

# Illiquid Banks, Financial Stability, and Interest Rate Policy

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Banks finance illiquid assets with demandable deposits, which discipline bankers but expose them to damaging runs. Authorities may not want to stand by and watch banks collapse. However, unconstrained direct bailouts undermine the disciplinary role of deposits. Moreover, competition forces banks to promise depositors more, increasing intervention and making the system worse off. By contrast, constrained central bank intervention to lower rates maintains private discipline, while offsetting contractual rigidity. It may still lead banks to make excessive liquidity promises. Anticipating this, central banks should raise rates in normal times to offset distortions from reducing rates in adverse times.

Should central banks alter interest rates to deal with episodes of illiquidity and financial fragility? For instance, Greenspan (2002, 5) has argued that while the Federal Reserve cannot recognize or prevent asset

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price booms, it can “mitigate the fallout when it occurs and, hopefully, ease the transition to the next expansion.” Others have responded that by following an asymmetric interest rate policy—colloquially known as the Greenspan put—a central bank can engender the kind of behavior that makes booms and busts more likely. In this paper, we ask why episodes of systemic financial sector illiquidity arise, why private arrangements may be insufficient to alleviate them, how central bank intervention to alleviate financial stress by lending to markets to reduce the short-term real interest rate may be better than other forms of intervention, and how even such intervention can create the potential for greater instability.

This paper takes as given a central bank that has developed the credibility to maintain price stability and, therefore, believes it has the room to stabilize (and enhance) real activity. We show that a willingness to recapitalize banks directly at times of stress will undermine the discipline induced by rigid private capital structures—it is not just that what is *ex post* optimal is not *ex ante* optimal (e.g., Kydland and Prescott 1977), but worse, intervention can undermine private commitment and make the system worse off. In this sense, undirected interest rate intervention, where the central bank lends to any solvent bank that needs funds, may be better because it preserves the commitment induced by private contracts even while restoring flexibility to the system by bringing down rates in a way that private contracts cannot achieve. Interest rate intervention to relieve stress may be better than the alternatives!

However, the central bank’s willingness to intervene when liquidity needs are high by pushing down interest rates *ex post* does not just affect expectations of the real interest rate but also encourages banks to make commitments that increase the need for intervention—because banks and depositors do not internalize the costs of interest rate intervention. Expectations of low real interest rates (colloquially the belief that the central bank will stay “low for long”) can increase the future need for low rates. To mitigate this, the central bank may have to commit to push the interest rate above the natural equilibrium rate in states where liquidity needs are low to offset the incentives created by its lowering them when needs are high. Central banks may need credibility in a new direction. Let us elaborate.

Our paper studies the effects of policy interventions to alter interest rates in an economy where short-term debt finances long-term illiquid projects. Banks themselves borrow from risk-averse households, who receive endowments every period. Households deposit their initial endowment in banks in return for demandable deposit claims (throughout the paper, we focus on demand deposits, although any form of overnight unsecured debt could be a close substitute). Competition for funds induces banks to offer deposit contracts that maximize *ex ante* house-

hold utility. There is no uncertainty about the average quality of a bank's projects in our model. However, there is uncertainty about future household endowments. This, coupled with the mismatch between the long gestation period for the projects and the demandable nature of deposits, is the source of banking sector difficulties.

Once households have deposited their initial endowment and projects have been started, some households may have an unexpectedly high need to withdraw deposits. One possibility is that they suffer an adverse shock to current endowment that causes them to want to run down financial assets to consume. But another is that they anticipate significantly higher income or endowments in the future and want to smooth consumption. Thus current adversity, as well as anticipated prosperity, can increase current household demand for consumption goods substantially, a source of financial sector stress when household claims on liquidity exceed available liquidity.

As households withdraw deposits to satisfy consumption needs, banks will have to call in loans to long gestation projects in order to generate the resources to pay them. The real interest rate will rise to equate the household demand for consumption goods and the supply of these goods from terminated projects. Thus greater consumption demand will lead to higher real rates and more projects being terminated. It also leads to lower bank net worth because the bank's loans pay off only in the long run and thus fall in value as real interest rates rise, while the bank's liabilities, that is, demandable deposits, do not fall in value. Eventually, if the rate rises enough, the bank may have negative net worth and experience runs, which are destructive of value because all manner of projects, including those viable at prevailing interest rates, are terminated.

The fundamental problem is that demandable deposits are noncontingent promises of liquidity, which are costly to service when the aggregate demand for liquidity is high. We show in Section III that these contracts are constrained efficient given the frictions described there. A key friction is the banker's inability to commit his human capital, an agency problem that explains why bank contracts must be demandable at all times so that the banker does not renegotiate households down (see Diamond and Rajan 2001). A second friction that interacts with the first is that it takes time to observe, verify, and implement contractual contingencies flowing from the aggregate state. If demandable contracts are to be state contingent, however, their terms will need to be adjusted instantaneously whenever the state becomes known to significant segments of the depositors, for a delay will cause informed depositors to run whenever they anticipate a reduction in face value. Frequent

changes in state and delays in contractual adjustments can explain why we do not see state-contingent demandable claims.<sup>1</sup>

Even if a social planner cannot overcome the frictions that lead to the use of deposit contracts, there may be room for improvement if the planner can use government powers that are not available to the private sector to restore useful state contingency. The planner could, for example, expropriate household endowments at date 1 through a uniform (across households) tax after seeing the state, and transfer the proceeds to the bank. This would make the bank's resources state contingent, which would help it avoid suboptimal bank runs and liquidation.

Unconstrained bank bailouts, however, undermine the discipline induced by private contracts. Banker rent extraction is usually limited by the banker's fear that he will precipitate runs if he takes too much. When the authorities are willing to intervene to prevent runs, they exacerbate ex post bank rent extraction. Ex ante, competitive banks will compensate by making even larger promises to households, making the system even more rigid and sometimes even exceeding the authorities' ability to save the system. We show that unconstrained intervention will make the system worse off.

However, *undirected lending* at market interest rates by the central bank to any solvent bank (funded by possibly forced borrowing from households) may help. Such "liquidity" intervention is still a bailout—the private banking sector is insolvent at rates that would prevail absent intervention, and it is the central bank's ability to lower overall market rates through its lending that allows it to "bail out" the banking system (a related point is stressed in Goodfriend and King [1988]). Also, the central bank's willingness to lend could leave it a hostage to market expectations—expectations that it will intervene substantially and push rates down very low could force it to intervene more. Nevertheless, the prohibition on direct support to insolvent banks prevents it from subsidizing banks even more and undermining the disciplinary effect of deposits. Moreover, we show that small nonpecuniary costs associated with borrowing from the central bank will result in unique interest rate equilibria that minimize ex post central bank intervention. This form of *interest rate intervention* by the central bank dominates unconstrained bailouts.

As in Diamond and Dybvig (1983) and Holmström and Tirole (1998), the planner/government/central bank's ability to partially access household endowments allows it to do more than the private sector. But the authorities' coercive power is essential. Intervention, even if restricted to borrowing and lending at market rates, which is seemingly respectful

<sup>1</sup> Relatively quick changes in the face value of claims may be possible for financial institutions such as mutual funds that hold only traded assets and offer quick but not immediate liquidity, but these will be harder for institutions that hold illiquid nontraded loans and offer promises of immediate liquidity.

of household property rights, has real effects only by forcibly changing household consumption opportunities and thus influencing real liquidation and investment decisions. For instance, interest rate intervention effectively forces some households to lend at rates they would not choose to voluntarily. If it prevents runs, though, everyone may be better off *ex post*.

Even interest rate intervention is not without problems, however. A lower *ex post* interest rate imposes a lower penalty on banks for having high leverage and reduces the value of maintaining liquidity. If the central bank is expected to reduce interest rates at times of financial stress, banks will take on more short-term leverage or make more illiquid loans, thus bringing about the very need for intervention. While the authorities may want to counter these incentives through *ex ante* regulations such as minimum capital requirements, regulations are easily evaded.

It may be better then for the central bank to change bank incentives by altering its interest-setting behavior. Knowing that it will be politically as well as economically undesirable to allow high rates in times of financial stress, the central bank may want to indicate that it will raise interest rates in normal times above the market-determined level in order to preserve bank incentives to maintain low leverage and high liquidity. Stability-focused central banks should also be reluctant to create expectations that real rates will be low for an extended period for fear that bank responses will make the system more fragile and force the central bank to continue keeping rates low (which could work against its goal of price stability).

In sum, then, even while central banks have become more credible on fighting inflation by binding themselves with inflation targets, they have created more room for themselves to vary real interest rates so as to affect real activity across periods (see, e.g., Borio and Lowe 2002, 2004). Central banks may now have to build credibility in a new direction so as to enhance financial stability.

The rest of the paper is as follows. In Section I, we lay out the basic model, in Section II, we solve it, and in Section III we describe the frictions that make demand deposits part of an optimal mechanism. In Section IV, we consider macroeconomic intervention on interest rates and its effect on *ex ante* choice of deposit level and asset liquidity. We then conclude.

## I. The Framework

### A. *Agents, Endowments, Technology, Preferences*

Consider an economy with risk-neutral entrepreneurs and bankers and risk-averse households, over three dates, 0, 1, and 2. Each household is

initially endowed with a unit of good at date 0. Households can invest their initial endowment in banks, which will lend the resources to entrepreneurs. At date 1, households will get endowment  $e_1$ . They also learn their date 2 endowment. Some households will get a high endowment,  $e_2^H$  at date 2 while the rest will get a low endowment  $e_2^L$ . Let the state at date 1 be indexed by  $s$ , with  $s \in \{\underline{s}, \underline{s} + 1, \underline{s} + 2, \dots, \bar{s}\}$ . States differ in the fraction of households  $\theta^s$  that expect a high endowment in that state, with  $\theta^{s+1} > \theta^s$ . Higher states thus reflect a higher degree of optimism about the economy. State  $s$  occurs with probability  $p^s$ . The state of nature and household types are not verifiable and, in addition, household types are private information.

Each entrepreneur has a project, which requires the investment of a unit of goods at date 0. The project produces an uncertain amount  $\tilde{Y}_2$  in goods at date 2 if it is not liquidated, with the realization becoming known at date 1. The amount  $\tilde{Y}_2$  is distributed uniformly over the range  $[0, \bar{Y}_2]$ . The project produces  $X_1$  at (or after) date 1 if the project is liquidated, where  $X_1$  is greater than the return on storage. Storage is a one-period constant returns to scale investment available to entrepreneurs and bankers each period. Entrepreneurs have no goods to begin with, and the demand from entrepreneurs looking for funds at date 0 is greater than household supply. Households maximize their expected utility of consumption,  $E(\log C_1 + \log C_2)$ . Risk-neutral entrepreneurs and bankers maximize  $E(C_1 + C_2)$ .

### B. *Financing Entrepreneurs*

Since entrepreneurs have no endowments, they need to borrow to invest. Each entrepreneur can borrow from a banker who has, or can acquire during the course of lending, knowledge about an alternative, but less effective, way to run the project. The banker's specific knowledge allows him to generate  $\gamma Y_2$  from a project that has just matured, with  $\gamma \leq 1$ . Once a banker has lent, no one else (including other bankers) can learn this alternative way to run the project.

Because there are more entrepreneurs than households, not all projects are funded. Banks will lend only if entrepreneurs promise to repay the maximum possible for a loan,  $\bar{Y}_2$ , on demand. If the entrepreneur fails to pay on demand, he can make a counteroffer to the bank. If that offer is rejected, the bank takes over the project and either harvests the date 2 cash flow  $\gamma Y_2$  or liquidates the project.<sup>2</sup> All financial contracts

<sup>2</sup> For a relaxation of these assumptions, see Diamond and Rajan (2001). In a more detailed model, we could think of the cash flows being generated over time, with the banker having a sufficient threat each period from his ability to seize assets and redeploy them that the entrepreneur will pay out some fraction of the cash flows that are generated.

are in real terms—we assume that the central bank has achieved a credible commitment to price level stability.

### C. *Financing Banks*

Since bankers have no resources initially, they have to raise them from households. But households have no collection skills, so how do banks commit to repaying households? By issuing deposits of face value  $D$ , repayable on demand. In our previous work (Diamond and Rajan 2001), we argued that the demandable nature of deposit contracts introduces a collective action problem for depositors that makes them run to demand repayment whenever they anticipate that the banker cannot, or will not, pay the promised amount. Because bankers will lose all rents when their bank is run, they will repay the promised amount on deposits whenever they can. Later we will detail plausible frictions that explain why demandable deposits are the optimal contract.

Deposit financing introduces rigidity into the bank's required repayments. Ex ante, this enables the banker to commit to repay if he can (i.e., avoid strategic defaults by passing through whatever he collects to depositors), but it exposes the bank to destructive runs if he truly cannot pay (it makes insolvency more costly): when depositors demand repayment before projects have matured and the bank does not have the means of payment, it will be forced to liquidate projects to get  $X_1$  immediately instead of allowing them to mature and generate  $\gamma Y_2$ .

We assume that each bank lends to enough entrepreneurs that the distribution of  $\tilde{Y}_2$  among the entrepreneurs it finances matches the aggregate distribution of entrepreneurs and there is no aggregate uncertainty about projects. Because the date 0 endowment is scarce relative to projects, banks will compete to offer the most attractive promised deposit payment  $D$  to households per unit of endowment deposited (henceforth, all values will be per unit of endowment). Since this is the face value repaid on a unit of good deposited, it is also the date 0 gross promised deposit interest rate. The time line is shown in figure 1.

## II. Solving the Basic Model

In what follows, we will start by solving the bank's decision vis-à-vis entrepreneurs at date 1, then the households' consumption and withdrawal decisions, and, finally, the bank's date 0 decision on what level of deposit repayment  $D$  to offer to maximize household willingness to deposit.

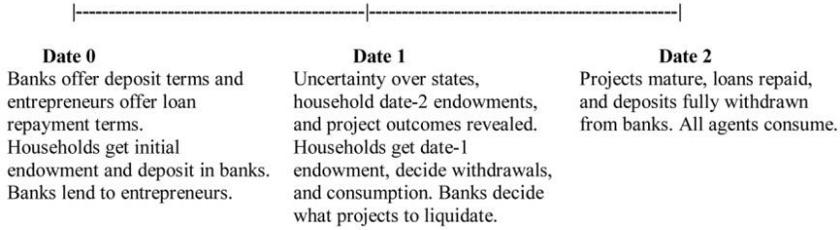


FIG. 1.—Time line

A. *Bank Decisions Vis-à-Vis Entrepreneurs*

Let us start our analysis at date 1, once uncertainty is revealed. Let the gross interest rate (total promised return on date 2 for a unit deposit on date 1) households demand in state  $s$  for redepositing between dates 1 and 2 be  $r_{12}^s$ . Through much of the paper, we will focus on the situation where the date 1 interest rate exceeds the return on storage, but storage is easily handled. The bank can get  $X_1$  at date 1 if the project is liquidated. The maximum it can collect from an entrepreneur with project realization  $Y_2$  is  $\gamma Y_2$  at date 2, and this determines how much the entrepreneur can commit to pay at date 1. So the bank liquidates projects with  $X_1 > \gamma Y_2 / r_{12}^s$ , that is, projects with  $Y_2 < Y_2^s = (r_{12}^s X_1) / \gamma$  and continues projects with  $Y_2 \geq Y_2^s$  in return for a promised payment of  $\gamma Y_2$ . Liquidated entrepreneurs get nothing, while entrepreneurs who are continued retain  $(1 - \gamma)Y_2$ . The present value of the bank's assets at date 1 (before withdrawals) is  $(1 / \bar{Y}_2) \int_0^{\bar{Y}_2} X_1 dY_2 + (1 / \bar{Y}_2) \int_{Y_2^s}^{\bar{Y}_2} (\gamma Y_2 / r_{12}^s) dY_2$ , which is easily shown to fall in  $r_{12}^s$ .

B. *Household Decisions*

Once uncertainty is revealed at date 1, households decide how much they want to withdraw and consume so as to maximize their expected utility of consumption. If they anticipate that the bank will not be able to meet its obligations, they will collectively run on the bank, in which case all projects will be liquidated to pay households. We assume initially that households can coordinate on a Pareto preferred Nash equilibrium and thus rule out other, panic-based, runs (it will turn out that interest rate policy can help eliminate panics). We start by considering situations where the bank will meet its obligations.

When a household does not withdraw all its deposit, its utility is maximized when the marginal rate of substitution between consumption at dates 1 and 2 is equal to the prevailing deposit rate,  $r_{12}^s$ , that is, when  $[U'(C_1)] / [U'(C_2)] = C_2 / C_1 = r_{12}^s$ . If household  $H$  (with high date 2 en-

dowment) withdraws amount  $w_1^{H,s}$  at date 1 then  $C_2^{H,s}/C_1^{H,s} = [e_2^H + (D - w_1^{H,s})r_{12}^s]/[e_1 + w_1^{H,s}]$ . A similar expression can be derived for household  $L$  with low date 2 endowment. For both households to have an equal marginal rate of substitution (and deposit at a common rate  $r_{12}^s$ ), it must be that  $H$  households withdraw more so that  $w_1^{H,s} > w_1^{L,s}$ .

LEMMA 1. If  $r_{12}^s \geq e_2^H/(e_1 - D)$ , both households leave all their money in the bank at date 1. If  $e_2^H/(e_1 - D) > r_{12}^s > e_2^H/(e_1 + D)$ , neither household withdraws fully from the banking system, but the  $H$  household withdraws more than the  $L$  household. If  $e_2^H/(e_1 + D) \geq r_{12}^s > e_2^L/(e_1 + D)$ , the  $H$  household withdraws fully, while the  $L$  household maintains some deposits. If  $e_2^L/(e_1 + D) \geq r_{12}^s$ , both households withdraw fully.

### C. Equilibrium

Assume first that the bank can repay depositors without failing. In equilibrium, markets for goods at date 1 and date 2 have to clear. At date 1, goods are produced when banks liquidate projects. Because all banks have the same distribution of projects and will liquidate projects with  $Y_2 < Y_2^s = r_{12}^s X_1/\gamma$ , date 1 liquidation proceeds are  $1/\bar{Y}_2 \int_0^{\bar{Y}_2} X_1 dY_2 = [r_{12}^s (X_1)^2]/\gamma \bar{Y}_2$ . Because this should equal the goods consumed by withdrawing households, it must be that

$$\theta^s w_1^{H,s} + (1 - \theta^s) w_1^{L,s} = \frac{r_{12}^s (X_1)^2}{\gamma \bar{Y}_2}, \tag{1}$$

where  $\theta^s$  is the fraction of  $H$  households in state  $s$ . An equilibrium at date 1 in state  $s$  is an interest rate  $r_{12}^s$  and withdrawals by the  $H$  and  $L$  households,  $w_1^{H,s}, w_1^{L,s}$  such that the date 1 supply of goods equals the date 1 consumption, banks liquidate enough projects to meet withdrawals, and households do not want to, or cannot, withdraw more.

#### THEOREM 1.

- i. If it exists, the equilibrium is unique.
- ii. If there is a set  $\{r_{12}^s, w_1^{H,s}, w_1^{L,s}\}$  that solves  $e_2^H/(e_1 + 2w_1^{H,s} - D) = r_{12}^s$ ,  $e_2^L/(e_1 + 2w_1^{L,s} - D) = r_{12}^s$ , and (1), with  $r_{12}^s >$  return on storage,  $w_1^{H,s} \in [0, D)$ ,  $w_1^{L,s} < D$ , then that is the equilibrium, else if there are  $r_{12}^s, w_1^{H,s} = D$ , and  $w_1^{L,s}$  that solve  $e_2^L/(e_1 + 2w_1^{L,s} - D) = r_{12}^s$ , and (1), with  $r_{12}^s >$  return on storage and  $w_1^{L,s} \in [0, D)$ , then  $\{r_{12}^s, D, w_1^{L,s}\}$  is the equilibrium, else  $\{(\gamma D \bar{Y}_2)/(X_1)^2, D, D\}$  is the equilibrium.<sup>3</sup>

<sup>3</sup> Note that we have assumed that the household marginal rates of substitution are not so low that they would be below the return on storage. Were that to be the case, we would have to examine the storage return as a candidate, with any excess deposits being stored by the bank. The three equations would now solve for the withdrawal by high households, the withdrawal by low households, and the amount invested by banks in the storage technology.

*Proof.* See Appendix.

COROLLARY 1.

- i. The interest rate  $r_{12}^s$ , total withdrawals,  $\theta^s w_1^{H,s} + (1 - \theta^s) w_1^{L,s}$ , and the fraction of projects liquidated all (weakly) increase in the fraction  $\theta^s$  of high endowment households and in the face value of deposits,  $D$ .
- ii. The net worth of banks (date 1 value of assets less deposits) decreases in the fraction  $\theta^s$  of high endowment households and in the face value of deposits,  $D$ .

*Proof.* See Appendix.

Since  $H$  households have more date 2 endowment than  $L$  households, at any given interest rate they will consume (weakly) more at date 1 and, hence, will withdraw more. This means that total withdrawals will (weakly) increase in the fraction  $\theta^s$  of high endowment households, which means that the interest rate and liquidation will have to increase to equilibrate the market. Because the present value of the bank's assets decreases with  $r_{12}^s$  while the value of its deposit liabilities does not, its net worth decreases with  $\theta^s$ . Note that all these implications would also hold if households differed, not in their date 2 endowments, but in their date 1 endowments, with  $H$  households receiving a lower date 1 endowment (facing poor current conditions) and, again, expecting higher growth in marginal utility. Much of past analysis has followed Diamond and Dybvig (1983) and focused on adverse liquidity shocks that are tantamount to poor current conditions, but it is useful to remember that because consumption and interest rates depend on anticipated consumption growth, exuberant views of the future could be equally problematic from the perspective of demands for liquidity. We will focus on this latter aspect through the paper.

*D. Bank Fragility and the Ex Ante Choice of Deposit Rate*

Let us now examine each bank's choice at date 0 of the promised payment  $D$  to offer for a deposit of a unit of good. Since the market is competitive and household endowment is scarce relative to entrepreneurs' projects, price-taking banks will have to offer a  $D$  that maximizes household utility, given the properly anticipated future interest rates and actions of households, banks, and entrepreneurs.

So long as the bank is not run in any state, a higher  $D$  makes households wealthier and better off. But the bank's net worth also falls by corollary 1, and for a given  $D$ , is lower in a state with higher  $\theta^s$ . When  $D$  is high enough that the bank's net worth is completely eroded in a state, the bank is run in that state—which means all projects are liquidated to generate funds to pay depositors, even though at the prevailing interest rate, some deserve to be continued. Runs are costly precisely because

depositors withdraw money independent of consumption needs or the prevailing rate. Because a depositor's place in line is random, each running household gets  $D$  with probability  $X_1/D$  and 0 with probability  $1 - (X_1/D)$ .

Let  $D^{\text{Max}}(s)$  be the promised payment above which a run will be precipitated in state  $s$ . From our discussion above,  $D^{\text{Max}}$  falls in  $s$ . If deposit repayments could be state contingent, then at date 0, the bank would offer to pay  $D^{\text{Max}}(s)$  in state  $s$  at date 1. But if state-contingent demandable deposit contracts are not possible (for reasons explained in Sec. III), promised payments will have to be fixed at date 0.

At date 0, the bank will choose the fixed promised payment to be among the set of  $D^{\text{Max}}(s)$  where  $s \in \{\underline{s}, \dots, \bar{s}\}$ .<sup>4</sup> If the promised payment is optimally set at  $D^{\text{Max}}(s^*)$ , it will be because the bank sees a net loss in household expected utility from having a run in state  $s^*$  (in addition to the run in all states  $s > s^*$ ) by setting the promised payment higher at  $D = D^{\text{Max}}(s^* - 1)$ , even though expected payments to households will be higher in all states  $s < s^*$ . This then is the basic trade-off banks face in setting  $D$  higher—commit to paying more in the less optimistic, lower-aggregate-consumption-growth states but have costly runs in more optimistic states. The higher the probability of state  $s$ , and the more costly the run, the less the desire to push  $D$  higher from  $D^{\text{Max}}(s)$  to  $D^{\text{Max}}(s - 1)$ . By contrast, the higher the probability of less optimistic states,  $\sum_{s=1}^{s^*-1} p_s$ , and the higher the increase in payout in those states,  $D^{\text{Max}}(s - 1) - D^{\text{Max}}(s)$ , the greater the incentive to push  $D$  higher. Let  $D^{\text{Max}}(s^*)$  be the promised deposit payout that maximizes ex ante household utility. We have lemma 2.

**LEMMA 2.** Consider an increase in the probability of state  $s^* + k$  accompanied by an equal decrease in the probability of state  $s^* - j + k$ , where  $j$  and  $k$  are both positive integers and  $j > k$ —that is, a rightward shift in probabilities from a state less optimistic than  $s^*$  to one more optimistic. Expected household utility from lower promised deposit payments  $\{D^{\text{Max}}(s^* + 1) \dots, D^{\text{Max}}(\bar{s})\}$  increases relative to expected utility from the old optimal  $D^{\text{Max}}(s^*)$ , while expected utility from higher promised deposit payments,  $\{D^{\text{Max}}(s^* + k - j) \dots, D^{\text{Max}}(s^* - 1)\}$ , decreases. If the probability of states  $\{\underline{s}, \dots, s^* + k - j - 1\}$  is small, the new bank-determined optimal promised payment will (weakly) decrease. Similarly, a leftward shift in probabilities will lead promises to (weakly) increase.

*Proof.* See Appendix.

The lemma suggests that higher probability weights on high consumption growth states, where liquidity is expected to be tight and

<sup>4</sup> Suppose not and the bank set the deposit rate at  $\hat{D}$  where  $D^{\text{Max}}(s + 1) < \hat{D} < D^{\text{Max}}(s)$  for two adjacent states. By increasing  $\hat{D}$  to  $D^{\text{Max}}(s)$ , the bank will not increase the probability of runs but will pay households more. Clearly,  $D^{\text{Max}}(s)$  dominates  $\hat{D}$ ;  $D^{\text{Max}}(1)$  will always be preferred to  $\hat{D} > D^{\text{Max}}(1)$ .

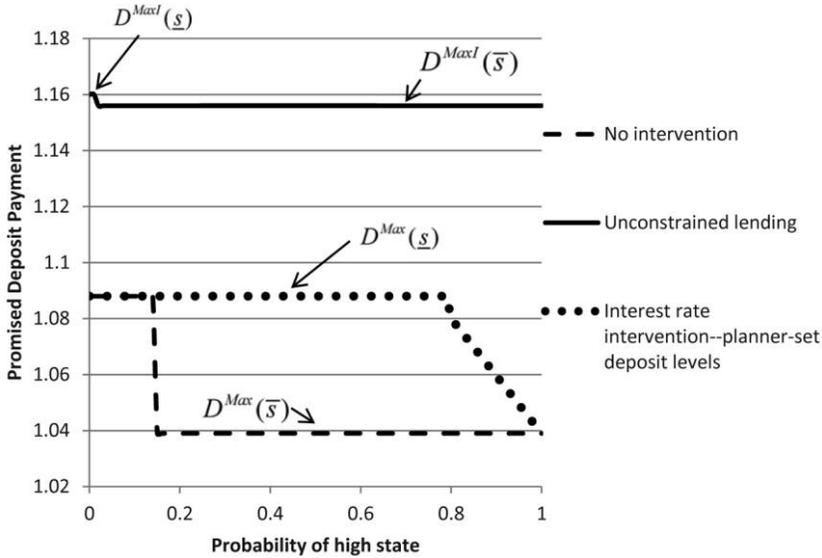


FIG. 2.—Deposit promises for different intervention regimes

interest rates high, will tend to push optimal promised payments lower. The implication is that in the absence of intervention by the authorities, banks will be more cautious about promising to supply liquidity when liquidity is scarce and the interest rate is likely to increase. Promised deposit payments are not procyclical because competition forces banks to internalize the cost of runs to their depositors.

E. Example with Comparative Statics

Consider an economy with two future states, low ( $\underline{s}$ ) and high ( $\bar{s}$ ). Let  $e_1 = 0.65$ ,  $e_2^L = 0.8$ ,  $e_2^H = 2.5$ ,  $X_1 = 0.95$ ,  $\underline{Y}_2 = 0.0$ ,  $\bar{Y}_2 = 3.5$ ,  $\gamma = 0.9$ ,  $\theta^{\bar{s}} = 0.6$ , and  $\theta^{\underline{s}} = 0.3$ . Let  $D$  be set at the maximum deposit face value with no run in either state,  $D^{\text{Max}}(\bar{s})$ , which is 1.039. In the  $\bar{s}$  state, the interest rate  $r_{12}^{\bar{s}}$  is 2.15,  $H$  households withdraw 0.77, while  $L$  households withdraw 0.38. In the  $\underline{s}$  state, the interest rate  $r_{12}^{\underline{s}}$  is 1.89, the  $H$  households withdraw 0.85, while  $L$  households withdraw 0.41. Now set  $D$  higher at  $D^{\text{Max}}(\underline{s}) = 1.088$ . The bank is run in the  $\bar{s}$  state, while in the  $\underline{s}$  state, the interest rate is 1.94, the  $H$  households withdraws 0.86 while the  $L$  households withdraw 0.43.

In figure 2, we plot the optimal ex ante choice of  $D$  (dashed line) for different ex ante probabilities  $p^{\bar{s}}$ . In this “no-intervention” case, the bank will set  $D$  high at 1.088 until the probability of the high state  $\bar{s}$  exceeds 0.14, at which point households are better off having safe banks

with  $D$  set lower at 1.039. Note from figure 4 that the expected utility of producers (the banks and entrepreneurs) shifts up substantially when the bank moves to setting a lower safe face value of deposits: the banker gets no rents when promised deposit payments are set at  $D^{\text{Max}}(\underline{s})$ , and many projects are liquidated at the high prevailing interest rate reducing the residual value the entrepreneurs expect. But banker cum entrepreneur expected utility jumps up when the promised payment is set lower at  $D^{\text{Max}}(\bar{s})$ , both because the banker is not forced to pay everything he collects in the  $\underline{s}$  state and because interest rates are lower so fewer projects are liquidated. Expected producer rents then decline as the probability of state  $\bar{s}$  increases.

In sum, then, in a competitive environment, the banking system can lever up to the point where it will fail with some probability when a significant fraction of households become exuberant about the future (or pessimistic about the present). Exuberance creates more pressure for current consumption, which the economy may be too illiquid to provide. Consequently, real interest rates rise to restore equilibrium, and projects are curtailed. The more levered the banking system is to begin with, the more projects are liquidated, and higher the likelihood of systemic bank failures.

### III. Optimal Contracts

Our analysis thus far takes demand deposits as given. Demand deposits allow depositors a choice of state-contingent amounts to withdraw, but in a way that could exacerbate banking system stress—deposits are firm promises of liquidity that are extremely costly to deliver when the overall demand for liquidity is high. Could the bank (or any other institution in the economy) offer contracts with better properties than deposit contracts? This section lists a set of frictions where demand deposit contracts are part of an optimal mechanism to implement constrained optimal allocations—the bank cannot do any better.

We begin with an environment with very few frictions, which is equivalent to an Arrow-Debreu complete markets allocation. The optimal full-information and full-commitment state- and household-type contingent contract would equate the marginal utilities of the households at each date and equate their marginal rates of substitution to the marginal rate of transformation between date 1 goods and date 2 goods. We would obtain a “first best” allocation (apart from the distortions caused by the entrepreneur’s inability to fully pledge his output). However, as we show in detail in the Appendix, three key frictions—that bankers need to commit to collect loan payments, that it is impossible to make the face value of demand deposits state contingent, and that trade between depositors or between depositors and other banks cannot be restricted—

ensure that there are no feasible contracts with better outcomes than the demandable deposit contract.

Specifically, following Diamond and Rajan (2001) and the logic in Hart and Moore (1994), consider what happens if the banker borrows long-term debt (or any other contract which can be renegotiated) rather than demand deposits. Following these papers, assume that the banker can make a credible threat to not collect the loans unless every lender to the bank makes concessions. Any longer term debt contract will be negotiated down to the debt holders' outside option—in this case, the liquidation value of project loans  $X_1$ . This limits how much the banker can commit to pay, making it impossible for him to borrow more than liquidation (collateral) value of loans. However, when the banker has issued demandable claims and then tries to renegotiate depositors down, because deposit claims are paid first come, first served, each depositor sees their outside option as running to get the face value of their claim whenever other depositors accept a reduced payment and do not run. Any attempt by the banker to renegotiate deposits will set off a run, lead to full liquidation of loans, and destroy the banker's residual claim. Demandable deposits thus allow the banker to pledge his collection skills to unskilled depositors, committing him to pay out  $D$ , which is more than the underlying collateral value of the claim  $X_1$ .<sup>5</sup>

Interestingly, demandability by itself does not constrain the set of feasible allocations—all that is required for the first best allocation for each household to be renegotiation proof is that at date 1, the present value of the optimal allocation should be equal the state-contingent demandable claim for that household type. The household can then achieve its optimal allocation by withdrawing what it needs for consumption at date 1 and the remainder at date 2 (Hellwig [1994] has a related result).

Demandability, however, can prevent the value of deposits from being state contingent. Deposit claims have to be demandable at every instant after contracting; otherwise households can be renegotiated down. Therefore, the initial promised payment on the deposit has to be a state-independent value, set *ex ante* at a level that makes households indifferent between taking out the deposit immediately (and potentially re-depositing elsewhere) and waiting for the realization of the state to redefine their face value. This means that the face value will sometimes

<sup>5</sup> We could also think of an alternative model where the banker can undertake actions that put assets or cash flows outside the reach of depositors, as in Calomiris and Kahn (1991) and Diamond (2004). The need to stop "crimes in progress" then forces deposits to be short term. Demandability and the first come, first served constraint gives depositors added incentive to monitor the bank. The key difference between Calomiris and Kahn (1991) and Diamond and Rajan (2001) is that the latter require demandability as a way to limit strategic default (also see Bolton and Scharfstein 1996), which enhances the bank's ability to borrow beyond the underlying collateral value of assets.

have to be reduced given the state.<sup>6</sup> In such states, the bank and the authorities enforcing claims would have to see the state and adjust the face value down before the households can withdraw, otherwise each household, anticipating a reduction with certainty, has an incentive to withdraw before it happens and collect the higher initial face value. If it takes a small amount of time to alter  $D$ , and a significant number of depositors can react faster than that, state-contingent deposits will precipitate the very runs they seek to avoid. We assume that the speed of face value adjustment is sufficiently slow that any state contingency in the face value of deposits is undesirable.

Could household claims be directly contingent on household type? Again, not if the type is private information and unverifiable, as we assume. The bank can offer households a menu of date 1 and date 2 payments to try to get them to self-select. However, because we assume that each household has access to the capital market (they can trade deposits with each other, as in Jacklin [1987], or trade with other banks, and this trade cannot be excluded by contract), all they care about when choosing a stream of payments is the present value of the claims they are offered, discounted at the market interest rate. Given a choice, households will pick the highest present value claim and transform it to their preferred payment stream. This makes it impossible to use their (possibly) different marginal rates of substitution to get them to self-select streams with different present values.

In sum then, the limited commitment by the banker, slow adjustment to the state, and the unverifiability of household types all imply that the demand deposit contract is the constrained optimal contract in the absence of intervention by a social planner with powers not possessed by the private sector.

#### IV. Macroeconomic Intervention

We have just seen that demand deposits entailing runs are the constrained efficient contract. We now examine how a government or planner that has the ability to tax household endowments can help (or hurt), taking as given the form of the deposit contract, although allowing the bank to change its promised payments in anticipation of intervention.

<sup>6</sup> If the deposit face value is set at the lowest possible desired future state-contingent present value, all revaluations would be up. But in that case, the banker would be able to negotiate depositors down to a state-contingent contract that promises a utility equivalent to the utility depositors get from the lowest state-contingent face value. To promise utility higher than this, the banker would have to set the initial face value higher than the lowest future state-contingent value (the state-contingent contract is renegotiation proof only if the utility the depositor obtains from withdrawing immediately equals the expected utility from the state-contingent payments). The face value will therefore have to be reduced in some situations.

*A. The Purpose of Intervention*

Assume initially that the planner (perhaps elected by households) wants to maximize expected household utility, subject to meeting a minimum threshold utility for the producers (the banks and entrepreneurs). If that threshold is low, the planner can be termed *household friendly*, which is the case we will focus on in much of this section—the detailed planner problem is laid out in the Appendix. Later, we will examine what happens when the planner might become *producer friendly*—when she maximizes the expected utility of producers subject to a low minimum threshold utility for households. In either case, we will assume that the planner wants to avoid the indiscriminate liquidation of entrepreneurs—the characteristic of a run—and therefore always wants to prevent runs *ex post*.

The bank “works” thus far because the rigidity of demand deposits commits the banker *ex ante* to pay out. However, if a social planner intervenes in a specific bank when depositors run because the bank is insolvent, she will be unable to commit to not intervene when they run because the banker is being opportunistic. Indeed, the banker is disciplined by the collective action problem inherent in demand deposits (i.e., no depositor internalizes the destruction in value, hence the threat of a run is credible), which will be undone because the social planner will internalize the externalities that depositors ignore. The bank will no longer be able to promise more than the collateral value of its assets, and the banker will collect rents *ex post*. In addition, *ex ante*, the banker will be pressured by competition to offset the effects of the anticipated *ex post* intervention. He will increase the bank’s commitments by promising to pay depositors even more, making everyone worse off.

This is why it is helpful to constrain the extent to which the planner can intervene *ex post*, for example, by limiting him to making loans only to solvent banks. Indeed, to limit intervention to the minimum needed, this constraint will need to be accompanied by small (nonpecuniary) penalties imposed on borrowings by banks from the planner. We now elaborate.

*B. Bailouts by the Social Planner without Commitment*

We assume the following:

- i. The planner knows the date 1 state of the world (perhaps after the passage of time) and can alter its actions accordingly. The planner cannot bind himself to a state-contingent plan, however, and thus has no advantage in this respect over private contracts.

- ii. The planner has no ability to collect cash flow from the entrepreneurs or distinguish between high  $Y$  and low  $Y$  entrepreneurs. She cannot observe household types and cannot tax them differentially based on their realized endowments.
- iii. The planner has no resources other than what can be obtained from households or banks.
- iv. Households and banks anticipate the actions of the planner and act accordingly.<sup>7</sup>

Knowing only what is public information, the planner can levy a tax  $\tau_1^s$  on household endowments at date  $t$  in state  $s$ , where  $\tau_1^s \leq e_1$  and  $\tau_2^s \leq e_2^L$ , and transfer these resources to the bank. Such a *direct bailout* would bolster the bank's net worth and could prevent a run. Unfortunately, though, the social planner cannot commit to intervene only in the states where the bank is actually insolvent given prevailing interest rates. Anticipating the willingness of the social planner to prevent runs, the banker could also threaten to not collect on loans even when the bank is solvent (a strategic default) unless the planner infuses funds into the bank. Demand deposits no longer serve to discipline the banker, for knowing that the social planner will do anything to prevent runs, depositors will not run—the planner can be relied on to internalize the externality depositors ignored. This means, however, that households will be taxed to pay for what the banker extracts.

How much will the banker extract? First suppose the social planner can tax only at date 1. The social planner will internalize not just the utility loss to depositors from a run but the entire loss in value of projects that should be continued at the prevailing interest rate. If the value of saving them is high, as we have assumed, the banker can extract everything the social planner can obtain in taxes from households. The banker will demand and obtain  $e_1$  at date 1.<sup>8</sup> In states where the bank is insolvent without intervention, some of that  $e_1$  will go to depositors. In states where the bank is solvent, all the  $e_1$  will go to the banker.

Knowing that he will get a direct additional rent in the future, each banker will promise it away in the ex ante competition for deposits. If  $D$  is the optimal level of deposits when there is no expectation of in-

<sup>7</sup> The planner's taxation authority plays a similar role in our model as the deposit insurance in Diamond and Dybvig (1983) and the government in Holmstrom and Tirole (1998) by making available for contracting household endowments that are inaccessible to the banking sector. However, our focus is very different from these models where the government provides new sources of collateral or explicit transfers of value. While the planner can alter household consumption decisions and influence interest rates in our model, it can also undo ex post private commitment and thereby alter ex ante decisions.

<sup>8</sup> One can consider cases where the banker's threat is less potent, and he extracts less than the taxation capacity of the social planner. This will require additional assumptions on relative bargaining power and outside options, and we do not pursue it further in this paper.

tervention,  $D' = D + e_1$  is the optimal level of deposits once the social planner is expected to intervene—the banker pays for the rents he expects to get by increasing the face value of promises to depositors. As a result, outcomes with lump sum taxes are exactly the same with intervention as without—indeed, now ex ante promised payments are so high that even after the maximum intervention, the social planner cannot save the banks. The bank is run in the same states as it was without intervention; the social planner taxes households and transfers to banks but the households withdraw an equivalent additional amount from their higher deposit holdings, so net withdrawals from the bank in each state do not change, nor does household consumption.<sup>9</sup>

If the banker can extract more from the planner in high  $\theta$  states than in low  $\theta$  states, then his postintervention net worth is more positively correlated with optimistic states than his net worth in the absence of intervention. Households are better off because the banker will promise them more ex ante (net of taxes) and/or fail in fewer states. The reverse is true if the banker can extract less in high  $\theta$  states than in low  $\theta$  states.

This suggests what happens when the planner can tax household endowments at both date 1 and date 2. She can give the bank  $e_1$  at date 1 and guarantee the bank  $e_2^L$  at date 2 (a guaranteed capital injection). The date 2 guaranteed inflow, however, has a present value that falls in high  $\theta$  states where the market clearing  $r_{12}^s$  is higher. The cum-transfer net worth of the bank relative to the no-intervention case now falls with  $\theta$ . So households are worse off because they are either promised less (net of taxes) than in the no-intervention case and/or the bank defaults more.

**THEOREM 2.** If the social planner can tax households and transfer to banks, limited only by the extent of the  $L$  household's endowments, households will be worse off than without intervention.

If the social planner is anticipated to intervene maximally ex post, ex ante promises by the banker do not affect the degree of intervention. A cap on  $D$  would then only preserve rents for bankers and make households worse off. Even if the social planner were to set  $D$ , anticipating her own intervention ex post she would favor the higher  $D$ .

<sup>9</sup> If the social planner cannot save a bank because its liabilities are too great given the planner's ability to tax and transfer, she will not have an ex post incentive to intervene. Intervention will not change the expected net flows that go to households. However, the distribution of run proceeds to households will be affected. If the authorities do intervene in a run (by providing all of  $e_1$  to banks) but cannot prevent it, households who do not succeed in withdrawing during the run will be left with zero consumption at date 1. This is a terrible outcome when households have log utility (or any preferences where the marginal utility of zero consumption is infinite).

C. *Arm's Length Central Bank Lending to Banks*

Could constraints on the social planner's ability to intervene ex post improve matters? One alternative is to allow the social planner to only make loans, not grants, with tax proceeds and only at a market interest rate (the equilibrium deposit rate) to banks that will be solvent at those rates. This could be accomplished by requiring that lending be contingent on simultaneous voluntary lending by households. We will see that small direct loans may be ineffective, but that larger loans can lower economy-wide interest rates and effectively bring insolvent (at the higher no-intervention rate) banks back to solvency. However, while arm's length lending can eliminate direct holdup by banks, banks can still obtain rents if the planner is anticipated to intervene heavily, so we may need to constrain the planner even more.

Specifically, suppose now the planner, henceforth called the central bank, borrows  $b_1^s$  (possibly forcibly) from household endowments at date 1 and lends the proceeds directly to the bank. It coerces repayment from the banker at date 2 and gives it to the households (or equivalently, distributes demand claims on banks of face value  $r_{12}^s b_1^s$  back to households at date 1). It is easily shown that a small intervention may have no effect on interest rates or bank solvency when neither household has fully withdrawn its deposits at date 1. Intuitively, because household wealth remains unchanged, households have no reason to change their consumption if they can withdraw an additional amount equivalent to the amount they are forced to lend out of endowment today. With unchanged consumption, the interest rate does not change nor does the amount the bank has to liquidate since the additional loan it receives from the central bank is completely exhausted by additional withdrawals. In short, because households have access to the capital market (through their deposits), household choices perfectly offset central bank actions, a form of Ricardian equivalence (see Barro 1974).

Central bank lending need not be neutral. Suppose that  $H$  households withdraw their entire deposit at date 1 even if their endowment is not taxed, or suppose that the central bank taxes a larger amount so that even if  $H$  households withdraw their deposits fully, they cannot compensate for the lost date 1 endowment. In either case,  $b_1^s > D - w_1^{H,S}$ . This means that  $H$  households' date 1 consumption will fall by  $b_1^s - (D - w_1^{H,S})$  relative to the no-intervention case, and their marginal rate of substitution will go up. But these households are on a corner: they no longer choose to hold deposits in the banking system and cannot borrow against their future endowment, so their marginal rate of substitution has no effect on the interest rate. The  $L$  households do hold deposits and affect the interest rate. To clear the market, their date 1 consumption will have to go up to absorb some of the consumption

given up by the  $H$  households. In the new equilibrium, their marginal rate of substitution falls, the interest rate falls, and  $L$  households do not make up entirely for the fall in consumption of  $H$  households.<sup>10</sup> Overall date 1 consumption falls, and the required liquidation to meet consumption needs falls, and bank net worth rises.

LEMMA 3. (i) If the prevailing no-intervention equilibrium has  $w_1^{H,S} < D$ , then a loan to the bank, raised from households, of  $b_1^s \leq D - w_1^{H,S}$  has no effect on the interest rate, on consumption, or bank net worth. (ii) A loan to the bank of  $b_1^s > D - w_1^{H,S}$ , raised by taxing households, will reduce the interest rate the bank faces, increasing bank net worth and reducing project liquidation.

The central bank in our model affects equilibrium rates, and hence bank solvency, because some of the participants who are taxed and forced to lend would not choose to lend in the capital market at market interest rates. Indeed,  $H$  households would like to withdraw more to offset the tax at prevailing interest rates, but they cannot once they have withdrawn their entire deposit. Compensation at market interest rates at date 2 does not fully make up their loss (because their marginal rate of substitution is higher). The  $L$  households that still participate in the market find an altered investment opportunity set—lower consumption by  $H$  households implies that projects with lower rates of return can be continued. A lower putative interest rate (than the earlier no-intervention equilibrium) gives  $L$  households the incentive to consume more, reduces their marginal rate of substitution, and allows the new equilibrium to establish at a lower interest rate than before. In a richer model where there is a continuum of types, the effects of intervention will typically be smoother, with a set of types on a corner and not lending voluntarily at market rates, and policies to affect real rates always having traction.

Intervention to lower rates makes all households worse off, unless the intervention is necessary to stop a run;  $H$  households (who have withdrawn all deposits in the range where the central bank can affect interest rates) are forced to consume less at date 1. Even though they are compensated for this at date 2, it is at an interest rate that is even lower than their already high marginal rate of substitution, so they are worse off on net. The  $L$  households use the financial system to save. For them, a reduction in interest rates reduces their investment opportunity set, making them worse off. Indeed it is easy to show the following.

LEMMA 4. Households are better off in state  $s$  if the deposit level is

<sup>10</sup> There are limits to how much effect planner lending can have. First, the planner cannot forcibly borrow more than the endowment,  $e_i$ . Second, once it pushes gross interest rates down to the storage return, it cannot push them down any further since the banks will simply store any funds they obtain and not pass on lower rates to borrowers.

set at  $D^{\text{Max}}(s)$  and the central bank does not intervene than if it is set above  $D^{\text{Max}}(s)$  and the central bank reduces rates to restore solvency.

*Proof.* See Appendix.

*D. Interest Rate Equilibria and Interest Rate Intervention*

Unfortunately, even under arm's length lending, the household-friendly central bank may be coerced by market expectations to lower the interest rate below the level that makes the banks just solvent, leaving rents for the banker at the expense of households. This is because interest rates are endogenous and depend on the extent to which the central bank is anticipated to intervene. Specifically, suppose all believe that the central bank will tax and lend the entire date 1 household endowment even if this is more than needed to restore solvency. The deposit market will then clear at interest rates that are the lowest feasible given anticipated central bank intervention, and the central bank will have to fulfill expectations or see banks run. Each banker will enjoy rents (because the bank is more than solvent at that low rate).

Put differently, if excess intervention is anticipated (multiple equilibria are now possible based on the extent to which intervention is anticipated), households will lend at a commensurate low rate so long as the central bank is expected to lend the necessary quantity to make that rate the equilibrium. Each banker has an implicit threat to suffer a run by not attracting sufficient deposits to meet withdrawals if the central bank does not come through.<sup>11</sup> The central bank's rule that it lends only at the market rate to solvent banks will not suffice to commit it to be household friendly. The banker gets a rent that rises as the equilibrium interest rate falls, even though bankers, who are price takers in the deposit market, make no explicit threats.

If, however, the central bank introduces a small nonpecuniary cost,

<sup>11</sup> Since  $b_1 \leq e_1$ , the lowest feasible interest rate is the maximum of the storage rate and the interest that would prevail without such a floor and with  $b_1 = e_1$ . The latter interest rate is obtained by solving

$$\frac{e_2^L + b_1^L r_{12}^{s,i} + (D - w_1^{L,s,i}) r_{12}^{s,i}}{e_1 - b_1^L + w_1^{L,s,i}} = r_{12}^{s,i}$$

and

$$\theta^S D + (1 - \theta^S) w_1^{L,s,i} = \frac{r_{12}^{s,i} (X_1)^2}{\gamma Y_2} + b_1^L$$

for  $\{r_{12}^{s,i}, w_1^{L,s,i}\}$  if there is a solution with  $w_1^{L,s,i} < D$  or solving

$$D = \frac{r_{12}^{s,i} (X_1)^2}{\gamma Y_2} + b_1^L$$

for  $r_{12}^{s,i}$  if there is not. In the first case, the social planner's intervention does not force the interest rate so low that the  $L$  household withdraws totally from the system; in the latter case, it does.

$\delta$ , of borrowing every additional dollar from it, over and above the market-established deposit rate, the unique ex post equilibrium will be the rate preferred by the household-friendly central bank.<sup>12</sup> Here is why.

Banks are price takers in the deposit market, and given the discrete additional nonpecuniary cost of borrowing from the central bank, will always prefer to exhaust the deposit market before turning to the central bank. So if banks are solvent without intervention, the equilibrium interest rate bid for deposits will be the no-intervention market clearing rate, and no banks will borrow from the central bank. Intuitively, if the equilibrium deposit rate were any lower, thus requiring some borrowing from the central bank, a banker would have the incentive to deviate and bid the deposit rate up rather than pay the higher all-in cost for central bank loans.

Following similar logic, the equilibrium deposit rate when banks are insolvent in the absence of intervention is the rate at which they are just solvent, and each bank will borrow just enough from the central bank to bridge the gap in its funding needs. Even though banks have to borrow from the central bank and incur the nonpecuniary cost, no bank can substitute for central bank funds by raising the rate it pays depositors, for it will become insolvent and not attract any deposits.

LEMMA 5. Let solvent banks be able to borrow from the central bank, with bankers paying a nonpecuniary markup  $\delta$  to the private market deposit rate. If banks are solvent at the no-intervention rate, the interest rate that will be bid on deposits will be the no-intervention equilibrium interest rate and no funds will be borrowed from the central bank. If banks are insolvent at the no-intervention rate and the central bank can borrow and lend endowments to lower interest rates enough to rescue them, the interest rate that will be bid on deposits will be the maximum rate that renders banks just solvent, and bankers will enjoy no rents.

### *E. Discussion of Ex Post Intervention*

We saw that the key to intervening without undoing the private commitment inherent in demand deposits is to require that the central bank only make arm's length loans, not grants, to solvent banks. This however, can result in a multiplicity of equilibria, some of which result in excess intervention and additional rents for the banks. The central bank can, however, eliminate multiplicity and ensure that the unique equilibrium is the household-friendly one where it lowers rates only if the banks are insolvent and, even so, only to the point at which banks are just solvent.

<sup>12</sup> Central banks typically frown upon bankers who borrow from its various facilities. The cost  $\delta$  could be thought of as a loss of banker reputation or an impairment to the banker's future career opportunities.

It does this by creating a small additional nonpecuniary cost for bankers of borrowing every additional dollar from the central bank. This ensures that banks exhaust the private deposit market before they turn to the central bank.

Bagehot's dictum to central banks, to lend freely but at a penal rate—which presumably means lending at a pecuniary markup to the market rate—achieves only some of what our nonpecuniary penalty achieves. If the system is solvent without central bank intervention, banks will indeed bid the deposit rate up to the no-intervention equilibrium rate, for the reasons described above. However, if the system is insolvent without intervention, there will be no equilibrium in which the central bank can restore solvency. Here is why: suppose there is one. This will entail some bank borrowing from the central bank. But any bank will have an incentive to deviate, pay a little more in the deposit market, and substitute entirely for central bank borrowing. This means that the deposit rate will be driven up to the point where a bank will be just solvent if it can get sufficient funds from the deposit market at that rate to meet its needs. But if this is the equilibrium deposit rate, there will be insufficient funds in the private deposit market at that rate for all banks, so banks will have to borrow some funds from the central bank at the penal rate. This, however, will render them insolvent.<sup>13</sup> Arm's length lending by the central bank with a nonpecuniary penalty results in effective interest rate intervention, allowing it to implement the ex post socially optimal household-friendly policy.

#### *F. Ex Ante Promises*

Let us turn finally to how anticipation of interest rate intervention will affect date 0 promises. If the banker knows that the central bank will drive rates lower, he has an incentive to set promised payments higher to attract depositors. There is an externality here. Each banker sets deposit rates considering only the benefits their depositors will get from higher promised payments—equally, in choosing a higher offered payment, depositors ignore the effect of the increased promise on the loans they, as taxpayers, will have to provide to the bank in the future, because

<sup>13</sup> Intuitively, the wedge between the central bank lending rate and the deposit market rate creates the incentive and ability for a bank to deviate from any equilibrium interest rate (where it is just solvent borrowing a blend of funds from the central bank and the deposit market) and still remain solvent. When the penalty from central bank borrowing is nonpecuniary, however, there is no wedge between the central bank rate and the deposit market rate, and no bank has an incentive to deviate (by paying a higher deposit rate) from an equilibrium where it is just solvent borrowing a blend of funds, because by doing so, it will render itself insolvent and attract no deposits. We conjecture that richer models of monopolistic competition for deposits can produce equilibria with Bagehot-like pecuniary penalties, although this is left for future research.

loans to a specific bank are small when averaged across all taxpayers. Of course, when every bank does this, the cost to the taxpayer, aggregated across all banks, is high.

Let  $D^{\text{Max}^l}(s)$  be the maximum deposit promises the bank can service at date 1 without a run in state  $s$  if the central bank is anticipated to reduce the rate to the lowest possible level. Clearly  $D^{\text{Max}^l}(s) \geq D^{\text{Max}}(s)$ . The promised deposit payment depends on how the central bank determines date 1 rates. One possibility is that the central bank sees every bank as too important to fail. If so, the unique equilibrium is that every bank will choose the same  $D^{\text{Max}^l}(s^*) \in \{D^{\text{Max}^l}(\bar{s}), \dots, D^{\text{Max}^l}(s)\}$  to maximize expected household utility, taking equilibrium central bank intervention as given. The central bank will not be able to prevent runs for  $s > s^*$ , the central bank will intervene maximally to just keep the system solvent if  $s = s^*$ , while it will intervene less to keep the system solvent (or not intervene at all if the system is solvent) if  $s < s^*$ .

Another possibility is that the central bank will intervene to lower rates only if sufficient banks pick deposit levels that would render them insolvent if interest rates are not reduced (a “too many to fail,” or strategic complementarity response). In that case, multiple equilibria are possible for promised deposit payments, one of which is the too important to fail level. This possibility is also emphasized in both Acharya and Yorulmazer (2007) and Farhi and Tirole (2012).

In contrast to the situation with ex post bailouts, the ex ante socially optimal promised deposit payment, anticipating interest rate intervention, is (weakly) lower than what would be set by banks if they know they are too important to fail. The central bank not only internalizes the cost to households of ex post intervention but also, because the extent of the intervention varies with the promised deposit payment, it would set deposit promises lower than would the market. Indeed, following lemma 4, the central bank would never set promised payments outside the interval  $[D^{\text{Max}}(\bar{s}), D^{\text{Max}}(s)]$ , though because runs are averted, it would set it (weakly) higher than the no-intervention level.

### G. Example (Continued)

First, consider the effects of central bank arm’s length lending with no penalty (this case is termed “unconstrained lending” in the figures). Market expectations can now drive outcomes. If the central bank is expected to borrow  $e_1$  and lend to the maximum extent possible to banks, competitive banks will promise even more at date 0. If the probability of state  $\bar{s}$  is below 0.017 (see the solid line in fig. 2) banks choose  $D = D^{\text{Max}^l}(\bar{s}) = 1.16$ , which ensures that even the authorities cannot tax and lend enough to save the banking system in the state  $\bar{s}$ . Clearly, as figure 3 suggests, the households are worse off (than with no interven-

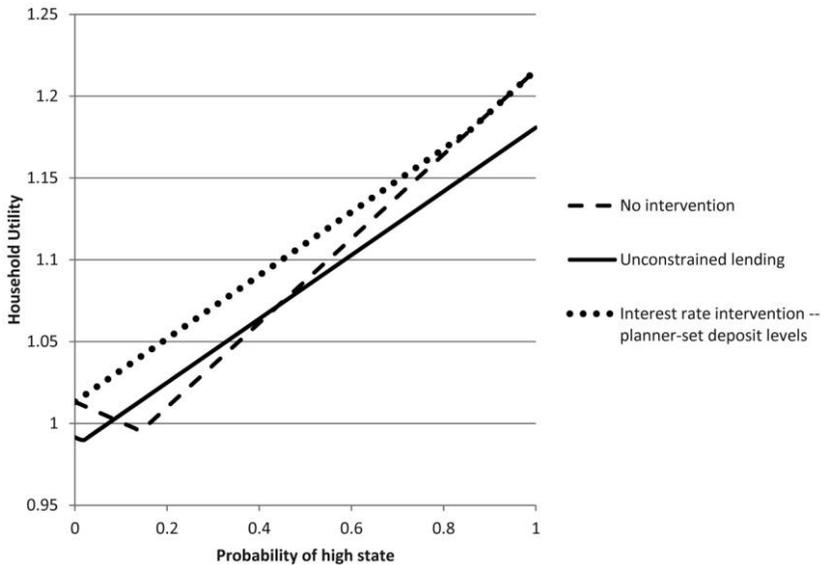


FIG. 3.—Expected household utility for different intervention regimes

tion) under these circumstances because not only is the system not safer than without intervention,  $H$  households lend against their wishes in state  $\underline{s}$ . As the probability of state  $\bar{s}$  increases, banks offer the lower  $D^{\text{Max}I}(\bar{s}) = 1.155$  and can be rescued in both states. Household expected utility now improves relative to the no-intervention case as the probability of the  $\bar{s}$  state increases, because the bank would have been run in that state under no intervention. Finally, when the probability of state  $\bar{s}$  is high, households are again better off with no intervention (following lemma 4) because the banker sets deposit rates at  $D^{\text{Max}I}(\bar{s}) = 1.039$  if he anticipates no intervention.

Now consider what happens under the more limited interest rate intervention. Banks will choose between  $D^{\text{Max}I}(\bar{s}) = 1.155$  and  $D^{\text{Max}I}(\underline{s}) = 1.16$ . When  $D = 1.155$  and  $s = \underline{s}$ , the central banker pushes down rates only to restore bank solvency and does not tax away the entire household endowment—this is the benefit of more limited intervention with the additional penalty to central bank borrowing. So household utility will be higher when  $D = 1.155$  under interest rate intervention. In fact, it is high enough that the banker never chooses to set  $D = 1.16$ , a level that would prompt a bank run in state  $\bar{s}$  even with maximal intervention. As discussed above, without the penalty, the banker would choose to set  $D = 1.16$  when the probability of the state  $\bar{s}$  is low. So the more limited ex post central bank intervention when it

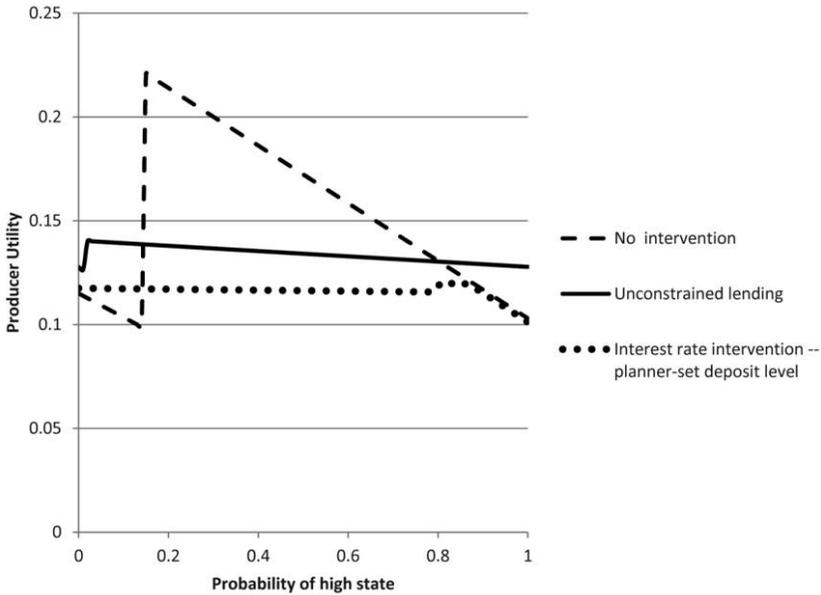


FIG. 4.—Expected producer utility for different intervention regimes

has to impose an additional nonpecuniary penalty reduces bank incentives to set deposit levels very high.

In figures 2–4, we also plot the promised deposit payments, household utility, and producer utility under interest rate intervention when the central bank rather than the banker sets promised deposit payments up front. Deposit promises are (weakly) higher than with no intervention, but lower than with unconstrained lending. Because payouts are effectively more state contingent than with no intervention, household expected utility goes up (see fig. 3). Finally, because interest rate intervention allows better state contingency and higher ex ante promises, producer rents are typically lower than with no interest rate intervention (see fig. 4).

#### H. *Producer-Friendly Central Bank*

Thus far, we have assumed that the central bank is household friendly—it maximizes household utility provided a (low) minimum threshold utility is met for producers. What if the central bank could become producer friendly at date 1—for example, if it is leaned on by producer-friendly politicians?

The producer-friendly central bank will want to deliver the lowest

date 1 rate possible, which both banks and entrepreneurs prefer. However, if restricted to lending with nonpecuniary penalties, it will be unable to guide the market rate lower than what a household-friendly central bank would achieve. Thus when the central bank is constrained to lend only when the market will, and with penalties, the lowest rate it achieves does not depend on its preferences. If, however, it is not required to impose the nonpecuniary penalty while lending, it will intervene to push down the interest rate to the maximum extent possible, and this will be the unique equilibrium rate.

Ex ante though, a permanently producer-friendly central bank, unlike the household-friendly central bank in similar circumstances, will want to curb ex ante bank liquidity promises drastically, since it recognizes that higher promises result in higher date 1 liquidation and higher rates that entrepreneurs and banks dislike.<sup>14</sup> Thus it might try to reduce deposit promises through some kind of ex ante leverage limit or, equivalently, a capital requirement. In our model, such regulations could hurt households. However, there are many ways banks can hide liquidity promises—raising funding through off-balance sheet vehicles that come back on balance sheet in times of stress, for example. If the extent of ex ante promises is hard to observe, ex ante regulation may not be effective. We now describe one more way ex ante liquidity promises may be hard to regulate before suggesting a potential resolution.

## V. Improving Ex Ante Incentives

### A. Ex Ante Liquidity Choice

Regulating bank liabilities alone may be ineffective because banks may react to expectations of low rates by reducing the liquidity of their assets. To see this, consider an extension of the model focusing on the liquidity of the bank's portfolio.

Let the liquidity of a project portfolio be given by a factor  $Z \geq 1$ , whereby portfolios pay  $\bar{Y}_2/Z$  in goods at date 2 if the project is not liquidated, and  $ZX_1$  at (or after) date 1 if the project is liquidated.<sup>15</sup> Thus more liquid portfolios have a greater liquidation value, while sacrificing long-run returns. Suppose that the bank has a choice between two assets with liquidity  $Z_1$  and  $Z_2$ , respectively, with  $Z_1 < Z_2$ . Suppose that the liquid asset is preferred by households as of date 0. This will

<sup>14</sup> A planner that was household friendly at date 0, and expects a high probability that a future planner would be producer friendly, would instead choose high leverage,  $D$ .

<sup>15</sup> The effect of the price of liquidity on the incentive of banks to hold liquid assets was first studied by Bhattacharya and Gale (1987) for banks with access to interbank markets for liquidity. Important work in this area includes Jacklin (1987), Diamond (1997), Lorenzoni (2001), Allen, Carletti, and Gale (2009), and Freixas, Martin, and Skeie (2011). These papers focus on the choice of the fraction of liquid asset holdings, such as cash.

be true if the unconditional probability of being a high type is large and  $e_2^H$  is large (implying a high value of consumption on date 1). Given  $D$ , how do date 1 rates affect liquidity choice?

A banker will choose the liquidity of his portfolio, taking future state-contingent interest rates as given, to maximize his residual claim. The total value (at date 1) of the bank's assets in state  $s$  is

$$\frac{Z}{\bar{Y}_2} \left[ \int_0^{Y_2(r_{12}^s)} X_1 dY_2 + \frac{\gamma}{Zr_{12}^s} \int_{Y_2(r_{12}^s)}^{\bar{Y}_2} Y_2 dY_2 \right] \equiv V_1(r_{12}^s, Z).$$

It is easily seen that  $\partial V_1 / \partial r_{12}^s \partial Z > 0$ , that is, a higher prospective date 1 interest rate makes it more valuable to have a more liquid project portfolio.

After refinancing deposits and meeting withdrawals at date 1, the date 2 value of the banker's claim is  $E[\max\{r_{12}^s(V_1(r_{12}^s, Z) - D), 0\}]$  and the banker chooses  $Z$  to maximize it. It is easily shown that there is an interest rate  $r_{12}^{IC}$  whereby the banker prefers, ex post, the portfolio with asset liquidity  $Z_1$  in state  $s$  iff  $r_{12}^s < r_{12}^{IC}$ . As a result, anticipated rate reductions that reduce interest rates below  $r_{12}^{IC}$  in a sufficient number of states will cause banks to choose the more illiquid portfolio with  $Z = Z_1$ .

Even interventions that merely reduce interest rates to prevent bank insolvency can cause banks to prefer to be illiquid. The intuition is as follows. As we have seen, intervention causes banks to set  $D$  higher. A higher  $D$  reduces the maximum interest rate at which the bank is solvent. By reducing interest rates in high  $\theta$  states to make the bank just solvent (zero equity value), the authorities make the value of the banker's claim depend only on the low interest rates states where the bank's residual value is substantial. The banker chooses ex ante liquidity to maximize value only conditional on the low  $\theta$  states where his stake is in the money, and if this occurs primarily when  $r_{12}^s < r_{12}^{IC}$ , the banker will choose lower liquidity than if there was no intervention and, in anticipation,  $D$  was set lower.

The broader point is that more illiquid assets effectively increase a bank's claim on date 1 liquidity, much as deposit promises do. We have already argued that deposit promises may be hard to regulate. The liquidity of assets may be even harder to regulate (the fraction of liquid cash on a bank's balance sheet is easy to monitor but it is only a small part of its overall liquidity position). This then suggests that concerned central banks should focus on altering bank incentives through ex post actions. Fortunately, they have an additional tool: raising rates.

*B. Intervention to Raise Rates*

The central bank can raise rates by essentially reversing the earlier mechanism to reduce rates; if (i) it can sell claims against date 2 taxes (i.e., sell government bonds) and transfer the receipts to households at date 1, (ii) it taxes household endowments at date 2 to repay bonds, (iii) the no-intervention date 1 equilibrium interest rate is low enough (and date 2 endowments different enough) that at least the  $H$  household withdraws its deposits entirely from the banking system in the absence of intervention.

This intervention works as follows. Because the marginal rate of substitution of the  $H$  household, prior to intervention, is higher than the pre-intervention market interest rate (which is why  $H$  households withdraw all deposits),  $H$  households would fully consume any additional date 1 resources they are given. By contrast,  $L$  households would not change consumption if the interest rate did not change. But this would mean that overall consumption would be higher than without intervention, which would require a higher market interest rate to clear the date 1 goods market and draw forth the necessary amount of liquidation. So it must be that in the new postintervention equilibrium, the interest rate is higher,  $L$  households consume less at date 1 than in the no-intervention equilibrium (so that their new marginal rate of substitution equals the higher interest rate), while  $H$  households consume more.

LEMMA 6. If the prevailing no-intervention equilibrium has  $w_1^{H,s} = D$ , then the central bank selling bonds to the bank (or  $L$  households) and paying the collected resources to households (with date 2 repayment of bonds financed by date 2 taxes on households) will increase the interest rate, decrease bank net worth, and increase project liquidation.

Households are better off when rates that can be increased are indeed increased, because it both improves the payoff to depositing as well as narrows the gap between marginal rates of substitution across households, thus making the interest rate a better measure of the representative household's marginal rate of substitution. Ex post, only a household-friendly central bank can be relied upon to raise rates when feasible (the producer-friendly central bank will not do so). Ex ante, if the household-friendly central bank is expected to raise rates sometimes when the system is solvent, the banking system will make lower promises of liquidity, holding more liquid assets or reducing the promises it makes to depositors. This will move the system toward the central bank's social optimum.

*C. Discussion*

Summarizing our analysis, a central bank that promises to cut interest rates conditional on stress or that is biased toward low interest rates favoring entrepreneurs and banks will induce banks to promise higher payouts or take more illiquid projects. This in turn can make the illiquidity crisis more severe and require a greater degree of intervention, a view reminiscent of the Austrian theory of cycles. According to this theory (Rothbard 2008, 11) the conditions for a crisis are put in place because “businessmen were misled by bank credit inflation to invest too much in higher-order capital goods [i.e., illiquid investment projects that take a long time to mature]. . . . Businessmen were led to this error by the credit expansion and its tampering with the free-market rate of interest.”

Bank credit inflation is, in turn, caused by an extremely accommodative central bank. The immediate trigger for the crisis is that (Rothbard 2008, 11) “new money percolates downwards from the business borrowers to the factors of production in wages, rents, and interest. . . . People will rush to spend the higher incomes . . . and demand will shift from the higher to the lower orders [i.e., from long dated capital projects to consumption goods]. Capital goods industries will find that their investments have been in error. . . . Higher orders of production have turned out to be wasteful, and the malinvestment must be liquidated.”

While the Austrian view seems to rely on banks that are excessively optimistic, our model can produce crises with rational optimizing banks. The point we make is quite general and transcends the specifics of the model. Private sector actions, such as financing with ex ante fragile demandable debt, have a purpose. The prospect of government intervention to prevent great societal losses can not only influence those actions—a point that is well understood in the literature—but also undermine their purpose. In order to prevent an escalating, and ultimately unproductive, sequence of anticipated intervention and response, authorities may want to commit to much more constrained intervention that respects the private sector purpose—for instance, undirected interest rate intervention may dominate more direct recapitalizations or lender of last resort loans to banks.

However, even with undirected interest rate intervention, competitive banks may have an incentive to make excessive promises of liquidity—either by promising depositors too much or by lending to excessively illiquid projects—because they do not internalize the costs of the intervention to taxpayers. Ex ante regulation to prevent such promises may not be effective. This is why central banks may want to offset the incentive effects of their inevitable and politically irresistible need to

reduce rates when the banking system is in trouble (a form of time inconsistency identified by Bagehot [1873]) by raising rates more quickly when they have the economic and political space to do so in normal times. Indeed, a central bank that is constituted to be household friendly in the context of our model will have the incentive to do so. Developing such credibility may require shielding the central bank from producer influences.

#### *D. Generalizations and Possible Extensions*

The specifics of our model can be generalized. The authorities need not force households to lend their endowment; any tax that households cannot avoid will do. For instance, expansionary interest rate policy by the central bank could be thought of as a combination of a seigniorage tax and lending to banks (offset eventually by contractionary policy). Alternatively, the authorities could simply cut back public spending temporarily (which effectively reduces household endowments) to generate the resources to lend to banks, and these cutbacks could be reversed in the future when banks repay loans. Similarly, policies to raise rates could involve additional spending at date 1, financed by government bond issuances, and cutbacks in the future. All these interventions are a combination of what is traditionally monetary and fiscal policy, because we focus not just on central bank actions but also where they get real resources from.

Also, the nature of bank moral hazard can be altered (see Diamond 2004). Instead of renegotiating down debt claims by threatening to withdraw its human capital, the bank can threaten to take, or actually take, actions that undermine the ability of investors to collect from it—for instance, by choosing low portfolio liquidity. The model can then be interpreted as including the shadow financial system (not just banks), where illiquid assets are funded by short-term debt other than demand deposits.

We could also ask why the government does not simply hand over claims at date 0 on taxable household endowments to the households, so that they can trade and write contingent contracts, so as to replicate some version of optimal interest rate policy with full commitment. One reason is that the government needs to retain future taxation authority to stop very bad things from happening—for instance, to stop panics or finance military defense to deter invasions. In the current model, the central bank's ability to conduct interest rate policy eliminates the possibility of sunspot-based panic runs. This is much like deposit insurance in the Diamond and Dybvig (1983) model with no uncertainty, where taxation is off the equilibrium path and the state usually retains some unused taxation authority so that it can prevent runs. Also, taxing

all future endowment all of the time would lead to unneeded distortions and would also give households the incentive to hide endowments from the government. Finally, fixed costs of intervention would again suggest intervening only when the benefits are high rather than all the time.

We have assumed throughout the paper that household endowments are independent of lending activity. An important extension would be to relate future endowments to the fraction of long dated projects that are allowed to continue, as in Diamond and Rajan (2005). On the one hand, project liquidation would reduce future household endowments and thus moderate the immediate demand for consumption. On the other hand, to the extent that liquidation set off financial panics as in Diamond and Dybvig (1983), it could create its own momentum for liquidity.

### *E. The Financial Crisis of 2008–9*

Ours is not intended as a model of the world financial crisis of 2008–9 (see Diamond and Rajan [2005, 2009] for more direct analysis). However, the perverse effects of anticipated low interest rates that we focus on do help us understand the build-up of illiquidity and leverage that contributed to the 2008–9 crisis. Indeed, there has been empirical work discussing the responsibility of low short-term interest rates in the United States from 2003 to 2006 (which were persistently below the level suggested by some versions of the Taylor rule), as well as the “Greenspan Put” (which we interpret as promising to maintain the low interest rates if the financial system comes under ex post stress) for the crisis (see Ioannidou, Ongena, and Peydró 2009; Maddaloni and Peydro 2010).

## **VI. Conclusion**

Our analysis shows why the structure of banks may necessitate ex post interest rate intervention, which may be better than the alternatives, but anticipation of persistently low short-term interest rates can lead to socially excessive short-term leverage and incentives to hold excessively illiquid assets relative to social optima. Even if regulatory liquidity and capital requirements constrain bank actions ex ante, unregulated parts of the financial system could benefit by setting up special vehicles that replicated bank structures. Rather than relying on ex ante regulation of banks, central banks may want to improve financial sector incentives to curtail promises of liquidity by raising interest rates in normal times more than strictly warranted by market conditions—central banks may need an additional form of credibility than just being averse to inflation. While our entire model is couched in terms of real goods, we hope in

future work to describe monetary implications, as well as explore the dynamic implications.

**Appendix**

*Why Demand Deposits Are the Constrained Optimal Contract*

Under what circumstances are demand deposits an optimal financing contract for the bank? Before we impose the constraints that imply their optimality, we begin by imposing only the resource constraints, the constraint that all contracts (including deposits) are voluntary, and the constraint that a central planner, like the bank, can only force each entrepreneur to pay a fraction  $\gamma$  of his realized value of  $\bar{Y}_2$  at date 2. The planner maximizes household utility, taking as given the need to give producers (bank plus firms) a minimum utility level  $\Lambda$ . By altering this minimum utility level, we can trace out the Pareto frontier. Start first with a contract that requires household  $H$  to receive (or pay if negative)  $V_1^{H,s}$  at date  $t$  in state  $s$  and similarly for the  $L$  household. The planner will never liquidate a project with higher realization of  $\bar{Y}_2$  before liquidating one with a lower realization, so the socially optimal liquidation policy is to choose a value  $Y^s$  such that all projects with realization less than or equal to  $Y^s$  are liquidated in state  $s$ , with total liquidation value  $(Y^s/\bar{Y})X_1$ . Let  $\Upsilon(Y^s) = \{[(\bar{Y})^2 - (Y^s)^2]/2\bar{Y}\}$  be the value of projects that are not liquidated (the precise functional form is not important, so we write it more generally below). The unconstrained planner's problem is

$$\begin{aligned} \max_{V_1^{L,s}, V_2^{L,s}, V_1^{H,s}, V_2^{H,s}, Y^s} & E\{\theta^s[U(e_1 + V_1^{H,s}) + U(e_2^H + V_2^{H,s})] \\ & + (1 - \theta^s)[U(e_1 + V_1^{L,s}) + U(e_2^L + V_2^{L,s})]\}, \end{aligned}$$

subject to

$$\begin{aligned} \theta_s V_2^{L,s} + (1 - \theta_s) V_2^{H,s} &\leq \gamma \Upsilon(Y^s) \quad (\text{solvency constraint; resources at date 2}), \\ \theta^s V_1^{H,s} + (1 - \theta^s) V_1^{L,s} &\leq (Y^s/\bar{Y})X_1 \quad (\text{liquidity constraint; resources at date 1}), \\ E\{\Upsilon(Y^s) - \theta^s V_2^{H,s} - (1 - \theta^s) V_2^{L,s} + [(Y^s/\bar{Y})X_1 - \theta^s V_1^{H,s} - (1 - \theta^s) V_1^{L,s}]\} &\geq \Lambda \quad (\text{floor on producer utility level}), \\ V_1^{L,s} \geq -e_1, V_1^{H,s} \geq -e_1, V_2^{L,s} \geq -e_2^L, V_2^{H,s} \geq -e_2^H & \quad (\text{feasibility constraints: household cannot pay more than its endowment}), \\ Y^s \in [0, \bar{Y}] & \quad (\text{feasible liquidation choices}). \end{aligned}$$

This is a standard problem equivalent to optimal competitive consumption, production, and risk sharing with complete markets (subject to the ability to commit to make payments). It is easily shown that the solution to this maximization problem is to equate the marginal utilities of each type of household in each state. Thus  $V_1^{H,s} = V_1^{L,s}$  and  $V_2^{H,s} = V_2^{L,s} - (e_2^H - e_2^L)$ . So we achieve perfect risk sharing. Also it is optimal to choose  $Y^s$  such that the marginal rate of transformation of date 2 consumption to date 1 consumption goods, which equals  $\gamma Y^s/X_1$ , is equated to the common marginal rate of substitution of the households. We refer to the solution as the first-best allocation.

We impose a number of additional constraints that prevent the planner from

achieving this outcome. To see how these will work, we add constraints in sequence.

(A1) Assume, as in Diamond and Rajan (2001), that only the bank can collect the loan, and the banker can threaten credibly at any point in time to not collect the loans he has made unless the households agree to a lower value of payments.

Given A1, the only way for households to force the banker to repay them more than the liquidation value,  $X_1$ , when the banker can make a credible threat not to collect more than this (unless households make concessions) is to commit themselves to reject offers that are in their collective interest to accept. If they choose demand deposits with face value  $D$ , they will reject any offer of less than  $D$  in favor of running and forcing liquidation.

We show that the first-best allocation can still generally be achieved with type- and state-contingent demand deposits. The demandable claim  $D^{H,s}$  for the  $H$  household (or  $D^{L,s}$  for the low type) puts a lower bound on the present value of what that household type will accept. These claims will be renegotiation proof because each household can withdraw its entire claim, consume a portion, and reinvest in some other bank at  $r_{12}^s$  (or trade with other households at this interest rate), which will be the marginal rate of transformation from the first-best allocation.<sup>16</sup> Let  $V_1^{H,s}, V_2^{H,s}$  be the renegotiation-proof payments the planner is trying to achieve with a demandable claim for the  $H$  household of  $D^{H,s}$ . It must be that for each type  $\tau \in \{H, L\}$ ,

$$U(e_1 + V_1^{\tau,s}) + U(e_2^{\tau} + V_2^{\tau,s}) \geq \max_{w_1^{\tau,s}} U(e_1 + w_1^{\tau,s}) + U[e_2^{\tau} + (D^{\tau,s} - w_1^{\tau,s})r_{12}^s].$$

Also, the banker has no incentive to pay more than he is forced to by the deposit claim, so

$$\max_{w_1^{\tau,s}} U(e_1 + w_1^{\tau,s}) + U[e_2^{\tau} + (D^{\tau,s} - w_1^{\tau,s})r_{12}^s] \geq U(e_1 + V_1^{\tau,s}) + U(e_2^{\tau} + V_2^{\tau,s}).$$

Thus the two weak inequalities imply equality, and the demandable claim determines the present value of what depositors get. Also feasibility requires  $w_1^{H,s} \geq -e_1, w_1^{L,s} \geq -e_1$ . It is clear then that with a state-contingent date 0 deposit contract that results in date 1 household-type-specific state-contingent deposit contracts  $D^{H,s} = V_1^{H,s} + (V_2^{H,s}/r_{12}^s)$  and  $D^{L,s} = V_1^{L,s} + (V_2^{L,s}/r_{12}^s)$ , the first-best allocation can be attained, even with limited banker commitment.

Here is why: each household with deposit claims will maximize utility when it withdraws enough that its marginal rate of substitution is equal to the marginal rate of transformation  $r_{12}^s$  in that state. But a withdrawal by the  $H$  household of  $w_1^{H,s} = V_1^{H,s}$  will achieve precisely the required marginal rate of substitution (since its date 2 demandable claim will then be  $(D^{H,s} - w_1^{H,s})r_{12}^s$ , which equals  $V_2^{H,s}$  by construction, and payments of  $V_1^{H,s}, V_2^{H,s}$  achieve the required marginal rate of substitution).<sup>17</sup> Therefore, if we have demandable state- and type-contingent deposits, we can address the banker's inability to commit to a stream of payments

<sup>16</sup> If depositors could not trade with other banks or households, then the bank could force a depositor to take only date 1 consumption if the depositor withdrew all of his deposit at date 1. Allowing trade improves the depositors' outside option but does not prevent type- and state-contingent deposits from implementing the first best.

<sup>17</sup> This is similar to the use of state-contingent deposit contracts in the Hellwig (1994) analysis of interest rate uncertainty in the Diamond and Dybvig (1983) model.

and still achieve the first best. Note that the demandable deposits, by allowing flexibility in withdrawals, offer a certain amount of type and state contingency.

(A2) There is a time delay between the arrival of information to households about the state and the quickest possible adjustment in the state-contingent value of claims held by households.

Given A2, we show that demandable deposit claim cannot be state contingent:  $D^{H,s} = D^H$ ,  $D^{L,s} = D^L$  for all  $s$ . Here is the rationale: deposit claims have to be demandable at every instant after contracting; otherwise households can be renegotiated down. Therefore, the initial promised payment on the deposit has to be a state-independent value, set ex ante at a level that makes households indifferent between withdrawing the deposit immediately (and potentially re-depositing elsewhere) and waiting for the state to redefine their face value. First, we show that there can never be a state-contingent reduction in face value below the initial value (or any subsequent state-contingent value). The bank and the legal authorities enforcing deposit claims can see the state and adjust the face value down more slowly than the households can withdraw. If the households know that a reduction is forthcoming, each household has an incentive to withdraw before it happens, to collect the higher initial face value. In other words, to avoid setting off a run, not only do the bank and the authorities have to learn the state before households can react, they have to also reflect it in the face values. Because it takes a given amount of time to alter  $D$ , and a significant number of depositors can react faster than that, any state-contingent reduction in face value will precipitate the very run it seeks to avoid. Therefore, any state-contingent changes in face value would need to specify only state-contingent increases in value.

Any contract that specifies only state-contingent increases in face value will be renegotiated. Suppose that the bank set a deposit face that specifies a state-contingent increase above an initial value  $\lambda$ . After receiving funding, the banker will be able to negotiate depositors down to a contract that promises a utility equivalent to the utility depositors get from withdrawing  $\lambda$  immediately. If the banker renegotiates and offers a contract with a fixed value of  $\lambda$  in all states, the depositors are as well off accepting as they are withdrawing and therefore will accept and not start a run. Demand deposit contracts cannot specify exclusively state-contingent increases.

A contract that cannot have an increase or decrease in face value is not state contingent. The time needed to adjust face values to the state of nature also rules out the possibility of making the value of demand deposits depend on the subsequently realized state-contingent interest rate, because the interest rate would need to be set in the market before any withdrawal is possible. For all these reasons, the first-best state-contingent demandable deposits are not feasible. Fixed face value demand deposits are constrained efficient.

(A3) Household type is private information (the bank or planner cannot distinguish  $H$ -type households from  $L$ -type households). In addition, trade between households is not observable.

Given that either household type has access to the capital market (they can trade with each other, as in Jacklin [1987], or trade with other banks), all they care about when choosing an interior amount to withdraw from a menu of choices is the present value of the claims they are offered, discounted at market

interest rates—given a choice, households will pick the highest present value and transform it to their preferred payment stream. This makes it impossible for the bank to get them to self-select the maximum amount that they are allowed to withdraw at date 1 based on their different marginal rates of substitution. Thus the possibility of trade in A3 rules out type-contingent deposit face value contracts. So  $D_H = D_L = D$ . Imposing all of these constraints implies that the demand deposit contract implements the constrained optimal allocations.

*Proof of Theorems and Lemmas*

Proof of Theorem 1

(i) The right-hand side of (1) is strictly increasing in  $r_{12}^S$ . The left-hand side is weakly decreasing—withdrawals decrease in the interest rate, but when  $D$  is withdrawn, the household cannot withdraw more. Thus there is a unique interest rate and withdrawals at which the supply of goods at date 1 equals demand, and the depositors do not want to, or cannot, withdraw more.

Suppose not. Can there, for instance, be another equilibrium with the same interest rate and different withdrawal levels? If the households are on their first-order conditions at the equilibrium interest rate, there is a unique withdrawal level consistent with the interest rate. If they are off their first-order condition, they have to withdraw  $D$ . So there cannot be another equilibrium with the same interest rate and different withdrawal levels. Can there be another equilibrium with a higher interest rate? No, because withdrawals would be (weakly) lower while liquidation would be strictly higher, so (1) would not hold. Similarly for a lower interest rate.

(ii) Follows by construction.

Proof of Corollary 1

When both households are on their first-order condition, the  $H$  household withdraws more than the  $L$  household. The  $H$  household withdraws  $D$  fully before the  $L$  household withdraws. Hence it (weakly) withdraws more and  $w_1^{H,S} \geq w_1^{L,S}$ .

(i) Totally differentiating (1) with respect to  $\theta^S$ , we get

$$(w_1^{H,S} - w_1^{L,S}) + \theta^S \frac{dw_1^{H,S}}{d\theta^S} + (1 - \theta^S) \frac{dw_1^{L,S}}{d\theta^S} - \frac{X_1^2}{\gamma(\bar{Y}_2)} \frac{dr_{12}^S}{d\theta^S} = 0.$$

Now

$$\frac{dw_1^{H,S}}{d\theta^S} = \frac{\partial w_1^{H,S}}{\partial r_{12}^S} \frac{dr_{12}^S}{d\theta^S},$$

where  $\partial w_1^{H,S} / \partial r_{12}^S \leq 0$  from the expressions for  $w_1^{H,S}$ . Similarly for  $dw_1^{L,S} / d\theta^S$ . Thus

$$\frac{dr_{12}^S}{d\theta^S} = \frac{(w_1^{H,S} - w_1^{L,S})}{[X_1^2 / \gamma(\bar{Y}_2)] - \theta^S (\partial w_1^{H,S} / \partial r_{12}^S) - (1 - \theta^S) (\partial w_1^{L,S} / \partial r_{12}^S)} \geq 0$$

because  $w_1^{H,S} \geq w_1^{L,S}$ .

Because  $dr_{12}^S / d\theta^S \geq 0$ , it must be that the right-hand side of (1) increases in  $\theta^S$ , so the left-hand side must also increase. Hence  $\theta^S w_1^{H,S} + (1 - \theta^S) w_1^{L,S}$  increases

in  $\theta^s$ . Finally, the fraction liquidated increases in  $r_{12}^s$ ; hence it must increase in  $\theta^s$ .

Turning to the effect of  $D$ , we have on totally differentiating (1)

$$\theta^s \frac{dw_1^{H,S}}{dD} + (1 - \theta^s) \frac{dw_1^{L,S}}{dD} - \frac{X_1^2}{\gamma(\bar{Y}_2)} \frac{dr_{12}^s}{dD} = 0.$$

Now either  $dw_1^S/dD = 1$  if the household is off its first-order condition and has withdrawn everything, or it is given by

$$\frac{1}{2} \left[ 1 - \dot{e}_2 \left( \frac{1}{r_{12}^s} \right)^2 \frac{dr_{12}^s}{dD} \right].$$

Substituting and simplifying, we get  $dr_{12}^s/dD \geq 0$ . Given  $r_{12}^s$  increases in  $D$ , the other comparative statics follow for the same reasons as above.

(iii) The date 1 asset value of banks is

$$(1/\bar{Y}_2) \int_{\bar{Y}_2}^{Y_2(r_{12}^s)} X_1 dY_2 + (1/\bar{Y}_2) \int_{Y_2(r_{12}^s)}^{\bar{Y}_2} (\gamma Y_2/r_{12}^s) dY_2$$

while the liabilities are  $D$ . The asset value falls in  $r_{12}^s$ , so it falls in  $\theta^s$  and  $D$ . The liabilities increase in  $D$ . Hence the net worth (= assets - liabilities) falls in  $\theta^s$  and  $D$ .

Proof of Lemma 2

Let  $U(p, D) = \sum_s p^s U^s(D)$ , where  $U^s(D)$  is the utility of households in state  $s$ . Then  $U(p, D^{\text{Max}}(s^*)) = \sum_s p^s U^s(D^{\text{Max}}(s^*)) + \sum_{s^*} p^{s^*} U^{s^*,\text{Run}}(D^{\text{Max}}(s^*))$ , where  $U^{s^*,\text{Run}}(D)$  is the utility of households in state  $s$  when their bank is run and they hold deposits  $D$ . Now consider a state  $t > s^*$ . We have

$$\begin{aligned} U(p, D^{\text{Max}}(s^*)) - U(p, D^{\text{Max}}(t)) &= \sum_s p^s [U^s(D^{\text{Max}}(s^*)) - U^s(D^{\text{Max}}(t))] \\ &+ \sum_{s^*+1}^t p^s [U^{s^*,\text{Run}}(D^{\text{Max}}(s^*)) - U^s(D^{\text{Max}}(t))] \\ &+ \sum_{t+1}^{\bar{s}} p^s [U^{s^*,\text{Run}}(D^{\text{Max}}(s^*)) - U^s(D^{\text{Max}}(t))]. \end{aligned}$$

Now consider a decrease  $\Delta$  in probability in one of the states  $s^* + k - j$  to the left of  $s^*$  (i.e., where  $j > k$ ) and a commensurate increase in probability in one of the states  $s^* + k$  to the right of  $s^*$ . Then when  $s^* + k \leq t$ , the change in  $U(p, D^{\text{Max}}(s^*)) - U(p, D^{\text{Max}}(t))$  is

$$\begin{aligned} &-\Delta [U^{s^*+k-j}(D^{\text{Max}}(s^*)) - U^{s^*+k-j}(D^{\text{Max}}(t))] \\ &+ \Delta [U^{s^*+k,\text{Run}}(D^{\text{Max}}(s^*)) - U^{s^*+k}(D^{\text{Max}}(t))]. \end{aligned}$$

The first term in square brackets is positive because  $D^{\text{Max}}(s^*) > D^{\text{Max}}(t)$  so  $U^{s^*+k-j}(D^{\text{Max}}(s^*)) > U^{s^*+k-j}(D^{\text{Max}}(t))$ . The second term in square brackets is negative because runs are costly and  $U^{s^*+k,\text{Run}}(D^{\text{Max}}(s^*)) < U^{s^*+k}(D^{\text{Max}}(t))$ . When  $s^* + k > t$ , the change in  $U(p, D^{\text{Max}}(s^*)) - U(p, D^{\text{Max}}(t))$  is

$$\begin{aligned}
 & - \Delta[U^{s^*+k-j}(D^{\text{Max}}(s^*)) - U^{s^*+k-j}(D^{\text{Max}}(t))] \\
 & + \Delta[U^{s^*+k,\text{Run}}(D^{\text{Max}}(s^*)) - U^{s^*+k,\text{Run}}(D^{\text{Max}}(t))].
 \end{aligned}$$

The first term in square brackets is positive as before. The second term in square brackets is negative because runs produce lower household utility when promised deposits are higher, and  $D^{\text{Max}}(s^*) > D^{\text{Max}}(t)$ .

Therefore, the change in probabilities makes the previous optimal deposit promise,  $D^{\text{Max}}(s^*)$ , less attractive compared to any lower deposit promise,  $D^{\text{Max}}(t)$  where  $t > s^*$ . It is also straightforward to show following a similar logic that any higher deposit promise  $D^{\text{Max}}(t)$  where  $s^* + k - j \leq t < s^*$  becomes less attractive compared to  $D^{\text{Max}}(s^*)$ . What is harder to sign is the change in attractiveness of deposit promises  $D^{\text{Max}}(t)$  where  $t < s^* + k - j$ . If, however, the probability weight on these states is small (and assuming finite values), they will not affect the calculations of optimality, and the banker-chosen optimal will (weakly) move to lower promised deposit payments.

Proof of Lemma 4

First, it is clear that both types of households are better off in state  $s$  if promised deposits are raised without precipitating a run, that is, to  $D^{\text{Max}}(s)$ . We then have

$$\frac{dU^{L,s}}{dD} = \frac{1}{e_1 + w_1^{L,s}} \left( \frac{dw_1^{L,s}}{dD} \right) - \frac{r_{12}^S}{e_2^L + (D - w_1^{L,s})r_{12}^S} \left( \frac{dw_1^{L,s}}{dD} \right) + \frac{r_{12}^S + (D - w_1^{L,s})(dr_{12}^S/dD)}{e_2^L + (D - w_1^{L,s})r_{12}^S}.$$

Because

$$\frac{1}{e_1 + w_1^{L,s}} - \frac{r_{12}^S}{e_2^L + (D - w_1^{L,s})r_{12}^S} = 0,$$

the first two terms are zero. The last term is positive. So  $dU^{L,s}/dD > 0$ . Similarly, if  $H$  households are on their first-order condition, we can show that  $dU^{H,s}/dD > 0$ . If they are not—either they withdraw everything or leave everything in the bank—their utility clearly increases in  $D$ .

Suppose now that we increase  $D$  from  $D^{\text{Max}}(s)$  while lowering  $r_{12}^s$  to keep the bank solvent.

Post intervention,  $U^{L,s} = \log(e_1 + w_1^{L,s} - t) + \log(e_2^L + (D - w_1^{L,s} + t)r_{12}^S)$  such that  $V(r_{12}^S) = D$ , where  $V$  is the value of the bank at date 1 (before withdrawals). We then have

$$\begin{aligned}
 \frac{dU^{L,s}}{dD} &= \frac{1}{e_1 + w_1^{L,s} - t} \left( \frac{dw_1^{L,s}}{dD} - \frac{dt}{dD} \right) \\
 & - \frac{r_{12}^S}{e_2^L + (D - w_1^{L,s} + t)r_{12}^S} \left( \frac{dw_1^{L,s}}{dD} - \frac{dt}{dD} \right) \\
 & + \frac{r_{12}^S + (D - w_1^{L,s} + t)(dr_{12}^S/dD)}{e_2^L + (D - w_1^{L,s} + t)r_{12}^S}.
 \end{aligned}$$

Because the  $L$  households are on their first-order condition, the first two terms sum to zero again. We also have from the requirement of solvency that

$$\frac{dr_{12}^S}{dD} = \frac{-1}{dV/dr_{12}^S}.$$

Simple algebra suggests that

$$\begin{aligned} \frac{dV}{dr_{12}^S} &= \frac{1}{(\bar{Y}_2)} \left( \frac{(X_1)^2}{2\gamma} - \frac{\bar{Y}_2^2 \gamma}{2(r_{12}^S)^2} \right) \\ &= \frac{-1}{r_{12}^S} (\text{Present value of bank (after withdrawals)}). \end{aligned}$$

This then means that

$$\frac{dU^{L,s}}{dD} = \frac{r_{12}^S(1 - [(D - w_1^{L,s} + t)/\text{Present value of bank}])}{e_2^L + (D - w_1^{L,s} + t)r_{12}^S} < 0$$

because if the bank is just solvent after withdrawals, the present value of the bank =  $(1 - \theta)(D - w_1^{L,s}) + t < D - w_1^{L,s} + t$ . What about the  $H$  household? With lower  $D$ , and compensation for taxes at a rate that is less than its marginal rate of substitution, it is worse off. So households are worse off if  $D$  is set above  $D^{\text{Max}}(s)$  and the planner intervenes to restore solvency.

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