Valuing Market Benefits and Costs in Related Output and Input Markets

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A frequent application of cost-benefit analysis is to measure the cost of a tax or the benefit of a technological improvement in some input market. In many instances data on the directly affected input market may not be available, but data on output markets or other input markets may be available. For example, to evaluate a tax on fertilizer, one would like data on the amount of fertilizer purchased. Such data may be unavailable, while data on bushels of wheat, tractors, or land purchased may be readily available. An important question is how well these related markets reflect costs and benefits of the directly affected input market. Being able to use data from related markets to measure benefits or costs can enable the analyst to overcome many data problems and thereby widen the applicability of his tools.

This paper argues that related output-market measures can perfectly reflect an input-market cost or benefit while related input-market measures can reflect this cost or benefit under special though plausible conditions. Section I proves that input-market costs and benefits are totally reflected in the output market for the general case when supply curves slope upwards (i.e., factors earn rents). This result was first proved by Daniel Wiser-carver for the special case of two inputs, and subsequently by James Anderson and by Richard Schmalensee for the special case of perfectly elastic supply curves. Section I emphasizes that the output-market measure can enable an analyst to perform precise measurements of social costs even when data on the directly affected input market are unavailable. Sections II and III discuss how information from other observable input and output markets can be used to measure costs and benefits that arise in an unobservable market.

I

I wish to show that an input-market distortion can be completely measured in the output market.1 Without loss of generality, let us examine the case where a tax is placed on only one factor, and assume inputs used to produce the output can be used to produce only this output. An obvious generalization of the proof establishes it for the case where many inputs are taxed and many outputs can be produced by the inputs. I assume competition in all markets.2 I do not require that factor supply curves be infinitely elastic and instead allow for the case where factor supply curves slope upwards. This means that factors can earn rents. There are two standard reasons why factor supply curves can slope upwards: the opportunity cost of the factor may not be constant as in the labor supply decision; second the marginal cost of producing the factor may be rising as, for example, in the case of mineral deposits of different extraction costs, or in the case of agricultural land that requires different amounts of fertilizer.

Let \( r_i(t) \) be the equilibrium wage received by factor \( i \) when the tax on factor \( 1 \) is \( t \). Therefore, a firm pays \( r_i(t) + t \) for factor \( 1 \), \( r_2(t) \) for factor 2, etc. Let \( X_i(t) \) be the amount of factor \( i \) demanded in equilibrium when the tax on factor 1 is \( t \). The deadweight loss \( S' \) of an increase in the tax on factor 1 from \( t_0 \) to \( t_1 \) is measured in the input market by

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1It is also true that an input-market benefit is completely reflected in the output market. This result follows from a proof almost identical to the one given below.

2The method of proof is similar to that used by Schmalensee (1976).
\[{2} \quad \bar{C}'(t) = C'(t) - \sum \frac{dr_i}{dt} X_i(t) - \sum r_i(t) \frac{dX_i}{dt}(t) \]

Denote the area under the (inverse) output demand curve, \(P(Q)\), between the quantities corresponding to the tax rates \(t_0\) and \(t_0\) by

\[{2} \quad CS(t) = \int_{Q(t_0)}^{Q(t)} P(Q) dQ \]

hence

\[{2} \quad \frac{dCS}{dt} = -Q'(t) P(Q(t)) \]

The measure of the input-market distortion of raising the tax from \(t_0\) to \(t_1\) can be measured as the change in the area under the output demand curve minus the change in "true" costs (i.e., excluding rent and tax revenue changes) of producing the output.

Let the costs (excluding tax revenue) of producing output \(Q(t)\) be denoted by\(^4\)

\[C(t) = \sum r_i(t) X_i(t)\]

The change in \(C(t)\) as the tax rate increases is

\[C'(t) = \sum r_i(t) \frac{dX_i}{dt}(t) + \sum \frac{dr_i}{dt} X_i(t)\]

The change in cost relevant for calculations of welfare loss is not \(C'(t)\) since \(C'(t)\) includes the term \(\Sigma(dr_i/dt)X_i(t)\) which is the change in rents that accrue to factor owners as taxes are altered. The cost change relevant for the cost-benefit calculation excludes rent and equals

\[{3}\text{Whenever cost-benefit analysis is done, compensated demand and supply curves must be used. The input demand curves are the demands for input that arise when the amount of output produced is given by a compensated demand. With a non-linear production-possibility frontier, the relevant curves to use are compensated "general equilibrium" demand curves which incorporate price changes in other markets into the demand curve. See Harberger and Peter Diamond and Daniel McFadden.}\]

\[{3}\text{For simplicity of notation, the cost of producing output } Q(t) \text{ is not written as } C(Q(t)) \text{ but as } C(t).\]

\[S^0 = \int_{t_0}^{t_1} \left[ \frac{dCS}{dt} + \bar{C}'(t) \right] dt \]

Since the output and factor markets are competitive, we have that

\[Q'(t) = \sum MPP_i \frac{dX_i}{dt} = \left[ \sum r_i(t) \frac{dX_i}{dt} + t \frac{dX_i}{dt} \right] \frac{1}{P(Q(t))} \]

where \(MPP_i\) is marginal physical of factor \(i\) and where we have used the relation that in competitive markets, the wage equals the value of the marginal physical product. Inserting \(Q'(t)\) into the expression for \(dCS/dt\), and using the expressions for \(S^0\) and \(C'\), we obtain

\[\frac{dS^0}{dt_1} = -\sum r_i(t_1) \frac{dX_i(t_1)}{dt} - t_1 \frac{dX_i(t_1)}{dt} + \sum r_i(t_1) \frac{dX_i(t_1)}{dt} \]

or

\[\frac{dS^0}{dt_1} = -t_1 \frac{dX_i}{dt} \]

which is the same as \(dS'/dt_1\) in (1). We therefore conclude that \(S'\) and \(S^0\) yield identical measures of the distortion.

It is useful to illustrate diagrammatically how one could calculate deadweight loss in an output market. Suppose that the tax on factor 1 is increased from 0 to some small \(\epsilon\). Let us
first calculate what the relevant cost change (\(\Delta C\)) equals. By definition (see (2)) \(\Delta C\) will approximately equal \(C'(0)\epsilon\), or the difference in total costs after taxes and rents have been excluded. Mathematically,

\[
\Delta C = (TC_1(Q) - RENT_1(Q)) - (TC_0(Q_0) - RENT_0(Q_0)),
\]

where \(Q_0\) = output when tax is 0
\(\bar{Q}\) = output when tax is \(\epsilon\)
\(TC_0(Q)\) = total cost of producing \(Q\) when tax is zero
\(TC_1(Q)\) = total cost of producing \(Q\) when tax is \(\epsilon\) (does not include tax payments)
\(RENT_0(Q)\) = rent earned when \(Q\) is produced and tax is 0
\(RENT_1(Q)\) = rent earned when \(Q\) is produced and tax is \(\epsilon\).

Rewrite the above expression for \(\Delta C\) as

\[
\Delta \bar{C} = [TC_1(Q) - RENT_1(Q)] - [TC_0(Q) - RENT_0(Q)] + [(TC_0(Q) - RENT_0(Q)) - (TC_0(Q) - RENT_0(Q))].
\]

Let \(C_0(Q)\) be the supply (unit cost) curve when the tax is zero and \(C_1(Q)\) be the supply (unit cost) curve when the tax on factor 1 is \(\epsilon\) and when tax revenues have been subtracted from costs. \(C_1(Q)\) lies above \(C_0(Q)\) because the tax causes output to be produced with inefficient factor proportions. Notice that the first term in (3), \([TC_1(Q) - RENT_1(Q)]\), is given exactly by the area beneath the \(C_1(Q)\) curve between 0 and \(\bar{Q}\). Similarly the second term in (3), \([TC_0(Q) - RENT_0(Q)]\), is given by the area beneath the \(C_0(Q)\) curve between 0 and \(Q\). The difference between these first two terms is then the area between the \(C_1(Q)\) and \(C_0(Q)\) curves from 0 to \(Q\). The last expression in (3) is something more than the incremental social cost of producing \(Q\) - \(Q_0\), which approximately equals \([Q - \bar{Q}]\)

Using the above analysis we can decompose \(\Delta C\) into the difference between two areas in Figure 1. The first is a banana-shaped area representing the difference between the first two expressions in (3). The banana is the increased cost of production caused by input distortions that arise from the tax on the factor. The second area is a box that represents the last term in (3). The box represents the decrease in cost that comes from a reduction in output. By construction, the difference between the banana and the box is precisely what \(C'(0)\Delta t\) measures for \(\Delta t = \epsilon\) in (2).

If we let \((\epsilon_1, \epsilon_2, \ldots, \epsilon_n)\) be a sequence whose sum converges to \(t\), and if we let \(C_t(Q)\) be the supply (unit cost) curve when the tax on factor 1 is \(t\) and taxes are excluded from costs, then we can build up the area relevant for consideration of the cost side for a tax \(t\) on factor 1 that reduces output from \(Q_0\) to \(Q\). That area is given as the sum of all the infinitesimal bananas minus boxes as \(\Sigma \epsilon_i \rightarrow t\). It equals \(ABCD\) minus \(DEFG\) in Figure 2. In Figure 2 the curve \(C_0^*(Q)\) is the unit cost curve including tax revenues when the tax on input 1 is \(t\). The demand curve is denoted as \(Q(p)\). The intersection of \(C_0^*(Q)\) and \(Q(p)\) at \(H\) determines the post tax quantity of output \(Q\). The area under the demand curve “lost” as a result of the tax is \(HEFG\). The net loss equals \(ABCD + HEFG - DEFG = ABCD + HGD\) and is illustrated as the shaded area in Figure 2. The tax revenues are given by the area of the rectangle \(HIJC\).
Since \( \bar{C}' \) was defined as the "relevant" area (i.e., a banana minus a box), (see (2) and (3)), an alternative diagram to use in the benefit-cost analysis is Figure 3 above. The deadweight loss is given as a "triangle" \( ABC \). The reason this diagram is not as useful as Figure 2 is because \( C' \) is a complicated function (see (2)) that must be derived from the information contained in the cost curves of Figure 2.

The rather complicated construction of loss in Figure 2 should be contrasted with the simple determination of loss in the directly affected factor market. From our previous proof we are guaranteed that the shaded area in Figure 2 equals the familiar triangle \( ABC \) of Figure 4, where \( S \) and \( D \) in Figure 4 refer to the supply and demand for the factor on which the tax is placed. The tax revenues in Figure 4 are given by the area of the rectangle \( ABDF \).

When information about supply of all inputs, prices of all inputs, and demand for output are available, then there is no reason to prefer using Figure 4 or Figure 2. On practical grounds, Figure 4 may often be the preferred alternative since its calculation requires knowledge only of one supply and demand curve. Information on other factor markets or on the output market is not needed.

Is it ever the case that information on the affected input market is so poor that Figure 2 can be used but not Figure 4? The answer is yes, and this result provides the sole justification for trying to measure input distortions (or benefits) in output markets. A simple example is the easiest way to illustrate this point.

Suppose that an undistorted output market is observed for a long time. Observable shifts in exogenous (for example, income, weather) variables enable the analyst to identify the relevant pretax supply and demand curves.

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1Here we draw \( \bar{C}' \) as a function of output. The notation in Figure 1 is the same as in Figure 2.

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8This statement appears to conflict with Wisecarver’s belief that “practical measurement of the substitution component in the output market is intractable” (p. 367) and that output measures “do not fit naturally into the framework of applied welfare economics” (p. 367). Wisecarver is, however, correct to insist on the simpler input market measure when it is available. The recent paper by John Panzar and Robert Willig also appears to imply that only direct input-market measures are correct. However, a careful reading of their paper shows that this implication would be a misrepresentation of their views. Panzar and Willig argue correctly that consumer's surplus in input and output markets will differ, but do not discuss how taking account of a banana-shaped area (that represents increased production costs caused by factor-market distortions) can enable use of output-market measures. This paper is in agreement then with Panzar and Willig. Without taking the banana into account, it is fruitless to perform cost-benefit analysis in the output market.
Data on certain input prices and quantities demanded are not available. A tax is imposed on the input and the policy remains in effect for several years. Observable shifts in exogenous variables enable the analyst to identify the posttax supply curve. Information on taxes collected \( T(Q) \) is available. If information on pretax factor prices and quantities is unavailable, then the distortion cannot be measured in the input market. However, using Figure 2, the distortion can readily be measured in the output market. We let \( C_0(Q) \) be the pretax supply curve, \( C^*(Q) \) be the posttax supply curve, and \( C^*(Q) - (T(Q)/Q) \). The shaded area in Figure 2 gives the correct measure of deadweight loss.

Measuring an input-market distortion in an output market is more tedious than measuring the distortion in the directly affected input market. However, it may be possible to

\[ \text{use the output-market distortion measure in cases where data unavailability precludes the use of the input-market distortion measure. In any practical application one must keep in mind the usual caveats about using compensated demand curves and "reduced-form general equilibrium" demand curves (see Harberger and Diamond-McFadden) and giving proper attention to distributional issues. Moreover, if inputs are used to produce several outputs then a tax on an input(s), when not measured in its own market(s), must be measured in all the affected output markets. The need to look at several markets when output distortion measures are used emphasizes the simplicity of using input distortion measures when data permit. However, when data are lacking, the more complicated output-market distortion measures can be used to accurately reflect costs and benefits in input markets. It is this property that should make the technique of measuring input-market distortions or benefits in the output market a powerful and useful one for cost-benefit analysts.} \]

**II**

Can the analyst ever use information on other input markets to evaluate an input-market benefit or distortion? A closely related question is whether the analyst can ever use information on other output markets to evaluate an output market benefit or distortion. The answer to both questions is yes but only with special though plausible assumptions. The intuition is that if the price of one input (output) is affected, the demand curves (see fn. 3) for other inputs (outputs) should also be affected. This section analyzes the use of data from other input markets, while Section III analyzes the use of data from other output markets.

If one has prior knowledge about production, then it is sometimes possible that information from only some input markets can provide complete information about all input markets. The issue is whether the prior plus observable information allows the derivation of the missing data. For example, suppose it is known that output \( (Q) \) is produced by a
Cobb-Douglas technology using two factors, labor \( L \) and capital \( K \), according to \( Q = F(L, K) = AL^a K^{1-a} \). Suppose that factor prices and \( L \) are observed and that \( \alpha \) is known. Then, knowledge of \( L \) together with the ratio of factor prices is sufficient to construct the demand for \( K \). This demand relation can then be used to measure any benefits or distortions in the capital market.

Another class of cases in which cost-benefit analysis can be done on unobservable markets are those in which prior information falls short of identifying the whole production process, but still enables calculation of the desired magnitudes. Suppose that for simplicity we assume constant returns to scale, perfectly elastic factor supplies, and a technological improvement that lowers the cost of input 1 from \( r_0 \) to \( r_1 \). How can we measure this benefit? First, we could measure it directly in the input market as a trapezoidal area between \( r_0 \) and \( r_1 \) and the derived demand curve for input 1 (see, for example, the second diagram in Figure 6). Alternatively, from Section 1 we know that we could measure it in the output market by the area composed of a banana\(^{11}\) (the difference between the new and old marginal cost curve) plus a triangle representing the additional consumer’s surplus.

Can we ever use information on other observable inputs to derive the relevant measure? The answer is yes—but only under a special though perfectly reasonable assumption. The special assumption is that as the price of the observable input becomes arbitrarily large, the price of the final product rises enough to reduce the value of consumption to zero.\(^{12}\) Under this assumption it is possible to show that the appropriate difference in areas bounded by the new and old input demand curve for any related input can be used to exactly measure the gain to society.

To prove this point suppose that input 1 and input 2 produce output 1. Let the wages of inputs 1 and 2 be denoted by \( r \) and \( s \), respectively. Let \( C(r, s) \) be the unit cost function for producing output. Let \( Q(u, p_1, p_2) \) be the individual’s compensated demand curve for output 1 at utility level \( u \) facing prices \( p_1 \) for output 1 and \( p_2 \) for other outputs. Let \( r \) fall from \( r_0 \) to \( r_1 \) because of a technological change. Let \( s_0 \) be the wage received by input 2. The direct measure of benefit is given in the input market by\(^{13}\)

\[
(4) \quad - \int_{r_0}^{r_1} Q(u, C(r, s_0), p_2) \frac{\partial C}{\partial r}(r, s_0) dr
\]

Using the reasoning from the previous section, the indirect measure of benefit measured in the output market is

\[
(5) \quad - \int_{C(r_0, s_0)}^{C(r_1, s_0)} Q(u, p_1, p_2) dp_1
\]

which immediately reduces to (4) once the change of the variable \( r \) for \( p_1 \) is performed using the relation \( p_1 = C(r, s_0) \).

Now let us look at the demand for input 2. Consider Figure 5 below. The initial derived demand for input 2 shifts in response to the shift in \( r \). We want to prove that the shaded area is the relevant area for the measure of benefit and is identical to (4) or (5). To prove this, notice that the shaded area equals\(^{14}\)

\[
\int_{r_0}^{r_1} \int_{s_0}^{s_1} \frac{\partial}{\partial r} [Q \frac{\partial C}{\partial s}] ds \, dr
\]

\[
= \int_{s_0}^{s_1} Q(u, C(r_1, s), p_2) \frac{\partial C}{\partial s}(r_1, s) ds
\]

\[
- \int_{s_0}^{s_1} Q(u, C(r_0, s), p_2) \frac{\partial C}{\partial s}(r_0, s) ds
\]

Now using the relation \( C(r, s) = p_1 \) we can rewrite the above expression as

\[
(6) \quad \int_{C(r_1, s_0)}^{C(r_1, s_0)} Q(u, p_1, p_2) dp_1
\]

\[
- \int_{C(r_0, s_0)}^{C(r_1, s_0)} Q(u, p_1, p_2) dp_1
\]

\[
- \int_{C(r_0, s_0)}^{C(r_1, s_0)} Q(u, p_1, p_2) dp_1
\]

\[
11\text{We use Shepherd’s Lemma that the demand curve for input 1 equals } Q \partial C(r, s)/\partial r. \text{ For simplicity I am assuming here that inputs 1 and 2 are used only in the production of output 1, and that the factors are in perfectly elastic supply.}
\]

\[
12\text{Again we are making use of Shepherd’s Lemma that the demand curve for input 2 is } Q \partial C(r, s)/\partial s.
\]

\[
13\text{With no rents, this banana is a rectangle.}
\]

\[
14\text{A Cobb-Douglas and a constant elasticity of substitution production function with a substitution elasticity below one satisfy this criterion.}
\]
III

The preceding discussion has shown how prior information on observable input markets can reveal the desired information about an unobservable input market. I now give an example where prior information on observable output markets yields the desired information about an unobservable output market. The example thus shows how information about other output markets can be used to measure benefits in an unobservable output market.

Suppose that we wish to evaluate the benefit of a technological improvement that lowers the price of the nth commodity from $q_0$ to $q_1$. Assume that the price vector $p$ of commodities 1 through $n-1$ are unchanged. The analyst knows the (compensated) demand curve for commodities 1, ..., $n-1$. Information on demand for commodity $n$ is unavailable. If $m(u, p, q)$ represents the expenditure required to achieve utility level $u$ at prices $p, q$, then we wish to calculate $m(u, p_0, q_1)$ or

$$\int_{q_0}^{q_1} \frac{\partial m}{\partial q} dq$$

which is just the area under a compensated demand curve for commodity $n$ between prices $q_0$ and $q_1$ (the derivative of the expenditure function with respect to price yields the compensated demand curve). Since no information on the compensated demand for commodity $n$ is available, the calculation cannot be done.

Now let the analyst be given just a little bit of information about the demand for commodity $n$. Suppose he knows that at prices $p^*$ for commodities 1, ..., $n-1$, the quantity demanded of commodity $n$ is zero at prices $q_0$ and $q_1$. For example, the demand for

of a new harvesting technology? It is possible to show (the proof is available on request) that when a related factor is in perfectly inelastic supply and when the technology is fixed coefficients, then the rent change in a related factor market will reflect the benefits of a technological change in some other factor. This last case appears to be the only one in which the relation between rent change and benefit measure holds in general.
tennis rackets can be driven to zero by sufficiently lowering the price of substitutes (golf clubs, squash rackets, etc.) and sufficiently raising the price of complements (tennis balls, sneakers, tennis court fees). The implication of the analyst’s information is that \( m(u, p^*, q_0) = m(u, p^*, q_1) \) or equivalently that the price of commodity \( n \) does not influence consumer expenditure when commodity \( n \) is not purchased. Our analyst can use his little bit of information to perform the desired measurement.\(^{17}\)

Using the definition of a line integral and the fact that \( m(u, p^*, q_0) = m(u, p^*, q_1) \) we can write that

\[
(7) \quad m(u, p_0, q_0) - m(u, p_0, q_1) = \left[ m(u, p_0, q_0) - m(u, p^*, q_0) \right] \\
+ \left[ m(u, p^*, q_1) - m(u, p_0, q_1) \right] \\
= \int_{p_0}^{p^*} \frac{\partial m}{\partial p} (u, p, q_0) \, dp + \int_{p_0}^{p^*} \frac{\partial m}{\partial p} (u, p, q_1) \, dp
\]

By assumption, the analyst knows the compensated demand curves \( (\partial m/\partial p) \) for commodities 1, \ldots, \( n-1 \). Therefore both integrals in (7) are calculable since they are just integrals under these \( n-1 \) compensated demand curves. Moreover, it follows that if the analyst has enough prior information about \( p^* \) so that he can calculate (7) for any \( q_0, q_1 \), then the analyst can let \( q_1 = q_0 + \epsilon \) for small \( \epsilon \) and thereby calculate \( \partial m/\partial q \) which is precisely the compensated demand curve for commodity \( n \).

IV

Missing data hamper many practical applications of cost-benefit analysis. This paper has shown that even when data on directly affected markets are unavailable, it is often still possible to perform the correct calculations by using available data from related output or other input markets. Such “indirect” calculations of costs and benefits should be a powerful and useful tool in applied cost-benefit analysis.

REFERENCES


A. C. Harberger, “The Measurement of


