplus but may also include the ability to extract rents from the agents, which complicates the analysis.

[3] It is assumed that the firm cannot base its contracts with the manager on the contracts offered by the manager to the worker, in keeping with the arm's-length relationship between the firm and the manager being studied here.

exertion for the marginal cost of that effort for any (known) task assignment.

As tasks are ordered such that higher tasks offer lower rents, the manager will choose a fraction of tasks α, where she carries out all tasks from 0 to α, and the worker carries out all tasks from α to 1. Let the manager's equilibrium selection of tasks be given by α^{**}. Then begin by considering the contracts offered by the manager to the worker for task $i > \alpha^{**}$. Suppose that the manager wishes to induce an effort level e^i. The manager can never induce the worker to exert that effort for a marginal cost of less than $C^i(e^i)$; otherwise, the choice of effort is not incentive compatible.[4] But the manager can simply devise a forcing contract which induces effort of e^i for a marginal cost of (almost) $C^i(e^i)$. This is done by offering a transfer that satisfies

$$\tau^i(y^i) = \begin{cases} x^i + C^i(e^i) + \varepsilon & \text{if } y^i = e^i - \mu_w(1 - \alpha^{**}), \\ x^i & \text{otherwise,} \end{cases} \tag{3}$$

for arbitrarily small ε. The worker's individual rationality constraint is then satisfied by

$$\int_{\alpha^{**}}^{1} x^i \, di = w. \tag{4}$$

This compensation scheme induces effort of $\{e^i\}$ for the marginal cost of that effort. Note that this contract does so without violating the worker's liquidity constraint as (4) holds even if no effort is exerted.

The manager can therefore induce the worker to exert efforts $\{e^i\}$, $i > \alpha^{**}$, at a wage cost of $w + \int_{\alpha^{**}}^{1} C^i(e^i)di$. But it also follows that the firm can write a contract with the manager that induces efforts of $\{e^i\}$ for *all* i at a wage cost of $m + w + \int_0^1 C^i(e^i)di$ for any known $\{\alpha^{**}\}$. This is done by offering the manager a contract where (i) for all tasks $i > \alpha^{**}$, $t^i(y^i) = \tau^i(y^i)$ and (ii) for all $i < \alpha^{**}$,

$$t^i(y^i) = \begin{cases} z^i + C^i(e^i) + \varepsilon & \text{if } y^i = e^i - \mu_m(\alpha^{**}), \\ z^i & \text{otherwise,} \end{cases} \tag{5}$$

for the manager's tasks, where

$$\int_0^{\alpha^{**}} z^i \, di = m. \tag{6}$$

[4] Remember that the worker can exert zero effort and receive at least w.

Because the manager's choice of α^{**} will be induced from her incentives below, this section implies that the manager can induce any level of effort for the marginal cost of that effort, conditional on α^{**} being optimally chosen by the manager.[5]

II. Distortions

The analysis thus far carries two useful points for the design of optimal contracts. First, the manager earns rents of ν^i from task i, which provides the incentives for overextending herself. Second, forcing contracts can be used to induce any level of effort from either agent for the marginal cost of that effort. Consequently, when the firm designs its contracts, it will do so in order to maximize total surplus (as there are no direct costs to inducing effort exertion) subject to the manager's incentives when allocating tasks.

Begin by considering the first-best. This is characterized by

$$1 - C^{i\prime}(e^{i*}) = 0 \quad \text{for all } i, \tag{7}$$

and[6]

$$\mu_{\mathrm{m}}(\alpha^*) + \int_0^{\alpha^*} \mu'_{\mathrm{m}}(\alpha^*)di = \mu_{\mathrm{w}}(1 - \alpha^*) + \int_{\alpha^*}^{1} \mu'_{\mathrm{w}}(1 - \alpha^*)di. \tag{8}$$

However, the manager holds discretion over task assignment so this combination of efforts and task assignment may not be incentive compatible. For notational simplicity, let $V(\alpha) = \int_0^\alpha \nu^i di$ be the rents obtained by the manager, where $V' > 0$, $V'' \leq 0$.

The manager can be rewarded only on the basis of output. This implies that all combinations of α and $\{e^i\}$ that produce the same level of output carry the same contractual rewards. Given this, I now show that the first-best is impossible. To see this, consider any contract where the first-best outputs $\{y^{i*}\}$ are produced, where y^{i*} is the output produced if (7) and (8) are satisfied. The manager then chooses α and $\{e^i\}$ to maximize her welfare

[5] Note therefore that the distortions that arise below do not occur as a result of effort exertion being costly to induce. Instead, they are caused by the manager's ability to use two instruments to attain any required level of output.

[6] The second-order condition on task assignments,

$$-\mu'_{\mathrm{m}} - \int_0^{\alpha^*} \mu''_{\mathrm{m}}\, di - \mu'_{\mathrm{w}} - \int_{\alpha^*}^{1} \mu''_{\mathrm{w}}\, di < 0,$$

holds through the convexity of the μ_i functions.

subject to expected output being $\{y^{i*}\}$.[7] While keeping output constant, the manager can vary α and $\{e^i\}$ such that

$$\frac{de^i}{d\alpha} = -\frac{\partial y^i/\partial \alpha}{\partial y^i/\partial e^i} = \mu'_{\mathrm{m}}(-\mu'_{\mathrm{w}}) \tag{9}$$

if the manager (worker) does the task, $i \neq \alpha$. This measures the slope of the isoquant facing the manager for the ith task. The manager earns rents of $V(\alpha)$ from carrying out tasks (the manager cannot extract any of the worker's rents due to the liquidity constraints) but at a cost of $\int_0^1 C^i(e^i)\,di$ (her own effort costs plus the monetary cost of inducing effort from the worker using the forcing contracts described above). The manager's objective is then to

$$\max V(\alpha) - \int_0^1 C^i(e^i)di \tag{10}$$

subject to $y^i = y^{i*}$ for all i and (9). Pointwise maximization yields the first-order condition

$$V'(\alpha^{**}) + \int_0^1 C^{i\prime}(e^{i**})\frac{\partial y^i/\partial \alpha}{\partial y^i/\partial e^i}\,di = 0, \tag{11}$$

which implies that[8]

$$\begin{aligned} &V'(\alpha^{**}) - C^{\alpha^{**}\prime}(e^{\alpha^{**}})[\mu_{\mathrm{m}}(\alpha^{**}) - \mu_{\mathrm{w}}(1-\alpha^{**})] \\ &- \int_0^{\alpha^{**}} C^{i\prime}(e^{i**})\mu'_{\mathrm{m}}(\alpha^{**})di + \int_{\alpha^{**}}^1 C^{i\prime}(e^{i**})\mu'_{\mathrm{w}}(1-\alpha^{**})di = 0, \end{aligned} \tag{12}$$

where $\{e^{i**}\}$ and α^{**} are the manager's equilibrium choice of efforts and tasks. Then if $e^{i**} = e^{i*}$, (12) simplifies to

[7] Remember that the manager also "chooses" the worker's effort through her choice of contracts.

[8] The second-order condition,

$$V'' + \int\left[-C^{j\prime\prime}\left(\frac{\partial y^j}{\partial \alpha}\right) + C^{j\prime}\frac{\partial^2 y^j}{\partial \alpha^2}\right]dj < 0,$$

is assumed to hold.

$$
\begin{aligned}
&-\mu_m(\alpha^{**}) - \int_0^{\alpha^{**}} \mu'_m(\alpha^{**})di + \mu_w(1-\alpha^{**}) \\
&\quad + \int_{\alpha^{**}}^{1} \mu'_w(1-\alpha^{**})di + V'(\alpha^{**}) = 0,
\end{aligned} \tag{13}
$$

violating the conditions for the first best. Proposition 1 immediately follows.

PROPOSITION 1. Efficient contracts do not exist.

III. The Optimal Contract

With forcing contracts, the firm's wage bill is $m + w + \int_0^1 C^i(e^i)\, di$. In effect, the firm chooses output levels $\{y^i\}$ for all i subject to the manager's incentives. The optimal contract then chooses $\{y^i\}$ to maximize

$$
\int_0^1 [y^i - C^i(e^i)]di - \int_0^{\alpha} \mu_m(\alpha)di - \int_{\alpha}^{1} \mu_w(1-\alpha)di - m - w, \tag{14}
$$

subject to (11).

PROPOSITION 2. The optimal contract involves (i) the manager carrying out too many tasks, (ii) the manager working harder than optimal on her tasks, and (iii) the worker exerting inefficiently little effort on his tasks.

Proof. See the appendix.

The intuition for this result is as follows. If the contract induces the manager to produce the first-best levels of output, the manager carries out too many tasks (as she collects rents from doing so), exerts too much effort (as she must work harder than optimal to cover for spreading herself too thin), while the worker does not work hard enough (as he need not work optimally to reach the first-best level of output, since he has "too few" tasks to concentrate on). The firm can manipulate the manager's incentives by requiring that she produce output at other than the first-best level. More specifically, by increasing the output level y^i at which the manager gets rewarded in the forcing contract, she distorts less on task assignment because reaching that higher output requirement takes more effort, the marginal cost of which is increasing. In the appendix, it is shown that

$$
\frac{d\alpha}{dy^i} = \frac{-C^{i\prime\prime}(\partial y^i/\partial\alpha)}{\int [-C^{j\prime\prime}(\partial y^j/\partial\alpha)^2 + C^{j\prime}(\partial^2 y^j/\partial\alpha^2)]dj + V''}. \tag{15}
$$

For $\alpha > \alpha^*$, $(\partial y^i/\partial\alpha)$ is negative for the manager's tasks and positive for the worker's tasks. As the denominator of (15) is negative from the worker's second-order condition, this implies that increasing required output on the manager's tasks reduces α, while increasing output on the worker's tasks increases α. Then the firm can reduce the incentive for the manager to overextend herself by increasing output requirements on her tasks and by reducing output requirements on the worker's tasks. In more familiar terms, if the manager is required to produce more on her tasks, she becomes more efficient in task assignment as her alternative is to work even harder, which is costly. However, if the required output on the worker's tasks rise, the manager assigns tasks even less efficiently, so that the worker has fewer to concentrate on.

To see why the firm does not completely eliminate the manager's desire to overextend herself, consider the manager's tasks. As output requirements rise above the optimal level, the manager assigns tasks more efficiently but also exerts more effort, as both inputs are normal. While assigning tasks more efficiently is good, the manager is already exerting too much effort so that there are costs associated with increasing output as the manager works even harder. The optimal contract trades off these costs and benefits, where an interior solution implies that the marginal benefit of improved task allocation is traded off against the (strictly positive) welfare costs of excess effort. This implies that the manager carries out too many tasks in equilibrium. A similar argument can be made for the worker's tasks, but where here the costs are incurred in terms of too little effort.

One point worth noting is that although the worker can be induced to exert the optimal level of effort, he is not required to do so, although he has no role in choosing task assignments. The reason for this is that the firm can use the manager's accountability for the worker's output to its benefit. By requiring that the worker produce "too little" output, so that in some sense there is excess supply of the worker's time, the manager can be offered incentives to delegate tasks more efficiently. Note also that the model predicts that the manager will offer the worker the most routine tasks in the sense that they provide the least opportunity to collect skills.

A caveat is necessary here. In the argument given above, the firm trades off the benefits of decreases in α with increases in effort when it increases output on the manager's tasks. This yields the result that the optimal contract offers the manager too many tasks, partly because the marginal effect of task assignment and effort are technologically independent. However, consider a case where output in task i is $y^i(\alpha, e^i)$ where y^i_{12} is nonzero. Then the analogue to (15) is

$$\frac{d\alpha}{dy^i} = \frac{-C^{i\prime\prime}(\partial y^i/\partial\alpha) - C^{i\prime}(\partial^2 y^i/\partial e^i\partial\alpha)}{\int [-C^{j\prime\prime}(\partial y^j/\partial\alpha)^2 + C^{j\prime}(\partial^2 y^j/\partial e^j\partial\alpha)(\partial y^j/\partial\alpha) + C^{j\prime}(\partial^2 y^j/\partial\alpha^2)]dj + V''}. \tag{16}$$

Unlike (15), this expression cannot be unambiguously signed. The implication of this is that the results on effort exertion do not generalize to other technologies, for the reason that the returns to effort exertion may depend on the task allocation itself.

Before concluding, note the importance of the liquidity constraints. On the one hand, if neither party is liquidity constrained, the manager can extract rents from the worker without distorting task assignments so that the first best is attainable. On the other hand, if only one party is liquidity constrained (or the liquidity constraints are asymmetric), the problem is more complicated. For example, consider the case where only the worker is credit constrained. Then the firm will no longer attempt to maximize surplus as it does above but instead will want to give "too many" tasks to the manager, as the firm can extract rents from the manager but not from the worker. Here the manager would still take on too many tasks: the only difference is that the firm wants it to.

IV. Conclusion

This article should be seen as identifying a possible inefficiency that can arise from allowing managers discretion over designing how production is organized in their departments. The key assumptions generating this result are (i) managers receive returns from carrying out tasks, (ii) firms only use arm's-length contracting over output rather than "micro managing," and (iii) rents cannot be extracted up front. In this environment, it was shown that managers allocate too many tasks to themselves, offer routine tasks to their subordinates, exert too much effort on their own tasks, while workers exert too little effort on their activities.

To conclude, note that it has been assumed that task assignment cannot be contracted on. Realistically, managers are offered considerable latitude over allocating resources within their sphere of influence, not because it is impossible to observe what they are doing, but rather because monitoring who does what is costly, so that firms may prefer to evaluate on the basis of the "bottom line," that is, output. This is probably the more appropriate interpretation of the assumption on noncontractibility of tasks.

Another restrictive assumption is that the cost of effort exerted is independent of (i) the agent carrying out the job and (ii) other tasks carried out by the agent. This is unrealistic in many circumstances. Yet this simplification allows me to consider distortions caused by a manager overextending herself, while ignoring the problem that the manager may carry out the wrong tasks independent of the number carried out. As such, it appears to be a reasonable first step in analyzing this complex problem.

Appendix

Proof of Proposition 2

The first-order condition to maximizing (14) is given by

$$\int_0^1 \frac{\partial y^k}{\partial \alpha}\, dk \frac{d\alpha}{dy^i} + \int_0^1 (1 - C^{k\prime}) \frac{de^k}{dy^i}\, dk = 0. \tag{A1}$$

But since $y^k = e^k - \mu_i$, $i = \mathrm{m}, \mathrm{w}$, this implies that

$$\frac{de^i}{dy^i} = 1 - \frac{\partial y^i}{\partial \alpha}\frac{d\alpha}{dy^i} \tag{A2}$$

and

$$\frac{de^k}{dy^i} = -\frac{\partial y^k}{\partial \alpha}\frac{d\alpha}{dy^i}, \quad i \neq k. \tag{A3}$$

Totally differentiating (11) yields

$$\begin{aligned} &C^{i\prime\prime}\frac{\partial y^i}{\partial \alpha}\frac{de^i}{dy^i} + C^{i\prime}\frac{\partial^2 y^i}{\partial \alpha^2}\frac{d\alpha}{dy^i} \\ &+ \int_{i+j}\left(C^{j\prime\prime}\frac{\partial y^j}{\partial \alpha}\frac{de^j}{dy^i} + C^{j\prime}\frac{\partial^2 y^j}{\partial \alpha^2}\frac{d\alpha}{dy^i}\right) dj + V''\frac{d\alpha}{dy^i} = 0. \end{aligned} \tag{A4}$$

Substituting for de^i/dy^i and de^j/dy^i gives

$$\frac{d\alpha}{dy^i} = -\frac{C^{i\prime\prime}(\partial y^i/\partial \alpha)}{\int [-C^{j\prime\prime}(\partial y^j/\partial \alpha)^2 + C^{j\prime}(\partial^2 y^j/\partial \alpha^2)]dj + V''}, \tag{A5}$$

which has the same sign because $\partial y^i/\partial \alpha$ as the denominator is negative from the second-order condition. Also note by substitution that

$$\frac{de^i}{dy^i} = 1 + \frac{C^{i\prime\prime}(\partial y^i/\partial \alpha)^2}{\int [-C^{j\prime\prime}(\partial y^j/\partial \alpha)^2 + C^{j\prime}(\partial^2 y^j/\partial \alpha^2)]dj + V''}, \tag{A6}$$

which lies strictly between 0 and 1, and that

$$\frac{de^k}{dy^i} = \frac{C^{i\prime\prime}(\partial y^i/\partial \alpha)\partial y^k/\partial \alpha}{\int [-C^{j\prime\prime}(\partial y^j/\partial \alpha)^2 + C^{j\prime}(\partial^2 y^j/\partial \alpha^2)]dj + V''}, \tag{A7}$$

which is negative if i and k are carried out by the same agent and positive if they are carried out by different agents. The manager's first-order condition is given by

$$V' + \int_0^1 \frac{\partial y^k}{\partial \alpha} C^{k\prime}\, dk = 0. \tag{A8}$$

Also note that substituting (A2) and (A3) into (A1) yields

$$\int_0^1 \frac{\partial y^k}{\partial \alpha} dk \frac{d\alpha}{dy^i} - \int_0^1 (1 - C^{k\prime}) \frac{d\alpha}{dy^i} \frac{\partial y^k}{\partial \alpha} dk + (1 - C^{i\prime}) = 0. \tag{A9}$$

Then substituting (A8) into (A9) gives

$$1 - C^{i\prime}(e^i) = V'(\alpha) \frac{d\alpha}{dy^i}. \tag{A10}$$

Note from (A5) that $d\alpha/dy^i < 0$ for the $i < \alpha$ but is positive for $i > \alpha$. This implies that $C^{i\prime} > 1$ for the manager's tasks and $C^{i\prime} < 1$ on the worker's tasks. Thus, the manager works harder than is optimal, and the worker exerts less effort than is optimal.

This proves (ii) and (iii) of proposition 2. To show that the manager carries out too many tasks in equilibrium, suppose the contrary, so that $\int_0^1 [\partial y^k/\partial \alpha]\, dk \geq 0$. Then consider the effect of increasing all y^i that the manager carries out by a small amount such that all e^i, $i < \alpha$ rise. To do this, let the firm increase all $\{y^i, y^j\}$ pairs in the ratio $dy^i C^{i\prime\prime} = dy^j C^{j\prime\prime}$. Then the change in effort on the manager's tasks induced by this change in the contract is given by

$$\Delta e^k = dy^k \left(1 - \frac{\int_0^\alpha C^{j\prime\prime} (\partial y^j/\partial \alpha)^2 dj}{\int [C^{j\prime\prime}(\partial y^j/\partial \alpha)^2 + C^{j\prime}(\partial^2 y^j/\partial \alpha^2) dj] + V''} \right) > 0. \tag{A11}$$

Similarly, the change induced on the worker's efforts ($i \geq \alpha$) is given by

$$\Delta e^k = \frac{\int_0^\alpha C^{j\prime\prime}(\partial y^j/\partial \alpha)(\partial y^k/\partial \alpha) dy^j dj}{\int [-C^{j\prime\prime}(\partial y^j/\partial \alpha)^2 + C^{j\prime}(\partial^2 y^j/\partial \alpha^2)]\, dj + V''} < 0. \tag{A12}$$

Then the change in welfare from this change in the contracts is given by

$$\Delta = \int_0^1 \frac{\partial y^k}{\partial \alpha}\, dk \Delta\alpha + \int_0^1 (1 - C^{k'}) \Delta e^k\, dk, \tag{A13}$$

where

$$\Delta\alpha = -\frac{\int_0^1 C^{j''}(\partial y^j/\partial\alpha) dj}{\int [-C^{j''}(\partial y^j/\partial\alpha)^2 + C^{j'}(\partial^2 y^j/\partial\alpha^2)]\, dj + V''} < 0. \tag{A14}$$

Then $\Delta < 0$ because the first term in (A13) is nonpositive if $\int_0^1 \{\partial y^k/\partial\alpha\} dk \geq 0$, as has been supposed above. But $\Delta e^k > (<)\ 0$ if $k < (\geq)\ \alpha$ and $1 - C^{k'} < (>)\ 0$ for $k < (\geq)\ \alpha$ from the previous part of the proof. Therefore, surplus falls on effort exertion, but if $\int_0^1 \{\partial y^k/\partial\alpha\} dk \geq 0$, then this change in the contracts also reduces welfare on task assignment. Consequently, this cannot be an equilibrium as there is a deviation that yields higher profits for the firm. Thus, it must be the case that $\int_0^1 \{\partial y^k/\partial\alpha\} dk > 0$, thus completing part (i) of the proof.

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