THE SPATIAL EFFECTS OF A TAX ON HOUSING AND LAND

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This paper analytically investigates the spatial consequences of a tax on housing and land. In general, a property tax is not spatially neutral; instead it disproportionately affects certain parts of the city. The property tax can therefore create distributional inequities and can distort the pattern of residential and industrial location. We derive conditions on locational preferences and housing production that determine which parts of a city will be disproportionately affected by a property tax. Empirical estimates suggest that central locations will be disproportionately affected by property taxes.

1. Introduction

The property tax is a widely used instrument of local governments. Numerous studies [e.g., Aaron (1975), Arnott and MacKinnon (1971), Haurin (1980), Polinsky and Rubinfeld (1978)] have been devoted to investigating the incidence of the property tax, whether capital owners or tenants bear the burden of the tax. A neglected area of study is how the property tax affects different locations within a city. This neglect is unusual since the housing market is intimately related to spatial considerations, and hence one would expect that the effects of the most common form of the property tax, a housing value tax, would also be strongly related to spatial considerations. At a time when concern about the centers of our large cities command national attention, it is surprising that the potent effect of property taxes on land and housing values, especially those at the center of the city, have been overlooked.

This paper examines the spatial consequence of property taxes. It derives analytic conditions, and then uses existing empirical estimates to illustrate that property taxes can be expected to disproportionately fall on housing and land values near the center of a city. A sudden increase in the property tax causes after-tax rents to fall by a greater percentage in the central city than in the outskirts of the city. The fact that the relative rents of different

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locations can be affected by a property tax implies that inefficient spatial allocation within a city can result with a property tax. That the effects of a property tax will be concentrated in the central city implies that land and housing owners near the center of the city will suffer greater declines in wealth than those far from the central city as a result of increases in the property tax. The spatial consequences of a property tax thus involve both efficiency and distributional effects which are made explicit in this paper.

We derive the spatial effects of taxes for three models of urban location. The models become increasingly realistic in their assumptions. The first and simplest model examines the spatial consequences of a land tax when consumers desire only land and other goods. The second model assumes that consumers desire housing and other goods, where housing is produced from land and capital. This second model is used to investigate the spatial incidence of a tax on housing value. A housing tax is the usual method of administering the property tax in the U.S. The first two models calculate the spatial incidence of a property tax in long-run equilibrium. The third model relaxes this assumption of long-run equilibrium and assumes instead that the housing stock is slow to adjust. For each model, conditions on preferences and housing production are derived which determine the spatial incidence of the tax. In the discussion following the models, existing empirical estimates of key parameters are used to argue that property taxes are likely to disproportionately affect the central locations within a city. Further implications of the spatial incidence of taxes are then discussed.

As the preceding discussion emphasizes, the spatial effects of a property tax entail both efficiency and distributional effects. These spatial effects of a property tax should definitely be taken into account in any investigation of the effects of property taxes.

2. The spatial effects of a land tax in a long-run model of the land market

We use the bid rent model, as developed by Alonso (1964), and Wheaton (1974), to examine the spatial effects of a tax on land. There are two goods in the economy, land, q, and all other goods, x. There are N consumers with identical incomes, y, and tastes. Both q and x are normal goods. All workers commute to the central business district (CBD) and pay the proportional commuting cost. We make the simplifying assumption that money collected from land is not recirculated into this urban economy.¹ In the spirit of other examinations [see, e.g., Feldstein (1977)] of the property tax, we do not allow the tax to finance public services that change a community's perception of its living environment. These assumptions are made for simplicity and for

¹This is a common assumption [e.g., Polinsky and Rubinfeld (1978)]. This assumption rules out wealth effects from the tax affecting the equilibrium. See Feldstein (1977) for an extensive examination of wealth effects on tax incidence.
consistency with earlier analyses. If either public services or income from rent induce strong spatial effects, then the conclusions below (as well as in earlier studies) would need to be altered.

In the bid rent model, consumers maximize their bid for land subject to the constraint that their utility be at some prespecified level. Equilibrium is completely determined by the level of utility \( \tilde{u} \) and the city's boundary, \( m \).

The individual's problem is, given \( t \) and \( \tilde{u} \),

\[
\max_{q} \frac{y - x - st}{q} = R \quad \text{subject to} \quad u(x, q) = \tilde{u},
\]

where \( R \) is the rent bid, \( t \) is the distance from CBD, \( s \) is the cost per mile of commuting and \( \tilde{u} \) is the level of utility. Introducing the Lagrange multiplier, \( \lambda \), the first-order conditions are (subscripts denote partial derivatives)

\[
\lambda u_x = 1/q, \quad \lambda u_q = R/q, \quad u(x, q) = \tilde{u}.
\]

These conditions define \( q, x \) and \( \lambda \) as functions of \( t \) and \( \tilde{u} \).

To determine the equilibrium utility, \( \tilde{u} \), and boundary, \( m \), we use the fact that the fixed population, \( N \), must fit into the city, and that at the boundary, \( m \), the bid must equal the value of land \( \tilde{R} \) used for non-residential purposes. These two equilibrium conditions can be written as

\[
\int_0^\infty \frac{1}{2 \pi t \sqrt{t - \tilde{u}}} \, dt = N \quad \text{and} \quad R(m, \tilde{u}) = \tilde{R}.
\]

Now, we impose a tax \( \tau \) on all land used for residential purposes. Since the tax applies only to residential land, the last condition in (2) is replaced by

\[
(1 - \tau)R(m, \tilde{u}) = \tilde{R}.
\]

A clear effect of imposition of the tax is to cause \( m \) to fall so that the city boundaries contract, and \( \tilde{u} \) to fall so that the population is worse off (Wheaton). The rent schedule within the new boundaries is everywhere higher than before. We now focus our attention on the spatial effect of the land tax on values of land within the new boundary. The spatial effect of the tax is measured by calculating the percentage change in the after-tax rent, \( \lambda(1/R(1)) \), which as we will soon see, is the same term that appears in the expression for percentage change in after-tax rent [see eq. (3)]. In the text, we focus on after-tax rents to stress the effects of a tax on the net returns to landowners and homeowners. If we are interested in the change in before-tax values, then in the statement of the results, the reader should replace phrases like 'landowners near the CBD are disproportionately hurt' with phrases like 'the rent on land near the CBD is disproportionately affected'.

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\(^2\)If we measure the spatial effect of a tax as the percentage change in the before-tax rent, all the results of the paper still apply. The percentage change in the before-tax rent will be a function of \( t \) through the term \( \lambda(1/R(1)) \), which as we will soon see, is the same term that appears in the expression for percentage change in after-tax rent [see eq. (3)]. In the text, we focus on after-tax rents to stress the effects of a tax on the net returns to landowners and homeowners. If we are interested in the change in before-tax values, then in the statement of the results, the reader should replace phrases like 'landowners near the CBD are disproportionately hurt' with phrases like 'the rent on land near the CBD is disproportionately affected'.
and asking what happens to this percentage change as a function of distance. If the percentage fall in after-tax rent is higher in the central city than at the city's outskirts, then we say that the tax has disproportionately affected the central city. In this case, the owners of land in the central city suffer a greater percent decline in their wealth than those owners near the outskirts of the city.

Suppose \( R(t) \) is the equilibrium rent at distance \( t \) paid when the tax rate is \( 1 - \tau \). The after-tax rent or net return to the landlord is

\[
\text{Return}(\tau) = (1 - \tau)R(t).
\]

As \( \tau \) increases, we have

\[
\frac{d\text{Return}(\tau)}{dt} = -R(t) + (1 - \tau) \frac{\partial R(t)}{\partial \tilde{u}} \frac{\partial \tilde{u}}{\partial \tau}.
\]

The percent change (PC) is defined as \( 1/\text{Return}(\tau) \cdot d\text{Return}(\tau)/dt \), or

\[
PC = \frac{1}{(1 - \tau)R(t)} \left[ -R(t) + (1 - \tau) \frac{\partial R(t)}{\partial \tilde{u}} \frac{\partial \tilde{u}}{\partial \tau} \right],
\]

or

\[
PC = -\frac{1}{1 - \tau} \frac{\partial R}{R \partial \tilde{u}} \frac{\partial \tilde{u}}{\partial \tau}.
\]

From the individual's first-order conditions (1), we have

\[
\frac{\partial R(t)}{\partial \tilde{u}} = -\lambda(t).
\]

So,

\[
PC = -\frac{1}{1 - \tau} \frac{\lambda(t)}{R(t)} \frac{\partial \tilde{u}}{\partial \tau}.
\]

\( PC \) is a function of \( t \) through the term \( \lambda(t)/R(t) \). How does \( PC(t) \) vary as a function of \( t \)? It is not possible to sign the derivative of \( PC(t) \) for all utility functions. Therefore it is possible that an increase in the land tax can be disproportionately concentrated on either landowners close to the CBD or on those further away. If \( PC'(t) > 0 \), then landowners near the CBD are hurt relatively more than those further away by an increase in the land tax. The reverse is true if \( PC''(t) < 0 \).
The first issue is to determine when the effect of a land tax is spatially neutral. The requirement for spatial neutrality is that \( PC'(t) = 0 \), which, from (3), is equivalent to the condition that \( \lambda(t)/R(t) \) be independent of \( t \). Using the individual's first-order conditions, we see that requiring \( \lambda(t)/R(t) \) to be independent of \( t \) is equivalent to requiring \( 1/\eta u \) to be independent of \( t \) whenever \( u(x, q) = \tilde{u} \), the equilibrium utility level.

**Theorem 1.** The property tax is spatially neutral if and only if utility can be expressed as \( u(q, x) = A(qg(x)) \) for \( A \) invertible, and positively monotonic (Cobb-Douglas utility function).

**Proof.** If \( \tilde{u} = A(qg(x)) \), then \( qu(q, x) = qg(x)A'(qg(x)) \) which is a function only of \( qg(x) \) which is uniquely determined by \( \tilde{u} \). Since \( qu \) is constant along \( u(x, q) = \tilde{u} \), it follows from the previous argument that the property tax is spatially invariant.

Now suppose that the property tax is spatially invariant, so that \( qu(q, x) = h(u(x, q)) \) for some \( h \). Let \( F^*(u) = \int^u du/h(u) \). Since \( h > 0 \), \( F^* \) is positively monotonic and is invertible. The solution to the differential equation \( du/h(u) = dq/q \) is \( F(u) = qg(x) \), where \( F(u) = \exp F^*(u) \), or since \( F^* \) and hence \( F \) are invertible positively monotonic functions, we have \( u = A(qg(x)) \) for \( A \) invertible and positively monotonic. Q.E.D.

We now want to determine the conditions under which \( PC(t) \) will be increasing or decreasing in \( t \). From (3), we have
\[
\frac{\partial u}{\partial \tau} = \text{sign} PC(t) = \text{sign} (d\lambda/dt)(\lambda/R)
\]
since \( \partial u/\partial \tau < 0 \). Therefore, we need to determine when \( \lambda/R \) is increasing or decreasing as a function of \( t \). Intuition leads us to suspect that the elasticity of substitution of land for other goods will be of crucial importance. This leads us to focus attention on the case of a constant elasticity of substitution (CES) utility function:

\[
u(x, q) = [(1-\delta)x^{-\rho} + \delta q^{-\rho}]^{-1/\rho}, \quad \rho \geq -1.\]

The parameter \( \rho \) measures the degree of substitutability between land and other goods. The elasticity of substitution between land and all other goods is \( 1/(1+\rho) \). The consumer has a greater willingness to substitute land for other goods when \( \rho < 0 \) than when \( \rho > 0 \). The price elasticity is below \( -1 \) if \( \rho < 0 \) and between \( 0 \) and \( -1 \) if \( \rho > 0 \).

**Theorem 2.** If \( u(x, q) = [(1-\delta)x^{-\rho} + \delta q^{-\rho}]^{-1/\rho} \), then the land tax disproportionately hurts land owners near the CBD if \( \rho > 0 \), and disproportionately

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\(^3\) Obviously, any positive monotonic transformation of this utility function has the same properties.
hurts suburban owners of land for $\rho<0$. (From theorem 1, it follows that the land tax is spatially neutral for $\rho=0$.)

Proof. From the individual's first-order conditions (1),

$$\dot{\lambda}/R = 1/\theta u_q.$$

It is possible to prove (see Lemma 1 in the appendix) that

$$\text{sign} \frac{d[q \cdot u_q]^{-1}}{dt} = -\text{sign} \rho \quad \text{so} \quad \text{sign} \frac{d[q \cdot u_q]^{-1}}{dt} = \text{sign} \rho.$$

From the expression in (3), we know that $\text{sign} \ PC'(t) = \text{sign}(d/dt)(\lambda/R)$, so $\text{sign} \ PC'(t) = \text{sign} \rho$.

Since $PC'(t)>0$ implies landowners near the CBD are disproportionately hurt by an increase in the land tax, and $PC'(t)<0$ implies that landowners far from the CBD are disproportionately hurt, the theorem is proved. Q.E.D.

The greater the rate that consumers substitute for other goods as the price of land rises, the worse off are land holders far from the CBD relative to those near the CBD as the land tax increases. The less flexible that consumers are in their substitution possibilities, the worse off are land holders near the CBD.

Initially, holding land rents constant, tax increases cause the same proportional decline in returns to all landowners, regardless of location. However, as the land market readjusts to the new tax rate, these returns will be increased by varying amounts depending on the location of the land. Therefore, taxes, or simply uncertainty about tax rates, could have enormously different effects on the capitalized value of land depending on the location of the land. It immediately follows that increases in property taxes could have enormously different effects on the income of land owners depending on the location of their land. Since taxes alter the relative prices between two parcels of land, inefficient spatial allocation of activity between different areas could result as taxes are imposed.

Even if the members of the population were not identical in their initial incomes a variant of Theorem 2 may hold. For example, one possible distributional consequence of a land tax would be that the poor who own land are hurt disproportionately more than the rich who own land. This could happen if the rich live further away from the CBD than the poor, if people own their own land, and if the elasticity of substitution between land and other goods is less than unity for both the rich and the poor.
3. The spatial effects of the property tax in a long-run model with housing

The first model, following in the tradition of previous urban land models [e.g., Alonso (1964)] ignores housing as a separate argument in the utility function. Housing capital, as distinct from land, is taken into account through the variable \( x \). We now wish to examine the spatial effects of a tax on housing value, not on land value. Such a housing tax is the way the property tax is usually administered in the urban areas of the U.S. We consider models, where the utility function is of the form \( u(h, x) \) where \( h \) represents 'housing' which is some function of land \( l \) and capital \( k \). We assume that \( h \) and \( x \) are normal goods. We seek to obtain equilibrium housing and land value functions that take account of the substitutability of land and capital in the production of housing services. This equilibrium is a long-run solution, assuming that both land and capital have sufficient time to adjust to their long-run values. We examine the spatial effects of a tax on housing.

The same approach as before can be used to solve for equilibrium. Substitute \( h \) for \( q \) in the first-order conditions (1). Differentiating the Lagrangean in the consumers' maximization problem with respect to \( t \) yields

\[
p_h(t) = -s/h. \tag{4}
\]

which is a differential equation that determines the price of housing \( p_h(t) \) as a function of distance. Let \( h = g(l, k) \), and let the price of capital be \( p_k \), and the price of land at distance \( t \) be \( p_l(t) \). Under the assumption that housing is competitively supplied, the following conditions hold:

\[
p_l g_l = p_l, \quad p_h g_h = p_h, \quad h p_h = l p_l + k p_k. \tag{5}
\]

Conditions (4), (5) and (2) are sufficient to determine all the equilibrium characteristics of the city.

As is clear from the previous model, we will be able to reach conclusions about the spatial effect of a property tax (a tax on \( p_h \)) only for specific functional forms. The first thing to notice is that, except for the equilibrium condition to determine \( \hat{u} \), the formal analysis in the first model is completely unchanged if we everywhere replace \( q \) by \( h \). In particular, as a restatement of Theorem 1, we have

Theorem 3. The percent loss in the after tax price of housing \( (i.e., \frac{d(1 - \tau)p_h}{dt}(1 - \tau)p_h) \) is spatially invariant as the tax rate rises if and only if the utility can be expressed as \( u(h, x) = A(h g(x)) \), \( A \) invertible and positively monotonic (Cobb–Douglas utility function).

Proof. Identical to proof of Theorem 1. Q.E.D.
It is not true however that a similar theorem holds regarding the price of land, which is only one component of the price of housing. Since in the long run capital is bought and sold on the open market at price \( p_k \), and since we are assuming adjustment of the capital stock to its long-run equilibrium values, housing suppliers (i.e., landowners) are not concerned with the price of housing \( p_h \) but with the price of land \( p_l \). How \( p_l \) is affected by changes in the tax rate depends on the form of the production function for housing.

We will restrict our attention to two types of constant returns to scale production functions, one a CES, and the other a Cobb–Douglas, which of course is just a special case of a CES production function. Before proceeding we will invoke two results from production theory, that are proved in the appendix.

Lemma 2. Under competition, if \( h = (l)^{\sigma}(k)^{1-\sigma} \), then \( p_l = c_1 p_h^{1/1-\sigma} \). Also, if \( h = (\delta l^{-\rho} + (1-\delta)k^{-\rho})^{-1/\rho} \), then \( p_l = c_2 (p_h^{1-\sigma} - c_3)^{1/1-\sigma} \), where \( c_1, c_2, c_3 \) are constants, \( c_3 < p_h^{1-\sigma} \), positive \( \sigma = 1/(\rho + 1) \), and \( \rho \geq -1 \).

Proof. See appendix. Q.E.D.

Let a dot over a variable represent its derivative with respect to a change in the tax rate \( \tau \). Then, from Lemma 2, it is easy to see that for the Cobb–Douglas production function

\[
\frac{\dot{p}_l}{p_l} = \left( \frac{1}{1-\sigma} \right) \frac{\dot{p}_h}{p_h},
\]

while for the CES production function,

\[
\frac{\dot{p}_l}{p_l} \frac{1}{p_h^{1-\sigma} - c_3} \frac{\dot{p}_h}{p_h}.
\]

Using (5) and (6), we can now address the spatial effects of a housing tax on the value of land.

Theorem 4. If \( u(h,x) = h g(x) \), then an increase in the housing tax on the value of land

(a) is spatially neutral if the housing production function is Cobb–Douglas. In

\footnote{The \( \rho \) here is unrelated to the \( \rho \) of the consumer's utility function. Henceforth, \( \rho \) refers only to the consumer's utility function, and the elasticity of substitution, \( \sigma \), refers only to the housing production function.}

\footnote{See footnote 3.}
other words, all land owners regardless of location suffer the same percent loss in the value of their land,

(b) more severely affects landowners near the CBD if the housing production function is CES and $\sigma < 1$,

(c) more severely affects landowners far away from the CBD if the housing production function is CES and $\sigma > 1$.

Proof. From Theorem 3, we know that $[(1-\tau)p_h]/(1-\tau)p_h$ is constant as a function of distance $t$ (remember the dot means $\partial/\partial t$, where $\tau$ is the tax on housing value). [Notice that the after tax price of housing $(1-\tau)p_h$ is the relevant price variable to use in (6) and (7).] If the production function is Cobb–Douglas, from (6), it immediately follows that $\hat{p}_i/p_i$ is unchanging over distance.

If the production function is CES, then from (7) it immediately follows that the behavior of $\hat{p}_i/p_i$ as a function of $t$ depends on the $[(1-\tau)p_h]^{-\sigma}/[(1-\tau)p_h]^{1-\sigma} - c_3$ as a function of $t$. Since $c_3/[(1-\tau)p_h]^{1-\sigma}$ is increasing as a function of $t$ for $\sigma < 1$ and decreasing for $\sigma > 1$, we have that $[(1-\tau)p_h]^{-\sigma}/[(1-\tau)p_h]^{1-\sigma} - c_3$ is increasing in $t$ for $\sigma < 1$ and decreasing for $\sigma > 1$. Parts (b) and (c) of the theorem follow immediately from this last observation and from Theorem 2 and (7). Q.E.D.

Suppose the utility function $u(h,x)$ is not multiplicative but as in Theorem 2, if of the CES variety. Then the analogue to Theorem 2 is

**Theorem 5.** If $u(h,x) = [(1-\delta)x^{-\rho} + \delta h^{-\rho}]^{-1/\rho}$, then the housing tax disproportionately affects the after-tax price of housing near the CBD if $\rho > 0$ [i.e., $[(1-\tau)p_h]/(1-\tau)p_h$ increases with $t$] is spatially neutral if $\rho = 0$, and disproportionately affects the after-tax price of housing far from the CBD if $\rho < 0$.

Proof. Identical to proof of Theorem 2. Q.E.D.

Using (6), (7) and Theorem 5, the following is true:

**Theorem 6.** If $u(h,x) = [(1-\delta)x^{-\rho} + \delta h^{-\rho}]^{-1/\rho}$, then landowners near the CBD are disproportionately hurt by an increase in the housing tax (i.e., $\hat{p}_i/p_i$ increases with $t$) when $\rho > 0$ and when

(a) housing production is Cobb–Douglas,
(b) housing production is CES with $\sigma < 1$. 
Land owners far from the CBD are disproportionately hurt by an increase in
the housing tax when \( \rho < 0 \) and when

(c) housing production is Cobb–Douglas,
(d) housing production is CES with \( \sigma > 1 \).

Proof

(a) From Theorem 5, \( \frac{([1-\tau]p_h)/(1-\tau)p_h}{(1-\tau)p_h} \) is increasing with distance, hence
from (6), so is \( \hat{p}_i/p_i \).
(b) From Theorem 5, \( \frac{([1-\tau]p_h)/(1-\tau)p_h}{(1-\tau)p_h} \) is increasing with distance, and so
is \( \frac{([1-\tau]p_h)^1-\sigma}{[[([1-\tau]p_h)^1-\sigma - c_3]} \), hence from (7), so is \( \hat{p}_i \cdot p_i \).
(c) From Theorem 5, \( \frac{([1-\tau]p_h)/(1-\tau)p_h}{(1-\tau)p_h} \) is decreasing with distance, hence
from (6) so is \( \hat{p}_i/p_i \).
(d) From Theorem 5, \( \frac{([1-\tau]p_h)/(1-\tau)p_h}{(1-\tau)p_h} \) is decreasing with distance, and so
is \( \frac{([1-\tau]p_h)^1-\sigma}{[[([1-\tau]p_h)^1-\sigma - c_3]} \), hence from (7) so is
\( \hat{p}_i/p_i \). Q.E.D.

Table 1 summarizes the results of the above theorems.

<table>
<thead>
<tr>
<th>( \rho ) of consumer's utility function</th>
<th>Elasticity of substitution, ( \sigma ), in housing production function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho &gt; 0 )</td>
<td>( \sigma &lt; 1 ) CBD</td>
</tr>
<tr>
<td>( \rho = 0 )</td>
<td>( \sigma = 1 ) CBD (Cobb–Douglas)</td>
</tr>
<tr>
<td></td>
<td>( \sigma &gt; 1 ) uncertain</td>
</tr>
<tr>
<td>( \rho &lt; 0 )</td>
<td>( \rho ) (Cobb–Douglas)</td>
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<tr>
<td></td>
<td>all affected equally</td>
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<tr>
<td></td>
<td>suburb</td>
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<tr>
<td></td>
<td>uncertain</td>
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<tr>
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<td>suburb</td>
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</tbody>
</table>

Unlike before, we now have ambiguous situations (\( \rho < 0 \), \( \sigma < 1 \) and \( \rho > 0 \),
\( \sigma > 1 \)). In these cases, the spatial effect could go either way depending on the value of the constant \( c_3 \). The main conclusion is as before, that the less the willingness of consumers to substitute between housing and other goods, the more the landowners near the CBD are hurt, relative to those further from the CBD. This effect is mitigated and possibly reversed when there is a high elasticity of substitution between land and capital in producing housing and is accentuated when the substitution elasticity is below 1.
4. Spatial effects of a property tax in a short- to medium-run model with housing

The two previous models have examined the spatial consequences of a land or housing tax in the long run when both the land and capital can be adjusted freely. In reality, the adjustment of housing capital is a very time consuming process. In fact, a distinguishing feature of the housing stock is its durability. Long-run analysis can be useful in telling where the market is heading. On the other hand, if the transition period to long-run equilibrium is very time consuming, the effects of a housing tax in this short to medium run may be of most interest to policy makers. In this section, we specifically address the effects of a housing tax in a short- to medium-run context.

The housing stock is composed of units of widely differing ages. In response to a change in economic conditions, the city will gradually replace the housing stock with those levels that represent equilibrium long-run values for the capital stock. In the short to medium run, the housing capital at certain locations will be changing only very slowly toward its long-run equilibrium values. It is these housing units that we focus attention on. (The first two models can be thought of as focusing attention on units that adjust rapidly to long-run equilibrium.)

Imagine the following situation. At two different distances \( t_1, t_2 \) \((t_1 < t_2)\), two different amounts of housing \( h_1 \) and \( h_2 \) are provided with a newly built capital structure and land. The prices \( p_{h1} \) and \( p_{h2} \) of the housing satisfy the conditions of long-run equilibrium in the housing market. A tax on housing value is now imposed. Because the two structures are newly built, the amount of housing provided is assumed not to change in the short to medium run. Other older obsolete structures are gradually replaced by structures corresponding to the new long-run equilibrium. Gradually the equilibrium level of utility falls as a result of the imposition of the tax, and hence \( p_{h1} \) and \( p_{h2} \) change. To examine the spatial incidence of the tax we seek to determine whether

\[
\frac{\left[ (1 - \tau) p_{h1} \right]}{(1 - \tau) p_{h1}} \mid_{h_1 \text{ fixed}} \quad \geq \quad \frac{\left[ (1 - \tau) p_{h2} \right]}{(1 - \tau) p_{h2}} \mid_{h_2 \text{ fixed}},
\]

where, as before, a dot over a variable indicates differentiation with respect to the tax \( \tau \).

In contrast to the two earlier models, in this model the provider of housing is concerned not with the price of land \( p_l \) but with the price of housing \( p_h \). Unlike the previous model, capital is not easily adjustable, so that the amount of capital used to produce housing becomes a fixed cost whose value to the landlord is not \( p_h \cdot k \), since the capital cannot be sold on
the open market. In unchanging circumstances, the price paid for capital will 
be exactly recouped by part of the rental payments. However, once the 
property tax rate unexpectedly changes, the amount of capital in place 
affects the profitability of the fixed investment. Because capital changes 
slowly, there is a cost (not examined in the second model) of being caught 
with the wrong level of capital stock.

To determine the direction of the inequality in (8), we proceed as follows. 
Initially, \( u(h_i, y - st_i - p_{h_i} \cdot h_i) = \bar{u}, \quad i = 1, 2 \). Notice that as \( p_{h_i} \) changes, while 
keeping \( h_i \) fixed, we obtain \( \partial u / \partial p_{h_i} = u_x \cdot (-h_i) \). Let \( \frac{d\bar{u}}{dt} \) represent the drop 
in the equilibrium level of utility that occurs in this short to medium run in 
which \( h_1 \) and \( h_2 \) are fixed after the imposition of the tax. Then, to maintain 
equilibrium in the housing market at \( t_1, t_2 \), we require

\[
\frac{dp_{h_i}}{dt} = \frac{1}{-h_i u_x} \frac{d\bar{u}}{dt}.
\] (9)

As the tax rate is increased, the percent change in the value of housing is

\[
\frac{d}{dt} \frac{(1 - \tau) p_h}{(1 - \tau) p_h} = -1 + \frac{dp_h}{dt} \quad \text{or} \quad PC(t_i) = -\frac{1}{1 - \tau} + \frac{-1}{p_h h_i u_x} \frac{d\bar{u}}{dt}.
\] (10)

From the consumer's initial maximization of utility, we know that \( u_h / u_x = p_{h_i} \). Using this, rewrite (10) as

\[
PC(t_i) = -\frac{1}{1 - \tau} - \frac{1}{h_i u_{h_i}} \frac{d\bar{u}}{dt}.
\] (11)

\( PC(t) \) is increasing in \( t \) if \( 1 / h_i u_{h_i} \) is increasing in \( t \), and decreasing when 
\( 1 / h_i u_{h_i} \) is decreasing in \( t \) since \( d\bar{u}/dt < 0 \). But, we have already 
analyzed the behavior of an expression such as \( 1 / hu_h \) in the first model. 
Although in the first model, we were focusing attention on the price of land, 
\( p_1 \), the analytics of determining whether \( 1 / hu_h \) is increasing in \( t \) are identical 
to those that determined whether \( 1 / qu_x \) was increasing or decreasing in \( t \). 
We immediately obtain the following results.

**Theorem 7.** In the short- to medium-run when capital is not perfectly flexible, 
the tax on housing is spatially neutral in its effect on housing prices if and
only if \( u(h, x) = A(hg(x)), \) for \( A \) invertible and positively monotonic.

If \( u(h, x) = [\delta h^{-\rho} + (1 - \delta) x^{-\rho}]^{-1/\rho} \), then the tax on housing disproportionately affects housing owners near the CBD when \( \rho > 0 \), and disproportionately affects housing owners far from the CBD when \( \rho < 0 \).

**Proof.** Identical to those of Theorems 1 and 2. Q.E.D.

Notice that the production function for housing does not enter Theorem 7. Since it is assumed that capital is fixed for the units under consideration, the way in which housing can be produced is not relevant. An interesting and ambitious extension of the analysis is to relax this assumption and analyze the spatial incidence of the property tax in a dynamic model with growing demand where the presence of the tax affects the rate at which housing capital depreciates and where the presence of the tax affects the rate of development and demolition. Such an extension could draw on recent work in housing dynamics such as Arnott and Lewis (1979), Arnott (1980), Anas (1978) and Brueckner and Von Rabeneau (1981).

5. Discussion

The incidence of the property tax has received a great deal of attention in the literature. In many discussions, researchers focus on land or housing taxes, and try to determine who pays the tax. From a theoretical viewpoint, it is possible to deal with the question of incidence. With untaxed alternative uses of land and a fixed population, part of a land tax is shifted forward to consumers. With no untaxed alternative uses of land, owners of land bear the entire tax. [But see Feldstein (1977) for an analysis emphasizing the importance of no wealth effects for this conclusion.] The incidence of a housing tax depends on the elasticity of substitution between land and capital [see Leroy (1976)]. If we drop the assumption of a fixed population, and instead assume free migration to a world of constant utility, tenants (by definition) never suffer from the imposition of a property tax. [See Polinsky and Rubinfield (1978) for an analysis with this assumption.]

This paper proposes that an equally important consideration associated with housing or land taxes is their spatial incidence. In three models, it was possible to draw out the spatial effects of the property tax. Although the models differ in their assumptions about the adjustment of variables to their equilibrium levels, all the models point to the same conclusions. The less willing that consumers are to substitute housing for non-housing, and the less able that housing producers are to substitute capital for land, the more disproportionate is the burden of a tax on housing producers and landowners close to the CBD. Equivalently, under the conditions just cited,
the property tax disproportionately affects values of housing and land close to the CBD relative to values far from the CBD.

Empirical estimates of the elasticity of substitution between land and capital in housing production indicate values below 1. For example, in a recent survey by McDonald (1981) of 13 studies, 12 estimated the substitution elasticity to be below one. None found an elasticity of substitution significantly above 1, while 9 found the estimated elasticity to be significantly below one. Estimates of income and price elasticities seem to suggest values near one [De Leeuw (1971)]. The recent work of Polinsky and Ellwood (1979) and Rosen (1979) suggest an income elasticity in excess of 0.8 [Polinsky and Ellwood (1979)] and a price elasticity of about 1 [Rosen (1979)]. A Cobb-Douglas utility function therefore seems like a reasonable approximation to use. [A Cobb-Douglas is what Arnott and MacKinnon (1971) used in their simulation study]. Alternatively if we do not use the Cobb-Douglas utility function and instead use the more complicated utility function (i.e., \( \rho \neq 0 \)) in the text, then we need to determine if \( \rho \) exceeds zero. A \( \rho > 0 \) implies a price elasticity of demand below 1. Recent empirical studies of price elasticity tend to find a price elasticity below 1 [e.g., Polinsky and Ellwood (1978), Rosen (1979)] though not necessarily statistically different from 1.\(^6\) Combining these empirical estimates with the theorems just proved implies that increases in property taxes will disproportionately affect the values of land and housing close to the center of the city. Property owners near the city's center will be most hurt by sudden increases in property taxes.

The fact that the property tax is not spatially neutral has several important implications for policy makers. First, changing property tax rates could drastically affect property values in one part of the city but not the other. Distributional inequities could result. By affecting relative land values, inefficient spatial location or inefficient use of property relative to other goods could result — with the greatest inefficiencies likely to occur in those parts of the city affected most by the taxes. Moreover, the mere uncertainty about future property tax rates could be sufficient to cause considerably different effects on the capitalized value of property in different parts of the city.

Although we have couched the discussion in terms of housing, it applies equally well to business location. The models have shown that the property tax can have spatially different effects on property and land values. As relative factor prices change, certain industries may find different locations desirable. The greatest changes in the capital-labor ratio are likely to occur in those places in the city where the price of property is most severely changed by the property tax. In fact, in response to uncertainty about

\(^6\)Murray (1975) has estimated CES type utility functions and finds \( \rho < 0 \). However, Murray's results imply price elasticities that are much higher than those found in the literature.
property tax rates, industries with flexible technologies (i.e., capital can be varied quickly) will have an advantage, provided there are non-negligible moving costs. This means that these flexible industries will be more likely to locate in areas where the spatial effects of property taxes are greatest. An inefficient spatial location of industry could result. An important effect of the property tax (or even uncertainty about the property tax) is to alter (distort) the spatial industrial structure of the city.

Under what circumstances will a property tax cause the least deadweight loss? Clearly, the answer to this question depends on how fast variables can adjust to their long-run values. A policy dictum in public finance has always been to tax most heavily those goods that are inelastically supplied and demanded. Comparable reasoning would suggest that the property tax will have the smallest efficiency effects in the short to medium run, provided that either housing or industry with inflexible and immobile technologies are located in those areas that will bear the disproportionate part of the tax.

The property tax is a widely used policy instrument whose effects are difficult to disentangle. Previous research has focused on the non-spatial question of who pays the property tax — landowners or tenants. In an urban setting, an equally important question is how does the property tax affect different parts of the city. As this paper has shown, the property tax is not in general spatially neutral. Empirical estimates suggest that the property tax falls disproportionately on the central city. Serious distributional consequences of property taxes could thereby result. By changing the relative attractiveness of different parts of a city, a property tax can have undesirable efficiency effects by drastically altering the spatial configuration of a city.

The spatial effects of a property tax are likely to be as important to policy makers as the non-spatial effects. The spatial effects of property taxes definitely serve attention.

Appendix

Lemma 1. If the utility function is of the CES variety, then
\[ \text{sign } dq \cdot u_q / dt = - \text{sign } \rho. \]

Proof. For this proof only, let a dot indicate differentiation with respect to distance \( t \).

\[ (\dot{q} \cdot u_q) - \dot{u}_q q + q[\dot{u}_{qq} \dot{q} + u_{qq} \dot{q}]. \]  \( (A \ 1) \)

From Wheaton (1974, theorem 2), we know that provided \( q \) and \( x \) are
normal goods, that \( x - - RMUq \), and \( q > 0 \), where \( RMU = u_q/u_x \). Substituting into (A.1), we obtain

\[
(q \cdot u) = \dot{q}[u_q + q[u_{qq} - RMU u_{xq}]], \quad \text{or} \quad (A.2)
\]

\[
\text{sign} (q \cdot u_q) = \text{sign}[u_q + q[u_{qq} - RMU u_{xq}]]. \quad (A.3)
\]

For the CES type utility function we have

\[
u(q, x) = [\delta q^{-\rho} + (1 - \delta)x^{-\rho}]^{-1/\rho} \equiv A^{-1/\rho},
\]

\[
q = \delta q^{-\rho - 1} A^{-1/\rho - 1},
\]

\[
u_{qq} = \delta (-\rho - 1)q^{-\rho - 2} A^{-1/\rho - 1}
\]

\[
+ \delta^2 q^{(-\rho - 1)^2}(-\rho)A^{2-1/\rho}(-1/\rho - 1),
\]

\[
u_x = (1 - \delta)x^{-\rho - 1} A^{-1-1/\rho},
\]

\[
u_{xq} = \delta (1 - \delta)q^{-\rho - 1}x^{-\rho - 1} A^{-2-1/\rho} (-\rho) (-1/\rho - 1),
\]

\[
RMU = u_q/u_x = \frac{\delta}{1 - \delta} \left(\frac{q}{x}\right)^{-\rho - 1}.
\]

Using the above, we can rewrite the expression in brackets in (A.3) as

\[
\delta q^{-\rho - 1} A^{-1/\rho - 1} + q \left[\delta (-\rho - 1)q^{-\rho - 2} A^{-1/\rho - 1}
\right.
\]

\[
+ \delta^2 (-\rho)(-1/\rho - 1)q^{-2\rho - 2} A^{-2-1/\rho}
\]

\[
- \frac{\delta}{1 - \delta} \left(\frac{q}{x}\right)^{-\rho - 1} \delta (1 - \delta)q^{-\rho - 1}x^{-\rho - 1}(-\rho)
\]

\[
x(-1/\rho - 1)A^{-2-1/\rho}
\]

or

\[
\delta q^{-\rho - 1} A^{-1/\rho - 1} + q[\delta (-\rho - 1)q^{-\rho - 2} A^{-1/\rho - 1}
\]

\[
+ \delta^2 (-\rho)(-1/\rho - 1)q^{-2\rho - 2} A^{-2-1/\rho}
\]

\[
- \delta^2 (-\rho)(-1/\rho - 1)q^{-2\rho - 2} A^{-2-1/\rho}]
\]
or
\[
\delta q^{-\rho -1} A^{-1/\rho -1} + \delta (-\rho -1) q^{-\rho -1} A^{-1/\rho -1} = (-\rho \delta) A^{-1/\rho -1} q^{-\rho -1}.
\]

Since \( \delta > 0, A^{-1/\rho -1} > 0, \) and \( q^{-\rho -1} > 0, \) we obtain
\[
\text{sign}(qu_q) = -\text{sign} \rho.
\]

Q.E.D.

**Lemma 2.** Under competition, if \( h = l^a k^{1-a} \), then \( p_l = c_1 p_h^{1/a} \). Also, if \( h = [\delta l^{-\rho} + (1-\delta)k^{-\rho}]^{-1/\rho} \), then \( p_l = c_2 [p_h^{-a} - c_3]^{-1/1-a} \), where \( c_1, c_2, c_3 \) are positive constants, and \( \sigma = 1/(\rho + 1) \).

**Proof.** We make use of the following conditions. If \( h = h(l, k) \), then \( p_h \cdot h_i = p_l, p_h \cdot h_k = p_k, \) and if \( h \) is homogeneous of degree 1 then
\[
p_h \cdot h = p_l \cdot l + p_k \cdot k.
\]

Since both production functions exhibit constant returns to scale, it suffices to consider the case \( h = 1 \). First, consider the Cobb–Douglas case. From the production function, we have
\[
l = l^a k^{1-a}.
\]

while the marginal product conditions give
\[
\frac{\alpha}{1-\alpha} \frac{h}{l} = \frac{p_l}{p_h}.
\]

Solving (A.5) gives \( l = k^{a-1/a} \) which when plugged into (A.6) yields
\[
h = \left(\frac{1-a}{a}\right)\left(\frac{p_l}{p_h}\right)^a \quad \text{and hence} \quad l = \left(\frac{1-a}{a}\right)^{a-1} \left(\frac{p_l}{p_h}\right)^{a-1}.
\]

Plugging these last two equations into (A.3) for \( h = 1 \) yields
\[
p_h = \left(\frac{1-a}{a}\right)^{a-1} \frac{p_l^a}{p_h^{a-1}} + \left(\frac{1-a}{a}\right)^a \frac{p_l^a}{p_h^{a-1}} \quad \text{or} \quad p_h = \left(\frac{1-a}{a}\right)^a \left(\frac{\alpha}{\alpha + 1}\right)
\]
\[
p_k = \frac{p_l^a}{p_h^{a-1}} \left(\frac{1-a}{a}\right)^a \left(\frac{\alpha}{\alpha + 1}\right) \quad \text{or} \quad p_l = c_1 p_h^{1/a},
\]

where \( c_1 \) is a constant.
Now, consider the CES case. From the production function,

$$1 = \left[ \delta l^{-\rho} + (1 - \delta)k^{-\rho} \right]^{-1/\rho}, \tag{A7}$$

while the marginal productivity conditions yield

$$\left( \frac{\delta}{1 - \delta} \right)^{l^{-\rho} - 1} \left( \frac{k^{-\rho} - 1}{p_l}{p_k} \right) = \frac{p_l}{p_k} \quad \text{or} \quad \tag{A.8}$$

$$l = \left( \frac{1 - \delta}{\delta} \right)^{-1/\rho + 1} \left( \frac{p_l}{p_k} \right)^{-1/\rho + 1} k. \tag{A.9}$$

Plugging this last expression into (A.7) yields

$$1 - k \left[ \delta \left( \frac{1 - \delta}{\delta} \right)^{\rho \rho + 1} \left( \frac{p_l}{p_k} \right)^{\rho \rho + 1} + (1 - \delta) \right]^{-1/\rho} \quad \text{or} \quad \tag{A.10}$$

$$k = \left[ \delta \left( \frac{1 - \delta}{\delta} \right)^{\rho \rho + 1} \left( \frac{p_l}{p_k} \right)^{\rho \rho + 1} + (1 - \delta) \right]^{1/\rho}.$$

Plugging (A.9) and (A.10) into (A.4) for $h = 1$ yields

$$p_h = \left[ p_l \left( \frac{1 - \delta}{\delta} \right)^{-1/\rho + 1} \left( \frac{p_l}{p_k} \right)^{-1/\rho + 1} + p_k \right] \times \left[ \delta \left( \frac{1 - \delta}{\delta} \right)^{\rho \rho + 1} \left( \frac{p_l}{p_k} \right)^{\rho \rho + 1} + (1 - \delta) \right]^{1/\rho},$$

or

$$p_h = \frac{p_k}{1 - \delta} \left[ \delta \left( \frac{1 - \delta}{\delta} \right)^{1 - 1/\rho + 1} \left( \frac{p_l}{p_k} \right)^{\rho \rho + 1} + (1 - \delta) \right] \times \left[ \delta \left( \frac{1 - \delta}{\delta} \right)^{\rho \rho + 1} \left( \frac{p_l}{p_k} \right)^{\rho \rho + 1} + (1 - \delta) \right]^{1/\rho},$$

or using $\sigma = 1/(\rho + 1)$,

$$p_h = c_4 \left[ c_5 p_l^{1-\sigma} + c_6 \right]^{1/1-\sigma},$$
where the subscripted $c$'s are constants. So

$$p_i = \left[ \frac{1}{c_5} \left( \frac{p_h}{c_4} \right)^{1-\sigma} - c_6 \right]^{1/1-\sigma}$$

or

$$p_i = c_2 \left[ p_h^{1-\sigma} - c_3 \right]^{1/(1-\sigma)}.$$

Q.E.D.

References


