Market Behavior with Demand Uncertainty and Price Inflexibility

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Most economists would agree that the large majority of markets do not precisely fit the classical assumptions of competition. For many markets, prices do not adjust at each instant of the day to balance supply and demand. Moreover, firms often do not know how much of their product will be demanded each day.

There are good reasons why most markets depart from the strict classical assumptions (see Armen Alchian). Changing prices frequently is time consuming and costly. Consumers may dislike price fluctuations. More importantly, prices may have to remain in effect for some time if their "signal" is to be received. The demand that an individual firm sees is random because the number of customers that frequent the firm will generally vary from day to day. In formulating its operating policy, a firm must take into account the randomness of its demand. Firms do not feel that they can sell all they want at the current market price and are concerned with overproducing or having excess capacity. Firms are also concerned with underproducing or having too little capacity. In these markets, it is an outcome of the market process that occasionally some customers will be unable to purchase the good instantly.

For these uncertain markets, the amount that a firm is willing to supply depends not only on the current market price, but also on the entire stochastic structure of demand that it faces. In this environment, supply cannot be defined without first specifying the random structure of demand.

There will be three essential features of market operation that we will study: price inflexibility, demand uncertainty, and timing considerations. By price inflexibility, I do not mean that prices do not respond to permanent shifts in the underlying supply and demand factors, but only that prices cannot be adjusting at each instant of time. An important feature of the analysis will be to determine exactly how prices are endogenously determined by market forces. Demand uncertainty means that, at the beginning of any market period after prices have been set, firms do not know for sure what their demand will be, although they do know what the random distribution of demand looks like. Demand is uncertain over the period for which prices are inflexible. Timing considerations refer to the need to have produced or to have made some prior commitment to production, such as the purchase of equipment, before the unknown customer demand is observed.

It is not immediately clear what the consequences of these three nonclassical features of market operation are, even though these three features would appear to be realistic characterizations of many market operations. In this paper, I address the following questions: How do firms compete in such markets? Can equilibrium be meaningfully defined, and if so, how does it compare to the classical equilibrium when the uncertainty is removed from the demand side? What are the properties of the competitive equilibrium as the size of the market increases? Will this equilibrium be Pareto optimal? In this equilibrium, do firms produce too little of the good in question? Would society benefit if the government paid lump sum subsidies to firms so as to encourage them to expand their production of the good?

For the markets under study, it will be a natural feature to have some customers being unable to purchase the good, and

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some firms being unable to sell all of their stock, or equivalently use all their capacity. Each good will have two characteristics associated with it, namely its price and the probability that it can be purchased. Customers will have preferences not only for the price of the good, but also for the probability of obtaining it. Firms will compete amongst themselves until an equilibrium is reached. Market clearing will require equilibrium along the dimensions of both price and probability of obtaining the good. In equilibrium, supply will not in general equal demand, and there will always be some customers who are unable to purchase the good. The "customers" can also be interpreted as being other firms who are trying to buy factor inputs for their production process. With this interpretation, we obtain a model where it is perfectly natural for firms to be concerned with obtaining an "assured" supply of the input, a concern that appears uppermost in the minds of businessmen (see Alfred Chandler).

In the special case where social welfare is measured by expected surplus, we show that a competitive equilibrium is optimal. This result stands in sharp contrast to previous models in the literature on optimal pricing under uncertainty. However, in general, a competitive equilibrium will not lead to the socially optimal point. The social optimum will, under a plausible set of assumptions, involve paying lump sum subsidies to encourage firms to expand their production.

The model is applicable to any market where availability of the good or of the means to produce the good is important. It does appear that in the private sector for many industries demand fluctuations are not always absorbed by price changes and that changes in the probability of whether or when the good can be obtained is often an important equilibrating mechanism. Some examples include retail stores, hotels, restaurants, and manufacturing. In the regulated and government sector the model also seems to have wide-spread application. For example, for airlines, railroads, public parks, and electric utilities, prices do not vary continuously and uncertainty in demand is absorbed by changes in rationing frequency.

I. Competitive Market Clearing

There is a large literature on the effects of uncertainty on firm behavior.\(^1\) Analyses of competitive markets focus on the effect of having uncertainty in price, and they maintain the assumption that firms can always sell all they want at the future uncertain market price.\(^2\) There are never any shortages in equilibrium. In his pioneering works, Edwin Mills (1959, 1962) has examined the effect of demand uncertainty and price inflexibility on the behavior of a monopolist who must decide what price to charge and how much to produce before demand can be observed. Surprisingly, despite the realism of the assumptions of demand uncertainty, price inflexibility, and a lead time necessary for production, there has been no attempt to examine the implication of these assumptions within a competitive environment.\(^3\) The purpose of this paper is to provide such an examination, and to derive and investigate the properties of an equilibrium in which it is natural to have supply not equal to demand. A simple model is presented which attempts to capture the essential features of the markets under study.

There are \(N\) identical firms who compete with each other. To make the assumption of competition plausible, the number of firms \(N\) will be considered to be large enough to prevent firms from having any monopoly power. Firms maximize expected profits.

At the beginning of each period, each firm sets price, which remains in effect for the entire period, and it decides how much of the good to produce and stock for the period.\(^4\) No additional production or de-

\(^1\) See Michael Rothschild and John McCall and the references cited therein.
\(^2\) See, for example, Edward Zabel.
\(^3\) Since this paper was written, John Gould and Arthur Devany and Thomas Saving have investigated issues closely related to those of this paper.
\(^4\) It is not necessary for the good to be produced at the very beginning of the market period. All that is required is that some commitment to production, such
livery of the good can occur during the period. The production cost per unit of the good is \( c \), where \( c \) is strictly positive. To keep the model simple, it is assumed that the good is perishable so that it is impossible to hold inventories between periods. Provided holding inventory is a costly activity, the qualitative results derived below will be unchanged.

I now wish to generate a simple characterization of random demand per firm in which the number of customers that a firm sees is random. One way to generate such a random process would be to have three random states of nature for each consumer: a) one in which the consumer does not desire the good (the occurrence of this state depends on exogenous random variables); b) one in which the consumer desires to purchase some amount of the good but cannot; and c) one in which the consumer desires to purchase some amount of the good and does so. (For simplicity, let each consumer have the same per capita demand curve for the good when the good is desired.) The utility of the consumer in each of the three states would influence the consumer’s total expected utility. Conditional on not being in state a), the amount that firms produce will influence whether a consumer winds up in state b) or c). Since it is this last set of events that we wish to analyze, I suppress discussion of state a) for the remainder of the paper (except for footnotes), not because it is unimportant but because its inclusion, though straightforward, would be cumbersome, and would obscure the main thrust of the analysis. It is crucial to stress that the main ideas and conclusions of the paper apply to any demand process that generates random demands per firm.

I adopt the following simple process to generate random demand per firm. Suppose that each period there are \( L \) identical consumers who desire to purchase the good, each with per capita demand \( x(p) \). During each period, each customer randomly fre-

5In equilibrium the demand to a firm is a random variable from a binomial process of size \( L \) and probability \( 1/N \). For large \( N \), which is assumed, this binomial process converges to the Poisson which in turn converges to the normal for moderate size \( L \). (It is this normal approximation that is used in the Appendix, Section A.) The reader can think of the demand arrival process as either Poisson or equivalently (since the number of firms can be finite but large) as binomial with large \( N \). (See William Feller, ch. 7.) As stressed above, what is of crucial importance is that demand to the individual firm be random. Whether total industry demand is random (as is natural with a Poisson interpretation) or nonrandom (as is natural with a binomial interpretation) is irrelevant. The referee has noted that nothing in the model requires \( L \) or \( N \) to be finite, only that \( L/N \) be finite.

6Just as in the case of inventory holding, consumer search behavior, providing it is costly, would not alter the main features of the model. This point is discussed more fully below.

7This equilibrium concept is closely related to the equilibrium concept in hedonic markets. See Sherwin Rosen.
with both the probability $1 - \lambda$ of obtaining the good and the price $p$ charged for the good. We can write this expected utility as $U(1 - \lambda, p)$. The function $U$ defines the iso-utility contours between $1 - \lambda$ and $p$ that leave a consumer indifferent.

Typical iso-utility contours are drawn in Figure 1.

The diagram shows that along any iso-utility curve, as price rises, the probability of satisfaction must rise if consumers are to remain indifferent. Also, for any fixed probability of satisfaction, consumers always prefer lower prices.

Consumers will always try to reach their highest iso-utility contour, and will only go to a firm that they think will provide this highest iso-utility level. If the buyer believes that several firms provide this highest utility level, then he will choose among them randomly.

No strong conclusion about the shape of the iso-utility curves are justified. We impose the very weak assumptions that the iso-utility curves exist over the relevant range in $(1 - \lambda, p)$ space, that they are continuous, and that they satisfy an upper and lower Lipschitz condition. This latter condition postulates that there exist two numbers, $b$ and $B$, such that $0 < b < B < \infty$ and such that the slope along any iso-utility curve always lies between them. The Lipschitz requirements insure that the consumer is never willing to make infinite tradeoffs in either the $p$ or $1 - \lambda$ directions.

III. Behavior of the Firm

Since consumers will wind up going only to those firms that provide the highest utility level in the market, competition forces firms to take the utility level as given. (If instantaneous production were possible so that no shortages could occur, then each good would have only one characteristic (price) associated with it. In that case, utility-taking behavior is equivalent to price-taking behavior and this market would behave exactly as a classical supply and demand analysis would indicate.) At the beginning of each period, firms have to decide on a price and production policy so as to maximize their profits subject to the constraint that they provide at least the given level of utility to consumers. Firms know that if they remain competitive with the other firms, then they will receive their random share of demand.

We can write the total amount that the firm decides to produce at price $p$ as $s \cdot x(p)$. The variable $s$ can be interpreted as the maximum number of customers that a firm can satisfy in that period. Henceforth, we will refer to $s$ as customer capacity. Clearly, the amount that a firm decides to produce affects the probability that a customer will be able to obtain the good from that firm.

Let us examine the relation between the expected number of customers $M$, who will

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9By this assumption I simply mean that there is some range of prices, which includes $p = c$, the cost of production, for which the consumer is interested in purchasing the good. In other words, if the consumer does not have positive demand for prices near $c$, then the market for the good will not exist, and there is nothing to analyze.
find the firm out of the good, and the customer capacity \( s \) that the firm provides. Let \( pr(i) \) stand for the probability that \( i \) customers will arrive at the firm. Then, we can write that

\[
M(s) = \sum_{i=1}^{s} (i - s)pr(i)
\]

If all \( N \) firms follow the same operating policies, then the total expected number of customers who will be dissatisfied is \( N \cdot M \), and the fraction of dissatisfied customers will equal \( NM/L \). The fraction \( 1 - \lambda \) of customers who are able to obtain the good can be written as

\[
1 - \lambda(s) = 1 - \frac{N \cdot M(s)}{L}
\]

(In my 1975 paper I explain how (2) can be interpreted as applying to a firm even when all firms do not follow the same policies.)

In the Appendix, Section A, I show that using the normal distribution to approximate the discrete binomial process of customer arrival, the probability of satisfaction function, \( 1 - \lambda(s) \) can be written as

\[
1 - \lambda(s) = \frac{\sigma I(u) + s}{\sigma^2}
\]

where \( \sigma^2 = L/N \),

\[
I(u) = \int_{-\infty}^{u} [t - u] f(u) du
\]

\( f(u) \) is the normal density function, and \( u = (s - \sigma^2)/\sigma \).

For a given level of utility, firms want to choose a price \( p \) and a customer capacity \( s \), so that profits are maximized and the consumer is able to achieve the given level of utility. When firms remain competitive by offering the given level of utility, they randomly receive their equal share of the \( L \) customers. Letting \( pr(i) \) stand once again for the probability that \( i \) of the \( L \) customers visit a firm this period, we can write that expected profits equal

\[
\pi(s, p) = p \cdot x(p) \sum_{i=0}^{s} i pr(i) + px(p)s \sum_{i=s+1}^{s} pr(i) - cspx(p)
\]

The first term in (4) represents expected sales revenue when \( i \leq s \) customers come to the firm, while the second term represents expected sales revenue when more than \( s \) customers come to the firm. The last term in (4) is the cost of being able to service \( s \) customers. Since (3) expresses a one-to-one relation between the probability of satisfaction \( 1 - \lambda \) and the customer capacity \( s \), we can interpret (4) as expressing profits as a function of \( 1 - \lambda \) and \( p \).

Regarding profits as a function of \( 1 - \lambda \) and \( p \), we can draw iso-profit curves in \((1 - \lambda, p)\) space. A typical family of such curves is depicted in Figure 2. The two iso-profit curves at the far right of the diagram are drawn to illustrate that each iso-profit curve involving positive profits can “turn around” on itself as price rises sufficiently high to drive demand to zero. Since consumers always prefer to be on the northwest boundary of the iso-profit curves, competition will insulate that the “dotted” segments of the iso-profit curves are never observed. The heavy dotted line in the diagram represents the \( \pi_i \) curve which is derived by setting \( \partial \pi / \partial s = 0 \) in (4). Iso-profit curves cross the \( \pi_i \) curve vertically, and so the relevant portions of all iso-profit curves emanate from the \( \pi_i \) curve. For the relevant portions of the curves, with fixed price, profits decrease as probability of satisfaction increases. Hence, in the diagram \( \pi_1 < \pi_2 \). The curve on the far left of Figure 2 represents the zero-profit \((\pi = 0)\) curve, which is the only one this paper will be interested in.

For any given iso-utility level, \( \bar{u} \), the firm will choose to operate at a point of tangency between the iso-utility curve representing iso-utility level, \( \bar{u} \), and the highest

![Figure 2. Iso-Profit Curves](image-url)
iso-profit curve. No firm ever chooses to operate to the left of the \( \pi = 0 \) curve since that represents negative expected profits.

The properties of the zero-profit curve play a key role in determining the behavior of equilibrium as the number of customers per firm increases. I now describe the relevant properties of the zero-profit curve, proved in a separate appendix available on request from the author.

The \( \pi = 0 \) curve is concave (i.e., \( d^2(1 - \lambda)/dp^2 < 0 \)), starts off with a very large slope at a point a little to the right of \( p = c \) on the horizontal axis, rises to 1 as price increases, and has a very flat slope for sufficiently high prices. The curve always lies to the right of the vertical line \( p = c \), since price must cover not only production costs, but also the cost of unsold goods. As price rises, firms can afford to provide a larger customer capacity \( s \). Hence along the \( \pi = 0 \) curve the probability of satisfaction increases to 1 as price increases.

As the customer per firm ratio \( L/N \) increases, the \( \pi = 0 \) curve is affected in several ways. First, the entire curve shifts up, indicating that for fixed price as the number of customers per firm increases firms can afford to increase their customer capacity in such a way that there is a higher probability of satisfying customers. Basically, this result occurs because there are economies of scale in serving a stochastic market. The proportional risk of having unsold goods declines as the customer per firm ratio increases. In other words, to achieve a satisfaction probability of .5 in a market with 100 customers per firm requires a \( s/100 \) figure that is larger than the \( s/1,000 \) figure in a market with 1,000 customers per firm.\(^{11}\) As the customer per firm ratio continues to increase, the \( \pi = 0 \) curve shifts up to the \( 1 - \lambda = 1 \) line.

How does the slope of the \( \pi(1 - \lambda, p) = 0 \) curve behave as the customer per firm ratio increases? For any fixed price \( p > c \), the slope \( d(1 - \lambda)/dp \) falls monotonically to zero as \( L/N \) increases. Furthermore, for any fixed probability of satisfaction, \( 1 - \lambda \), below 1, the slope \( d(1 - \lambda)/dp \) approaches infinity as \( L/N \) increases.

IV. Market Equilibrium

In the diagram of the iso-profit curves, superimpose the iso-utility curves of customers. We can define a contract curve as the locus of tangencies between the iso-utility and iso-profit curves. Firms always operate on this contract curve.

In a classical market, firms compete with each other by offering to consumers lower prices (i.e., higher utilities) than other firms. Prices (or consumer utilities) continue falling (rising) until firms have no incentive to lower price (raise utility) any more. Analogously, in this market, firms compete with each other by offering better (i.e., higher utility) combinations of price and probability of satisfaction to consumers. The utility level is "bid" up until there is no incentive for any firm to continue to alter its price-probability of satisfaction combination. This point occurs when the contract curve intersects the zero-profit (\( \pi = 0 \)) curve. At this point, firms would prefer to go out of business rather than offer a higher utility combination to consumers and earn negative expected profits. Hence, competition on the utility level forces the market equilibrium up the contract curve, until the zero-profit curve is reached. Equilibrium can be regarded as a tangency\(^{12,13}\) between the zero-profit curve and the highest attainable iso-profit curve.

\(^{10}\)Since I am using a continuous random variable to approximate a discrete positive random variable, there is a slight error involved. By the Central Limit Theorem, we know that any such approximation errors become insignificant for even moderate (i.e., 15-20) values of the customer per firm ratio, \( L/N \). In the subsequent analysis, I ignore such approximation errors.

\(^{11}\)Recall that \( s \) refers to customer capacity.

\(^{12}\)Note the similarity between this equilibrium and the hedonic good equilibrium of Rosen.

\(^{13}\)Ignore the uninteresting case of corner solutions in which either a) the firms produce nothing or b) each firm by itself stocks an amount of the good to satisfy the entire customer population by itself. Notice that multiple tangencies are possible, since the iso-utility curves can be convex. The dynamic arguments justifying the establishment of this equilibrium are outlined in the author (1975). See the author (1977a) for a discussion of possible instabilities in these markets.
tendable iso-utility curve. This equilibrium is depicted in Figure 3.

There are several noteworthy features of this equilibrium. In general, there will be a positive probability of being unable to obtain the good. Second, in equilibrium the price will exceed the constant cost of production. This occurs because the revenue from sold goods must compensate not only for the cost of producing those goods, but also for the cost of producing the unsold goods. Equivalently, it is necessary to pay for available but unused capacity. Third, there is no reason why supply should equal demand even in expected values since equilibrium depends in part on consumers' willingness to take risk.

V. Behavior of Market Equilibrium as the Customer Per Firm Ratio Increases

Armed with the properties of the $\pi = 0$ curve, we can investigate the behavior of equilibrium as $L/N$, the customer per firm ratio, increases. It will be useful for the reader to recall that $b$ and $B$ are the lower and upper bounds on the slope of the iso-utility curves, respectively.

THEOREM 1: As $L/N$, the customer per firm ratio, increases, the equilibrium price associated with the market-clearing point approaches the deterministic market-clearing price $c$.

\[ \lim_{L/N \to \infty} p^* \to c \]

THEOREM 2: As $L/N$, the customer per firm ratio, increases, the equilibrium probability of satisfaction approaches 1.

PROOF:
The method of proof will be to show that as $L/N$ increases, any equilibrium point $(p^*, 1 - \lambda^*)$ of the market clearing under uncertainty will eventually lie to the left of the vertical line $p = c + e$ for every positive $e$.

Choose the point $p = c + e$ for any positive $e$. Equilibrium in the uncertain market is defined as a point of tangency between the $\pi = 0$ curve and an iso-utility curve. Now, increase $L/N$. As $L/N$ increases, the slope of the $\pi = 0$ curve declines to zero for any fixed $p > c$. Increase $L/N$ so that the slope of the $\pi = 0$ curve is less than $b$ at $p = c + e$. This implies that the slope of $\pi = 0$ is less than $b$ for all $p \geq c + e$ since the $\pi = 0$ curve is concave. But then it is impossible for any iso-utility curve to be tangent to the $\pi = 0$ curve at any price above $c + e$. Hence, any market equilibrium price $p^*$ is less than $c + e$. Since $p^*$ must be greater than $c$ for any production to occur at all, and since $p^*$ is less than $c + e$ for any positive $e$, it follows that

With instantaneous production, the model becomes identical to the classical supply and demand model. For that case, the $\pi = 0$ curve is a vertical line at $p = c$, and equilibrium as defined above coincides with the classical equilibrium of price $= c$, probability of satisfaction $= 1$. We see then that the classical model is a special case of this model.
ciently large $L/N$ it is impossible for any iso-utility curve to be tangent to the $\pi = 0$ curve for a probability of satisfaction less than or equal to $1 - \lambda$. Since the equilibrium probability of satisfaction is bounded above by 1, and lies above every $1 - \lambda$ less than 1, it follows that

$$\lim_{L/N \to \infty} 1 - \lambda^* \to 1$$

It immediately follows from Theorems 1 and 2 that the equilibrium level of expected utility achievable by consumers in equilibrium approaches the level of utility achievable in the deterministic market, where price equals $c$ and the probability of satisfaction equals 1.

**THEOREM 3:** As $L/N$, the customer per firm ratio, increases, the percent discrepancy between the amount supplied and the amount demanded approaches 0.

**PROOF:**

The total amount demanded equals the number of customers times the per capita demand $L \cdot x(p)$, while the total amount supplied equals the number of firms times the customer capacity per firm times the per capita demand $N \cdot s \cdot x(p)$. To prove the theorem it is sufficient to show that $N \cdot s/L \to 1$ as $L/N$ increases.

Write the zero-profit condition as

$$1 - \lambda) p \cdot L = N c \cdot s$$

From the previous two theorems we know that in equilibrium $p \to c$ and $1 - \lambda \to 1$ as $L/N$ increases. Hence the theorem follows immediately.

Theorem 3 dealt with the percent discrepancy between supply and demand. What about the absolute discrepancy, $[L - N \cdot s]x(p)$—does that too approach zero as the customer per firm ratio $L/N$ increases? The answer in general is no. Usually the absolute discrepancy will approach either plus or minus infinity as $L/N$ increases. In other words, equilibrium is possible even though the number of dissatisfied customers is arbitrarily large.

The reason why the market equilibrium does not converge to the deterministic one in all respects as the customer per firm ratio $L/N$ increases can be explained as follows. As $L/N$ increases, the total uncertainty in the market increases, so that market operation under uncertainty differs considerably from that under certainty. On the other hand, by the law of large numbers, the proportional risks caused by the uncertainty vanish as $L/N$ increases. Therefore, percentage-wise concepts (for example, supply + demand), or concepts that apply to individual units of the good (for example, price) or individual customers (for example, probability of satisfaction) approach their values in the corresponding deterministic market as $L/N$ increases. However, aggregate concepts such as supply, demand, and total number of customers dissatisfied do not in general approach their values in the deterministic market as the customer per firm ratio increases.\(^{15,16}\)

VI. Different Types of Customers

It is perfectly natural to imagine a market with two types of customers who have different preferences between price and

\(^{15}\)How much do markets under uncertainty differ from those under certainty? As seen below, social welfare implications and incentives for vertical integration are different. See the author (1977a) for further differences. It is possible (see the author, 1975) to calculate lower bounds on the customer per firm ratio necessary to achieve any given level of convergence of price and probability to the certainty equilibrium $p = c, 1 - \lambda = 1$. For convergence to the 1 percent level, $L/N$ must exceed 6,500.

\(^{16}\)The number of firms $N$ and the total amount demanded at any price $p$, $L x(p)$ have been taken as fixed. The assumptions were made for analytical tractability. It is not necessary that total demand be fixed. For example, total demand could be random and each firm could obtain some fixed share of total demand. As long as demand to an individual firm is random, the analysis developed above applies. See Section I for further discussion. Are there any incentives for merger in the model? When total demand is random and firms obtain fixed shares of total demand, no incentives for merger exist. When total demand is fixed, it appears that total merger is desirable. But this last result emerges only because considerations like spatial patterns of demand variation and costs of merger do not explicitly appear in the model.
probability of satisfaction. In such a situation, it is possible to have an equilibrium in which two types of firms are established, each of which caters only to the preferences of one type of consumer. An equilibrium involving firm specialization is depicted in Figure 4.17

As Figure 5 illustrates, such specialized equilibrium may not always exist.18 The specialized equilibrium cannot exist because all the type 2 customers are better off at type 1 stores than at type 2 stores.

When only one equilibrium can exist in the market, the question of where it is established will be determined by the tastes of the majority. If any firm does not cater to the tastes of the majority, it will lose a majority of its business and will have to specialize in the minority’s tastes. But, by assumption, specialized equilibrium is impossible, so the firm could not profitably attract just the minority types to its firm.

VII. Search Behavior

So far, the model has restricted consumer’s search to only one firm per period.

17 For the case of equilibrium involving firm specialization an outside observer might incorrectly conclude that there was a distribution of prices for an identical good and attribute it to consumer ignorance. As this paper emphasizes, since each type of firm offers a different probability of satisfaction, the “goods” at different types of firms are not identical.

18 The nonexistence of such equilibria occurs for reasons similar to those studied by Rothschild and Joseph Stiglitz. See their article for further discussion. As that discussion makes clear, the above analysis of specialized equilibrium is different from that of a hedonic market (see Rosen). The key difference is that in the above model one of the characteristics of the good is endogenous and cannot be specified independently of consumer behavior as is true in a hedonic market.

Unlike other models of search, in this model consumers have information about the characteristics (i.e., 1 – λ, ρ) of each firm. Consumers simply do not know which firms have the good. If we allow consumers more than one search, then the main features of market behavior are unchanged in the sense that firms and consumers will still take into account the probability they cannot sell or buy a good. A new feature that does emerge is that all firms may not behave identically. Some firms could charge low prices and run out frequently, while others could charge high prices but run out infrequently. Customers might go to the low-price firm first and then, if unable to satisfy their demands, go to the high-priced firm. Price distributions arise naturally. Notice, though, that the higher prices compensate firms for the higher risk of having unsold capacity. The different price firms sell different products.19

VIII. Social Welfare Implications

The previous section examined how markets operate when the production decision must be made before the uncertain demand for the product can be observed, and when prices, once set, cannot vary over the market period. An important question is whether a competitive equilibrium involves a combination of price and probability of satisfaction that is optimal in the sense of maximizing some measure of social welfare.

19 As the referee has pointed out, if consumers had imperfect information about price and availability, then just as in the other search models (see Gerard Butters), firms offering the same product (i.e., availability) could receive different prices.
This is the issue that is examined in the next two sections.

Throughout this examination, I do not allow insurance markets to develop. There are well-known reasons why such markets may not exist. For example, in the present case, there would be the problem of ascertaining that someone actually attempted to purchase the good. Such insurance markets rarely exist in practice. (If a customer finds that a firm is out of a good, there is not a market to compensate him.)

The first question to be asked is when, if ever, will the market equilibrium maximize the expected value of the total surplus to society. This question is motivated by two considerations. First, deterministic markets in competition maximize surplus, so it is natural to see if uncertain markets do also. Second, expected surplus is often used as a measure of social welfare. I will show that, in the special case where expected consumer’s surplus is derived from an individual’s preferences between price and probability of satisfaction, a competitive equilibrium does indeed maximize the expected value of surplus to society.

Consumer’s surplus is an appropriate measure of welfare only under very narrow assumptions. Moreover, in an uncertain setting expected consumer’s surplus may not properly reflect consumer attitudes toward risk. To avoid the defects associated with consumer surplus, I also examine the social welfare question in a simple two-good model. A two-good model is set up by introducing an alternative (nonrationed) good, and asking how a social planner who takes both markets into account would operate this economy so as to maximize the expected utility of a representative consumer. It will be shown that the socially optimal solution will usually be different from a competitive equilibrium. The socially optimal solution will, under a plausible assumption, involve paying lump sum subsidies to the firms that deal with the good that is subject to shortages. Compared to the social optimum, a competitive equilibrium will usually not devote sufficient resources to production of the good that is subject to shortages.

IX. Maximizing Expected Surplus

As mentioned above, consumer surplus is not generally a good measure of social welfare for uncertain markets because, aside from problems associated with its use in a deterministic setting, it may not reflect consumer preferences between the probability of obtaining the good and the price of the good. For the special case where expected consumer surplus does reflect consumer attitudes toward risk, we want to examine whether a competitive equilibrium maximizes expected surplus. The main result of this section is that for this special case a competitive equilibrium does indeed maximize the expected surplus to society.

The model is the same as before. Let us consider the expression for expected surplus to society when all N firms follow the same price and stocking policy. Expected surplus to society (ESS) equals the per capita consumer surplus times the number of customers times the expected fraction of customers that are satisfied minus the cost of the goods. Expressed mathematically, we have that

$$ESS = (1 - \lambda(s)) \int_0^{x(s)} x^{-1}(q) dq \cdot L - c_s N x(\rho)$$

where, to refresh the reader’s mind, we repeat the definitions:

$$L = \text{number of (identical) customers in the market}$$

21This special case occurs when, in the notation of fn. 8, \( u(x_1, x_2) = g(x_1) + x_2 \), where \( u \) is the von Neumann-Morgenstern utility function, \( x_1 \) is the amount of the good that is subject to shortages, and \( x_2 \) represents all other goods which are assumed to be available at a price of one. The above analysis also applies to the more general formulation of random demand in which consumers have the possibility of not desiring the good (see the discussion in Section I about state of nature a) provided utility in that state of nature is equal to \( x_2 \).

20See, for example, Gardiner Brown and M. Bruce Johnson.
\( N \) = number of firms
\( x(p) \) = the per capita demand curve\(^{22}\)
\( x^{-1}(q) \) = the inverse per capita demand function
\( p \) = price of the good
\( c \) = cost per unit of the good
\( s \) = the number of customers that can be serviced per firm
\( 1 - \lambda \) = the probability of satisfaction as a function of \( s \)

The government wishes to determine an operating policy (i.e., \( s \) and \( p \)), so that \( ESI \) is maximized when all firms behave according to this operating policy. To maximize \( ESI \) with respect to \( p \) and \( s \), take derivatives of (6) to obtain the following first-order conditions:

\[
(1 - \lambda)x'(p)pL - csNx'(p) = 0
\]

or equivalently

\[
(1 - \lambda)pL - csN = 0
\] (7)

and

\[
\frac{d(1 - \lambda)}{ds} \cdot L \int_0^{x(p)} x^{-1}(q) dq - c \cdot x(p)N = 0
\] (8)

Equations (7) and (8) determine the \( s \) and \( p \) of the operating policy for each firm that the government should follow to maximize the expected total surplus to society.\(^ {23} \)

Using the expression for profits derived earlier, it can be seen that (7) is equivalent to the condition that expected profits per firm equal zero. Equation (8) determines the point along the zero-profit (\( \pi = 0 \)) curve at which the government should operate.

The question then arises as to whether a competitive market equilibrium would maximize the expected value of surplus to society if consumers' tradeoffs between the price of the good and the probability of obtaining the good were adequately represented by the expected value of their consumer surplus. At first glance, the answer to this question appears obvious. If expected consumer surplus reflects consumer preferences, then we know from the properties of equilibrium in an uncertain market that the expected consumer surplus of each individual (\( ESI \)) is maximized. Hence, the social planner will maximize the surplus to society at this point. This reasoning is faulty although the conclusion turns out to be correct. The sum of individual consumer surpluses does not equal the surplus to society for the markets under study. There are unsold goods at the end of each period which must enter the government's calculation of surplus but not that of any individual.

More specifically, suppose each of the \( L \) consumers seeks to maximize\(^ {24} \)

\[
ESI = (1 - \lambda) \int_0^{x(p)} x^{-1}(q) dq - px(p)
\]

where the notation was just defined beneath (6). Summing \( ESI \) over all \( L \) consumers and comparing this sum to the objective function \( ESS \) of the government, we see that the two expressions differ by

\[
(1 - \lambda)pLx(p) - csx(p)N = x(p)[p(1 - \lambda)L - sNe]
\]

This last expression is the difference between the expected revenue to be received and the cost of all the goods, sold and unsold. In view of the differences in the objective functions between the individual and the government, it is interesting that the following theorem holds.

**THEOREM 4:** Suppose \( ESI \), as defined in (9), represents consumer preferences between the price \( p \) and probability of satisfaction \( 1 - \lambda \). Then, a competitive equilibrium maximizes the expected value of surplus to society, \( ESS \).

\(^{22}\)Recall from fn. 8 that \( x(p) \) is simply the per capita demand for the good when it is available. The relation between \( x(p) \), and \( u(1 - \lambda, p) \) is explained in that same footnote. We always assume \( x'(p) < 0 \).

\(^{23}\)As before I disregard the uninteresting case of boundary solutions and assume that (7) and (8) have a solution that represents an interior maximum (i.e., second-order conditions for a maximum are fulfilled).

\(^{24}\)Recall that consumers will maximize \( ESI \) if their von Neumann-Morgenstern utility functions are of the form \( u(x_1, x_2) = g(x_1) + x_2 \) where \( x_1 \) = the good under analysis, and \( x_2 \) = all other goods always available at a price of 1.
PROOF:
If $ESI$ reflects consumer preferences, then from the definition of competitive equilibrium, we know that competitive equilibrium occurs at that point along the zero-profit curve that maximizes $ESI$. From (7), we know that the point that maximizes $ESS$ also occurs along the zero-profit curve.

The difference between surplus to society $ESS$, and the sum of consumer surpluses to an individual $L\cdot ESI$, was derived above and equals $x(p)(1 - \lambda)pL - Nsc$. However, from (7), we see that along the zero-profit curve, this difference equals zero. Therefore, along the zero-profit curve, the two measures $ESS$ and $L\cdot ESI$ attain their maximum values at the same points.

We see then that if individual consumer preferences are represented by $ESI$, then just as in deterministic markets, a competitive equilibrium will maximize the expected value of the total $ESS$. Notice that price exceeds $c$ and firms earn zero-expected profits when expected surplus is maximized. These results contrast sharply with those of other models that appear in the public finance literature (see Brown and Johnson; Michael Visscher), and deal with a similar type of problem. The results of those other models imply that to maximize expected surplus to society, price should in general be less than or equal to $c$, and hence firms should operate at an expected loss.

The reason for the difference in results stems from the manner in which the randomness is introduced into the demand curve and the way goods are rationed. In the model under study, a firm’s demand is multiplicative and equals $x(p) \cdot i$ where $x(p)$ equals per capita demand and $i$ equals the random number of consumers who visit the firm. All customers face the same probability of being rationed. In Brown and Johnson, rationing is done by willingness to pay with the demand for units that generate large consumer surplus being satisfied first. As Visscher points out, it is difficult to imagine how such a rationing scheme could be implemented without using a recontracting market. In Visscher’s models, more realistic rationing schemes are introduced, however only the case of additive demand uncertainty is analyzed. For most purposes, the multiplicative formulation would appear more plausible. See the author (1977b) for further discussion.

If $ESI$ does not represent consumer preferences for the probability of satisfaction $1 - \lambda$, and the price $p$, then Theorem 4 will not hold. However, if $ESI$ does not represent consumers’ preferences toward risk, then expected surplus is a very poor criterion to use as a measure of market performance in an uncertain environment. In the next section, I allow the consumer to have quite general preferences for the probability of satisfaction and the price, and examine how the introduction of an alternative good affects the analysis of the social optimum.

X. The Social Optimum in a Simple Two-Good Model

Let there be two goods on which each of the $L$ consumers can spend their identical endowment $Y$. Good 1 is subject to shortages, while good 2 is always available from the outside world at a constant price. The price of good 1 is $p$, while the price of good 2 is $\beta$. As before, each unit of good 1 uses up $c$ units of resources and must be produced before any firm observes its random demand. Demand is random in the same manner as discussed previously. As usual, no firm can receive delivery of the good

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25 It should be mentioned that the result that expected profits equal zero at the social optimum does not depend on the particular random process for demand. See the author (1977b).

26 To see the relation of the model of this paper to the peak load problem under uncertainty, reinterpret $c$ as the fixed cost per unit. In the model the marginal variable cost $\beta$ is taken as 0. If $\beta$ is nonzero, prices would rise by $\beta$. The model above suggests that under certain assumptions $p > c + \beta$ is optimal. The previous models in the literature suggest that $p \leq c + \beta$ is optimal.

27 This is one reason why econometric equations are specified so often in log-log form.

28 This point is not addressed by either Brown and Johnson or Visscher.
once a market period has begun. The government owns each of the \( N \) firms that dispense good 1, and wishes to choose the same tax policy and operating policy for each of the \( N \) firms so as to maximize the expected utility of a representative consumer. The government faces the budget constraint that the sum of the firms’ expected profits plus the total taxes collected or dispersed must equal zero.

Let \( u(x, z) \) represent the von Neumann-Morgenstern utility function of each consumer where \( x \) denotes good 1 and \( z \) denotes good 2. When good 1 is obtainable at price \( p \), the utility of each consumer is given by \( V(p, Y) \), the indirect utility function. When good 1 is not obtainable, the utility of each consumer is given by \( u(0, Y) \). If \( 1 - \lambda \) represents the probability of obtaining good 1, then the expected utility of a representative consumer can be written as

\[
U(1 - \lambda, p) = (1 - \lambda)V(p, Y) + u(0, Y)
\]

The government seeks to determine a transfer \( T \) for each individual,\(^{29}\) a price \( p \), and a customer capacity \( s \) (recall that \( s \) refers to the maximum number of customers that can be serviced at any firm in any market period), so that the expected utility of each (identical) consumer is maximized. The government’s problem can be written as

\[
\max_{s, p, T} (1 - \lambda(s))V(p, Y + T) + \lambda(s) u(0, Y + T)
\]

subject to the budget constraint,

\[
\pi(s, p) - (L/N)T = 0
\]

where \( \pi(s, p) \) is the expression for expected profits per firm, which can be written as

\[
\pi(s, p) = (1 - \lambda(s))p \frac{L}{N} x(p) - csx(p)
\]

where \( 1 - \lambda(s) \) is the expression for the probability of satisfaction as a function of customer capacity \( s \), and is given by (3).

From the statement of the problem, we see that if (and only if) the transfer \( T \) equals 0 in the socially optimal solution, then it follows that a competitive equilibrium will also be the socially optimal point since both points maximize expected utility subject to the constraint that expected profits are zero. In general, there is no reason to expect that the optimal solution to the above problem will have \( T = 0 \), so that a competitive equilibrium will usually not represent the social optimum. The social optimum will usually involve either taxes or subsidies for the firms who sell good 1, the good subject to shortages. In such cases government intervention into a competitive market may be called for.

In order to investigate the conditions under which either taxes or subsidies will be paid in the social optimum, it is necessary to make an assumption about consumers’ preferences.

**Assumption 1:** The marginal utility of an extra dollar, when good 1 is obtainable, is higher than the corresponding marginal utility when good 1 is unobtainable. More precisely, \( V_2(p, Y) > u_2(0, Y) \) for all \( p, Y \), where the subscripts denote partial differentiation.

The assumption reflects the idea that the greater the variety of goods that can be purchased, the higher is the marginal utility of an extra dollar. (One sufficient condition for this assumption is that \( u_{21} \geq 0 \).) Given the above assumption, the following theorem holds.

**THEOREM 5:** Under Assumption 1 and the assumption that per capita demand depends positively on income, the social optimum involves operating the \( N \) firms that sell good 1 at a loss and using lump sum taxes to subsidize their operation.

**PROOF:**

See the Appendix, Section B.

Under Assumption 1, the socially optimal solution involves operating the \( N \) firms that produce good 1 at a loss, and
using lump sum taxes to subsidize these firms' revenues. Since a single price competitive equilibrium involves zero profits, we see that government intervention into a competitive market may be necessary to achieve the social optimum.\footnote{As should be clear from the proof of the theorem, if we replace Assumption 1 with the (less plausible) assumption that the marginal utility of income declines as the variety of goods increases, then in the social optimum firms would be taxed and consumers subsidized. Under the assumption that $V_2(p, Y) = u_3(0, Y)$ (as in Section IX) the optimal tax is zero (see the Appendix, Section B). Competitive equilibrium is optimal in this special case. In the more general formulation of random demand in which the consumer has the possibility of not desiring the good (see the discussion in Section I about state of nature a)), there is another term in the expression for $E(U)$ for this additional state of nature. An assumption that the marginal utility of income is lower in this state of nature than in those where the consumer desires the good is then sufficient for Theorem 5 to hold.}

The heuristic reason why, under Assumption 1, it is optimal to tax consumers and pay subsidies to firms can be seen as follows. There are two states in which the consumer can wind up, one where he can purchase the good at the market price and one where he cannot. Under Assumption 1, the last dollar is more valuable in the state in which the good is obtainable than in the state in which the good is unobtainable. A person could increase his utility if he could in some way transfer part of his income between the two possible states. Such transfers of income are impossible in the problem under examination. (Remember no insurance or recontracting markets exist.) However, what is possible is that the government can use taxes to reduce the income of a consumer in both states and subsidize the operation of firms that produce the good, and thereby reduce the price of the good subject to shortages. In this way, a transfer of purchasing power can occur between the two possible states in which the consumer can find himself. It turns out that this price reduction is always sufficient to overwhelm the decline in income, so that under Assumption 1 imposing some taxes always raises expected consumer utility.

Theorem 5 tells us that a competitive equilibrium will not achieve the social optimum. Can we say whether, under Assumption 1, a competitive equilibrium will devote too few resources to the production of good 1 and/or will involve a higher price for good 1 than occurs in the social optimum? Without further restrictions, all that can be said is that in the social optimum either the probability of satisfaction (or equivalently the customer capacity, $s$) will be higher and/or the price of good 1 will be lower than in a competitive equilibrium. We expect the normal case to involve an increase in the probability of satisfaction $1 - \Lambda$, and a decrease in the price $p$. For this normal case it immediately follows that under Assumption 1 a competitive equilibrium will involve devoting too few resources (i.e., $csN\pi(p)$) to the production of the good that is subject to shortages, when compared to the social optimum.

\section{Summary and Conclusions}

This paper has examined the behavior of markets characterized by price inflexibility, demand uncertainty, and production lags. It appears that many markets in the private, regulated, and government sectors are better described by the model examined here than by the traditional supply and demand model. Examples of markets described by the models of this paper include transportation, manufacturing, retailing, electric utilities, restaurants, hotels, and public parks.

Natural features of the markets studied here are that buyers and sellers will always have some probability of being unable to buy or sell all they want of the good. Supply will not in general equal demand. Price will exceed average production cost of the goods that are sold. It was possible to prove that as the size of the market increased, the equilibrium under uncertainty converged percentage-wise to that under certainty. Numerical calculations suggested that the customer per firm ratio might have to be unrealistically large before close convergence occurred. The social welfare implications of markets under uncertainty differ from those under certainty. In the special
case where expected consumer surplus reflects consumer's preferences, a competitive equilibrium is optimal. This result contrasts with previous results in the literature. In general though, the competitive equilibrium is not socially optimal, and the conditions under which subsidization would occur were derived.

The models examined here are rich in their implications. Demand uncertainty imposes costs on a firm in the form of potential idle capacity. Firms have an incentive to stabilize their random demand by finding “loyal” customers. Such loyal customers get a price discount because they enable a firm to better plan its production. In terms of the model, any customer willing to order in advance need pay only c for the product. Customers not willing to order in advance pay a higher price for the privilege of only buying occasionally. Newspaper subscriptions illustrate this point nicely. Coupons in packages are another example where random buyers pay a higher price than repeat buyers. Special favors and discounts for steady customers provide a final important example.

When the customers are interpreted as firms purchasing inputs, we obtain a model where firms might consider vertically integrating (or signing long-term contracts) to better assure themselves of supply at a lower than market price. Any firm with certain demand could supply its own needs at a price below the market price. In the market under study, externalities can occur when supplying firms cannot distinguish among customers with differing demand uncertainties. Firms might vertically integrate to escape paying for the costs that someone else's uncertainty imposes on the market. (See the author, 1976, for an examination of vertical integration.)

This paper emphasizes that the behavior of markets characterized by price inflexibility, demand uncertainty, and production lags differs in important respects from that of traditional deterministic markets. Because of the prevalence of these “nontraditional” markets in the economy, their further study definitely seems warranted.

### Appendix

#### A

Define the expected shortage $M$ for one firm with customer capacity $s$ as

$$M(s) = \sum_{i=1}^{\infty} (i - s)p r(i)$$

or

$$M(s) = \sum_{i=1}^{\infty} (i - s)p r(i) + (\bar{s} - \bar{r})p r(i)$$

where

$$\bar{r} = -\sigma N^L(u) - \sigma(1 - F(u))$$

and

$$u = (s - \bar{s})/\sigma, \bar{s} = L/N, \quad \sigma = \sqrt{\frac{L}{N}}, \quad \sigma^2 = N^L(u) = \frac{l}{\sqrt{2\pi}} \int_{-\infty}^{u} te^{-t^2/2} dt$$

$\bar{s} = E(i), pr(i) = \text{binomial probability that i of the L customers come to one store.}$

Notice that $\bar{s}$ and $\sigma^2$ are the mean and approximate variance of this binomial process. Hence $(i - \bar{s})/\sigma$ is approximately normally distributed with mean 0, and variance 1.\(^{31}\) If all firms follow the same operating policy then

$$1 - \lambda(s) = 1 - N \cdot M/L \quad \text{or}$$

$$1 - \lambda(s) = 1 - M/\sigma^2 \quad \text{or}$$

$$1 - \lambda(s) = [\sigma^2 + \sigma N^L(u)]/[\sigma^2 + \sigma(1 - F(u))]/\sigma^2$$

or defining $I(s) = N^L(u) - uF(u)$, we have

$$1 - \lambda(s) = (\sigma I + s)/\sigma^2.$$  

#### B

**Proof of Theorem 2:**

The Lagrangian for the government's maximization problem can be written as

$$(A1) \quad \mathcal{L}(p, T, s, \mu) = (1 - \lambda)V(p, Y + T) + \lambda U(0, Y + T) - \mu \left[ (1 - \lambda)p \frac{L}{N} - cs \right] x(p, Y + T) - \frac{L}{N} T$$

where $\mu$ is a negative Lagrange multiplier.\(^{32}\)

\(^{31}\) See Feller, ch. 7.

\(^{32}\) The notation was defined beneath (6). Recall that $\lambda$ is not a Lagrange multiplier, but is the probability of disappointment which is a function of $s$ given in (3).
The first-order conditions are:33

\[(A2)\]  
\[(1 - \lambda)V_1 = \mu \left[ (1 - \lambda) \frac{L}{N} x + \left( 1 - \lambda \right) \frac{L}{N} p - sc \right] x_1 \]

\[(A3)\]  
\[(1 - \lambda)V_2 + \lambda U_2 = \mu \left[ (1 - \lambda) \frac{L}{N} p - sc \right] x_2 - \frac{L}{N} \]

\[(A4)\]  
\[\frac{d(1 - \lambda)}{ds} \frac{N}{L} [V - U] = \mu \left[ (1 - \lambda) \frac{d(1 - \lambda)}{ds} p - c \right] x \]

\[(A5)\]  
\[\left( 1 - \lambda \right) \frac{L}{N} p - cs \right] x = \frac{L}{N} T \]

Substituting (A5) into (A2) and (A3), we obtain

\[(A6)\]  
\[(1 - \lambda)V_1 = \mu \left[ (1 - \lambda) \frac{L}{N} x + \frac{L}{N} \frac{T}{x} x_1 \right] \]

and

\[(A7)\]  
\[(1 - \lambda)V_2 + \lambda U_2 = \mu \left[ \frac{L}{N} \frac{T}{x} x_2 - \frac{L}{N} \right] \]

Since \(V\) is an indirect utility function, we have that \(x = -V_1/V_2\). Using this relation, rewrite (A6) as

\[\begin{align*}
(1 - \lambda)V_1 &= \mu \left[ (1 - \lambda) \frac{L}{N} x + \frac{L}{N} \frac{T}{x} x_1 \right] \\
\text{or} & \\
(1 - \lambda)V_2 &= (-\mu) \left[ (1 - \lambda) \frac{L}{N} x + \frac{L}{N} \frac{T}{x} x_1 \right] \\
\text{or} & \\
(A8) \quad V_2 &= (-\mu) \left[ \frac{L}{N} + \frac{L}{N} \frac{T}{x} \frac{x_1}{x} \frac{1}{1 - \lambda} \right]
\end{align*}\]

From (A7) and the above assumption, it follows that

\[\begin{align*}
\text{(A9)} \quad \mu \left[ \frac{L}{N} \frac{T}{x} x_2 - \frac{L}{N} \right] < V_2 \\
\text{Substituting the expression for } V_2 \text{ from (A8) into (A9), we have that} \\
\mu \left[ \frac{L}{N} \frac{T}{x} x_2 - \frac{L}{N} \right] < (-\mu) \left[ \frac{L}{N} + \frac{L}{N} \frac{T}{x} \frac{x_1}{x} \frac{1}{1 - \lambda} \right] \\
\text{or} \\
(-1)(-\mu) \frac{L}{N} \frac{T}{x} x_2 - \mu \frac{L}{N} < -\mu \frac{L}{N} \\
-\frac{L}{N} \frac{T}{x} \frac{x_1}{x} \frac{1}{1 - \lambda} < -\mu \frac{L}{N} \\
\text{or since } -\mu > 0, \\
(A10) \quad (-1)T \frac{x_2}{x} (1 - \lambda) < \frac{T}{x} x_1 \\
\text{or} \\
T \left( -\frac{x_1}{x} \frac{x_2}{x} \right) < T(1 - \lambda) \\
\text{If } T > 0, \text{ then } -\frac{x_1}{x} \frac{x_2}{x} < 1 - \lambda < 1, \text{ while if } T < 0, \text{ then } -\frac{x_1}{x} \frac{x_2}{x} > 1 - \lambda. \text{ But from the Slutsky equation, we know that} \\
x_1 + x_2 \leq 0 \text{ or } -x_1/x_2 \geq 1. \text{ Therefore if} \\
T > 0, \text{ we obtain a contradiction. Hence} \\
only T < 0 \text{ is possible in the optimal solution.}^{34,35}
\]

\[34\] I am implicitly assuming that the optimal solution is an interior solution and satisfies the first- and second-order conditions. Corner solutions \((T = -Y)\) or \((s = 0)\) are assumed not to be optimal.

\[35\] If we assume that \(x_2 > 0\) and \(V_2(p, Y) = \nu_2(0, Y)\), then (A10) (with an equality sign) implies that the optimal \(T = 0\). This result follows from the fact that if \(x_2(1 - \lambda)\) equaled \(-x_1/x\), the Slutsky condition \(x_1 + x \cdot x_2 \leq 0\) would be violated.

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