PRODUCT VARIETY AND DEMAND UNCERTAINTY: WHY MARKUPS VARY WITH QUALITY*

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We demonstrate that demand uncertainty can explain equilibrium product variety in the presence of sunk costs. Product variety is an efficient response to uncertainty because it reduces the expected costs associated with excess capacity. We find that within the firm’s product line, the highest quality product has the highest profit margin but the lowest percentage margin, while the lowest quality product has the highest percentage margin but the lowest absolute margin. Both of these relationships are consistent with evidence available from marketing studies.

I. INTRODUCTION

What determines the breadth of a firm’s product line? We show that when production costs must be sunk in anticipation of demand, demand uncertainty can lead to an increase in product variety. By limiting their inventory of high quality goods and selling low quality goods once their high quality goods have stocked out, firms reduce the costs associated with demand uncertainty. The model helps explain the extent of product differentiation and suggests a rationale for many common retailing practices such as the use of private labels and full product line forcing by manufacturers. The model also yields the testable empirical prediction that higher quality products earn higher absolute margins and lower percentage margins.

While differences in preferences provide a convincing explanation in many cases for product variety in retail stores, we argue that another

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important explanation in many cases is that firms face uncertain demand and their products are costly to store. Specific retail examples include grocery stores’ offerings of national brands and private labels, their offerings of fresh and frozen meats and seafood, their offerings of fresh and packaged baked goods and produce, and restaurants’ offerings of special entrées in addition to the regular dinner menu. The model also implies that toy retailers will stock fewer high quality toys when lower quality substitutes are available, and similarly, clothing retailers will limit the availability and variety of designer fashion items when they can offer consumers lower quality and less fashion sensitive alternatives.

Similarly, the model can also help to explain product differentiation in travel and other service industries. For example, while an airline’s decision about how many first class, business class, and coach class seats to put in a passenger plane often reflects the mix of customers between business and leisure passengers, our model suggests another important benefit from the observed variety and can explain why coach seats are more plentiful and sell out less frequently. Similarly, hotels can be designed so that every room has a view (a long, narrow design) or so that some rooms have views and others don’t (by using a wider design); rooms with views clearly cost more given the shadow (land) cost of the scarce view (i.e., the water can only be viewed from exterior rooms on the side of the building facing it). Stadiums can be built with both permanent seats (more comfortable) and temporary seats (less comfortable). And finally, universities must decide how many faculty members versus Ph.D. students to use in staffing their undergraduate classes, and medical centers must decide how many physician’s aids versus doctor’s aids to use in an office that accepts walk-in patients.

The common elements in these examples are that firms sink their costs of production before demand is realized and that firms choose their capacity (or inventory) of high and low quality products with the expectation that the high quality products will be consumed more frequently than low quality products. While empirically distinguishing between demand uncertainty and demand heterogeneity as explanations for product variety in these applications may sometimes be challenging, our model’s predictions about relative and absolute margins are directly testable. Specifically, we find that relative or percentage margins decline with quality, which is in contrast to some models of monopoly price discrimination in settings with multiple products. Moreover, we are able to devise experiments where changes in uncertainty or in sunk costs change the pricing in our model but not necessarily in models of price discrimination.

The economics and marketing literatures have made big steps in understanding product differentiation, but both literatures have focused on consumer preferences as the reason for variety (see surveys by Eaton and Lipsey [1989], and Lancaster [1990]). An important part of this literature looks at the product line and pricing decisions of a monopolist engaging in
second degree price discrimination when consumers’ valuations are correlated with their preference for product quality (see Mussa and Rosen [1978], Johnson and Myatt [2003] and the empirical work of Shepard [1991]). This literature emphasizes the role of asymmetric information as opposed to sunk costs of production. As discussed later, some of our insights are similar to those in that literature (especially Johnson and Myatt) but others are quite different. However, unlike the price discrimination literature, our insights about product variety arise from the fact that costs are sunk before anyone knows demand and not because firms have any information disadvantage.

Our paper is related to research on how firms’ ex ante production decisions depend on demand uncertainty. Research beginning with Sheshinski and Dreze [1976] has shown that firms facing demand uncertainty will use a range of production technologies to meet demand and allocate their production across these production technologies to minimize costs given the realization of demand. Plants with higher fixed costs and lower variable costs are utilized to meet base demand and plants with higher variable costs and lower fixed costs are utilized only to meet peak demand. Firms are willing to pay on average higher costs (as a consequence of producing with a range of production technologies) to reduce the production costs they must sink ex ante before demand is known. Similarly, we find that firms are willing to sacrifice expected revenues (as a consequence of producing a range of product qualities) in order to reduce the production costs they must sink before demand is known.

Our work is also related to work in the operations management literature on the inventory problem for a multi-product firm when consumers can substitute products in response to stockouts (see Mahajan and van Ryzin [2001] and Bassok, Anupindi and Akella [1999] and the references therein). However, this literature typically treats the firm’s product line as exogenous. One exception is van Ryzin and Mahajan [1999], who consider a model of optimal product assortment with stockouts, but they assume stockouts result in lost sales rather than substitution. Smith and Agrawal [2000] and Honhon, Gaur and Seshadri [2006] analyze product assortment decisions with substitution, but treat prices as exogenous. Similarly, our work is related to the revenue management literature (see Talluri and van Ryzin [2004]). Some work in this literature looks at consumer substitution between products in response to stockouts; however, this literature generally assumes that the firm has a common capacity constraint for its products (a traveler who books seven days ahead occupies the same seat as one who books one day ahead) and treats both prices and product characteristics as exogenous.

In the paper we characterize the optimal product line for a monopolist in the presence of ex ante sunk costs and uncertain demand. We show that when demand is certain, a single product is produced, but when demand is uncertain, selling heterogeneous products is more profitable and more efficient. The highest quality product produced is the one that generates the
greatest social surplus; it yields the highest absolute margin and the lowest percentage margin of the firm’s products. The lowest quality product produced yields the lowest absolute margin and the highest percentage margin of the firm’s products.

After presenting the formal model, we consider several important extensions including the introduction of competition, consumer heterogeneity, and a mixture of sunk and non-sunk costs. We also discuss empirical implications of the model, analyze how those implications differ from other theories of product differentiation, and discuss the implications of the model for manufacturer-retailer relationships and full-line forcing.

II. OPTIMAL PRODUCT LINE WITH DEMAND UNCERTAINTY

In this section, we analyze the product line, inventory, and pricing decisions of a monopolist that can produce an arbitrary number of vertically differentiated varieties of its product.

II(i). The Model

Consumers have unit demands and maximize their surplus given the set of available products and prices. Let \( v \) denote consumers’ willingness to pay for a product of quality \( v \) and \( p(v) \) denote the price of product \( v \). So consumers’ purchase decisions maximize \( v - p(v) \) subject to current product availability.

The firm can produce any product or set of products in \([v, \bar{v}] \subseteq \mathbb{R}^+\). Let \( c(v) \) denote the sunk cost of producing one unit of a product with quality \( v \), that is, the cost that is incurred whether or not the good is sold. We normalize the non-sunk costs to zero, though we relax this assumption later in the paper. We assume \( c(v) \) is continuously differentiable, \( c(v) > 0 \), \( c'(v) > 0 \), and \( c''(v) > 0 \). We also assume that \( v - c(v) < 0 \) and \( \bar{v} - c(\bar{v}) < 0 \), so production is inefficient at the boundaries of the firm’s choice set.\(^1\) Naturally we assume that \( v - c(v) > 0 \) for some \( v \in (v, \bar{v}) \), so production is efficient for some level of quality. We define \( v^* = \arg \max (v - c(v)) \) to be the product quality for which the total surplus is the greatest.

When demand is uncertain, let the random variable \( x \) denote the measure of identical consumers willing to buy one unit of output. Let \( f(x) \) denote the probability density function associated with \( x \). We assume \( f(x) > 0 \) on \([\underline{x}, \bar{x}]\), where \( \underline{x} > 0 \), and \( f(x) = 0 \) otherwise. Let \( F(x) \) denote the associated cumulative distribution function. When demand is certain \( \underline{x} = \bar{x} \), \( F(x) = 1 \) for all \( x \geq \bar{x} \), and \( F(x) = 0 \) for all \( x < \bar{x} \).

We begin with the simple case in which demand is certain.

\(^1\) We could have allowed the range of feasible product qualities to be \([0, \infty]\), however in this case the cost function would need to be discontinuous at zero since clearly producing nothing is more efficient than producing a product of quality zero when \( c(0) > 0 \).
Proposition 1: When demand is certain to be $x$, the monopolist chooses product quality $v^*$, produces $q = x$ units, and sets its price equal to $v^*$.

The proof of Proposition 1 follows immediately from the definition of $v^*$ and profit maximization. We now turn to the case in which demand is uncertain.

II(ii). Single Product Monopolist

Suppose that the firm is able to produce only one product. Which level of product quality $\tilde{v}$, would the firm choose? The firm chooses its price, quantity and quality to maximize profits. Clearly the firm sets $p = \tilde{v}$, whether prices are set before or after demand is realized, so we can write the firm’s problem as

$$
\max_{\tilde{v}, q} \int_{x}^{q} \tilde{v}xf(x)dx + \int_{q}^{\infty} \tilde{v}f(x)dx - qc(\tilde{v}).
$$

The first order conditions are

$$
\tilde{v}[1 - F(q)] - c(\tilde{v}) = 0,
$$

and

$$
\int_{x}^{q} xf(x)dx + \int_{q}^{\infty} qf(x)dx - qc'(\tilde{v}) = 0.
$$

Rewriting these expressions, the firm’s optimal inventory and quality are given by

$$
[1 - F(q)] = \frac{c(\tilde{v})}{\tilde{v}}
$$

and

$$
(1) \quad c'(\tilde{v}) = \frac{\int_{x}^{q} xf(x)dx}{q} + [1 - F(q)].
$$

It is obvious that the profit maximizing inventory and quality must lie in the interior of $[x, \tilde{x}] \times [\tilde{v}, \tilde{v}]$, and since the profit is twice continuously differentiable, the profit maximizing inventory and quality must satisfy these conditions.

So equation (1) implies the following result:

Proposition 2: When demand is uncertain and the firm is constrained to choose a single product variety, then the product quality chosen satisfies $\tilde{v} < v^*$.
The first order conditions also imply that as demand becomes more certain, the firm’s quality rises. Consider the case where \( f(x) \) is uniform on \([\underline{x}, \bar{x}]\). It is easy to verify that \( \hat{v} \) is increasing in \( \underline{x} \) and decreasing in \( \bar{x} \), and as \( \bar{x} \to \underline{x} \) or \( \underline{x} \to \bar{x} \), \( \hat{v} \to v^* \).

III(iii). The Multi-Product Monopolist

We now suppose that the monopolist can produce a range of vertically differentiated products and produce and sell an arbitrary quantity of each product. The following lemma simplifies the statement of the firm’s optimization problem. We show that the monopolist sets \( p(v) = v \) for all of its products, consumers’ purchase decisions are ex post efficient, and the firm captures the entire surplus. Therefore it is impossible for the monopolist to achieve any higher ex post profits.

**Proposition 3:** Given its inventory, the monopolist sets \( p(v) = v \) for all of the products it produces, and in equilibrium consumers buy goods in decreasing order of quality (highest quality goods stock out first).

**Proof:** If \( p(v) \) is not equal to \( v \) across the firm’s products, then consumers will have a strict preference for some products over others. If so, then the firm could increase the prices of some of its goods, and in so doing increase its profits, without affecting the order in which consumers make their purchases or the total volume of their purchases. Hence at the profit maximizing solution, consumers must be indifferent between all goods, and so \( p(v) \) must equal \( v \) across all qualities produced.

Though consumers are indifferent, it also must be the case that consumers buy the highest priced (highest quality) good first and consume the goods in decreasing order of price. If not, then the firm could strictly increase its profit by lowering the prices of all of its goods by a small, systematically different, amount, and inducing consumers to make their purchases in decreasing order of price, which would strictly increase the firm’s expected revenues.

Finally, the price of the firm’s lowest quality good must be equal to consumers’ valuations since otherwise raising this price would have no impact on the volume or order of product sales and would strictly increase profits. And since consumers are indifferent between the goods, it follows that the prices for all of the firm’s products must equal consumers’ valuations.

The firm’s problem is further simplified if we let the firm choose the quality of each unit of its output rather than the amount of output to offer at each quality level. Define \( v(x) \) to be the quality of the good purchased and consumed by the marginal consumer when the realized demand is \( x \). So \( c(v(x)) \) is the ex ante cost of producing the marginal unit consumed when
demand is \( x \). It follows directly from the Lemma that \( v(x) \) is non-increasing in \( x \) and, therefore, that \( c(v(x)) \) is non-increasing in \( x \).

The firm’s problem is to choose its total inventory and the product quality of each unit of its inventory in order to maximize its expected profit. Let \( Q \) denote the firm’s inventory, which is finite, so the firm chooses \( Q \) and \( v(x) \) on \([0, Q]\) to maximize

\[
\int_0^Q v(x)(1 - F(x))dx - \int_0^Q c(v(x))dx
\]

subject to the constraint that \( v(x) \) is non-increasing. The first term represents expected revenues while the second term represents sunk costs.

**Proposition 4:** When demand is uncertain, the monopolist produces multiple quality products, and the range of product qualities for the firm is \([\hat{v}, v^*]\) where \( \hat{v} = \arg \min_v c(v)/v \), \( v^* = \arg \max_v v - c(v) \), and \( \hat{v} < v^* \).

**Proof:** The firm’s optimization problem can be solved by pointwise maximization if we ignore the constraint that \( v(x) \) is non-increasing. The resulting first order conditions are

\[
(2) \quad 1 - F(x) - c'(v(x)) = 0
\]

for all \( x \in [0, Q] \), and

\[
(3) \quad v(Q)(1 - F(Q)) - c(v(Q)) = 0
\]

Equation (2) implies \( v'(x) < 0 \), so the solution to (2) and (3) is non-increasing and hence is a solution to the constrained optimization problem as well.

Note that since \(-c''(v) < 0\) for all \( v \) and

\[
-f'(Q)v(Q) + v'(Q)[1 - F(Q)] - c'(v(Q))v'(Q) = -f'(Q)v(Q) < 0,
\]

it follows that the solution, \( \{v(x), Q\} \), to the first order conditions, (2) and (3), is unique and is a maximum.

Consider the quality associated with the first unit of inventory, \( v(0) \). Equation (2) implies that \( 1 - c'(v(0)) = 0 \), so \( v(0) = v^* = \arg \max (v - c(v)) \). Intuitively, since consumers buy the highest quality product first, and since \( p(v) = v \) (by Proposition 3), it follows that \( v(0) \) must be the same as the quality under certainty, that is, \( v(0) = v^* \).

Combining (2) and (3) yields

\[
(4) \quad c'(v(Q)) = \frac{c(v(Q))}{v(Q)}
\]
The following problem,

\[ \hat{v} = \arg \min_v c(v) - \frac{c'(v)}{v}, \]

has the following first order condition,

\[ c'(v)/v - c(v)/v^2 = 0, \]

and the following second order condition,

\[ c(v)/v - 2(c'(v)/v - c(v)/v^2)/v > 0. \]

Since the second order condition holds whenever the first order condition is satisfied (using the fact that \( c''(v) > 0 \)), this means that every solution to the first order condition is a local minimum. Since \( \hat{v} \) is the unique solution to the first order condition, \( \hat{v} \) must also be the unique solution to (4), so \( v(Q) = \hat{v} \).

Finally, because \( v^* \) maximizes \( v - c(v) \), there necessarily exists some \( v < v^* \) such that

\[ \frac{c(v)}{v} < \frac{c(v^*)}{v^*}, \]

so \( \hat{v} \) is strictly less than \( v^* \). And \( v'(x) < 0 \) implies \( \hat{v} = v(Q) \leq v(x) \leq v(0) = v^* \) for all \( x \), so the optimal range of qualities is \([\hat{v}, v^*]\).

Proposition 4 establishes that the highest quality product that the firm produces is the product that would be offered if demand were certain. That is, the highest quality offered is the one that maximizes surplus (and the monopolist’s margin) conditional on sale. The lowest quality product that the firm produces is the product that minimizes the average cost of quality.

Interestingly, the range of products produced by the firm in our model is the same as the range of products produced by a monopolist selling to privately informed consumers in Johnson and Myatt’s [2003] model of second degree price discrimination. While our theories of product variety are very different (heterogeneous preferences and asymmetric information versus demand uncertainty and sunk costs), both models predict that selling a product of lower quality than \( \hat{v} = \arg \min_v c(v)/v \) is never profitable because the firm can increase its profit by selling the same consumer a product of quality \( \hat{v} \).\(^2\)

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\(^2\) In our model, the reason the firm does not want to sell lower quality products than \( \hat{v} \) is because if the firm can make money selling a product of quality \( \tilde{v} < \hat{v} \) at a price \( \tilde{v} \) with probability \( \eta \) at cost \( c(\tilde{v}) \) (i.e., \( \eta \tilde{v} \geq c(\tilde{v}) \)), then the firm can make even more money by also selling an upgrade to quality \( \hat{v} \) at a price \( \hat{v} - \tilde{v} \) with probability \( \eta \). Clearly \( \eta(\hat{v} - \tilde{v}) \geq c(\hat{v}) - c(\tilde{v}) \) since the average cost of quality is falling for all \( v \leq \hat{v} \), and the cost of the upgrade is \( \int_{\hat{v}}^{\tilde{v}} c'(v) dv \), which is strictly less than \( c(\hat{v}) - c(\tilde{v}) \) since the marginal cost is less than the average cost of quality for all.
The following proposition summarizes the relationship between product quality and the firm’s absolute and percentage markups.

**Proposition 5:** When demand is uncertain, the monopolist’s highest quality product earns the highest absolute margin and lowest percentage margin of the products produced, while the monopolist’s lowest quality product earns the lowest absolute margin and the highest percentage margin of the products produced.

**Proof:** Proposition 4 established that the firm’s highest quality product, \( v^* \), has the highest absolute margin. Because \( 1 - c'(v^*) = 0 \) and \( c'(v) \) is strictly increasing, for all \( v < v^* \). Since \( p(v) = v \), this implies that the firm’s lowest quality product, \( \hat{v} \), earns the lowest absolute margin of the products produced.

The proof of Proposition 4 also established that the firm’s lowest quality product is uniquely defined by

\[
\hat{v} = \arg \min_v \frac{c(v)}{v}.
\]

Since the associated first order condition has a unique solution, it follows that \( c'(v)/v - c'(v)/v^2 > 0 \), or equivalently that \( c(v)/v \) is strictly increasing, on \([\hat{v}, v^*]\). Since \( p(v) = v \), this implies that the firm’s lowest quality product, \( \hat{v} \), has the highest percentage margin, and that the firm’s highest quality product, \( v^* \), has the lowest percentage margin of the products produced.

In our model, as demand becomes more certain, the range of product qualities offered remains the same (i.e., \( \hat{v} = \arg \min c(v)/v \) is independent of \( f(x) \)). However, as the demand becomes more certain, in particular as \( \bar{x} \to x \), the distribution of products offered converges to a mass point at \( v^* \). That is, the firm’s production of every other quality product converges to zero and its production of quality \( v^* \) converges to \( \bar{x} \).

The fact that the product range collapses only in the limit is empirically counterintuitive. However, recall that we are ignoring the fixed costs associated with product introductions. With strictly positive fixed costs, in the limit as demand becomes certain, the firm’s optimal product line is a single product with quality \( v^* \).

### III. Extensions

Our results are quite robust to changes in the model. For example, since the monopolist extracts the entire consumer surplus, it is clear that a social planner would make the same product line decision as the monopolist. In

\[
v \leq \hat{v}, \text{ so } \eta(\hat{v} - \hat{v}) \geq \int_\hat{v}^{v^*} c'(v)dv.
\]

Johnson and Myatt [2003] use a similar argument, but it only holds when the preferences for quality are multiplicative in types.
other words, demand uncertainty and sunk costs of production induce product variety because it is efficient and not simply because the firm has market power.

III(i).  **Sunk and Non-Sunk Costs**

We can also generalize the model to include both sunk and non-sunk costs and show how the extent of product variety is related to the fraction of costs that are sunk. Let $s(v)$ denote the portion of the firm’s costs of production for a product of quality $v$ that are sunk before demand is known, and let $r(v)$ denote the portion of the firm’s costs that are expended after demand is learned. Equivalently, one can think of $s(v)$ as representing the portion of the final product that is perishable or the time cost of holding the good to the next sale period if it fails to sell this period. We assume $s(v)$ and $r(v)$ are continuously differentiable, $r'(v) > 0$, $r''(v) > 0$, and $r'''(v) > 0$.\(^3\)

As before, the monopoly seller will price every product at $p(v) = v$ and sell its products in decreasing order of quality. By a similar argument, we can write the firm’s problem as choosing $Q$ and $v(x)$ on $[0, Q]$ to maximize

$$\int_0^Q (v(x) - r(v(x)))(1 - F(x))dx - \int_0^Q s(v(x))dx$$

subject to the constraint that $v(x)$ is non-increasing. The first order conditions are

$$\frac{(1 - r'(v(x)))(1 - F(x)) - s'(v(x))}{1 - F(x)} = 0$$

for all $x \in [0, Q]$ and

$$\frac{(v(Q) - r(v(Q)))(1 - F(Q)) - s(v(Q))}{1 - F(Q)} = 0.$$  

From (5), as before, $v(0) = v^*$, where $v^* = \arg \max_v v - s(v) - r(v)$. More generally, we can write (5) as

$$r'(v(x)) + \frac{s'(v(x))}{(1 - F(x))} = 1,$$

that is, the marginal non-sunk cost of quality plus the marginal sunk cost of quality divided by the probability of sale is equal to the marginal benefit of quality. As before, uncertainty increases the marginal cost of quality for units that are less likely to sell and implies the firm has an incentive to offer lower quality for goods that are less likely to sell.

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\(^3\) In some cases, the firm may be able to influence the ratio of sunk to non-sunk costs. This idea first appeared in Stigler [1939], but has yet to be studied in detail.
From (5) and (6), it follows that

\[
\frac{s'(v(Q))}{1 - r'(v(Q))} = \frac{s(v(Q))}{v(Q) - r(v(Q))}.
\]

The ratio of the sunk cost to net revenue from an additional sale is equal to the ratio of incremental sunk costs to incremental net revenue.

Now suppose \(s(v) = \alpha c(v)\) and \(r(v) = (1 - \alpha)c(v)\) where \(\alpha \in (0, 1)\) is a constant, so the proportion \(\alpha\) of costs which are sunk is independent of quality. In this case, (7) becomes

\[
\frac{\alpha c'(v(Q))}{1 - (1 - \alpha)c'(v(Q))} = \frac{\alpha c(v(Q))}{v(Q) - (1 - \alpha)c(v(Q))},
\]

or

\[
c'(v(Q)) = \frac{c(v(Q))}{v(Q)},
\]

so the range of products offered, \([v(Q), v(0)]\), and more generally, the distribution of inventory over qualities is independent of \(\alpha\), or independent of the share of the costs that are sunk.

In many instances, all of the costs are incurred \textit{ex ante}, but storage is feasible, so the sunk costs represent the time value of money, that is, the cost of producing in period \(t\) when the sale is not made until period \(t + 1\). In this case, the assumption that \(s(v) = \alpha c(v)\) seems reasonable. The implication is that the distribution of qualities offered is independent of the interest rate (which is a determinant of \(\alpha\)).

In other instances, the feasibility of storage varies with quality. For example, the highest quality prepared foods, such as bakery items, typically contain the fewest preservatives and have a shorter shelf life than lower quality items in the same category. That is, the share of costs that are sunk, \(\alpha(v) = s(v)/c(v)\), is increasing in \(v\). So (7) becomes

\[
\frac{\alpha'c'(v(Q))c(v(Q)) + \alpha(v(Q))c'(v(Q))}{1 + \alpha'(v(Q))c(v(Q)) - (1 - \alpha(v(Q)))c'(v(Q))} = \frac{\alpha(v(Q))c(v(Q))}{v(Q) - (1 - \alpha(v(Q)))c(v(Q))}
\]

or

\[
\frac{\alpha'(v(Q))c(v(Q))}{\alpha(v(Q))}c(v(Q)) \left(1 - \frac{c(v(Q))}{v(Q)}\right) + c'(v(Q)) = \frac{c(v(Q))}{v(Q)}.
\]

If \(\alpha'(v) > 0\), then (8) implies that \(v(Q) < \text{arg min}_v c(v)/v\). That is, if the share of sunk costs rises as quality rises, then the product range is increased—the firm offers even lower quality products. So an important determinant
of the extent of product variety when demand is uncertain is whether or not the share of sunk costs can be reduced through a reduction in quality.

III(ii). Competition

Firms in competitive markets face the same type of product line decisions as the monopolist. In an earlier version of this paper (Carlton and Dana [2004]), we extend our results by considering three competitive models: ex post market clearing, ex ante market clearing with costless ex post search for availability (Prescott [1975]), and ex ante market clearing with infinitely costly search for availability (Carlton [1978]).

In the ex post market clearing model, firms decide how much of each quality good to produce before observing demand. After demand is realized, prices of each quality good are determined so as to clear markets. In the ex ante market clearing model of Prescott [1975], see also Eden [1990] and Dana [2000], firms decide how much of each quality good to produce and what price to charge before demand is observed. After demand is realized, consumers choose costlessly among the alternatives. In the ex ante market clearing model of Carlton [1978] (see also Deneckere and Peck [1995] and Dana [2001]) firms decide how much of each quality good to produce and what price to charge, and consumers choose a single firm from which to purchase (randomizing if the firms offer equal expected consumer surplus) before learning demand. In each case, the product line decision is identical to the monopolist’s.

Consider briefly the ex post market-clearing model. Let \( p(x) \) and \( v(x) \) denote the price and quality of the marginal unit sold when demand is \( x \). Under market clearing, when the realized demand exceeds supply, i.e., \( \hat{x} > Q \), the price of every product is equal to the consumers’ valuation for that product, i.e., \( p(x) = v(x) \). When the realized demand is less than supply, i.e., \( \hat{x} < Q \), the price of every unsold unit is equal to zero. This implies that the price of the marginal unit sold, which is of quality \( v(\hat{x}) \), is also equal to zero, so \( p(x) = 0, \forall x \geq \hat{x} \). And since consumers must be indifferent in equilibrium between every good sold, the price of the inframarginal units that are sold (those of higher quality than \( v(\hat{x}) \)) must be \( p(\hat{x}) = v(x) - v(\hat{x}) \).

In the competitive equilibrium for this ex post market-clearing model, the two conditions that must hold for all \( x \) are the first order condition on quality,

\[
1 - F(x) - \epsilon'(v(x)) = 0,
\]

which implies that the expected marginal benefit of an increase in the quality of the goods that are sold only when realized demand is \( x \) or above
is equal to the marginal cost of quality, and a zero expected profit condition,

\[
\int_{x}^{Q} (v(x) - v(y))f(y)dy + (1 - F(Q))v(x) = c(v(x))
\]

which implies that the expected revenues from the goods that are sold only in states \(x\) or above are equal to the expected costs. The former clearly implies that goods that are sold with probability one have quality equal to \(v^*\), the same as in the monopoly model. Similarly, the later implies

\[
v(Q)(1 - F(Q)) - c(v(Q)) = 0
\]

so the competitive inventory is the same as the monopoly inventory. And finally, the latter also implies

\[
1 - F(Q) - c'(v(Q)) = 0,
\]

so the lowest quality product offered has quality equal to \(\hat{v}\), the same as in the monopoly model. Hence this model of competition yields the same product offerings as monopoly, but of course lower average prices (see Carlton and Dana [2004]) for more details on the robustness of our results across a wider range of models).

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III(iii). Oligopoly

The general economic forces that arise from demand uncertainty and sunk costs, and affect product variety, also apply to oligopoly. However, demand uncertainty and sunk costs increase product variety and make it more likely that an individual firm will stock out of the good. Hence firms may be more differentiated which may soften competition and lead to higher prices than one would otherwise expect. On the other hand (see e.g. Carlton and Perloff [2005] and Tirole [1988]) product differentiation makes it more difficult to support prices above the static Nash equilibrium levels in a dynamic game.

III(iv). Heterogeneous Valuations

In the working paper version of this paper (Carlton and Dana [2004]) we allowed for heterogeneous consumer valuations. When consumers have heterogeneous valuations, the equivalence between competition, monopoly, and the social optimum no longer holds, and both demand uncertainty and sunk costs have potentially different implications for product variety. However, as in the previous section, an increase in uncertainty or a decrease in the fraction of sunk cost tends to increase product variety.
IV. EMPIRICAL PREDICTIONS

We believe that this work should have a direct impact on the empirical literature on product differentiation and price discrimination. The model is relevant for any market in which consumers substitute between high and low quality, demand is uncertain, production costs are sunk before demand is known, and firms’ inventory or capacity is not necessarily fully utilized (e.g., prices are rigid or excess capacity exists even at equilibrium, market clearing prices). We predict higher absolute markups and lower percentage markups for high quality products in both competitive and monopoly markets, even when consumers have \textit{ex post} identical preferences for product quality. We also predict greater product variety when demand is more uncertain (if there are fixed costs of product introductions) and when the share of sunk costs (or the degree to which the product is perishable) is positively correlated with quality. Finally, since underutilization of inventory is much more likely when firms’ prices are rigid, all else equal, we expect to see a greater range of product qualities when prices are constrained to be more rigid.

Empirical testing of the model would require careful attention to the measurement of margins, turnover, shelf space restrictions, product shelf life, competitive conditions in both retailing and manufacturing, the extent to which customers vary in their relative valuations over quality. Perhaps the most direct existing studies relevant to our model are studies of grocery stores, which, for a wide variety of products, stock both high quality national brands and low quality private labels.

The evidence from grocery stores seems to support our model’s prediction that margins are higher on higher quality products. For a wide variety of products (e.g., tooth brushes, toothpaste, soft drinks, crackers, soups, cereals, etc.) grocery stores earn higher percentage margins on private labels than on national brands, while the absolute margin (especially after adjusting for turnover) is generally higher on the national brands.\footnote{See Barsky \textit{et al.} [2001], Hoch and Banerji [1993], Ailawadi and Harlam [2002], Salmon and Cmar [1987], \textit{Supermarket Strategic Alert} [2002], Brady \textit{et al.} [2003], Berges-Sennou \textit{et al.} [2003].} But these studies should be viewed as only suggestive of the model’s applicability, and more carefully designed studies across a variety of different industries would be necessary to fully test the applicability of the model’s predictions.

One relevant question is whether empirical tests can be devised to distinguish the impact of the economic forces we have identified from the impact of those economic forces identified in other models, such as Mussa and Rosen [1978] and Lancaster [1990]. In Mussa and Rosen [1978] asymmetric information combined with profit maximization lead to a range of product variety that is greater in monopoly markets than competitive markets. Their model, like ours, predicts higher absolute margins on high quality products in both competitive and monopoly markets. However, our model predicts that absolute margins are lower in competitive markets, while Mussa and Rosen predict the opposite. This suggests that empirical tests could be developed to distinguish between the two models.

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quality goods than on low quality goods, however their model predicts increasing relative margins. More specifically, Mussa and Rosen’s [1978] model (see also the monopoly model in Johnson and Myatt [2003]) assumes heterogeneous consumers with types \( \theta \) choose the product that maximizes \( \theta v - p \). They assume firms know the distribution of consumers (i.e., can accurately forecast sales), but they do not observe consumers’ individual types. They also assume firms have increasing and convex costs of quality, but have constant returns to sale. Under these assumptions, relative margins are always increasing, though this important result is not reported in Mussa and Rosen’s paper and also seems to have escaped notice in subsequent work on their model (a proof is provided in the appendix).

Also, since Mussa and Rosen’s model assumes asymmetric information, but no aggregate uncertainty, it makes no predictions about the effect of uncertainty or sunk costs on product variety, while ours does. Similarly, the Lancaster model is silent with respect to the effect of increases in sunk costs on margins or product variety. Each of these differences could be the basis for an empirical test attempting to figure out the relative importance of the multiple economic forces affecting any one market.

Another difference between our theory and others is our prediction that product variety will be greater when the proportion of production costs that are sunk is increasing in product quality. For many products, the most perishable products are also higher quality (more specifically, the technology for increasing shelf life reduces product quality). So, all else equal, product variety should be greater for products that are more perishable, but can be modified through the use of preservatives to increase shelf life.

V. CONCLUSION

Demand uncertainty can lead to product variety where it would not have existed otherwise. In particular, demand uncertainty makes it possible that not all inventory or capacity is utilized, and, as a consequence, firms respond by stocking low cost, low quality products as an alternative to their high cost, high quality products that are occasionally stocked out. Our model describes new economic forces influencing equilibrium product variety and produces unambiguous and testable predictions about how the relationship of absolute and percentage margins should depend on quality, and how the market equilibrium should change as demand uncertainty and share of sunk costs changes. We find that within the firm’s product line, the highest quality product has the highest profit margin but the lowest percentage margin, while the lowest quality product has the highest percentage margin but the lowest absolute margin. Both of these relationships are consistent with evidence available from marketing studies.
Future work should consider product line choice when the manufacturer and the retailer are different firms. In this case, a monopoly manufacturer of a high quality good can choose to extract rents from his retailer either directly through a higher price, or indirectly by being the sole supplier of the low quality good and earning an additional margin when a low quality sale is made. This second approach is likely to be more efficient than the first, because it avoids a marginal price distortion. This suggests that simple extensions of our model can provide an explanation for manufacturers’ use of full-line forcing.

APPENDIX

**Proposition.** In Mussa and Rosen’s [1978] model of monopoly second degree price discrimination, relative margins are strictly increasing.

**Proof.** The monopolist’s problem can be written

$$\max_{\theta, q(\theta)} \int_{\theta_l}^{\theta_u} [\theta q(\theta) - J(\theta)q(\theta) - c(q(\theta))] dF(\theta)$$

subject to the constraint that $q(\theta)$ is non-decreasing, where $c(q(\theta))$ is increasing and convex, $\theta$ is distributed with cumulative distribution function $F(\theta)$, and $J(\theta) = \frac{1 - F(\theta)}{f(\theta)}$ is assumed to be monotonically decreasing.

The first order condition (obtained by pointwise maximization) is

$$\theta - J(\theta) - c_q(q(\theta)) = 0, \forall \theta,$$

and implies that $q(\theta)$ is non-decreasing.

The relative margin is increasing if

$$\frac{\theta q(\theta) - J(\theta)q(\theta) - c(q(\theta))}{c(q(\theta))}$$

is increasing in $\theta$. Or equivalently, if

$$\frac{\theta q(\theta) - J(\theta)q(\theta)}{c(q(\theta))}$$

is increasing in $\theta$. Or equivalently, if

$$\frac{\theta - J(\theta)}{c(q(\theta))/q(\theta)}$$

is increasing in $\theta$. Or equivalently, using the first order condition, if

$$\frac{c_q(q(\theta))}{c(q(\theta))/q(\theta)}$$

is increasing in $\theta$, which holds because for any increasing and convex cost function, marginal cost increases faster than average cost.

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The same result holds in Johnson and Myatt [2003] when the assumption that average costs are increasing is relaxed because they find that the monopolist only produces on the increasing average cost portion of the cost function (or equivalently, for a discrete set of products, the monopolist produces at most one product on the decreasing average cost portion of the cost function).

REFERENCES


