Equilibrium fluctuations when price and delivery lag clear the market

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To say that the price of some good is inflexible over time has little meaning if the “good” is changing over time. In this article, I concentrate on delivery lags as the only dimension other than price that varies. It is shown how one can predict the relative importance of price and delivery lag fluctuations as equilibrating mechanisms. These fluctuations are related to underlying supply and demand elasticities, and some surprising results are derived. For example, the importance of price fluctuations increases as the absolute value of the price elasticity increases. The surprising results underscore the complexity of predicting price behavior when the characteristics of the good are endogenous. Relatively inflexible prices combined with relatively flexible delivery lags may be the predicted market-clearing response for many industries to fluctuations in supply and demand conditions.

1. Introduction

The idea that prices, especially in manufacturing, are inflexible and nonmarket-clearing appears persistently in the economic literature. (See Carlton 1979b, Section 2, for a brief survey.) Evidence often used to support the nonmarket-clearing hypothesis has been the observation that large quantity adjustments, either through inventory changes or delivery lag changes, and not large price adjustments, characterize the response of many markets to supply and demand changes. This article builds on the ideas in Carlton (1979b) to develop an equilibrium model that is capable of explaining how markets will respond in the short run to changes in supply and demand.

The basic idea of the article is a simple one. Consumers care not only about price but also about quality attributes of a good. In response to supply and demand shocks, adjustment in quality of the good may be more important than adjustment in price. To say that price appears inflexible over time for some good has little meaning if the “good” is changing over time. In this article I concentrate on delivery lags as the only dimension of the good other than price that varies. This is, of course, a simplification, but delivery lag is often one of the easiest (least costly) quality attributes to adjust. In Section 2, I extend the theory developed by Rosen (1974) and Zarnowitz (1962)1 to show how one can predict the relative importance of price and delivery lag fluctuations as equilibrating mechanisms in a competitive market. The case of monopoly is also discussed in Section 2.

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I thank Robert Hall, Alvin Klevorick, Sherwin Rosen, and an anonymous referee for helpful comments and John Geweke and Blaine Roberts for providing me with data. I thank NSF and the Law and Economics Program of the University of Chicago for research support.

1 Zarnowitz (1962) seems to be the first author to stress the use of delivery lags as an equilibrium mechanism. This article owes much to his pioneering work. Other work related to that in this article includes Becker and Lewis (1974), Edlefon (1981), Houthakker (1952), and Theil (1952).
TABLE 1  
Price and Delivery Lag Fluctuations

<table>
<thead>
<tr>
<th>SIC Code</th>
<th>Industry</th>
<th>Standard Deviation of Log of Price</th>
<th>Standard Deviation of Log of Delivery Lag</th>
<th>Median Delivery Lag (months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>Textile Mill Products</td>
<td>.06</td>
<td>.17</td>
<td>1.26</td>
</tr>
<tr>
<td>26</td>
<td>Paper and Allied Products</td>
<td>.05</td>
<td>.08</td>
<td>.46</td>
</tr>
<tr>
<td>331</td>
<td>Steel</td>
<td>.03</td>
<td>.25</td>
<td>1.95</td>
</tr>
<tr>
<td>34</td>
<td>Fabricated Metals</td>
<td>.03</td>
<td>.18</td>
<td>3.06</td>
</tr>
<tr>
<td>35</td>
<td>Nonelectrical Machinery</td>
<td>.04</td>
<td>.25</td>
<td>3.63</td>
</tr>
<tr>
<td>36</td>
<td>Electrical Machinery</td>
<td>.05</td>
<td>.10</td>
<td>3.86</td>
</tr>
</tbody>
</table>

1 Based on a quarterly price series constructed by Al-Samarrie, Kraft, and Roberts (1977) by using BLS data and regrouped to be compatible with Census SIC code definitions. I thank John Geweke for providing me with these data.

2 Based on Census of Manufactures, Bureau of the Census. The delivery lag is calculated as the amount of time it would have to fill the first order based on existing backlog and existing production.

3 Assumes delivery lags are log normally distributed.

The surprising results emerging from the theory underscore the complexity involved in predicting price behavior when the characteristics of the “good” are endogenous. Not only may simple supply-equals-demand models be overly simple, but they might provide the wrong intuition.

Table 1 illustrates just how important fluctuations in delivery time and price are for some selected manufacturing industries. For each of the industries, (percentage) fluctuations in delivery lags are usually much larger than those in prices. The industries range considerably in the size and variability of delivery lags. For example, SIC 26 typically has delivery lags of under 1 month, while SIC 36 typically has delivery lags of almost 4 months. SIC 35 and 331 have the greatest fluctuation in delivery lags while SIC 26 has the least. For SIC 35, delivery lags will typically vary between 2 and 5 months. The purpose of this article is to explain how fluctuations in price and delivery lags equilibrate markets in the short run.

2 Consumers could be other firms. In that case, the profit function of the buyer firm, not an indirect utility function, would express the tradeoffs of buyers between p and k. Just as a derived input demand reflects the total input demand, so too the tradeoffs between p and k of the buyer firm would reflect the tradeoffs of the final consumer. It is straightforward, though tedious, (see Carlton (1979a)) to derive the correspondence between final consumer preferences and buyer firm preferences, even in the complicated cases where the buyer firm’s technology uses many inputs, each of which is subject to a delivery delay. We focus here on the simplest case where there is only one good subject to delivery delay and the buyer is the final consumer. This last case is identical to one involving a buyer firm whose technology is fixed coefficients, and whose other inputs are not subject to delivery delay.
from the list of arguments in \( V \). \(^2\) I further assume that the marginal utility of income, \( V_p \), is constant and equal to 1. The indirect utility function determines those \((p, k)\) combinations that leave the consumer indifferent. Clearly, as \( p \) is raised, \( k \) must fall along any indifference curve, and utility falls as \( p \) or \( k \) increases. There is, however, no reason to suppose that the amount purchased along an indifference curve in \((p, k)\) space is constant.\(^3\) Indeed, one way for a buyer to take advantage of a lower price and longer delivery time is to purchase more of the “good.”

It is not possible \textit{a priori} to characterize the shape of the indifference curves in \((p, k)\) space, though it is possible to derive the conditions (available upon request) under which the indifference curves are concave and to relate the conditions to whether the quantity demanded rises or falls along an indifference curve as \( p \) rises \((k \) falls\). The consumer chooses that available \((p, k)\) combination that yields highest utility.

Firms have a production technology whose costs depend upon how quickly the good is produced as well as upon how much of the good is produced. Hence, for example, it is cheaper to produce 1 unit in 1 week than in 1 minute. The firm is assumed to have already optimized over inventory holdings and technology choice so that we are examining its short-run response to market changes. The firm has two margins with which to concern itself. First, holding delivery time constant, how much should be produced? Second, what delivery time should be chosen? Firms have a restricted profit function \( \pi(p, k) \) from which \((p, k)\) combinations yielding constant profit can be constructed. Clearly, profits increase the higher is \( p \) and the larger is \( k \). In general, along any isoprofit curve the quantity supplied by the firm will vary.\(^4\)

The equilibrium examined is the short-term equilibrium price and delivery delay that clear the market. It is the equilibrium conditioned on the level of inventory holding, advance planning, and choice of technology of the economic agents. [If the demand and supply conditions remained constant forever, (barring small numbers problems) the long-run solution would involve no delivery delays.]

A short-run equilibrium \((p, k)\) combination requires (a) that the marginal rate of substitution for consumers between \( p \) and \( k \) equal the marginal rate of technological transformation for suppliers, and (b) that the total amount demanded equal the total amount supplied. Condition (a) requires that the equilibrium be a point of tangency between the indifference and isoprofit curves. (I assume interior solutions.) This tangency is illustrated as point \( A \) in Figure 1 for concave indifference curves and convex isoprofit curves. Condition (b) determines which of the many possible tangencies between indifference and isoprofit curves is equilibrium.

As demand increases relative to supply, equilibrium moves to a tangency point between a higher isoprofit curve and an indifference curve of lower utility. I want to investigate how the short-run equilibrium price and delivery lag change in response to short-run market shocks in the amount supplied or demanded. There is no reason to believe that a shift in the amount demanded or supplied should necessarily change the underlying tradeoffs between \( p \) and \( k \) (i.e., the indifference and isoprofit curves) for either a consumer or a firm. Therefore, I model the market shocks as shifts in demand relative to supply at every \( p, k \), while holding consumer and firm preferences for \( p \) and \( k \) constant. This seems to be the most neutral type of shift which does not create an obvious bias in favor of either prices or delivery lags as equilibrating devices, but which does cause supply and demand to change. I model this shock by assuming that the number of buyers relative

\(^3\) Zarnowitz (1962) defines an indifference curve so that a constant amount is demanded along it.

\(^4\) As in the case of the indifference curves, it cannot be determined \textit{a priori} whether the isoprofit curves are convex, but conditions for the convexity of the isoprofit curves can be easily derived (available upon request) and depend, in part, upon whether the quantity supplied rises or falls along an isoprofit curve as \( p \) rises \((k \) falls\).
to the number of sellers randomly fluctuates. Without loss of generality I shall assume that the number of competitive firms is one, and the number of consumers is $N$.

The type of market shock being examined could be generated by fluctuations in either the number of consumers or the number of firms. It can also be generated by variations in factors that affect either the indirect utility function or the restricted profit function. For example, if prices (which have been held fixed) of other consumption goods or of other factor inputs vary or if stochastic variables affect production, the same type of shocks as I am examining can occur.\(^5\)

The condition that the consumer's marginal rate of substitution between $p$ and $k$ equals the marginal rate of technological transformation of the firm can be written as (subscripts denote partial derivatives)

$$-\frac{V_k}{V_p} = -\frac{\pi_k}{\pi_p}. \quad (1)$$

The second-order condition to guarantee that (1) represents an optimum is

$$\frac{d}{dk} \left[ -\frac{V_k}{V_p} + \frac{\pi_k}{\pi_p} \right] < 0. \quad (2)$$

The condition that supply equals demand can be written as

$$N(-V_p) = \pi_p \quad (3)$$

by using Roy's identity that quantity demanded equals $-V_p/V_y$, the fact that $V_y$ is assumed equal to 1, and Hotelling's lemma that the quantity supplied equals $\pi_p$. Equations (1) and (2)...

\(^5\)Technically, variation in any factor that enters multiplicatively in $V(p, k)$ or $\pi(p, k)$ will generate the random shocks we are examining. Such shocks are the only ones that shift demand relative to supply by an equal percentage at all $p$ and $k$ but do not alter preferences for $p$ and $k$. 
and (3) are two equations in two unknowns, \( p \) and \( k \). By dividing (3) by (1), it is possible to rewrite the equilibrium conditions as (3) and

\[
N(-V_k) = \pi_k. \tag{4}
\]

We now want to see how the equilibrium price, \( p \), and delivery lag, \( k \), will be altered if \( N \), the number of demanders relative to the number of sellers, suddenly increases. To determine the percentage fluctuations in \( p \) and \( k \) in response to percentage changes in \( N \), we perform comparative statics on (3) and (4) to obtain

\[
\begin{bmatrix}
\frac{\partial \ln (-V_k) - \ln \pi_k}{\partial \ln p} & \frac{\partial \ln (-V_k) - \ln \pi_k}{\partial \ln k} \\
\frac{\partial \ln (-V_p) - \ln \pi_p}{\partial \ln p} & \frac{\partial \ln (-V_p) - \ln \pi_p}{\partial \ln k}
\end{bmatrix}
\begin{bmatrix}
\frac{d \ln p}{d \ln N} \\
\frac{d \ln k}{d \ln N}
\end{bmatrix} = \begin{bmatrix}
-1 \\
-1
\end{bmatrix}. \tag{5}
\]

Equation (5) completely characterizes the equilibrium fluctuations in \( p \) and \( k \) that occur in response to short-run shifts in supply and demand. I now wish to express the quantities in (5) in terms of elasticities of supply and demand. Notice that since all firms are identical and all individuals are identical, elasticities of market curves will be the same as the corresponding elasticities of individual consumers or firms.

The following notation is adopted:

- \( n_p \) = price elasticity of demand \( (\partial \ln (-V_p)/\partial \ln p) \);
- \( n_k \) = delivery lag elasticity of demand \( (\partial \ln (-V_k)/\partial \ln k) \);
- \( \eta^*_p \) = price elasticity of supply \( (\partial \ln \pi_p/\partial \ln p) \);
- \( \eta^*_k \) = delivery lag elasticity of supply \( (\partial \ln \pi_k/\partial \ln k) \);
- \( \theta_k \) = (demand) elasticity of marginal disutility of delivery lag \( \frac{\partial \ln (-V_k)}{\partial \ln k} \);
- \( \theta^*_k \) = (supply) elasticity of marginal profit gain of delivery lag \( \frac{\partial \ln \pi_k}{\partial \ln k} \).

With the above notation and (1), (5) can be rewritten as:

\[
\begin{bmatrix}
\frac{\eta^*_p}{\pi_k} (n_k - n^*_k) & (\theta_k - \theta^*_k) \\
(n_p - n^*_p) & (n_k - n^*_k)
\end{bmatrix}
\begin{bmatrix}
\frac{d \ln p}{d \ln N} \\
\frac{d \ln k}{d \ln N}
\end{bmatrix} = \begin{bmatrix}
-1 \\
-1
\end{bmatrix}. \tag{6}
\]

If delivery lag \( k \) were set arbitrarily, then (3) above would determine equilibrium and only the second row of (6) would represent the comparative statics. In that situation, we would obtain the usual result that price fluctuations are smaller the larger is \( |n_p| \), the absolute value of the price elasticity of demand.

To solve (6) for \( d \ln p/d \ln N \) and \( d \ln k/d \ln N \) is straightforward. Let

\[
D = \left| \begin{array}{cc}
\frac{\eta^*_p}{k \pi_k} & \theta_k - \theta^*_k \\
(n_p - n^*_p) & (n_k - n^*_k)
\end{array} \right| = \frac{\eta^*_p}{k \pi_k} [n_k - n^*_k]^2 - [\theta_k - \theta^*_k][n_p - n^*_p], \tag{7}
\]

These elasticities are generated by looking at what happens to the quantity demanded as either price or delivery lag changes while the other is held fixed. An equivalent alternative interpretation for generating the price elasticities is to postulate that the entire schedule of price as a function of delivery lag shifts infinitesimally so as to leave the optimal delivery lag unchanged. A similar interpretation holds for delivery lag elasticities. Such interpretations are in the spirit of the work of Houthakker (1952) and Theil (1952).
\[
A = \begin{bmatrix}
-1 & \theta_k - \theta_k' \\
-1 & n_k - n_k'
\end{bmatrix} = [\theta_k - \theta_k'] - [n_k - n_k'],
\]
\[\text{and} \quad B = \begin{bmatrix}
p\pi \rho_{k} [n_k - n_k'] & -1 \\
n_p - n_p'
\end{bmatrix} = [n_p - n_p'] - \frac{p\pi \rho_{k}}{k\pi_k} [n_k - n_k'].
\]

It follows then that
\[
\frac{d \ln p}{d \ln N} = \frac{A}{D}.
\]
\[\text{and} \quad \frac{d \ln k}{d \ln N} = \frac{B}{D}.
\]

Therefore,
\[
\frac{d \ln p}{d \ln N} = \frac{A}{D}.
\]
\[
\frac{d \ln k}{d \ln N} = \frac{B}{D}.
\]

The left-hand side of equation (12) is the ratio of the percentage fluctuation in price to the percentage fluctuation in delivery lag. To see how (12) depends on the relevant underlying elasticities, we need to sign A, B, and D. It is not possible a priori to sign A, B, and D without further assumptions. We make the following assumption:

**Assumption 1:** In response to an increase in demand relative to supply, N, the equilibrium price, p, and delivery lag, k, both increase.

In a model where the good has two attributes, p and k, it is possible that in response to demand increases, one attribute of the good could increase while the other decreases. In general, however, price and delivery delays are positively correlated (Zarnowitz, 1973, p. 315), thereby making Assumption 1 quite a reasonable one. Assumption 1 together with (10) and (11) implies that A, B, and D are all of the same sign. The following lemma establishes that this common sign is positive.

**Lemma 1:** The second-order condition (see equation (2)) in conjunction with Assumption 1 and equations (10) and (11) establishes that A, B, and D are all positive.

**Proof:** See the Appendix.\(^7\)

Equations (10)-(12) relate the equilibrium price and delivery lag fluctuations to various elasticities of supply and demand. We now investigate how the underlying elasticities influence the equilibrium fluctuations in price and delivery lag.\(^8\) First, observe that in the expression for D, all quantities except \(\theta_k - \theta_k'\) are signed. By Lemma 1, \(D > 0\). If D is to be positive for all possible equilibrium values of \(n_k, n_k', n_p, n_p'\) across all industries,

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\(^7\) The condition \(B > 0\) is equivalent to the condition that at the equilibrium point, the rate at which total demand increases is greater (or its rate of decrease is less) than that at which supply increases, as \(p\) falls with utility and profit held constant. Curiously, then, under the assumption that equal quantities are demanded and supplied along one indifference and one isoprofit curve, \(B = 0\). It then follows from (11) that, to a first approximation, all equilibration takes place through \(p\).

\(^8\) Because it is possible to change \(V\) and \(\pi\) so as to change independently the elasticities at any one point, it makes sense to see how the comparative statics of the system change as a function of these elasticities. In the discussion below, I present proofs only for demand elasticities. Proofs for supply elasticities are straightforward adaptations of those proofs.
then \( \theta_k - \theta_k^* \) must be positive. If \( \theta_k - \theta_k^* \) is positive, then from (7) \( \partial D/\partial |n_p| > 0 \). Moreover, it follows from (8) and (9) that \( \partial A/\partial |n_p| = 0 \), and \( \partial B/\partial |n_p| < 0 \). Therefore, we have established:

**Proposition 1**: Under Assumption 1 and the condition \( \theta_k - \theta_k^* > 0 \), as the demand curve (or supply curve) becomes more price elastic, the equilibrium fluctuations in \( p \) and \( k \) fall in response to short-run shocks in supply and demand

\[
\left( \text{i.e.,} \frac{\partial}{\partial |n_p|} \frac{d \ln p}{d \ln N} < 0 \quad \text{and} \quad \frac{\partial}{\partial |n_p|} \frac{d \ln k}{d \ln N} < 0 \right).
\]

It is interesting to note that for any particular industry, \( D \) could be positive, yet \( \theta_k - \theta_k^* \) could be negative. In such a case, the equilibrium price fluctuations increase as demand becomes more price elastic.

As the delivery lag elasticity of demand increases, we see that \( D \) unambiguously increases, but both \( A \) and \( B \) also increase. It is possible to prove:

**Proposition 2**: Under the same assumptions as Proposition 1, as the delivery lag elasticity of demand increases in absolute value, the equilibrium price fluctuation falls

\[
\left( \text{i.e.,} \frac{\partial}{\partial |n_k|} \frac{d \ln p}{d \ln N} < 0 \right),
\]

while the equilibrium delivery lag fluctuation may either rise or fall

\[
\left( \text{i.e.,} \frac{\partial}{\partial |n_k|} \frac{d \ln k}{d \ln N} \right) \leq 0.
\]

**Proof**: The proof involves differentiation of (10) and (11) and is available from the author upon request.\(^9\)

Propositions 1 and 2 accord reasonably well with intuition gained from markets where quality is exogenous and price alone clears the market. In those markets too, as demand or supply becomes more price elastic, the equilibrium fluctuations in price in response to supply and demand shifts diminish. Intuition (mine at least) would also suggest that the relative price fluctuations versus relative delivery lag fluctuations would depend negatively on the absolute value of price elasticity of demand and positively on the absolute value of the delivery lag elasticity of demand. The reasoning behind this intuition is straightforward. If the demand curve is very price elastic, when delivery time is held fixed, and very delivery lag inelastic, when price is held fixed, one would expect price not to change much but delivery lag to change a lot in response to shifts in supply and demand. In fact, this intuition is incorrect, as shown in Proposition 3 below.

**Proposition 3**: Under the same assumptions as Proposition 1, the ratio, \( R \), of relative price fluctuation to relative delivery lag fluctuation depends positively on the absolute value of price elasticity and negatively on the absolute value of delivery lag demand elasticity.\(^10\)

**Proof**: From (12), we know that

\[
R = \frac{\frac{d \ln p}{d \ln N}}{\frac{d \ln k}{d \ln N}} = \frac{A}{B} = \frac{(\theta_k - \theta_k^*) - (n_k - n_k^*)}{(n_p - n_p^*) - \frac{p \pi e}{k \pi_k} (n_k - n_k^*)}.
\]

\(^9\) If we drop the assumption that \( \theta_k - \theta_k^* > 0 \), then it is no longer true that \( \frac{\partial}{\partial |n_k|} \frac{d \ln p}{d \ln N} < 0 \).

\(^10\) The reader may wonder why the symmetry present in (3) and (4) does not guarantee symmetry in Proposition 3. The reason is that \( n_p \) and \( n_k \) are not symmetric expressions; \( n_p \) and \( \theta_k \) are.
Clearly,
\[ \frac{\partial R}{\partial n_p} < 0 \quad \text{or} \quad \frac{\partial R}{\partial |n_p|} > 0. \]

Also,
\[
\frac{\partial A/B}{\partial n_k} = -\frac{1}{B} + \frac{A \frac{P_{\pi_p}}{k\pi_k}}{B^2} 
- \frac{(n_p - n^*_p) + \frac{P_{\pi_p}}{k\pi_k} (n_k - n^*_k) + \frac{P_{\pi_p}}{k\pi_k} (\theta_k - \theta^*_k) - \frac{P_{\pi_p}}{k\pi_k} (n_k - n^*_k)}{B^2} 
= \frac{-(n_p - n^*_p) + \frac{P_{\pi_p}}{k\pi_k} (\theta_k - \theta^*_k)}{B^2} > 0,
\]

provided \( \theta_k - \theta^*_k > 0. \) Therefore, \( \partial A/B/\partial |n_k| < 0. \) Q.E.D.

To explain why Proposition 3 is true, it will be useful first to explain why the simple intuition described above, based on supply and demand elasticities, is incorrect. The basic reason is that elasticities of supply and demand do not determine the tradeoffs consumers and firms are willing to make between price and delivery lag, and these tradeoffs play an important role in determining equilibrium. The distinguishing feature of markets with endogenous quality attributes is that the relative \((p, k)\) fluctuations at any equilibrium are determined not only by price and delivery lag elasticities of supply and demand at that point, but by the relative \textit{shapes} of indifference curves and isoprofit curves in \((p, k)\) space at that point. The tradeoffs that consumers and producers are willing to make between \(p\) and \(k\) are not in one-to-one correspondence with demand and supply elasticities at any one point. The relative equilibrium variation of \(p\) and \(k\) to demand and supply shocks will depend on the shapes of the indifference and isoprofit curves.

Proposition 3 is true because the elasticities interact with the relative shapes of the indifference and isoprofit curves in such a way as to contradict one's (my) simple intuition about the determinants of the ratio of relative price to relative delivery-lag changes. To provide some refined intuition on this point, suppose that initially there is an equilibrium at \(p^*, k^*\) as illustrated in Figure 2. Let demand \((N)\) increase, so that now the new equilibrium is on a higher isoprofit curve and a lower indifference curve. Let \(\pi^*\) be the new equilibrium profit level. Hold \(k\) at \(k^*\), and raise \(p\) until it reaches the \(\pi(p, k) = \pi^*\) curve at \(p = p^*\). The distance \(p^{**} - p^*\) will be the maximum that the equilibrium \(p\) can move (otherwise \(k\) would have to fall from \(k^*\) and that would violate Assumption 1). The amount of price decline from \(p^{**}\) and delivery lag increase beyond \(k^*\) that will occur in the new equilibrium will be positively related to the discrepancy in the slopes of the isoprofit and indifference curves at \((p^{**}, k^*)\). It can be shown that this discrepancy in slope decreases as \(|n_p|\) increases (or as \(|n_k|\) decreases).\(^\text{12}\) In other words, increases in \(|n_p|\) (decreases in \(|n_k|\)) affect the curvature of indifference surfaces so as to favor price over delivery lag fluctuations.

Econometric estimation of all the parameters in \(R\), the ratio of price fluctuation to delivery lag fluctuation, should enable prediction of the relative flexibility of prices across

\(^{11}\) If we dropped the assumption that \(\theta_k - \theta^*_k > 0\), then it would be possible that the ratio of relative price to relative delivery lag variability increases as \(|n_k|\) increases.

\(^{12}\) \textit{Proof:} Define the discrepancy in slope, \(D\), as \(\left(\frac{-V_k}{V_p}\right) - \left(\frac{-\pi_k}{-\pi_p}\right)\). The results in the text follow immediately from the fact that \(\frac{\partial D}{\partial p} = \frac{V_k}{pV_p} n_p - \frac{\pi_k}{p\pi_p} n_p - \frac{1}{k} (n_k - n^*_k).\)
industries. Without actual estimates, it is hard to predict the precise magnitude of \( R \). But, I expect that a 1% increase in price will typically be much more profitable than a 1% increase in delivery lag, and therefore the value of \( B \) in (9) will tend to be large (since \( p\pi_p/k\pi_k \) will be large) and hence \( R \) will tend to be low. This suggests one reason why prices may tend to be relatively rigid. Of course, I expect \( R \) to vary over industries, and any conclusions about the magnitude of \( R \) must await econometric estimates of the relevant parameters.

\[ \square \] Monopoly. The preceding analysis has investigated equilibrium fluctuations in price and delivery lag when competition prevailed. How will the results change if monopoly prevails? Are there any general relationships between equilibrium fluctuations under monopoly and those under competition?

I have not been able to derive the same types of results for monopoly as for competition. The difficulty arises because the comparative statics of the monopoly solution depend on a third derivative of the indirect utility function (derivative of marginal revenue), and I have not found a meaningful way to interpret the conditions in terms of supply and demand elasticities. I have also not been able to find any general result comparing equilibrium fluctuations under monopoly and under competition. In fact, it is not difficult to construct simple examples to show that no general result holds. (These are available upon request.) One cannot look at the pattern of price and delivery lag fluctuation alone and determine whether the market is competitive.

3. Conclusions

This article has investigated price flexibility in a model in which both price and quality of the good are endogenous. Although models of general hedonic equilibrium have been analyzed before, none has been used to investigate the comparative statics of such equi-
ilibrium and their implications for understanding the relation between price and delivery lag fluctuations. The complications of the theory as well as some surprising results flowing from the theory illustrate how some intuitions based on simple supply-equals-demand models can be misleading in trying to understand price behavior.

A main point of the analysis is that very different patterns of price flexibility relative to delivery lag flexibility are possible in equilibrium. When consumers are willing to wait, there is less need for large price fluctuations to clear the market. It may be wrong to regard delivery lags as quantity rationing induced by overly rigid prices. Instead, relatively inflexible prices combined with relatively flexible delivery lags may be the predicted market-clearing response to fluctuations in supply and demand conditions.

Appendix

The proof of Lemma 2 follows.

Proof of Lemma 2: The second-order condition is

\[
\frac{d}{dk} \left( \frac{-V_k}{V_p} + \frac{\pi_k}{\pi_p} \right) < 0,
\]

or

\[
- \frac{V_{kk}}{V_p} + \frac{V_k}{V_p^2} \frac{V_{pk}}{V_p} + \frac{\pi_{kk}}{\pi_p} - \frac{\pi_k \pi_{pk}}{\pi_p^2} + \frac{dp}{dk} \left[ \frac{-V_{kp}}{V_p} + \frac{V_k}{V_p^2} \frac{V_{pp}}{V_p} + \frac{\pi_{kp}}{\pi_p} - \frac{\pi_k \pi_{pp}}{\pi_p^2} \right] < 0. \quad (A1)
\]

Since from (1), \( \frac{\pi_k}{\pi_p} = \frac{V_p}{V_k} \) and \( \frac{dp}{dk} = -\frac{V_k}{V_p} \), (A1) can be written as

\[
- \frac{V_k}{kV_p} - V_{kk} k + \frac{V_k}{V_p} - V_{pk} k + \frac{\pi_k}{k \pi_p} \frac{\pi_{kk}}{\pi_k} k - \frac{\pi_k}{k \pi_p} \frac{\pi_{pk}}{\pi_k} k + \frac{\pi_k}{k \pi_p} \frac{1}{k} \left( \frac{V_{kp}}{V_p} \right) k - \frac{\pi_k}{\pi_p} \frac{1}{p} \left( \frac{V_{pp}}{V_p} \right) + \frac{\pi_k}{\pi_p} \frac{\pi_{kp}}{\pi_p} k + \frac{\pi_k}{\pi_p} \frac{\pi_{pp}}{\pi_p} k < 0
\]

or

\[
- \frac{V_k}{kV_p} \left[ (\theta_k - \theta_k^2) + \frac{V_k}{V_p} [n_k - n_k^2] + \frac{\pi_k}{k \pi_p} \frac{1}{p} \left( n_p - n_p^2 \right) \right] < 0,
\]

or

\[
- \frac{\pi_k}{k \pi_p} \left[ (\theta_k - \theta_k^2) - (n_k - n_k^2) \right] - \frac{\pi_k}{p \pi_p} \left[ (n_p - n_p^2) - \frac{p \pi_p}{k \pi_k} (n_k - n_k^2) \right] < 0,
\]

or

\[
- \frac{\pi_k}{k \pi_p} A - \left( \frac{\pi_k}{\pi_p} \right)^2 \frac{1}{p} B < 0,
\]

or

\[
\alpha_1 A + \alpha_2 B > 0, \quad \text{where} \quad \alpha_1, \alpha_2 > 0.
\]

If \( A \) and \( B \) are of the same sign (Assumption 1), then they must both be positive, which from (10) and (11) and Assumption 1, implies that \( D \) is positive. Q.E.D.

References


