Selecting Subsidy Strategies for Housing Allowance Programs

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Received April 14, 1975

The market effects of alternative housing payment formulas are analyzed and compared for a metropolitan housing market using measures of efficiency and distributional equity. The effects of “earmarking” allowance payments are considered. Estimated market effects are based on a model of housing market behavior over a 10-year period. The results differ significantly from what one might anticipate based on demand analyses of individual behavior. “Housing gap” formulas perform better than percent-of-rent formulas. Certain characteristics of the housing market together with particular income redistribution effects of the allowances appear to explain the market behavior.

The Federal government is now seriously considering the implementation of a direct cash assistance program as a way of subsidizing the low-income housing market. This program represents a continuation of the current philosophy to use the workings of the private market to improve the living standards of the poor. A difficult issue in the design of an effective housing allowance program revolves around the choice of an appropriate payment formula. Should the allowance depend on rent, income, or some hybrid of the two? In order to choose between alternative payment schemes, one must answer the following types of questions for each possible program formulation.

What types of incentives are created for households to relocate, and landlords to upgrade their dwellings? What types of housing improvement can be achieved for various costs? Will prices rise substantially in the housing market? Which segments of the population benefit the most under the plan? How would the setting of minimum rent levels, or quality standards affect the performance of the program?

This paper addresses the above questions by using a 10-year simulation model of a particular metropolitan housing market. The answers contrast

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sharply with what one might anticipate based on demand analyses of individual behavior.

All factors involved in designing a housing allowance program are not explicitly considered in this paper. Administrative considerations, work disincentives, and possibilities for fraud are examples. Their qualitative effects are discussed elsewhere \[4, 11, 12\]. This paper concentrates on the market effects of alternative payment formulas.

This research differs from other studies of housing allowance programs in several respects. Analyses by Rand \[11\], for example, have focused on the impact of allowances on housing demand and have not explicitly addressed the interaction of supply and demand. There are two primary distinctions between this research and ongoing work at the Urban Institute \[6, 7\]. First, we have used a modified version \[9\] of their model and have allowed for income elasticities below unity without distorting the purchasing power of an allowance dollar. Judging from a comparison with the Urban Institute's results \[7\] for one particular formula, our modification significantly reduces the predicted impact that a housing allowance program will have. Second, for a particular housing market, we consider a broad range of payment formulas and allowance levels and develop measures with which to compare them. To date, the Urban Institute has focused on intercity comparisons of only one type of payment formula. Other housing allowance studies, notably the analysis of HUD's demand, supply, and administrative experiments and the several ongoing studies designed to develop housing market models, have not yet progressed to the stage where they make comparisons of alternative housing allowance formulations.

This paper is organized as follows. Section 1 describes the payment formulas that will be analyzed. Section 2 describes the simulation model of the housing market, together with the modifications that we have made. Section 3 develops the criteria with which the payment formulas will be compared. Section 4 characterizes the overall performance of allowance programs. Section 5 compares the payment formulas and, based on these comparisons, makes recommendations concerning preferred formulas for a national housing allowance program.

1. PAYMENT FORMULAS

In this section, five payment formulas will be briefly described. Each formula differs in the type of incentives it provides to recipients, in how the allowance depends on income and/or rent, and in who receives the largest allowances. Except for the rent conditioned housing gap, the payment formulas considered here are similar to those discussed in \[11\].
1.1 Housing Gap Formulas

The housing gap (HG) payment formula gives an allowance according to the formula

$$\text{monthly allowance} = C^* - bY, \quad \text{for } Y < C^*/b,$$

where $Y = \text{monthly disposable permanent income}$, and $C^*$ and $b$ are the parameters of the plan. The formula has the feature that the allowance is independent of rent. The allowance declines to zero as the income of the recipients increases, and thereby avoids the "notch" effect whereby recipients of the allowance have a higher after-transfer income than some noneligible persons with a higher initial income.

1.2 Percent of Rent Formulas

For the percent of rent (PR) payment formula the form of the allowance is

$$\text{monthly allowance} = a \cdot R,$$

where $R = \text{monthly rent}$, and $a$ is a program parameter equal to the percent of subsidy. The parameter $a$ does not vary according to family size. This formula is independent of income, and encourages housing consumption by giving the highest allowance to those recipients who pay the highest rent. Percent of rent formulas are frequently called "self-earmarking."

1.3. Rent Conditioned Housing Gap Formulas

The basic form of the rent conditioned housing gap (RCHG) payment formula is

$$\text{monthly allowance} = \begin{cases} \frac{(R/R^*)(C^* - bY)}{C^* - bY} & \text{for } R < R^*, \text{ and } Y < C^*/b, \\ C^* - bY & \text{for } R \geq R^*, \text{ and } Y < C^*/b, \end{cases}$$

where $R = \text{monthly rent}$, $Y = \text{monthly permanent income}$, and $R^*$, $C^*$, and $b$ are parameters of the program. For rents above $R^*$, the formula behaves as a housing gap formula, while for rents below $R^*$, it behaves as a percent of rent formula. The allowance paid declines to zero as income increases.

1.4. Variable Percent of Rent Formulas

The allowance given by the variable percent of rent program (VPR) is

$$\text{monthly allowance} = \left[ 1 - \left( \frac{Y}{Y^*} \right) \right] R, \quad \text{for } Y < Y^*,$$

where $Y = \text{monthly permanent income}$, $R = \text{monthly rent}$, and $Y^*$ is a parameter of the program. Whereas the "pure" percent of rent formula pays a fixed percent of rent to all recipients, the VPR formula allows this percent to decrease to zero as household income rises.
Table 1: Housing Allowance Payment Formulas

<table>
<thead>
<tr>
<th>Program Name</th>
<th>Abbreviation</th>
<th>Allowance Formulas*</th>
<th>Parameters of the Formula</th>
<th>Allowance vs Income Diagram</th>
<th>Allowance vs Rent Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing Gap</td>
<td>HG</td>
<td>[ A = C* \cdot bY ]</td>
<td>( b, C* )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rent Conditioned Housing Gap</td>
<td>RC</td>
<td>( A = \frac{R}{b} (C* - bY) ) for ( R &lt; R^* ) ( A = C* - bY ) for ( R \geq R^* )</td>
<td>( b, C^<em>, R^</em> )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percent of Rent</td>
<td>PR</td>
<td>( A = \frac{R}{b} )</td>
<td>( \frac{R}{b} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable Percent of Rent</td>
<td>VPR</td>
<td>( A = \left(1 - \frac{Y}{\overline{I}}\right)) ( \frac{Y}{\overline{I}} )</td>
<td>( \frac{Y}{\overline{I}} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rent Triangle</td>
<td>RT</td>
<td>( A = a_1 R ) ( R &lt; R_1 ) ( A = a_2 (R_2 - R) ) ( R_1 &lt; R &lt; R_2 ) ( A = a_3 R_2 ) ( R \geq R_3 )</td>
<td>( a_1, a_2, a_3 ) ( R_1, R_2, R_3 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Notation: \( R = \) monthly rent rate \( Y = \) monthly permanent income \( A = \) monthly allowance

1.5. Rent Triangle Formulas

The final type of hybrid payment formula that is considered is the rent triangle (RT). For this housing allowance plan, the allowance is

\[
\text{monthly allowance} = \begin{cases} 
    a_1 R & \text{for } 0 \leq R < R_1, \\
    a_2 (R_2 - R) & \text{for } R_1 \leq R < R_2, \\
    0 & \text{for } R \geq R_3,
\end{cases}
\]

where \( R = \) monthly rent, and \( a_1, a_2, R_1, R_2, R_3 \) are parameters of the program. This formula behaves like a pure percent of rent for rents below \( R_1 \). The diagram in Table 1 illustrates the triangular relation between allowance and rent paid. As rent increases past \( R_1 \), the allowance declines to zero.

The table summarizes for easy reference each of the formulas and the general configurations of the allowance vs rent and vs income relations for the five payment formulas.

1.6. Earmarking Requirements

In an effort to stimulate or "earmark" additional housing expenditures, eligible households may be required to meet certain minimum rent or mini-
mum housing consumption requirements in order to participate in the allowance program. Earmarking in this paper always took the form of specifying some minimum level of housing consumption. With earmarking, some eligible households may choose not to participate in the allowance program. A household will elect not to participate if it prefers less housing and no allowance to housing at the minimum standard (at its market price) with the allowance.

From the point of view of the model, an equilibrium solution obtained using minimum consumption earmarking is virtually indistinguishable from an equilibrium solution obtained using minimum rent earmarking, where the specified minimum rent is that rent level needed to purchase housing at the minimum consumption standard. Because of this equivalence we chose to use a minimum consumption standard in our simulations to illustrate the effects of either minimum rent or minimum consumption standards.

1.7. Allowance Program Simulations

The effect of each payment formula was simulated under a variety of different assumptions about parameter values and earmarking levels so as to illustrate the performance of each type of allowance formula under widely varying circumstances. The parameter values and the earmarking requirements used for each plan are summarized in Appendix 1. The same 22% of the population was specified as eligible for each allowance program. For each allowance program, a distinction is made between the needs of single/elderly families and multiple families. Therefore, for any one program, the parameters of that program take on two different values, one for each type of family. For each formula, we usually chose three different mixes of parameter values to correspond to a high, medium, and low allowance level. To investigate the effects of earmarking, each of the payment formulas at each allowance level was run with the different earmarking requirements and also without earmarking. In order to ensure that the parameters that we used in the housing model were applicable to actual housing markets, we estimated the housing model for the particular SMSA of Pittsburgh. The specific parameter values for the allowance programs were then chosen in relation to the rents and income of Pittsburgh as reported in the 1970 Census [13].

We have not attempted to model the financing of housing allowances in any detail. As a rough approximation to the net effect of financing, we have reduced family income (in excess of a low minimum exemption level) at a rate of 1% for families not eligible for the housing allowance.

2. THE SIMULATION MODEL

To predict the effect of a housing allowance program on a metropolitan housing market, it is necessary to trace through the effects on the supply and
demand of housing in each of the many separate, though highly interdependent, housing submarkets. The housing market model that will be used for the simulation is a modified version [9] of the Urban Institute housing model [5].

The essence of the Urban Institute model [5] is a microeconomic picture of households and landlords contracting for housing at qualities and prices determined in several submarkets. Within a particular SMSA, several residential zones are distinguished which differ, insofar as the model is concerned, in their housing stock, racial composition, average income, and in the exogenously specified values for the average travel time to work for residents of each zone.

On the supply side, a housing unit is characterized by the "quantity of service" ($Q$) it provides (an assumed one-dimensional combination of size and quality) and the price per unit of service ($P$). Several different quality levels are distinguished to correspond to the many alternative quality submarkets in the actual housing market. Profit maximizing behavior on the part of landlords is assumed to imply linear supply curves (of varying slope for each type of housing unit) as an approximation to the price-quantity relations that govern landlord behavior during a 10-year period. An unlimited amount of new construction within separate homogeneous residential areas is allowed at a fixed price $P_N$. The only constraint on new construction is that housing providing less than a specified minimum level of housing services cannot be built.

On the demand side, model households are characterized by any one of four different preference schedules (reflecting age, family status, and race) and by income. These preference schedules are based on utility functions which compare various amounts of housing consumption, permanent disposable income, leisure time, neighborhood quality, and racial composition.

Most model parameters are estimated readily from census data. Certain other parameters related to the importance of neighborhood externalities and the slope of supply curves must be estimated by comparing model solutions using different parameter values with actual 1960 and 1970 data.

In using the model for a particular city, one must select a level of aggregation for the number of zones, households, and dwellings that are to be distinguished. The number of zones is generally predetermined to be four or five, to provide as much spatial distinction as possible without sacrificing computational ease. Each representative household and dwelling in the model corresponds to several thousand actual households and dwellings. For Pittsburgh the ratio is about 1 to 15,000.

For a particular set of supply and demand parameters and initial market conditions the model estimates housing market conditions 10 years later using a "tatonnement" procedure. These predicted market conditions have the property that households are assigned to dwelling units at a set of prices
for which no household has an incentive to relocate and landlords have no incentive to change the amount of housing services they are offering. (This version of the model does not explicitly distinguish between homeowners and renters.)

Though the model is conceptually simple and assumes convenient forms for household preferences and dwelling supply curves, it captures many important aspects of particular interest in formulating housing subsidy policies. In particular, it distinguishes several types of households and housing units; allows for racial, neighborhood, and travel time effects in household preferences; separates changes in price from changes in quantity of housing consumption; considers a continuous range of household incomes, thereby enabling straightforward modification to adjust for housing allowances. The model also allows for different supply behavior on the part of landlords located in different quality submarkets. The model makes explicit that the market equilibrium depends on the underlying distributions of purchasing power and quality of housing, and on the competition between neighboring quality submarkets.

In the Urban Institute model, an ad hoc method is used to obtain the permanent income of a household from its current income. The method implies a unitary income elasticity of housing demand with respect to permanent income, and an income elasticity of 0.6 with respect to housing allowance income. The method also results in a permanent $100 allowance being modeled as an increase in a household's permanent income of substantially less than $100. (See [1] for further discussion of this problem.)

The use of low-income elasticities is motivated by recent empirical findings using disaggregated data [3, 8, 10]. Although we believe that the income elasticity out of a permanent change in income is indeed below unity, we did not wish to distort the purchasing power of an allowance dollar. By modifying the household utility functions we were able to allow income elasticities to be specified independently of any permanent income adjustments.

A new econometric method was developed to estimate the distribution of permanent income from cross-sectional data. It produces realistic changes in the means and variances of the permanent income distributions for the various households, and yields model (i.e., permanent) incomes that result in plausible model parameters and low calibration errors. In the modified model, a $100 allowance that remains in effect for 10 years is modeled as a $100 change in permanent income while the income elasticity of demand for housing is 0.47. The theory underlying the model modifications and the results of the parameter estimation are fully described in [9]. A computer printout of the modified model is available on request from the authors.

2 In the unmodified version of the model a household's model income is defined to be $Y_m = (Y_c)^{0.6} (Y_{av})^{0.4}$, where $Y_c$ and $Y_{av}$ are current and peer group incomes, respectively.
To simulate the effects of a housing allowance program, the Pittsburgh housing model was rerun for the 1960-1970 decade with permanent incomes of eligible households altered to reflect the allowance. The difference between the predicted 1970 housing market conditions with and without the allowance was then interpreted as a measure of the change that would have resulted in the Pittsburgh housing market if particular housing allowances had been in effect during those 10 years.

The 1970 Pittsburgh estimation of the modified housing model indicated substantial price discounts (i.e., price below the replacement cost, $P^*_N$) per unit of housing service for the low-quality end of the existing housing stock in the absence of housing allowances. The 20–25% price discount in the low-quality submarket indicates that it will be difficult to stimulate demand for higher priced newly constructed housing units.

3. THE MEASURES OF EFFECTIVENESS

To interpret the effects of a housing allowance program, no one measure of performance is sufficient. Housing allowances may be designed to further a variety of sometimes conflicting goals. We will analyze the effects of the selected housing allowance programs using a variety of measures obtained from the model simulations.

Variations in the family size and in the marginal propensity to consume housing among different household types can cause a poorer family to consume more housing than a higher-income family. All the eligible households are included among households occupying that third of the housing stock that offered the lowest quantities of service in 1970. Hence changes in average conditions in this "bottom third" of the households reflect changes throughout that segment of the market that is the focus of the allowance programs. Average conditions in the "bottom sixth" indicate how the poorest half of the target market is affected. Approximately 86% of the households in the bottom sixth are eligible for the allowance. Within the bottom third the percentage is 64%.

3.1. Housing Improvement and Price Increases

The simulation model distinguishes price per unit of service from the quantity of housing services provided by any of the model dwellings representing the various submarkets. Hence it is possible to separate increased rental expenditures into components reflecting increased price and increased services.

For each simulation we distinguish several measures of housing change and price increases. These measures indicate the percent change in the average level of either the housing consumption or housing price within the entire housing market, the "bottom third," the "bottom sixth," and among the
eligibles and the poorest 20% of the noneligible households. Since many of
the price and quantity change measures are closely related and/or varied little
across allowance programs, there will be no need to report each of them
explicitly.

3.2. Program Costs

Allowance levels are measured in terms of the estimated total monthly
payments for the Pittsburgh metropolitan area. The costs are estimates of
what an allowance plan would have cost in 1970 if the program had been in
effect between 1960 and 1970. Administrative costs are not considered, but
the effect of price increases on allowance levels are. For housing gap payment
formulas, allowance levels are predetermined. For formulas which depend on
rent, allowance levels depend upon market conditions.

3.3. Efficiency

Three measures are used to indicate how effectively allowance dollars are
channeled into expenditures for improved housing. The recipient expenditure
or earmarking ratio (abbreviated ER) measures the change in housing expendi-
ture among recipients per dollar of allowance cost. It indicates the fraction of
allowance dollars that recipients use for increased housing expenditures. Since
this measure considers increases in rent, it increases with both housing price
and consumption.

The other two measures, henceforth called the market efficiency measures,
focus on a cost standardized measure of housing change within the poorest
submarkets. They measure the percent increase in housing consumption
among all households in the bottom third and bottom sixth, normalized by
dividing by the average net dollars transferred to households in the bottom
third. For example, a market efficiency measure of 0.30 for the bottom third
indicates the following. An allowance program that involved an average net
transfer of $20 per month to each household in the bottom third causes a
$20 \times 0.30 = 6\%$ rise in the average housing consumption within the bottom
third of the market. The range for these two market efficiency measures
depends on the particular characteristics of the housing market in Pittsburgh
for 1970. A program which produces no change in housing consumption
would have an efficiency of zero. A program which left housing prices un-
changed, and for which recipients spent their entire allowances on additional
increased housing consumption, would have an efficiency of approximately
2.0.

Note that the recipient expenditure ratio considers rent, recipients only,
and allowance costs, whereas the market efficiency measures consider housing
changes, all households in the bottom third (or bottom sixth), and the net
transfer of funds to those households in the bottom third.
3.4. Equity

Notions of equity involve comparisons of the allowances received by recipients with different incomes, the participation rate among the eligible population, and the differential effect of the allowance program on eligible and noneligible households. With regard to the distribution of the allowance, we make the value judgment that it is more equitable to give the largest allowances to the poorest recipients. Comparisons of the market efficiency measures for the poorest sixth and third of the housing market also give an indication of this equity consideration.

Though the same households are eligible under all programs, participation rates among the eligible population vary among plans. In almost all cases, those eligible households who elect to refuse the allowance are the poorest of the eligibles for whom the earmarking level is too high. In this case, low participation rates indicate a failure to help the neediest households. Between two plans with different participation rates, the plan with the higher one may be regarded as more equitable.

The final equity consideration deals with the differential effect of the allowance on recipients and noneligible households. With an allowance program, noneligible households are not only subject to financing taxes but may also face higher prices and increased competition for housing if they are poor enough to reside in low-quality submarkets. As an indicator of what happens to such households, we shall examine the percent increase in housing price among the poorest 20% of the noneligible households.

3.5. Interpreting Price Increases

Though the model identifies price increases, it is misleading to call these price increases "pure inflation." The model's supply assumptions require dwellings to operate on their long-run supply curves. To the extent that housing improvements come from upgrading the existing stock, some price increase is needed to maintain the improved housing. Since many of the units available to the poor are rented at levels less than what is required for maintenance of the unit at some target minimum standard, some price increases are desirable in the housing market. In evaluating the effect of a program on recipients, we are interested in how well the program translates allowance dollars into increased housing consumption. Price inflation is but one factor and, by itself, gives no indication of program performance. The market efficiency measures reflect the aggregate effect of all the factors on program performance. Price inflation is an important consideration when one considers the effect of the allowance program on nonrecipients. The average price increase to the poorest 20% of the noneligibles is a measure of this effect.
3.6. Limitations

A few cautions are in order concerning the interpretation of the simulation results. First, the price changes that are considered indicate long-term increases needed to maintain improved housing at higher quality levels. Short-run price changes that would influence the manner in which a program would be “phased in” are not considered. Second, in using the model to simulate allowance programs, decisions about how to regard the allowance arise. The position taken in this paper is that any transfer that will remain in effect for as long a period as 10 years, would be regarded as permanent income by the recipient in both his spending behavior and expectation formations.

The final limitations concern the confidence that should be placed in the model’s predictions. The level of aggregation of the model limits our ability to analyze allowance programs’ effects on neighborhood racial composition and the locational choices of allowance recipients. Furthermore, as with all models, this model contains many simplifying assumptions designed to make the model manageable to estimate and use. We do not expect any one simulation to tell us the absolute magnitude of the effect of a program with specific parameter values. On the other hand, we are confident that the model can provide useful information about the relative performances of the different allowance programs in housing markets similar to those that were simulated.

Finally, one should note that no few measures of performance are completely adequate to describe the results of each allowance program. We attempted to choose a large enough set of performance measures so that we could capture the most important effects of each program, and yet, keep the set of performance measures manageable enough so that the major effects of each program could readily be interpreted from our set of performance measures.

4. RESULTS

Before turning to a comparison of the different plans, it will be useful to discuss the general characteristics associated with the performance of the allowance plans. For specific results and a more detailed discussion see [1]. Recall that a housing allowance “program” or “plan” consists of a choice of payment formula and related parameter values, a specification of eligible households, and a level of earmarking.

4.1. General Results

In all of the housing allowance simulations that we considered, improvement in housing quality always came from rehabilitation of existing dwellings and not from the construction of new dwellings. The poorest households could not afford new housing and their increased demand was not sufficient to stimulate additional new construction by wealthier households.
The cost of the programs varied from 1 to $5 million per month. The typical cost figure was around $2.5 million per month. A typical program provided an average monthly allowance of about $20 per family. The programs differ considerably in who receives the allowance. For some housing gap programs, the allowance of the poorest recipients amounts to more than 50% of their income, while, for percent of rent programs, this figure was often less than 9%.

For unearmarked programs, the percent increases in housing ranged between 1.0 and 4.0% for the bottom third, between 1.5 and 8.0% for the bottom sixth, and between 3.5 and 6.6% for eligible households. The higher the cost of the program, the greater the increase in housing consumption. Typical market efficiency measures (i.e., cost standardized measures of housing change) were 0.17 and 0.31 for the bottom third and sixth, respectively. So, for example, an increase in the average net transfer per household of $10 per month would raise the percent change in housing in the bottom sixth from, say, 4.0% to 4.0 + 3.1 = 7.1%. The expenditure ratio, which measures the fraction of the allowance spent on rent, was typically 0.22.

Earmarking was able to substantially improve the performance of all programs, even those providing so-called "self-earmarking" percent of rent subsidies. Initially, the imposition of earmarking had a very substantial effect on program efficiency. However, as the level of earmarking increased, the added improvement in efficiency from the additional earmarking tapered off. With earmarking, the percent increase in housing ranged from between 1.8 and 4.5% for the bottom third, between 1.8 and 9.2% for the bottom sixth, and between 4.5 and 6.9% for all eligible households. Earmarked programs were able to achieve their improvement in housing consumption at lower cost than unearmarked programs. Hence, earmarked programs were considerably more efficient than unearmarked programs in channeling dollars into increased housing consumption. Typical market efficiency measures for earmarked programs were 0.28 and 0.53 for the bottom third and sixth, respectively, while the expenditure ratio was 0.35. However, even for earmarked programs the market efficiencies are considerably less than their maximum values of about 2.00.

Aside from its ability to improve the efficiency of any plan, earmarking has the undesirable effect of reducing the participation among the eligibles. The poorest eligible households often preferred not to participate in the allowance program rather than accept the allowance and meet the earmarking standards. This illustrates an important trade-off between equity and efficiency, and shows that the goal of designing an efficient allowance program can conflict with the goal of helping the neediest households.

For the rent triangle programs a particularly troublesome trade-off arose between efficiency and equity. The more efficient RT programs actually caused a drop in the average housing consumption of the noneligible households in the bottom third of the market. Apparently, the RT plans give money
to recipients in such a way that they bid away the housing of the nonrecipients and force these noneligibles into poorer quality housing. Since these noneligibles are themselves fairly poor (with income averaging $500 per month) such a result seems very undesirable.

One concern with allowance programs is that they will increase the long-run price of housing. The fear is that the allowance dollar will be absorbed by price increases and not by increases in housing consumed. The market efficiency and housing change measures, discussed earlier, were always greater than zero, indicating that an allowance program can effectively increase housing consumption, and not simply drive up rents. Nevertheless, the price increases were substantial. Typically in the bottom third and sixth, the percent change in price was roughly twice the percent change in housing consumption. For example, a 10% price increase accompanied a 5% housing increase in the bottom third. The effect of these price increases on the ability of an allowance program to increase housing consumption is fully reflected in the market efficiency measures. The rather high level of observed price increases keeps the market efficiency measures well below their maximum values of 2.00.

As discussed earlier, price increases are an important factor when one considers the effect of the allowance program on nonrecipients. To examine this effect, we computed the price increases that the bottom 20% of noneligibles experience as a result of each allowance plan. For the programs simulated, a price increase of between 0 and 5% occurred among this segment of the population. Programs which involved earmarking had higher rates of price increase among noneligibles than unearmarked programs, since one of the effects of earmarking is to increase competition in the poorer quality submarkets that meet the earmarking standard. As might be expected, these price increases are greater for those programs which give the high allowances to those higher-income eligibles who compete with the poorest noneligibles for housing.

One disturbing and recurrent effect of allowance programs was to increase the concentration of poverty and nonwhites within the poorest zone. This effect would be particularly bothersome if it occurred on a neighborhood level. However, the reader should recall that the aggregation level of the model prevents us from reaching conclusions about specific geographical areas small enough to be considered neighborhoods.

4.2. Scale Effects

One common theme running through the performance of the allowance programs was the lack of sizeable scale effects among programs using the same payment formula and yielding the same participation rate. More precisely, for any payment formula, the efficiency measures associated with
an allowance program do not change substantially as the allowance costs vary unless participation rates also change. In fact, for any payment formula, our results indicate that the efficiency measures are primarily a function of participation rates. (Of course, this function is different for each of the five different payment formulas that we examined.) In other words, for any particular payment formula, the efficiency measures are approximately constant among all programs whose allowance and earmarking levels bring about the same participation rate. For a fuller discussion of the minor scale effects that did occur, see [1]. Scale effects might have been more pronounced if new construction were stimulated when allowances exceeded some threshold level.

4.3. Explaining the Market Effects

The simulation results differ in a number of ways from what one might anticipate from demand analyses of one individual's behavior. See [1] for an extended discussion of the differences between the simulation results and a comparative static analysis.

A number of factors explain why the response of a metropolitan housing market would differ from what a comparative static analysis of one individual might suggest. One clear reason for the difference is that the model simulations reflect the net effect of an allowance program on several housing submarkets and on a mixture of households with different characteristics and income. Another obvious factor is the allowance's effect on the market prices of housing in various submarkets.

Other factors at work are: (1) the initial low price levels in the low-quality submarkets; and (2) the allowance vs income relationships of the payment formulas, and their effect on the distribution of income among recipients. Recall from Section 2 that, without a housing allowance, market conditions for Pittsburgh were such that housing in the low-quality submarkets was offered at a price discount (i.e., at a price considerably below the replacement cost). The discounting was most pronounced in the very poorest segment of the market. This discounting together with the supply parameters of the model make it more efficient to improve housing of the poorest quality than of a higher quality in the sense that equal increments in rent yield larger increments in housing services when spent on poor-quality housing rather than on high-quality housing. Moreover, the discounted price provides additional incentive to the poorest recipients in the lowest-quality submarkets to increase their housing expenditures. Hence, this discounting of price will tend to improve the efficiency measures of payment formulas which give the largest allowances to the poorest recipients.

The second factor involves the income redistribution effects of the allowance. All housing allowance programs redistribute income so that recipients have higher after-transfer incomes. In addition, housing gap formulas further
concentrate income among the poorest households, since the allowance level is larger for households with lower incomes. Percent of rent formulas, on the other hand, increase the variation in after-transfer incomes among recipients. (Higher-income households tend to spend more on rent, so that they receive higher allowances than poorer recipients under a percent of rent program.)

When the distribution of income becomes more concentrated in a particular submarket, households are better able to compete with each other and such competition tends to drive up both the price and quantity of housing consumed in the submarket. Interaction among submarkets can complicate this effect but in general, among equal cost allowance programs, those programs which increased the concentration of income tended to cause more housing to be consumed and price increases to be more severe than programs which dispersed the distribution of income.

The various factors discussed above indicate why comparative static analysis of the behavior of one individual might be misleading. Most of these factors tend to make housing gap programs look better relative to percent of rent plans than one might otherwise expect.

5. COMPARISONS AND CONCLUSIONS

We may now compare the relative advantages and disadvantages of each of the five payment formulas. One should recall that a housing allowance "program" or "plan" consists of a choice of payment formulas, parameter values, and level of earmarking. Each possible allowance program has associated with it three basic characteristics: cost, efficiency, and equity. Efficiency is measured by the three efficiency indicators described in Section 3, while judgments about equity involve measures of the participation rate among the eligibles, the housing change in the bottom sixth and third of the market, the price increases for the noneligible population, and whether the very poorest receive the largest subsidy.

To compare the performance of two payment formulas, it is necessary to compare for each formula (1) the achievable participation-efficiency combinations, (2) the distribution of allowance benefits among the eligibles, and (3) the price increases faced by the noneligible population. Figures 1 and 2 present the achievable efficiency-participation locus for each formula, using the simulated efficiency measures in the bottom sixth and third.\(^3\) In plotting the graphs for any one formula, we took an average efficiency measure over programs yielding the same participation rate. (The presence of some scale effects and the level of aggregation of the model explain why each formula

\(^3\) A graph of the recipient expenditure ratio vs the participation rate could also be drawn. This graph is not presented since it does not add to the conclusion developed from Figs. 1 and 2.
may have more than one value for efficiency at each participation rate. For more detailed results see [1],] The level of aggregation of the model makes it possible only to distinguish a discrete number of participation rates. The continuous curves in the graphs were made by joining together these discrete points.

A few comments will help in the interpretation of the graphs. The market efficiency measures, defined in Section 3.3, can range between 0 and 2, and represent a cost standardized measure of the percent increase in housing consumption among all households in the bottom third and bottom sixth. A
simple example will illustrate how to interpret differences in efficiency measures. A market efficiency measure of 0.5 in the bottom sixth would translate into a 5% increase in housing consumption in the bottom sixth for a program that transferred an average of $10 to each household in the poorest third of the market (i.e., $10 \times 0.5 = 5$). From the discussion in Section 4, this 5% increase in housing consumption entails a 10% price increase, so that rents rise by approximately 15%. A different program involving the same net transfer, but a market efficiency measure of 0.6, would translate into a 6% increase in housing consumption and an increase of about 18% in rent. For more expensive allowance programs, the difference in the percent increase in housing between the two programs would be greater.

Another issue needing clarification concerns the participation rate used in Figs. 1 and 2. For all except the RT programs, the measures of participation for each program were comparable in the sense that for a given participation rate, it is the same poorest households who chose not to participate in each program. For the RT program, 11% of the highest-income recipients always chose not to participate in the program, so that participation rates even for unearmarked RT programs do not exceed 89%. It seems reasonable that a policymaker might be more concerned about participation among the poorest eligibles than among the richest. Therefore, the graph of the RT program

![Graph showing distribution of allowance among recipients.](image)

**Fig. 3.** The distribution of allowance among recipients.
would look misleadingly bad if the nonparticipation of the highest-income eligibles were treated in the same way as that of the poorest. On the other hand, it is not appropriate to ignore this nonparticipation completely. As a compromise for graphical purposes, we chose to create an "effective" participation rate for RT programs that values participation among the richest at one-half that among the poorest. As seen below, most of the comparisons between programs will be true even for the two extreme viewpoints on the treatment of participation for RT programs.

The final graph, Fig. 3, indicates the distribution of allowance benefits among typical lower-, middle-, and upper-income recipients under each formula for representative participation rates. In Section 4.1, we noted that the rate of inflation among nonparticipants depended on whether the allowance program gave high allowances to higher-income recipients who can compete with some of the noneligibles. Except for the RT plan, which we shall discuss later, favorable judgments about the inflation rate among noneligibles coincided exactly with favorable equity judgements of whether the poor receive the largest allowances. Therefore in comparing plans, we will not explicitly make this inflation comparison between plans, but the reader should keep in mind that comparisons using Fig. 3 also reveal which plans create higher price inflation among the noneligibles.

5.1. The Choice among Payment Formulas

Perhaps the most striking characteristic of the graphs in Figs. 1 and 2 is the relatively low efficiencies of programs using the pure percent of rent formulas. Only in the case of no earmarking do percent of rent programs have higher market efficiencies than any of the other four programs. Even in this case, the percent of rent plans have higher efficiencies only in the bottom third. In addition, Fig. 3 indicates that pure percent of rent plans are least equitable in terms of giving the largest allowances to the very poor. In general, pure percent of rent programs are outperformed by the other four types of allowances plans.

Let us now consider the relative performance of the three hybrid formulas. The differences amongst the three hybrids arise from the relative magnitude of the allowances for households with different rents and incomes and from the relative slopes of the allowance vs rent and allowance vs income relations. To the poorest recipients who pay low rents, all three hybrids look like percent of rent formulas. For two of the hybrids, namely, the VPR and RCHG plans, the dependence of the allowance on income and rent is very similar.4

4 The only exception is for rent triangle programs where some of the higher-income recipients choose housing above the rent maximum (see Section 1.5) and receive no allowance. Nonparticipation for these households is likely to be of less concern to a policy-maker than nonparticipation among the poorest households.
Figures 1 and 2 indicate that at all participation rates, RCHG plans are slightly more efficient than VPR plans in the bottom third and significantly more efficient in the bottom sixth. Compared to RT programs, the RCHG plans are always more efficient in the bottom third and significantly less efficient in the bottom sixth only at low participation rates, which may well be regarded as inequitable. Figure 3 illustrates that RCHG plans are also more equitable than the other two hybrid plans in terms of giving the poorest the most allowance.

One relative disadvantage of the RCHG plans relative to the RT plans is that the RCHG plans usually cause more price increase (2–5%) among the noneligibles than comparable cost RT plans (2–3% price increase). However, the RT plans achieve the smaller price increases and the high efficiencies in the bottom sixth at the expense of those in the second sixth of the housing market. In fact, as mentioned earlier, the RT plans actually hurt those low-income nonrecipients in the bottom second sixth, in the sense that their housing consumption falls when the RT allowance plan is instituted. Such an occurrence of recipients benefiting at the expense of nonrecipients is highly undesirable, and could cause RT programs to be socially unacceptable. Based on the preceding arguments it follows that, among the three hybrid formulas, the RCHG plans are preferable when considerations of both equity and efficiency are taken into account.

The implication of the above discussion is that the choice among the five payment formulas is narrowed down to a choice between the HG and RCHG plans. The basic difference between pure housing gap formulas and rent conditioned housing gap formulas, is that the latter formula gives persons with rent $R$ below $R^*$ only $R/R^*$ times the full value of their housing gap allowance. For the same housing gap parameter values, the poorest recipients receive a smaller allowance under the RCHG plans than under pure housing gap forms, but have a stronger price incentive to increase their housing consumption since, below $R^*$, the RCHG formulas operate as percent of rent programs. Hence, HG plans are superior to RCHG plans in terms of giving the poorest households the largest allowances. On the other hand, as Figs. 1 and 2 illustrate, the stronger price incentives of the RCHG plans makes them more efficient than HG plans when unearmarked programs are considered.

As is indicated in Fig. 1 and 2, it is difficult to distinguish between the efficiencies of earmarked RCHG and HG programs. Initially earmarking has a greater impact on the efficiency measures of pure housing gap programs.

For households with rents below $R^*$, the RCHG and VPR allowances are related as follows. For RCHG plans, the percent subsidy to those with rents below $R^*$ is

$$\frac{1}{R^*}[C^* - bY] = \frac{C^*}{R^*}\frac{1}{1 - (Y/Y^*)},$$

since $Y^* = C^*/b$ in the simulations. Hence the RCHG percent subsidy (for those renting below $R^*$) is greater than the VPR percent subsidy as long as $R^* < C^*$, a condition that always held for the simulations.
This result occurs because HG plans give higher allowances to the poorest than do RCHG plans, hence housing consumption of these poorest can be pushed further upward (through earmarking) under HG than under RCHG programs before these recipients decide to withdraw from the allowance program. With low earmarking and participation rates between 70 and 90%, HG plans appear to be slightly more efficient than RCHG plans. For very high levels of earmarking RCHG and HG plans become indistinguishable.

5.2. Trade-offs for Any One Payment Formula

Once a particular type of payment formula has been selected, there still remain two questions. First, what allowance level should be chosen, and second, what participation-efficiency combination is preferred (among those available through the use of earmarking)? Answers to each of these questions depend on both the policymaker’s subjective preferences and the potential trade-offs that are available. For each question, the simulation results provided information on the potential trade-offs that are available. It was found that changing the allowance level of a program had minor effects on the cost standardized performance measures achievable with a particular formula. The graphs in Figs. 1 and 2 indicate the magnitude of the trade-offs between participation and efficiency that are possible for each payment formula. The trade-offs are summarized below.

(1) Earmarking greatly improves the efficiency of all programs, but reduces participation among eligible households and increases the price faced by poor noneligible households. The fraction of allowance dollars which recipients channeled into increased rents was below 25% for unearmarked programs but 30 to 40% for earmarked programs.

(2) As earmarking levels increase, participation continues to decline but the efficiency measures tend to level off. Raising earmarking levels so high that participation falls below 80% appears undesirable, since efficiency is often not significantly increased by such earmarking.

(3) Since it primarily affects the poorest households, earmarking is most effective for those plans giving the largest allowances to the very poor, (e.g., housing gap plans) and least effective for those plans that give the smallest allowance to the very poor (e.g., percent of rent plans).

(4) Flexible earmarking levels that varied according to the family income as well as the family size of the poorest recipients may improve the efficiency of programs without adversely affecting participation rates.

5.3. The Performance of Housing Allowance Programs

As was indicated in Section 4.4, the complicated set of forces at work in the interrelated housing submarkets makes it impossible and misleading to predict
the market effects of housing allowance programs from an examination of the demand behavior of a single individual. The simulation exercise helped identify some of the major market forces affecting the performance of housing allowance programs. The most important forces were: (1) the initial price discount in the low-quality submarkets; (2) the effect of the allowance on the distribution of income with its subsequent effect on competition across the interrelated housing submarkets; and (3) the effect of increased earmarking on participation rates, on the level of competition in various submarkets, and on the market efficiency measures.

As discussed earlier, the 1970 Pittsburgh housing market had a substantial price discount per unit of housing in the low-quality submarkets in the absence of housing allowances. One effect of this discount was that housing improvement resulted from the rehabilitation of existing dwellings rather than new construction followed by abandonment of the lowest-quality housing. The discount also affected program efficiencies. Because of the price discount and the shape of the estimated supply curves, an allowance dollar added to the rent of a poorer household could purchase a larger increase in housing consumption through rehabilitation than could a dollar added to the rent of higher-income recipients. This effect tended to increase the relative efficiencies of programs that gave most of their allowances to the very poor.

The allowance programs differed in their effect on the after-transfer distribution of income. Programs which concentrated the income distribution increased competition, especially in the low-quality submarkets, while programs that dispersed the income distribution had the opposite effect. As mentioned earlier, the effect of increased competition was to stimulate additional housing consumption.

Earmarking not only increases certain households' incentive to improve their housing but also increases competition in housing quality submarkets meeting the minimum standards. The effect is enhanced when those households for whom earmarking matters most receive the largest allowances. However, earmarking also results in reduced participation rates among eligible households and lessened competition in the substandard submarkets. Non-participants tend to be the poorest households, who tend to occupy the inexpensive low-quality housing that can be efficiently improved. Predicting the net effect of earmarking on efficiency as a result of these various market forces is difficult. The simulations indicated a net improvement in the efficiency of most programs as a result of earmarking, but the effect diminished and, in some cases, reversed for earmarking sufficiently high to drop participation below 80%.

The analysis of the simulations leads to the following general conclusions about the performance of housing allowance programs in housing markets where the lowest-quality housing is available at a price discount.
Improving the housing consumption of the poor is feasible, but expensive. All of the allowance programs had efficiency measures that were positive, but well below their maximum values. To a large extent the low efficiencies resulted because price increases, though confined to the bottom third of the housing market were substantial within that segment. The percent increase in prices in the bottom sixth and third and among the eligibles were 1.5 to 2 times the corresponding percent increases in housing consumption. For cities with price discounts in the low-quality market, it is inescapable that large price increases in the lower-quality submarkets will accompany increases in housing consumption, since higher prices are needed to maintain the improved housing at a higher-quality level. Moreover, for cities with these price discounts, housing improvement is likely to occur from rehabilitation and not from new construction. Substantial improvements in the low-quality housing stock are possible before the price would reach the new construction price and make new construction competitive at the low to medium quality levels.

The best programs could increase housing consumption by at most 9% in the bottom sixth, 4% in the bottom third, and 7% among the eligibles, using allowance levels that amounted to a monthly average of $20 per recipient (in 1970). Taking into account price increases, such programs increased average rental expenditures in the bottom sixth and third by 23 and 12%, respectively, and by 18% for the eligible households. The poorest noneligibles faced price increases of about 4%.

Despite income elasticities of one-half, income-related formulas can work relatively well. Income elasticities were assumed to be only half as large as price elasticities. Nevertheless the two rent-related payment formulas that acted as price subsidies (i.e., VPR and PR formulas) were never more efficient than earmarked housing gap programs and were more efficient than the un-earmarked housing gap program (which is equivalent to an income maintenance plan) only in the bottom third of the housing market. Moreover, housing gap formulas were always more equitable than rent related formulas.

The so-called “self-earmarking” incentive of formulas using percent of rent subsidies appear to matter much less than a static analysis would indicate. No formulas caused recipients to spend more than 40% of their allowance on increased rent. Even these percentages could be achieved only with the addition of minimum standards/minimum consumption earmarking. In fact, with earmarking both the recipient expenditure ratio and the market efficiency measures for pure housing gap programs were often higher than those for programs using any of the other four “self-earmarked” formulas.

5.4. The Generality of the Results

The comparisons of the various housing allowance plans were based on simulations for the city of Pittsburgh. In cities where new construction is a
more competitive alternative for low- to moderate-income households, housing allowances might work differently by stimulating new construction.

When new construction can be easily stimulated, it is possible that pure percent of rent programs might work better. The pure percent of rent programs might look better than before since they give a stronger stimulus than the other four plans to higher-income recipients—who might be able to exert enough competitive pressure in the housing market to cause new construction to become profitable at moderate quality levels. Rent triangle plans, on the other hand, might look worse than before, because they give very little incentive to these higher-income recipients in the second sixth of the market.

Cities that will tend to have sizable price discounts like Pittsburgh are the large Northeastern cities whose low-quality housing stock is old and decaying and often suffers from the problem of abandonment. (See [9, Part 7] for a discussion of why the presence of a housing stock that is being allowed to decay indicates that a price discount exists.) New cities of the Southeast, for example, will tend not to have these price discounts. For these cities, the conclusions of this paper about the design of an allowance would not necessarily apply.

Each payment formula was simulated under a wide range of earmarking levels and parameter values to develop general conclusions about the relative performance of the five specific payment formulas under market conditions similar to the simulated Pittsburgh housing market. Of course sufficient variations in the hybrid payment formula parameters destroy their hybrid nature and make then look like other forms. For low $R^*$ the RCHG formula is equivalent to the HG. Sufficiently large $R^*$ and $C^*$ parameters make the RCHG formula look like a pure percent of rent. The parameters that were used were selected to make the formulas look like true hybrids. The absence of scale effects suggests that our conclusions hold over a wide range of allowance levels.

Other more complicated payment formulas might conceivably dominate those considered here. Imposing an allowance maximum for the percent of rent formula might improve its performance by shifting more of the allowance dollars away from the highest-income recipients. It is possible that a RCHG formula for which $R^*$ increased with family income might perform well. Another possibility, designed to lessen the trade-off between increased efficiency and low participation, would be to have earmarking levels for the poorest recipients that vary by the income of the recipients (as well as by family size). With such a flexible scheme for earmarking, it should be possible to maintain high participation rates among the poorest households, but still encourage more housing consumption than in the no earmarking case. Studying such alternative, more elaborate hybrid forms, however, was beyond the scope of our effort.
Other caveats deal with the housing market model and its estimation for the city of Pittsburgh. Limitations associated with the model were discussed earlier in Section 3.5.

5.5. Designing Effective Allowance Programs

In designing an allowance programs, there are three basic questions that must be answered: (1) which payment formula should be used; (2) what scale of the program should be employed; and (3) what trade-off between participation and efficiency is desired. The simulations provide insight regarding each of these questions for housing markets with price discounts in low-quality submarkets. Rent conditioned and pure housing gap formulas are preferred based on considerations of equity and efficiency. Regarding the scale of the program, the results indicated no significant differences in the cost standardized efficiency measures of the five payment formulas as the allowance level was increased. On the trade-off between participation and efficiency, the graphs of Fig. 1 and 2 indicate that little was to be gained in terms of efficiency by earmarking levels so high that participation fell below 80%.

The relative performance of the different payment formulas in any city will depend on the characteristics of that city’s housing stock. Therefore it would be unwise to select a single formula for use in all cities participating in a national housing allowance program. The conclusions developed in this paper will tend to apply to the large cities of the Northeast. For cities without price discounts in the low-quality housing submarkets, different decisions about the design of the allowance may be appropriate.

The housing model provides a useful, consistent framework for tracing some of the complex housing market responses resulting from altered demand conditions. By modifying the Urban Institute housing market model, we were able to avoid distortions in purchasing power of an allowance dollar, and to explore the implications of income elasticities below unity. Although no model can address all of the questions involved in designing an effective housing allowance program, these simulation results provide valuable information about several of these difficult questions.

APPENDIX I

This Appendix contains the specific values for the parameters of the various housing allowance formulas. All sets of parameters for each formula were simulated with the four different earmarking levels that are described in Table A-6. As mentioned in Section 1, different parameter values for single/elderly and multiple families are used to distinguish between the needs of these two types of families.

Each of the programs described above was simulated with the following earmarking levels.
TABLE A-1

Pure Housing Gap Parameters\(^a\)

<table>
<thead>
<tr>
<th>Allowance level</th>
<th>Parameters</th>
<th>$C^*$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single/elderly families ($)</td>
<td>Multiple ($)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>100.0</td>
<td>125.0</td>
<td>0.3</td>
</tr>
<tr>
<td>Medium</td>
<td>83.2</td>
<td>104.0</td>
<td>0.25</td>
</tr>
<tr>
<td>Low</td>
<td>66.4</td>
<td>82.7</td>
<td>0.2</td>
</tr>
</tbody>
</table>

\(^a\) (HG) allowance = $C^* - bY$.

TABLE A-2

Pure Percent of Rent Parameters\(^a\)

<table>
<thead>
<tr>
<th>Allowance level (%)</th>
<th>Parameter $a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.30</td>
</tr>
<tr>
<td>40</td>
<td>0.40</td>
</tr>
</tbody>
</table>

\(^a\) (PR) allowance = $a \cdot R$.

TABLE A-3

Rent Conditioned Housing Gap Parameters\(^a\)

<table>
<thead>
<tr>
<th>Allowance level</th>
<th>Parameters</th>
<th>$R^*$</th>
<th>$C^*$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single/elderly families ($)</td>
<td>Multiple families ($)</td>
<td>Single/elderly families ($)</td>
<td>Multiple families ($)</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>54</td>
<td>65</td>
<td>100.0</td>
<td>125.0</td>
</tr>
<tr>
<td>Medium</td>
<td>54</td>
<td>65</td>
<td>83.2</td>
<td>104.0</td>
</tr>
<tr>
<td>Low</td>
<td>54</td>
<td>65</td>
<td>66.4</td>
<td>82.7</td>
</tr>
</tbody>
</table>

\(^a\) (RCHG) allowance = $(R/R^*)(C^* - bY)$, $R < R^*$; $C^* - bY$, $R \geq R^*$. 
### TABLE A-4

<table>
<thead>
<tr>
<th>Allowance level</th>
<th>Parameters (value of $Y^*$)</th>
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<tr>
<td></td>
<td>Single/elderly families ($)</td>
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<tr>
<td>Medium</td>
<td>333</td>
</tr>
</tbody>
</table>

* (VPR) allowance = $(1 - (Y/Y^*))R.$

### TABLE A-5

<table>
<thead>
<tr>
<th>Allowance level</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$R_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Single/elderly families ($)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Single/elderly families ($)</td>
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<tr>
<td>High</td>
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<td>0.65</td>
<td>42.5</td>
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<td>0.5</td>
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<tr>
<td>Low</td>
<td>0.5</td>
<td>2.5</td>
<td>41.5</td>
</tr>
</tbody>
</table>

* (RT) allowance = $\alpha_1 R$, $R < R_1$; $\alpha_2(R_2 - R)$, $R_1 < R < R_2$.

### TABLE A-6

<table>
<thead>
<tr>
<th>Earmarking requirements</th>
<th>Minimum quantity of housing services</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Single/elderly family</td>
</tr>
<tr>
<td>Medium</td>
<td>46</td>
</tr>
<tr>
<td>Low</td>
<td>41</td>
</tr>
<tr>
<td>Very low</td>
<td>37</td>
</tr>
<tr>
<td>None</td>
<td>0</td>
</tr>
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</table>
ACKNOWLEDGMENTS

This research was financed through grants from HUD to the Joint Center for Urban Studies and to Abt Associates, Inc., and through support at M.I.T. from a Class of 1922 Assistant Professor chair. We gratefully acknowledge Frank de Leeuw, Raymond Struyk, and the Urban Institute Housing Study Group for assistance in the initial stages of this research. We also thank Arthur Solomon for providing valuable comments and Dennis Fromholzer for computer assistance.

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