Distribution of F^* in Hansen … equation (10).

Let X_i ~ iid N(0, I_q), i = 1, …, n.

$$F^* \xrightarrow{d} X_n{}'\left[\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})(X_i - \bar{X})'\right]^{-1} X_n = Z_n{}'\left[\frac{1}{n}\sum_{i=1}^{n-1} Z_i Z_i{}'\right]^{-1} Z_n$$

where Z_i ~ iid N(0, I_q) (using the usual transformation).

Write this as

$$F^* = n\left[\frac{Z_n{}'Z_n}{Z_n\left(\sum_{i=1}^{n-1} Z_i Z_i{}'\right)^{-1} Z_n}\right]^{-1}(Z_n{}'Z_n).$$

Then, following Rao chapter 8b:

(i) $\dfrac{Z_n{}'Z_n}{Z_n\left(\sum_{i=1}^{n-1} Z_i Z_i{}'\right)^{-1} Z_n} \,|Z_n \sim \chi^2_{n-q}$ (which does not depend on Z_n)

(ii) $Z_n{}'Z_n \sim \chi^2_q$

So that $F^* \xrightarrow{d} \dfrac{\chi^2_q / q}{\chi^2_{n-q}/(n-q)} \dfrac{nq}{n-q} = \dfrac{nq}{n-q} F_{q,n-q}$

and (in more familiar F-stat form)

$$F^*/q \xrightarrow{d} \frac{n}{n-q} F_{q,n-q}$$