

Appendix: Extracting a robust U.S. business cycle using a
time-varying multivariate model-based bandpass filter

DREW CREAL

Department of Econometrics, VU University Amsterdam
De Boelelaan 1105, NL-1081 HV Amsterdam
dcreal@feweb.vu.nl

SIEM JAN KOOPMAN

Department of Econometrics, VU University Amsterdam
De Boelelaan 1105, NL-1081 HV Amsterdam
s.j.koopman@feweb.vu.nl

ERIC ZIVOT

Department of Economics, University of Washington
Box 353330, 302 Savery Hall, Seattle, WA 98195
ezivot@u.washington.edu

February 2009

	Prior δ_i		Posterior δ_i		Prior ξ_i		Posterior ξ_i	
	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
unemployment	0.00	2.00	-0.263	0.010	0.00	2.50	-0.958	0.182
manufacturing	0.00	2.00	0.368	0.014	0.00	2.50	4.001	0.242
inflation	0.00	2.00	0.332	0.035	0.00	2.50	-3.807	0.604
retail	0.00	2.00	0.699	0.036	0.00	2.50	2.969	0.354
retail/food	0.00	2.00	0.556	0.095	-	-	-	-
productivity	0.00	2.00	0.326	0.025	0.00	2.50	8.391	0.536
real GDP	0.00	2.00	0.577	0.024	0.00	2.50	3.409	0.250
hours	0.00	2.00	0.550	0.023	0.00	2.50	0.648	0.261
consumption	0.00	2.00	0.142	0.015	0.00	2.50	6.365	0.741
investment	0.00	2.00	2.239	0.091	0.00	2.50	3.349	0.231
	Prior ρ		Posterior ρ		Prior $2\pi/\lambda$		Posterior $2\pi/\lambda$	
	0.50	0.08	0.969	0.004	63.56	67.02	41.07	1.25

Table 1: Prior and posterior means and standard deviations for ρ , $2\pi/\lambda$, and for each series δ_i and ξ_i . Industrial production has $\delta = 1$ and $\xi = 0$.

1 Additional results for the main model

	Prior σ_ε^2		Posterior σ_ε^2	
	mean	st. dev.	mean	st. dev.
	$\times 10^{-4}$	$\times 10^{-5}$	$\times 10^{-4}$	$\times 10^{-5}$
unemployment	-	-	242.0	171.0
retail/food	-	-	6583.0	7230.0
productivity	-	-	3573.0	4070.0
hours	-	-	909.0	1220.0

Table 2: Prior and posterior means and standard deviations of σ_ε^2 for the remaining series with a constant variance for the irregular component.

	distribution	hyperparameter 1	hyperparameter 2	hyperparameter 3	hyperparameter 4
ρ	$\mathcal{U}(0,1)$	-	-	-	-
λ	$\mathcal{B}(\alpha,\beta)$	$\alpha = 2.6377$	$\beta = 15.0577$	-	-
δ_i	$\mathcal{N}(\mu,V)$	$\mu = 0$	$V = 4$	-	-
ξ_i	$\mathcal{TN}(\mu,V,a,b)$	$\mu = 0$	$V = 6.25$	$a = -\frac{\pi}{2\lambda}$	$b = \frac{\pi}{2\lambda}$
σ_ε^2	$\mathcal{IG}(c,d)$	$c = 0$	$d = 0$	-	-

Table 3: Prior distributions for the parameters of the main model. \mathcal{TN} denotes the truncated normal distribution. The inverse gamma distribution is in terms of the shape and scale parameters. The beta distribution has mean $\frac{\alpha}{\alpha+\beta}$.

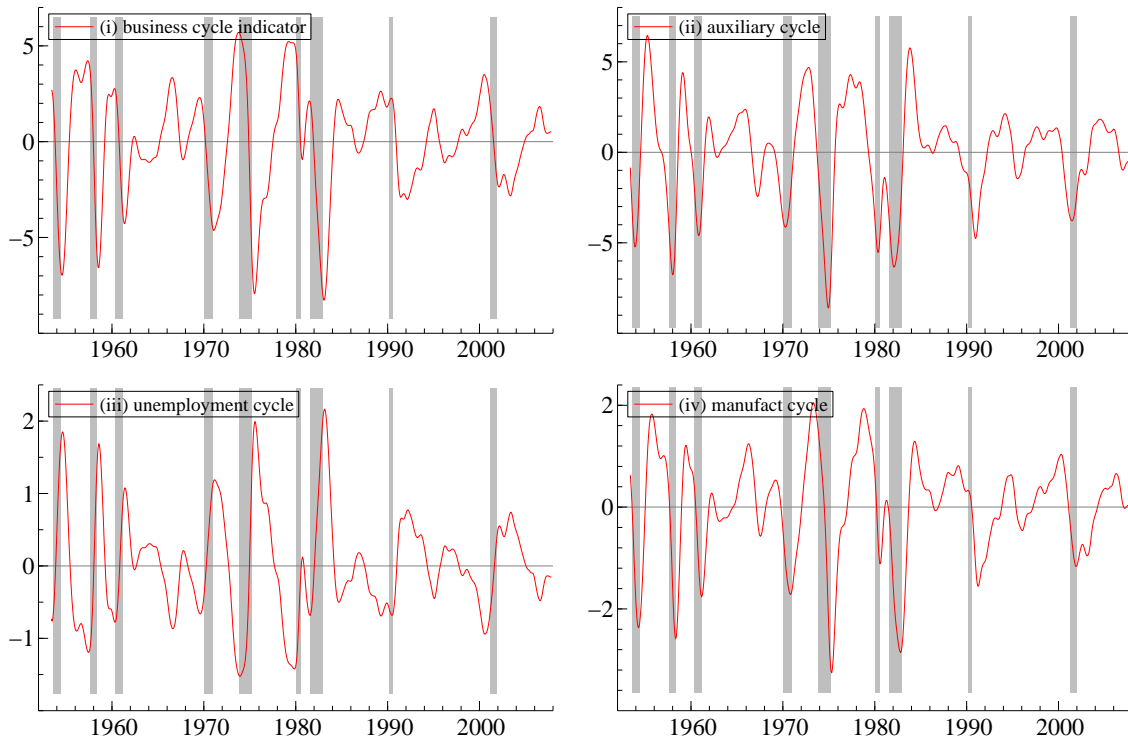


Figure 1: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) main indicator ψ_t ; (ii) auxiliary cycle ψ_t^+ ; (iii) unemployment; (iv) manufacturing.*

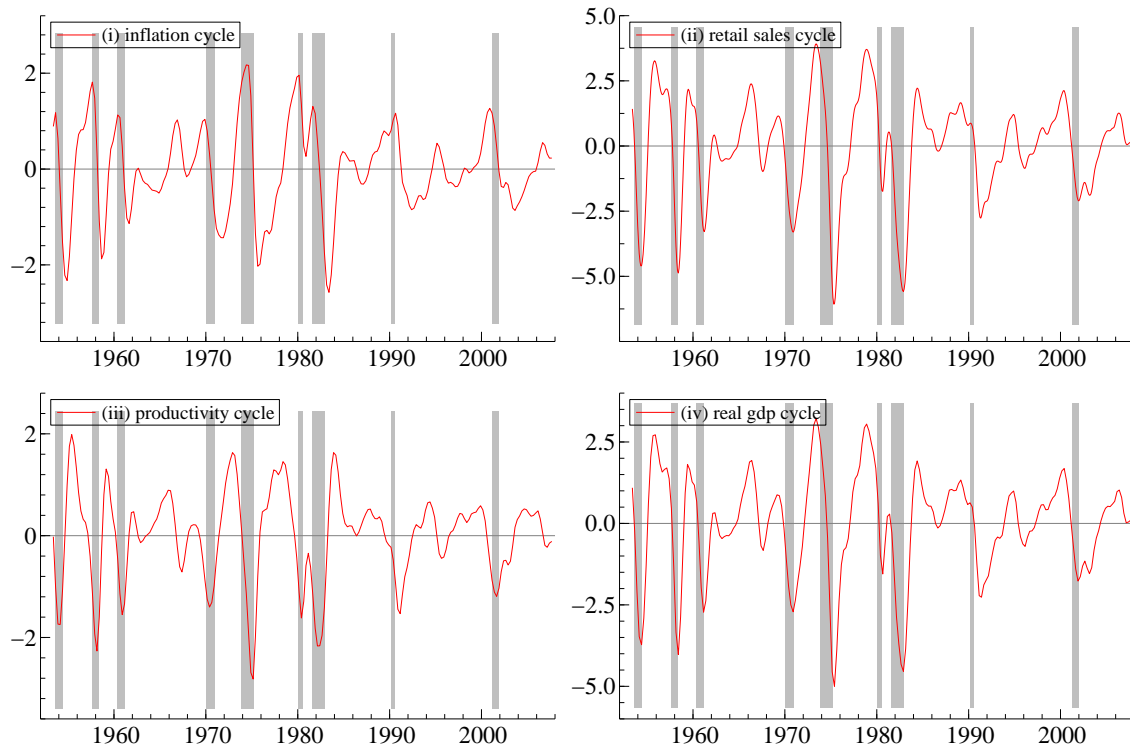


Figure 2: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) inflation; (ii) retail sales; (iii) productivity NFBS; (iv) real gdp.*

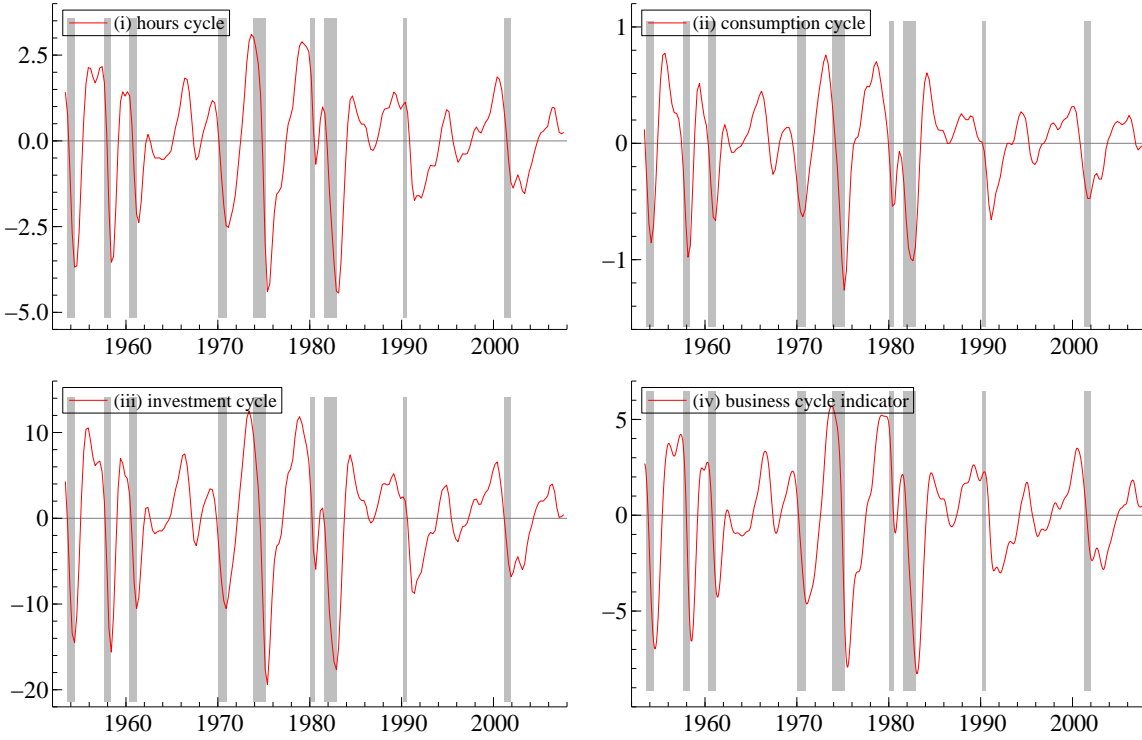


Figure 3: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) hours NFBS; (ii) real consumption of nondurables + services; (iii) real consumption of durables + real fixed investment; (iv) main indicator ψ_t .*

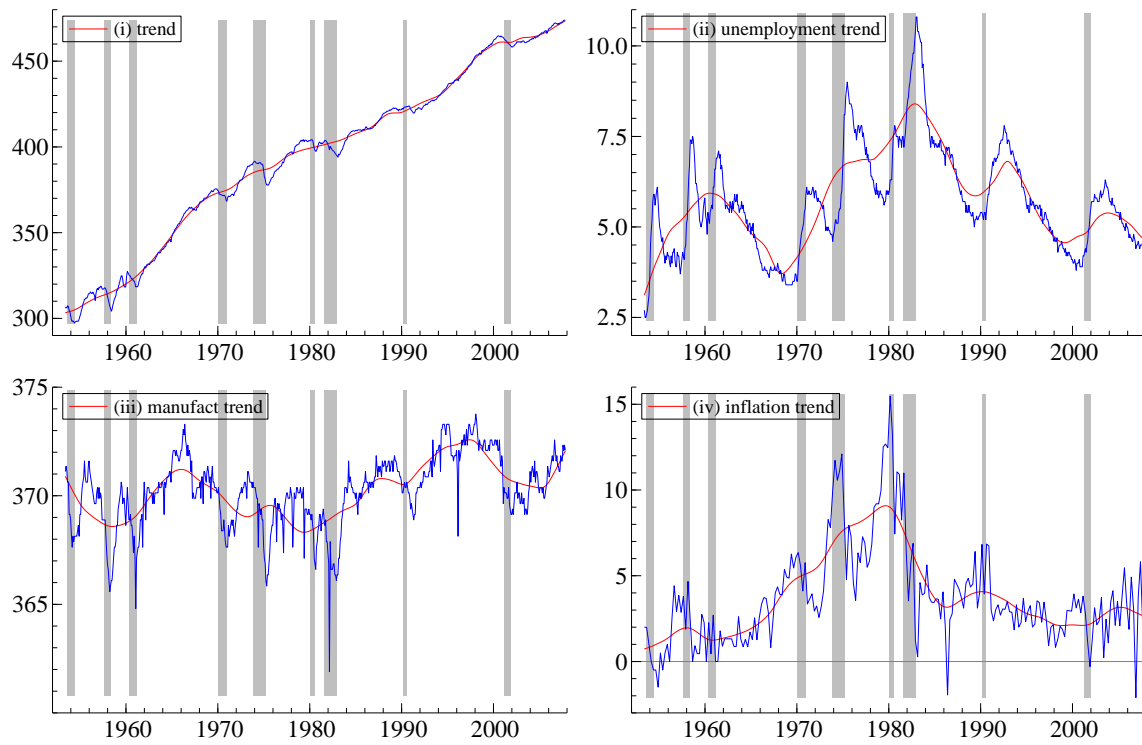


Figure 4: *Smoothed estimates of the trends. (i) industrial production; (ii) unemployment; (iii) manufacturing; (iv) inflation. The trends are plotted with each respective series.*

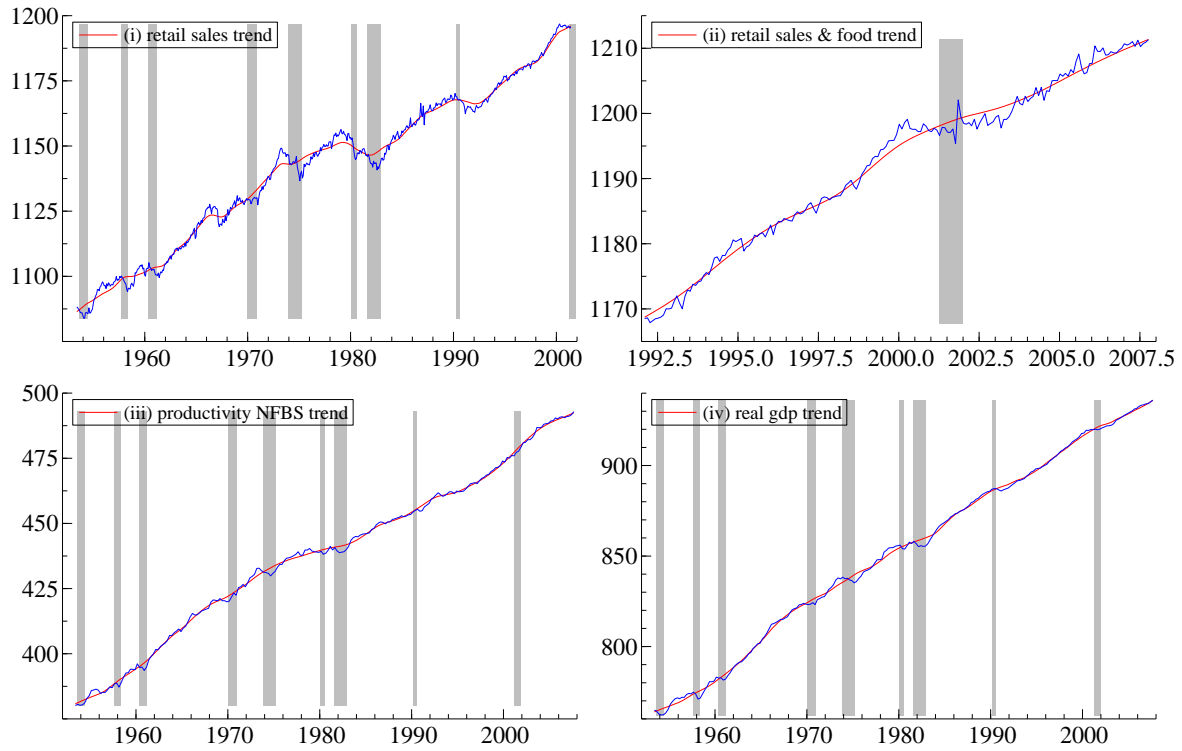


Figure 5: *Smoothed estimates of the trends. (i) retail sales; (ii) retail sales and food services; (iii) productivity of NFBS; (iv) real gdp. The trends are plotted with each respective series.*

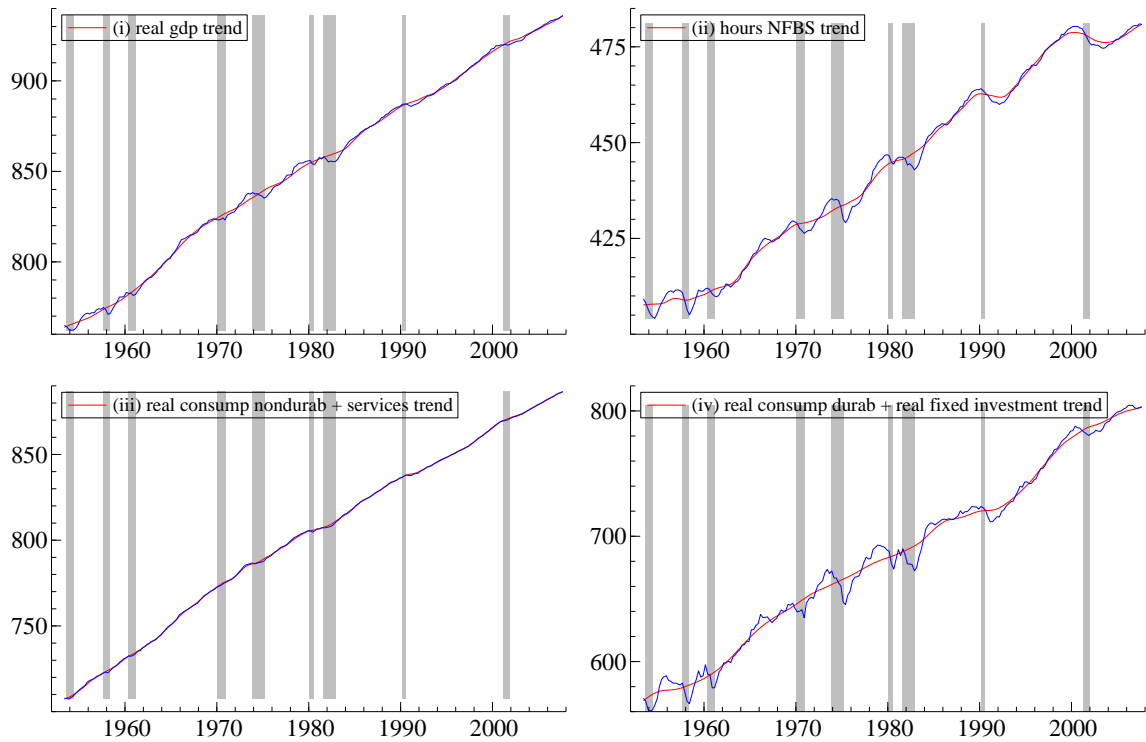


Figure 6: *Smoothed estimates of the trends. (i) real gdp; (ii) hours of NFBS; (iii) real consumption of nondurables + services; (iv) real consumption of durables + real fixed investment. The trends are plotted with each respective series.*

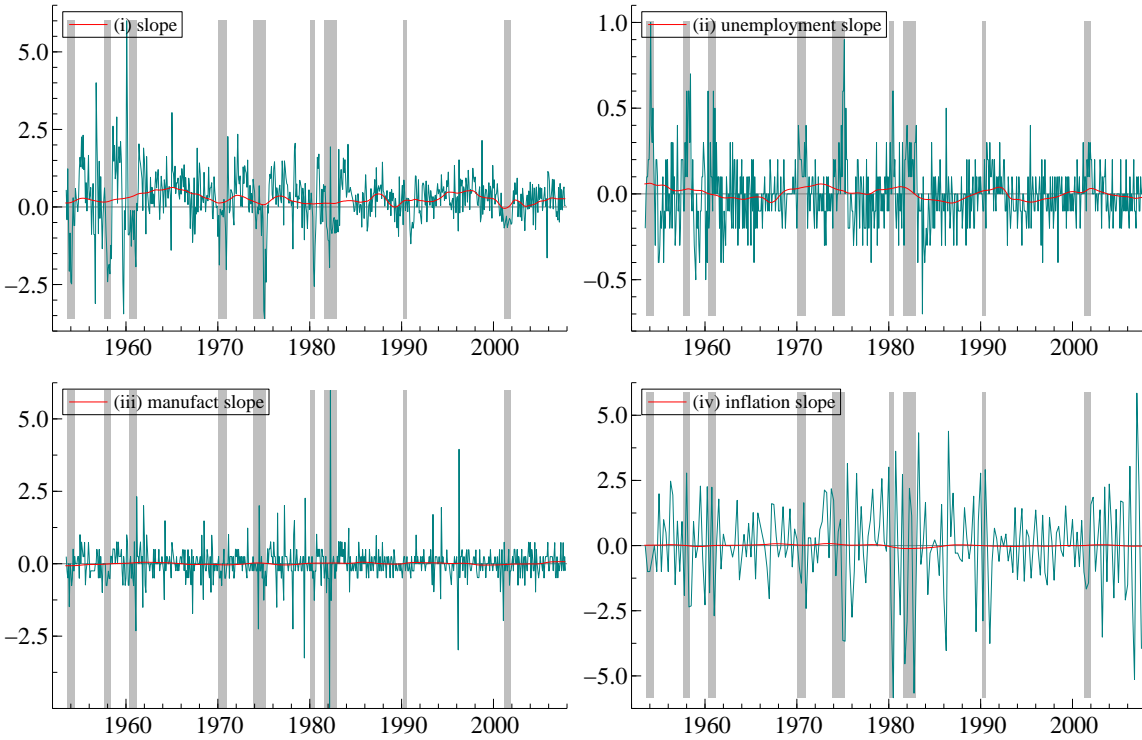


Figure 7: *Smoothed estimates of the slopes. (i) industrial production; (ii) unemployment; (iii) inflation; (iv) manufacturing. The slopes are plotted with the growth rates of the respective series.*

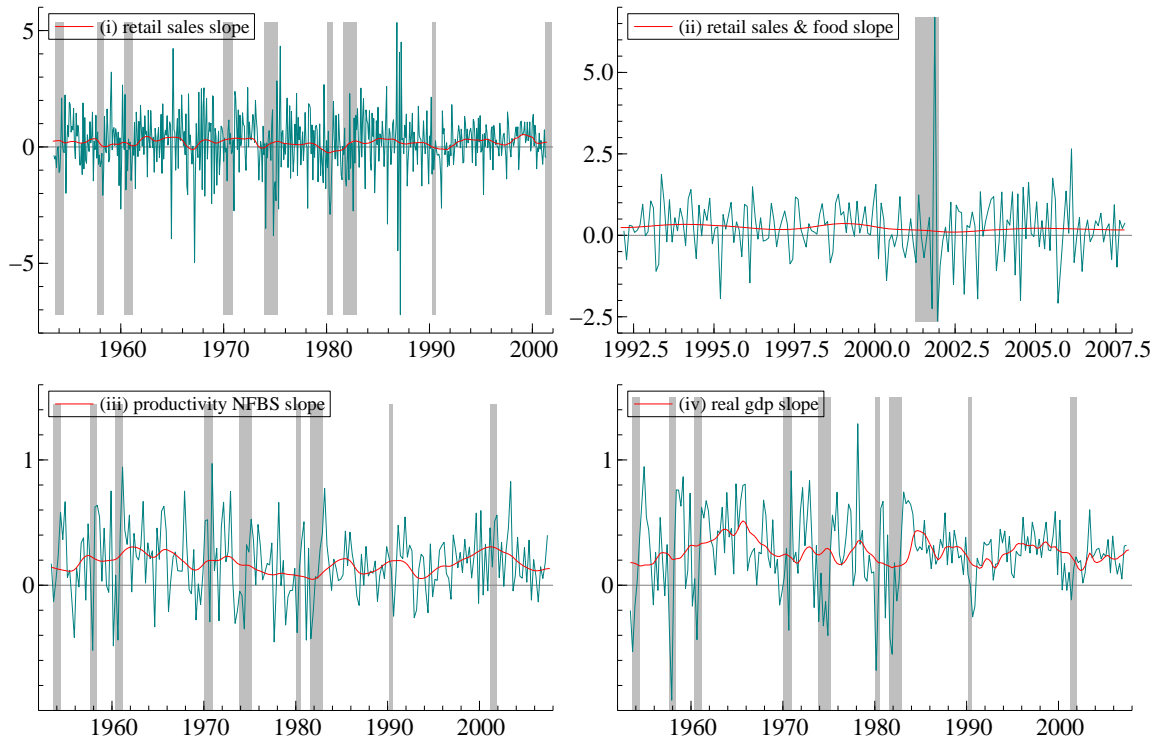


Figure 8: *Smoothed estimates of the slopes. (i) retail sales; (ii) retail sales and food services, (iii) real gdp; (iv) productivity of NFBS. The slopes are plotted with the growth rates of the respective series.*

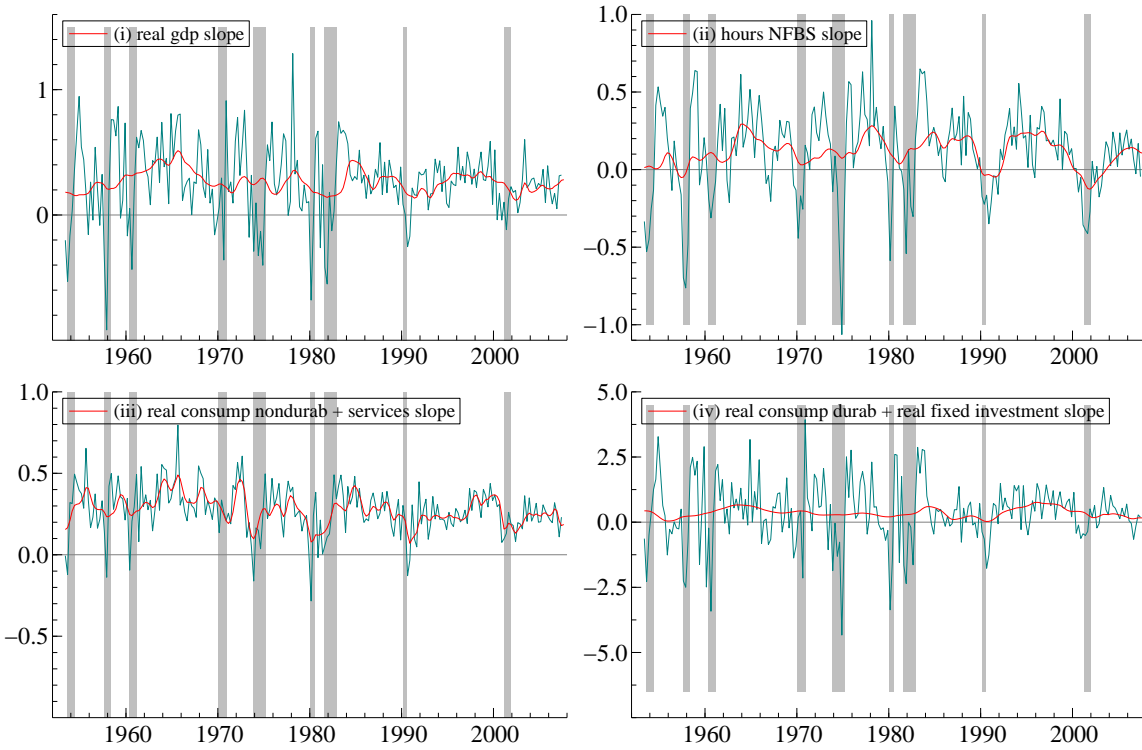


Figure 9: *Smoothed estimates of the slopes. (i) real gdp; (ii) hours of NFBS; (iii) real consumption of nondurables + services; (iv) real consumption of durables + real fixed investment. The slopes are plotted with the growth rates of the respective series.*

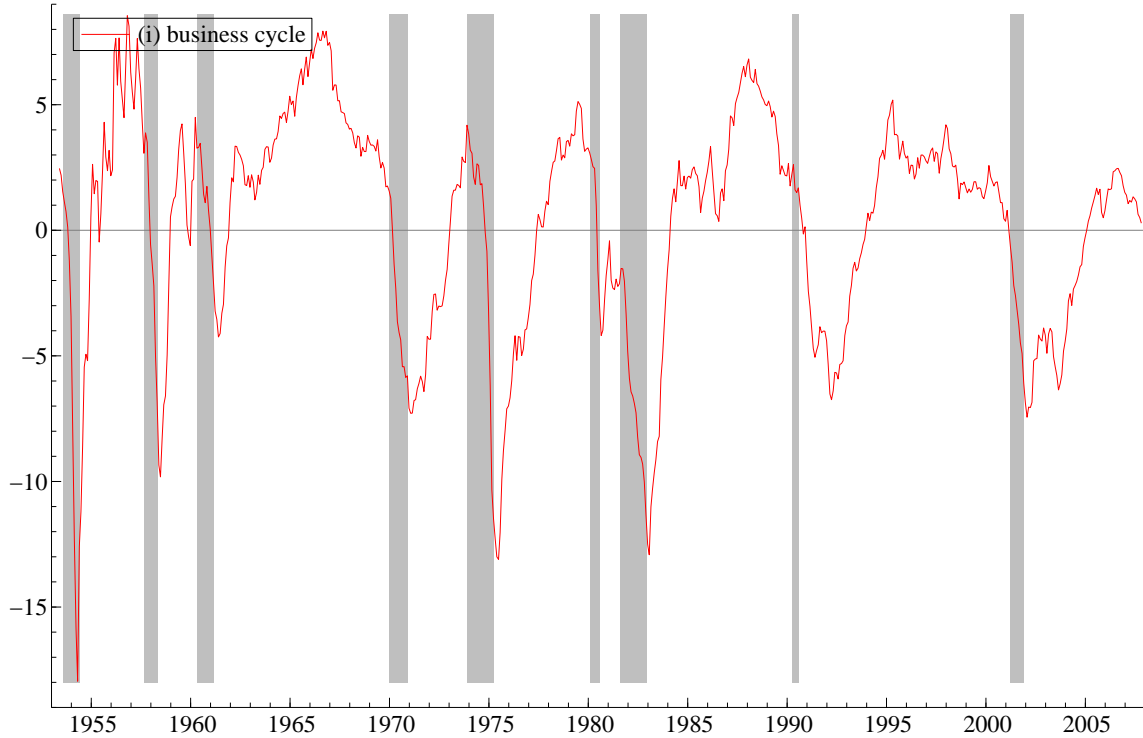


Figure 10: *Filtered estimates of the cycle computed by particle filter. This is for the main model.*

2 Additional results for the model with no stochastic volatility

	Prior δ_i		Posterior δ_i		Prior ξ_i		Posterior ξ_i	
	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
unemployment	0.00	2.00	-0.20	0.005	0.00	2.50	-2.39	0.33
manufacturing	0.00	2.00	0.30	0.015	0.00	2.50	4.32	0.41
inflation	0.00	2.00	0.29	0.030	0.00	2.50	-5.77	0.86
retail	0.00	2.00	0.60	0.036	0.00	2.50	3.79	0.45
retail/food	0.00	2.00	0.49	0.079	–	–	–	–
productivity	0.00	2.00	0.34	0.032	0.00	2.50	9.28	0.75
rgdp	0.00	2.00	0.49	0.023	0.00	2.50	4.02	0.39
hours	0.00	2.00	0.41	0.014	0.00	2.50	-0.20	0.33
consumption	0.00	2.00	0.13	0.014	0.00	2.50	5.69	0.81
investment	0.00	2.00	1.92	0.082	0.00	2.50	3.60	0.33
	Prior ρ		Posterior ρ		Prior λ		Posterior λ	
	0.50	0.08	0.866	0.009	63.56	67.02	52.01	0.009

Table 4: *Prior and posterior means and standard deviations for ρ , λ , and for each series δ_i and ξ_i . Industrial production has $\delta = 1$ and $\xi = 0$. This is for the model without stochastic volatility.*

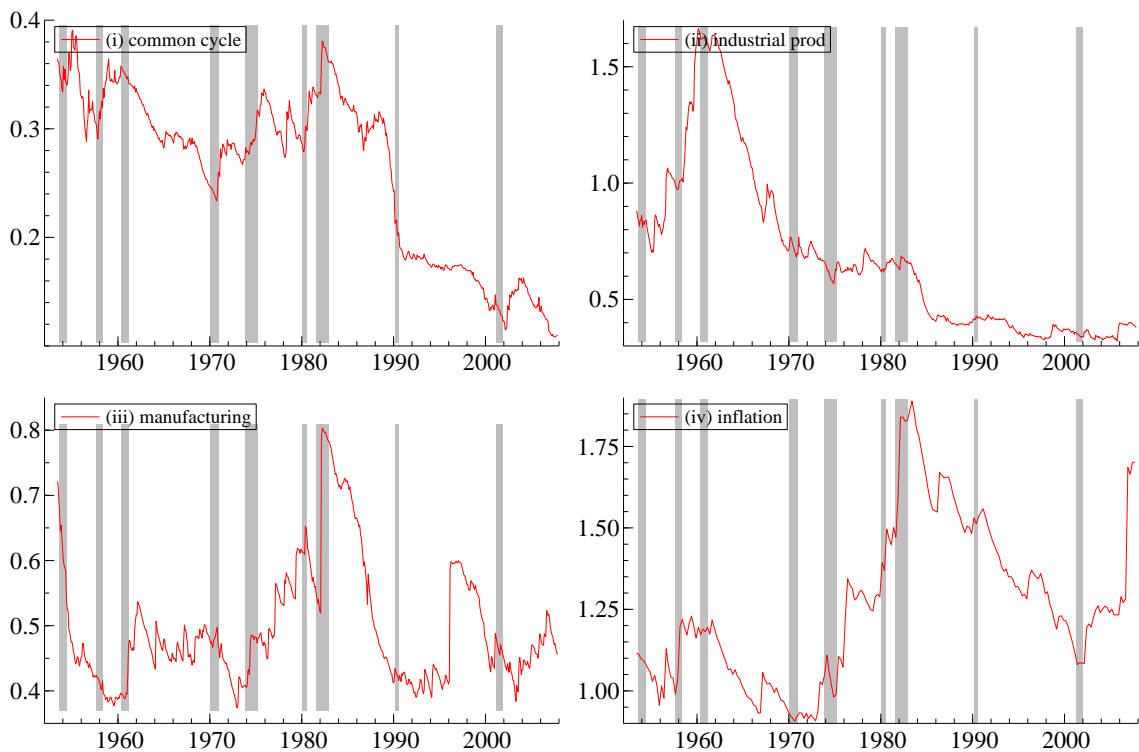


Figure 11: *Filtered estimates of the stochastic volatility computed by particle filter. This is for the main model.*

	Prior σ_{ε}^2		Posterior σ_{ε}^2	
	mean	st. dev.	mean	st. dev.
indust prod	–	–	0.3851	0.0358
unemployment	–	–	0.0141	0.0013
manufacturing	–	–	0.2720	0.0172
inflation	–	–	1.9272	0.2038
retail	–	–	1.0733	0.0736
retail/food	–	–	0.6132	0.0714
productivity	–	–	0.1890	0.0290
GDP	–	–	0.0535	0.0114
hours	–	–	0.0815	0.0113
consumption	–	–	0.0646	0.0087
investment	–	–	1.8865	0.2749
common cycle	–	–	0.1427	0.0158

Table 5: *Prior and posterior means and standard deviations of the variance of the irregular, $\sigma_{\varepsilon,i}^2$, for each series and the variance of the common cycle, σ_{κ}^2 .*

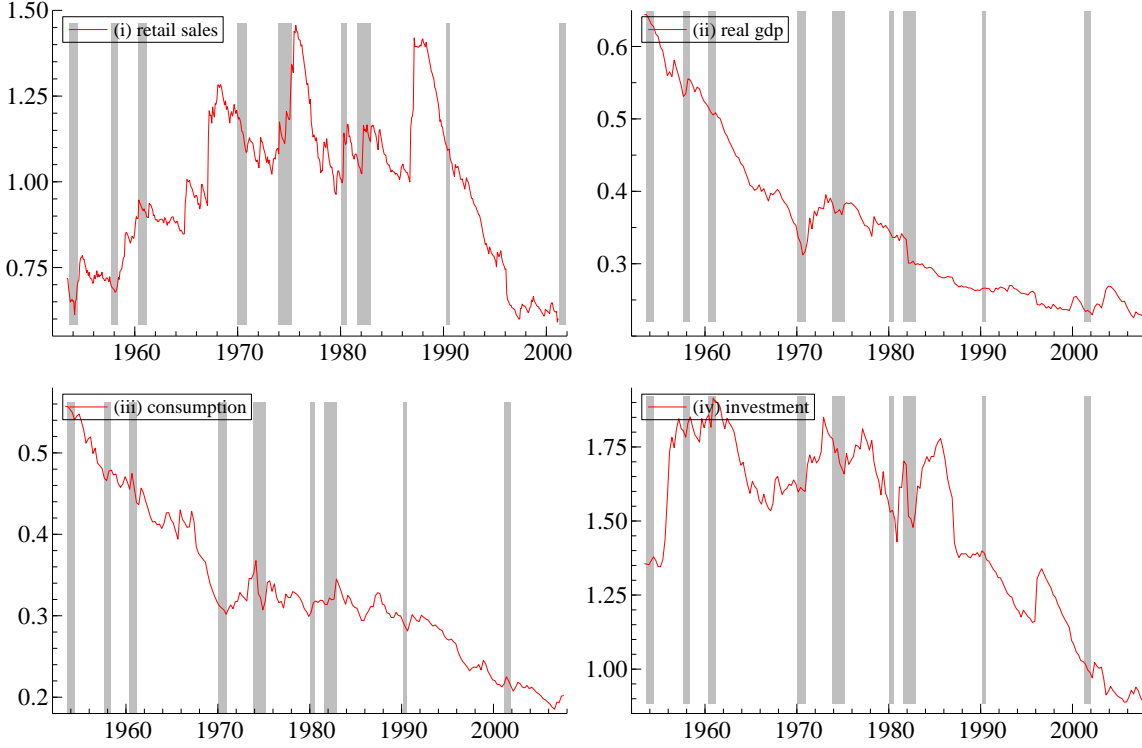


Figure 12: *Filtered estimates of the stochastic volatility computed by particle filter. This is for the main model.*

	distribution	hyperparameter 1	hyperparameter 2	hyperparameter 3	hyperparameter 4
ρ	$\mathcal{U}[0,1)$	-	-	-	-
λ	$\mathcal{B}(\alpha,\beta)$	$\alpha = 2.6377$	$\beta = 15.0577$	-	-
δ_i	$\mathcal{N}(\mu,V)$	$\mu = 0$	$V = 4$	-	-
ξ_i	$\mathcal{TN}(\mu,V,a,b)$	$\mu = 0$	$V = 6.25$	$a = -\frac{\pi}{2\lambda}$	$b = \frac{\pi}{2\lambda}$
σ_ε^2	$\mathcal{IG}(c,d)$	$c = 0$	$d = 0$	-	-
σ_κ^2	$\mathcal{IG}(e,f)$	$e = 0$	$f = 0$	-	-

Table 6: *Prior distributions for the parameters of the estimated model without stochastic volatility. \mathcal{TN} denotes the truncated normal distribution. The inverse gamma distribution is in terms of the shape and scale parameters. The beta distribution has mean $\frac{\alpha}{\alpha+\beta}$.*

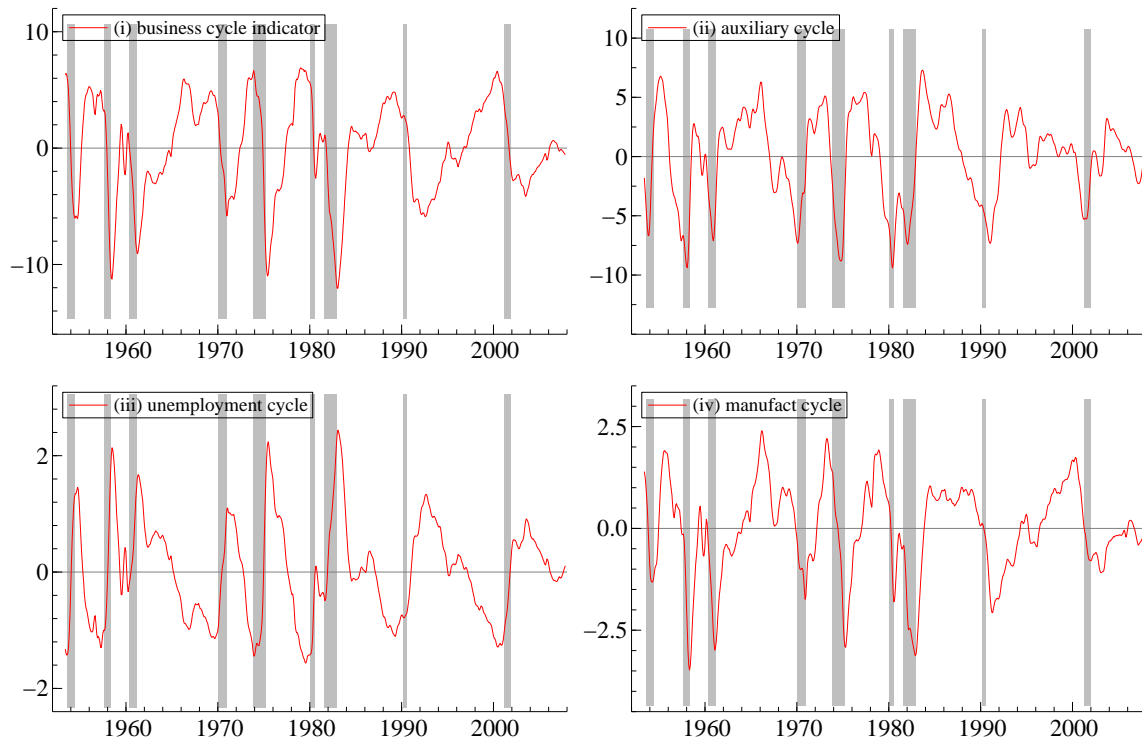


Figure 13: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) business cycle indicator; (ii) auxiliary cycle; (iii) unemployment; (iv) manufacturing. This is for the model with no stochastic volatility.*

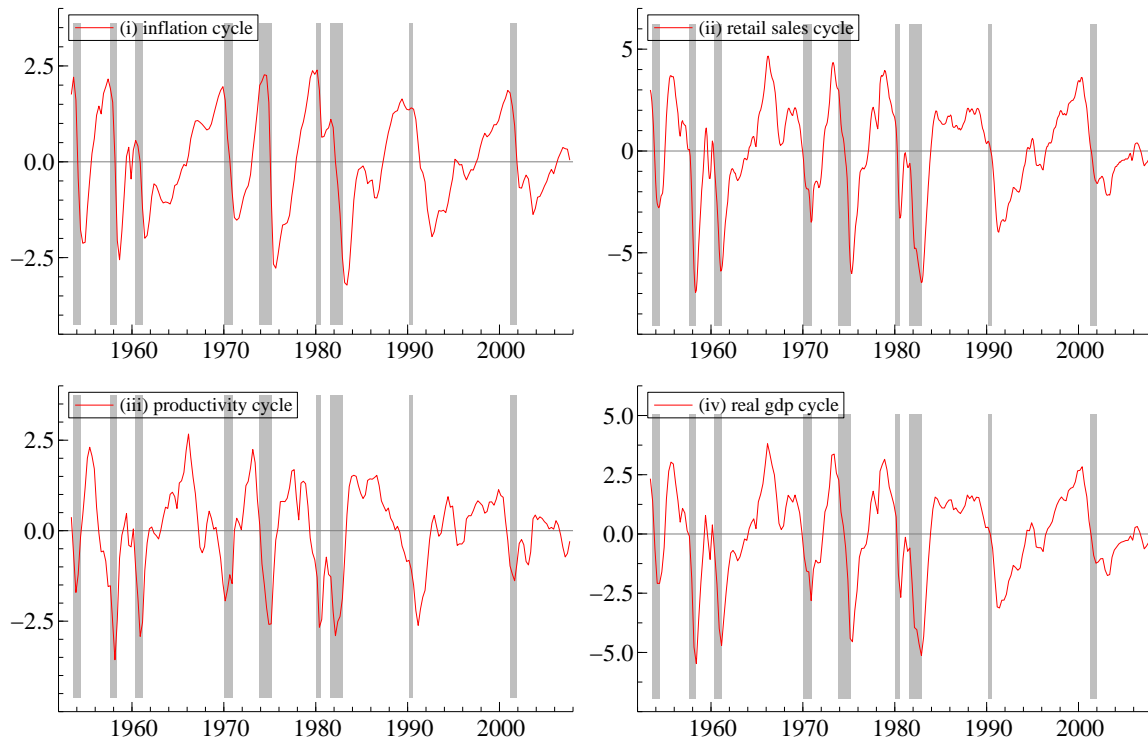


Figure 14: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) inflation; (ii) retail sales; (iii) productivity of NFBS; (iv) real gdp. This is for the model with no stochastic volatility.*

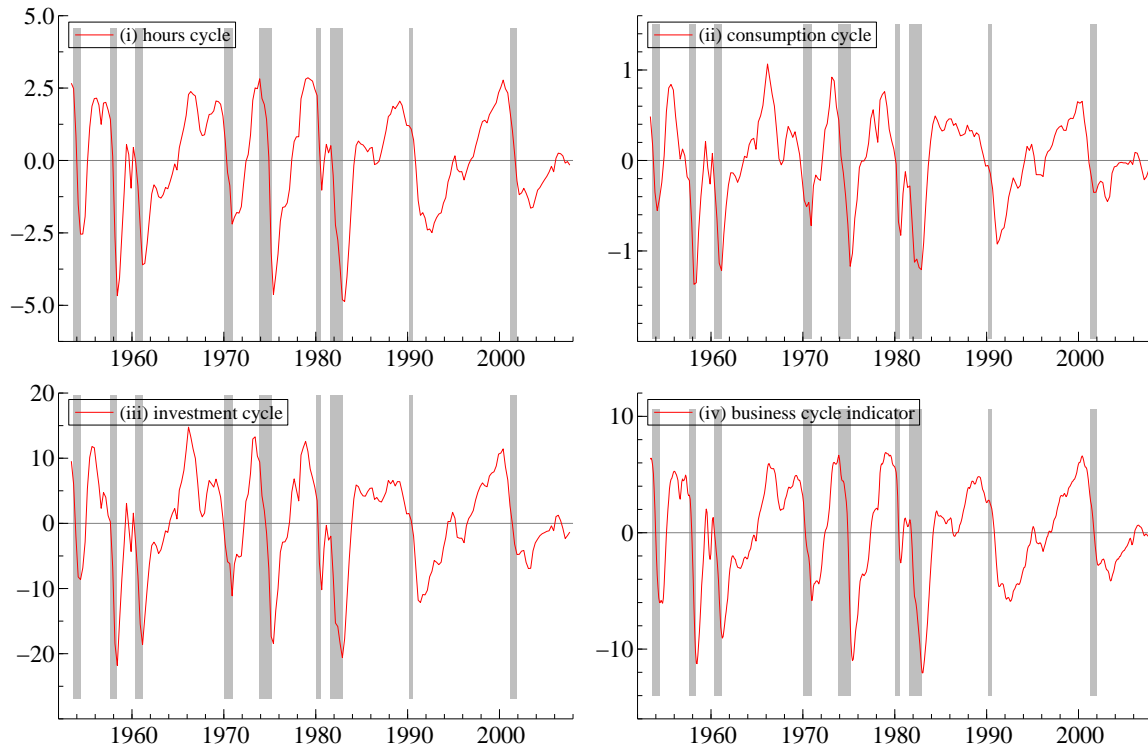


Figure 15: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) hours of NFBS; (ii) real consumption nondurables + services; (iii) real consumption of durables + real fixed investment; (iv) main indicator ψ_t . This is for the model with no stochastic volatility.*

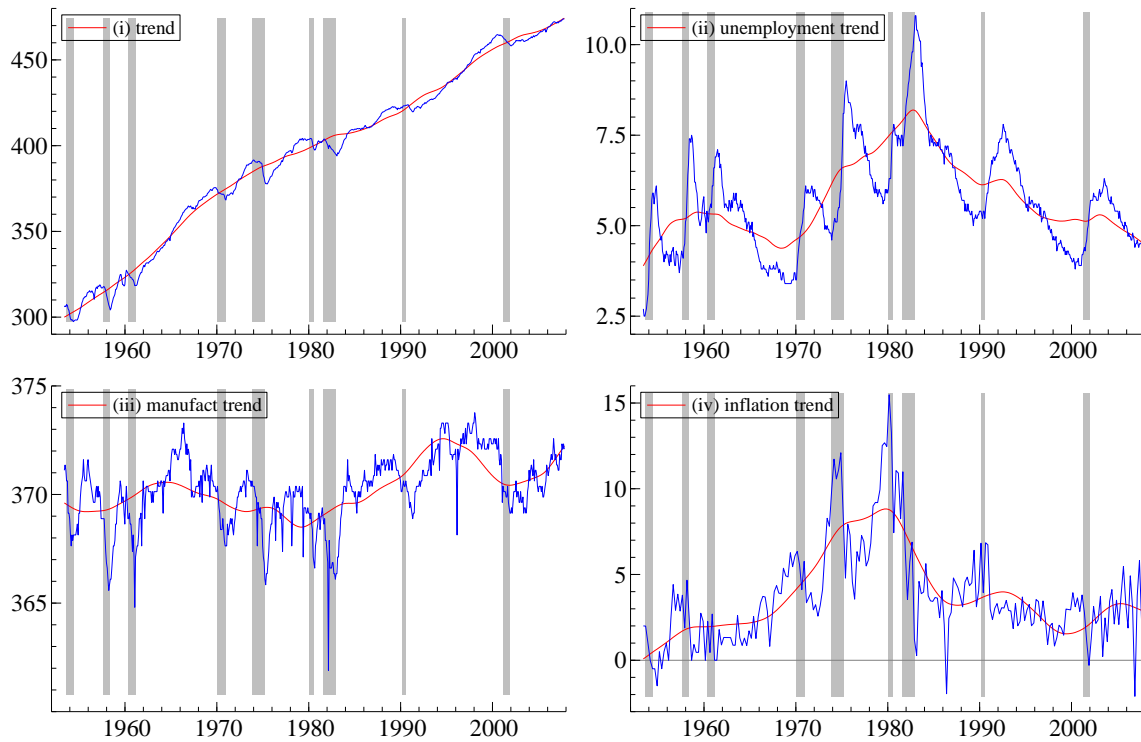


Figure 16: *Smoothed estimates of the trends. (i) industrial production; (ii) unemployment; (iii) manufacturing; (iv) inflation. The trends are plotted with each respective series. This is for the model with no stochastic volatility.*

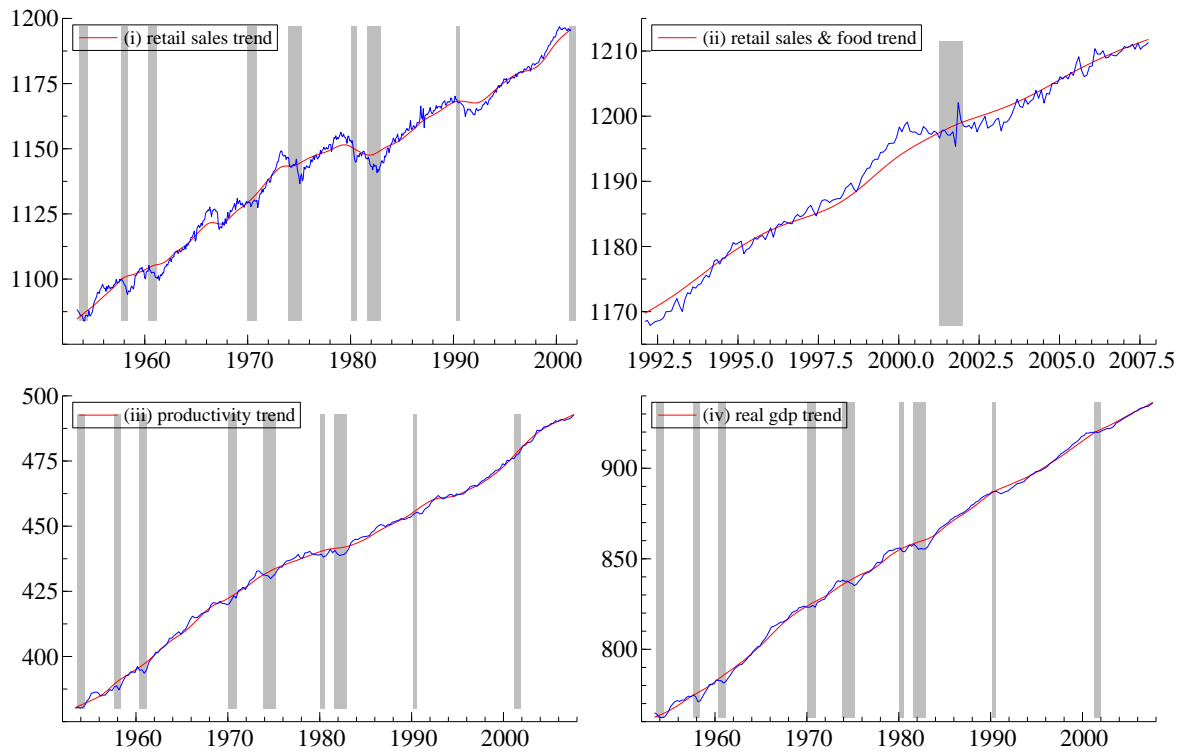


Figure 17: *Smoothed estimates of the trends. (i) retail sales; (ii) retail sales and food services; (iii) productivity of NFBS; (iv) real gdp. The trends are plotted with each respective series. This is for the model with no stochastic volatility.*

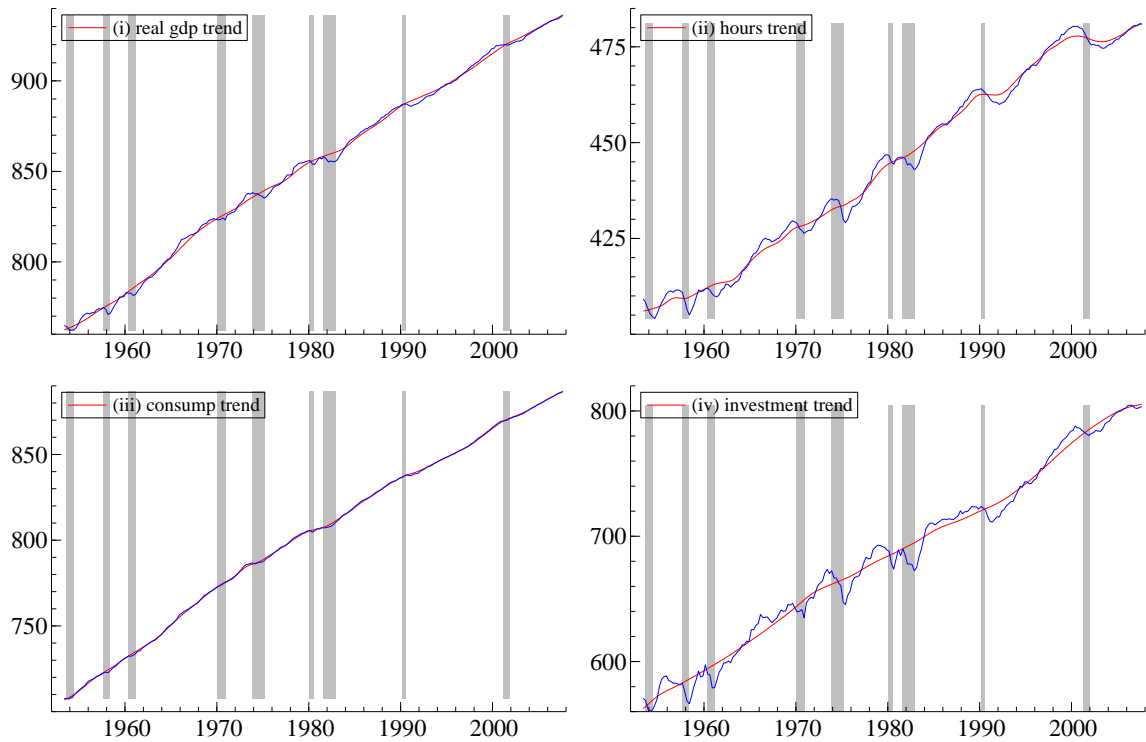


Figure 18: *Smoothed estimates of the trends. (i) real gdp; (ii) hours; (iii) real consumption of nondurables + services; (iv) real consumption of durables + real fixed investment. The trends are plotted with each respective series. This is for the model with no stochastic volatility.*

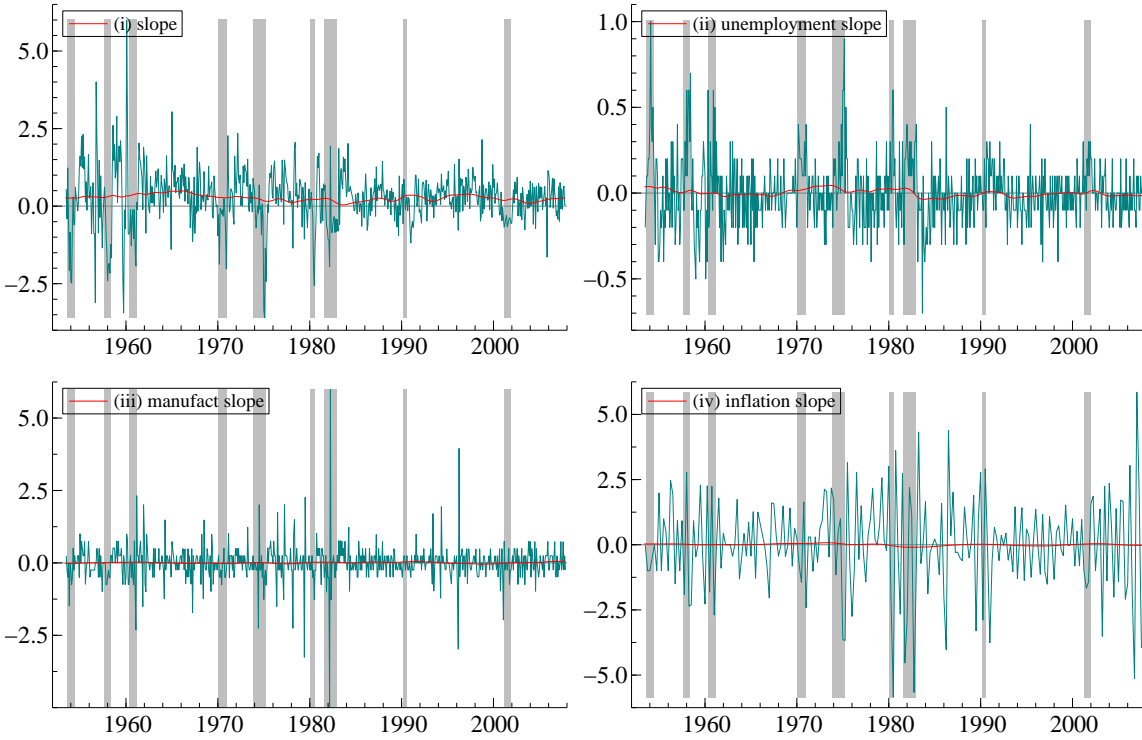


Figure 19: *Smoothed estimates of the slopes. (i) industrial production; (ii) unemployment; (iii) inflation; (iv) manufacturing. The slopes are plotted with the growth rates of the respective series. This is for the model with no stochastic volatility.*

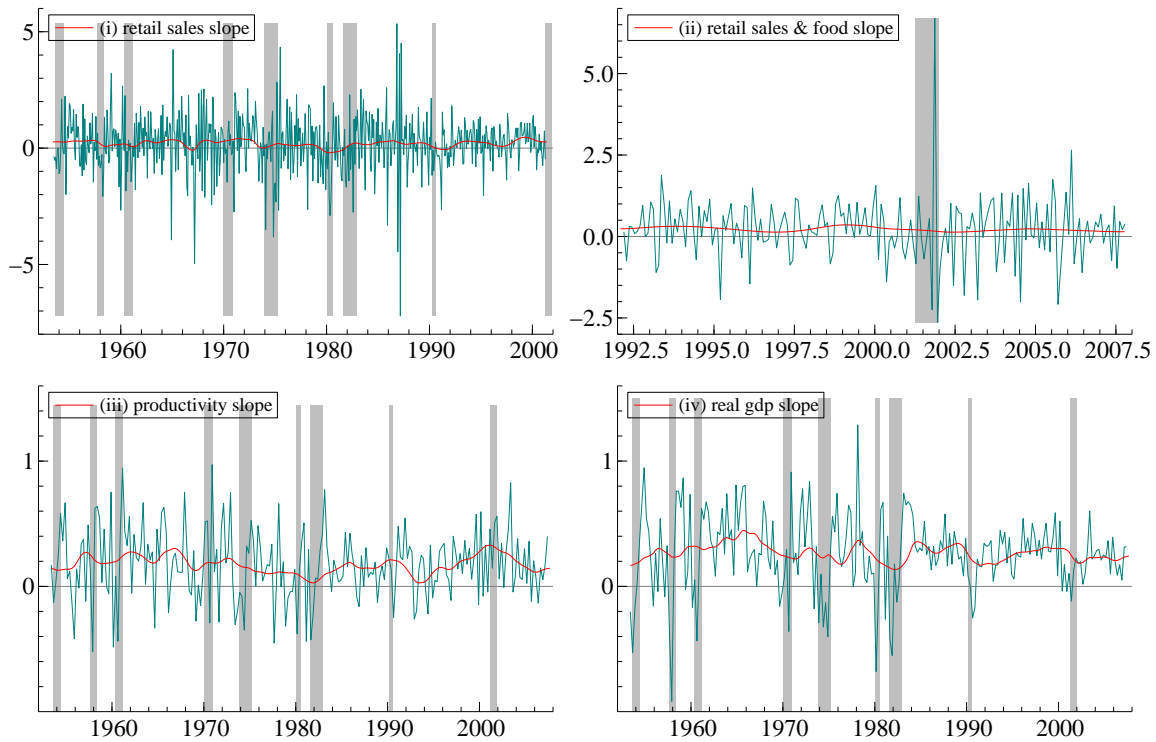


Figure 20: *Smoothed estimates of the slopes. (i) retail sales; (ii) retail sales and food services, (iii) real gdp; (iv) productivity of NFBS. The slopes are plotted with the growth rates of the respective series. This is for the model with no stochastic volatility.*

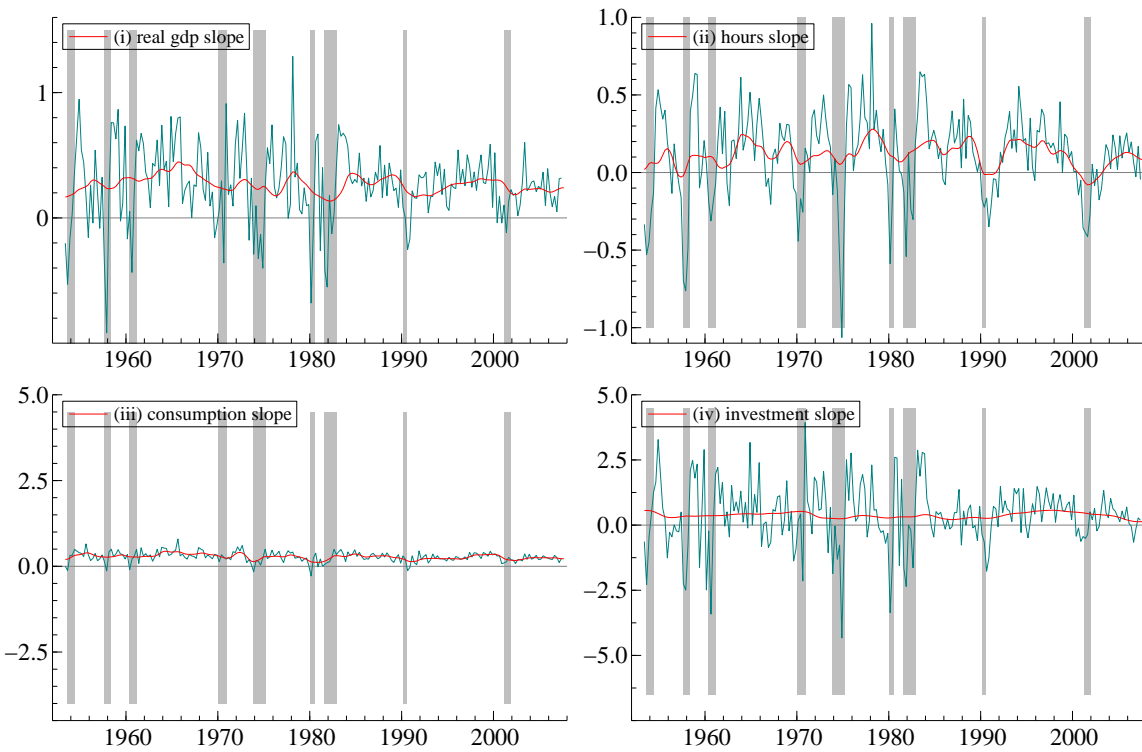


Figure 21: *Smoothed estimates of the slopes. (i) real gdp; (ii) hours NFBS; (iii) real consumption of nondurables + services; (iv) real consumption of durables + real fixed investment. The slopes are plotted with the growth rates of the respective series. This is for the model with no stochastic volatility.*

3 Additional results for the model with stochastic volatility and a known break in 1984 Q1

	Prior ξ_i		Posterior ξ_i pre-1984		Posterior ξ_i post-1984	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
unemployment	0.00	2.50	-1.082	0.164	-2.045	0.560
manufacturing	0.00	2.50	3.420	0.234	7.781	0.826
inflation	0.00	2.50	-4.535	0.590	-0.729	1.601
retail	0.00	2.50	2.470	0.367	4.893	1.079
retail/food	—	—	—	—	—	—
productivity	0.00	2.50	7.392	0.528	10.34	1.773
rgdp	0.00	2.50	2.752	0.239	6.261	0.822
hours	0.00	2.50	0.567	0.230	0.062	0.769
consumption	0.00	2.50	5.922	0.733	4.575	1.854
investment	0.00	2.50	2.766	0.211	5.422	0.621

Table 7: *Prior and posterior means and standard deviations for ξ_i . This is for the model with stochastic volatility and a known break in 1984 Q1.*

	Prior δ_i		Posterior δ_i	
	mean	st. dev.	mean	st. dev.
unemployment	0.00	2.00	-0.256	0.009
manufacturing	0.00	2.00	0.363	0.014
inflation	0.00	2.00	0.329	0.034
retail	0.00	2.00	0.689	0.040
retail/food	0.00	2.00	0.565	0.024
productivity	0.00	2.00	0.318	0.025
rgdp	0.00	2.00	0.572	0.020
hours	0.00	2.00	0.548	0.024
consumption	0.00	2.00	0.138	0.015
investment	0.00	2.00	2.234	0.090

Table 8: *Prior and posterior means and standard deviations for δ_i . This is for the model with stochastic volatility and a known break in 1984 Q1.*

	distribution	hyperparameter 1	hyperparameter 2	hyperparameter 3	hyperparameter 4
$\rho_{pre-1984}$	$\mathcal{U}[0,1]$	-	-	-	-
$\rho_{post-1984}$	$\mathcal{U}[0,1]$	-	-	-	-
$\lambda_{pre-194}$	$\mathcal{B}(\alpha,\beta)$	$\alpha = 2.6377$	$\beta = 15.0577$	-	-
$\lambda_{post-1984}$	$\mathcal{B}(\alpha,\beta)$	$\alpha = 2.6377$	$\beta = 15.0577$	-	-
δ_i	$\mathcal{N}(\mu,V)$	$\mu = 0$	$V = 4$	-	-
$\xi_{i,pre-1984}$	$\mathcal{TN}(\mu,V,a,b)$	$\mu = 0$	$V = 6.25$	$a = -\frac{\pi}{2\lambda}$	$b = \frac{\pi}{2\lambda}$
$\xi_{i,post-1984}$	$\mathcal{TN}(\mu,V,a,b)$	$\mu = 0$	$V = 6.25$	$a = -\frac{\pi}{2\lambda}$	$b = \frac{\pi}{2\lambda}$
σ_ε^2	$\mathcal{IG}(c,d)$	$c = 0$	$d = 0$	-	-

Table 9: Prior distributions for the parameters of the model with a known structural break in 1984 Q1. \mathcal{TN} denotes the truncated normal distribution. The inverse gamma distribution is in terms of the shape and scale parameters. The beta distribution has mean $\frac{\alpha}{\alpha+\beta}$.

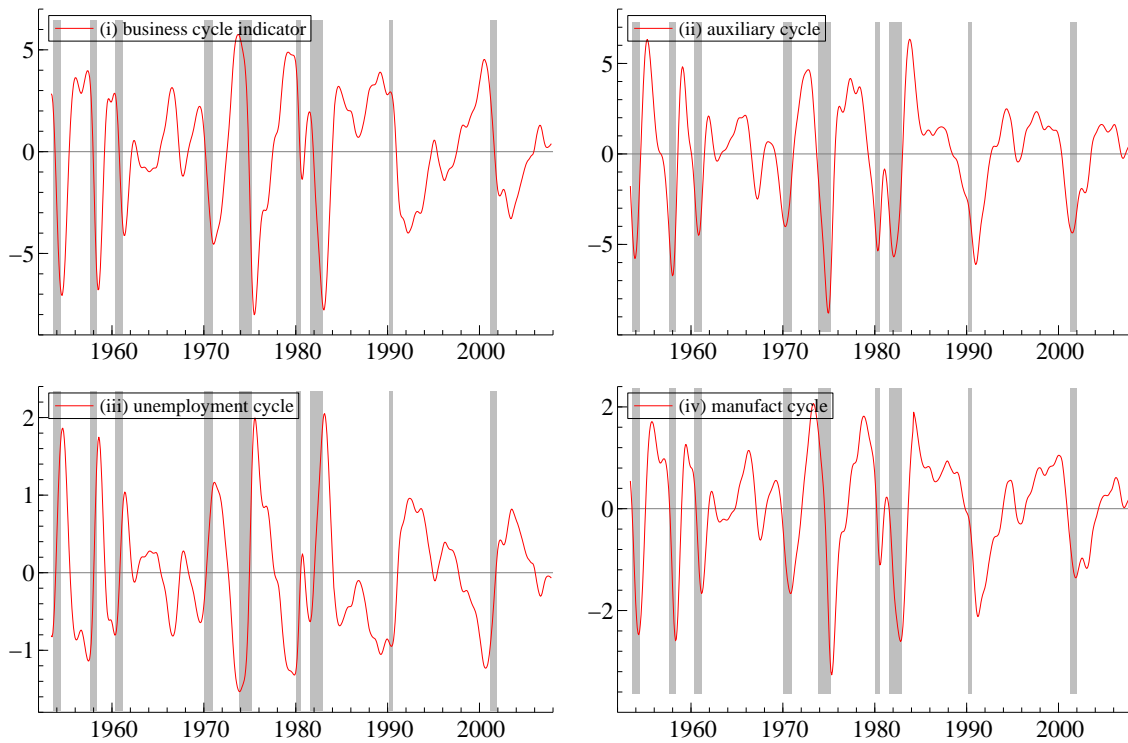


Figure 22: Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) business cycle indicator; (ii) auxiliary cycle; (iii) unemployment; (iv) manufacturing. This is for the model with a known break in 1984 Q1.

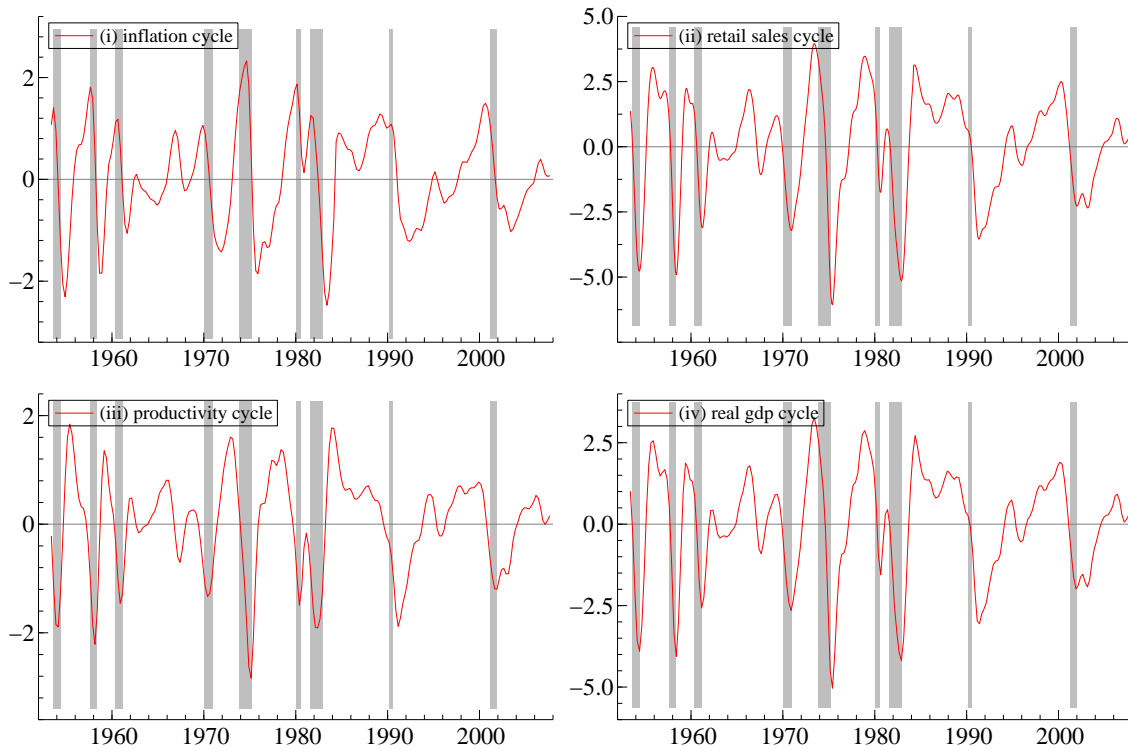


Figure 23: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) inflation; (ii) retail sales; (iii) productivity of NFBS; (iv) real gdp. This is for the model with a known break in 1984 Q1.*

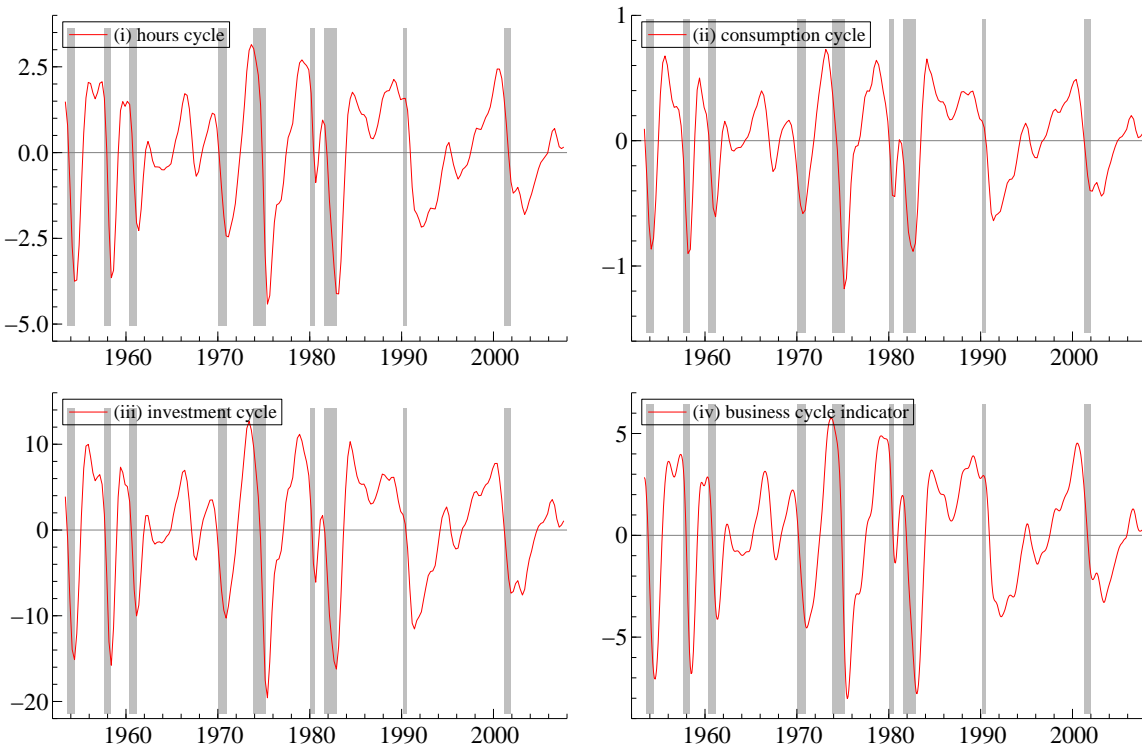


Figure 24: *Smoothed estimates of the cycles adjusted for phase shift and amplitude. (i) hours of NFBS; (ii) real consumption of nondurables + services; (iii) real consumption of durables + real fixed investment; (iv) main indicator ψ_t . This is for the model with a known break in 1984 Q1.*

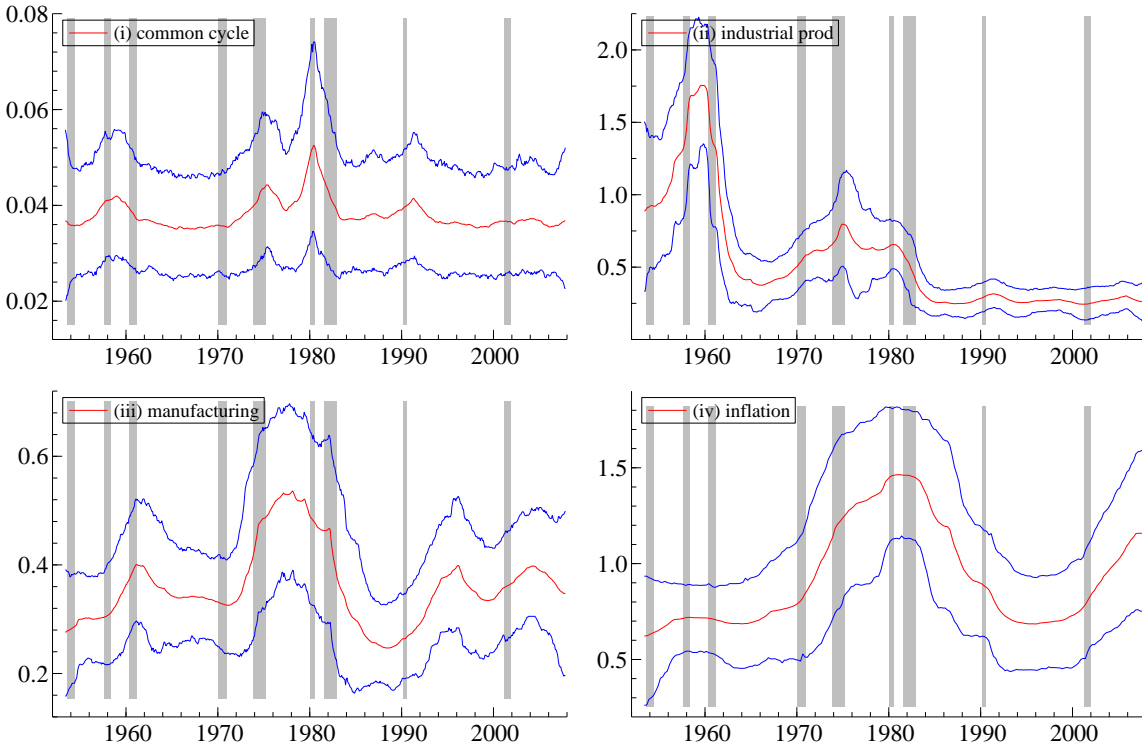


Figure 25: *Smoothed stochastic volatility* $\sigma_{t,i}$. (i) *common cycle*; (ii) *industrial production*; (iii) *manufacturing*; (iv) *inflation*. This is for the model with a known break in 1984 Q1.

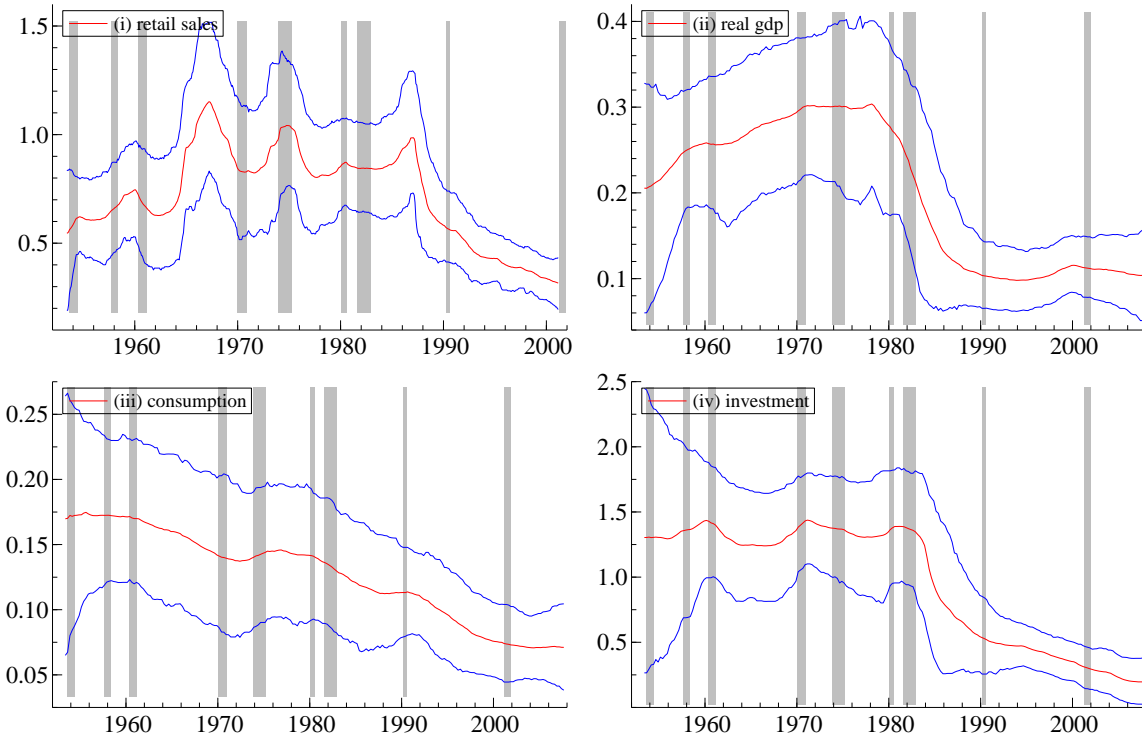


Figure 26: *Smoothed stochastic volatility $\sigma_{t,i}$. (i) retail sales; (ii) real gdp; (iii) real consumption of nondurables + services; (iv) real consumption of durables + real fixed investment. This is for the model with a known break in 1984 Q1.*

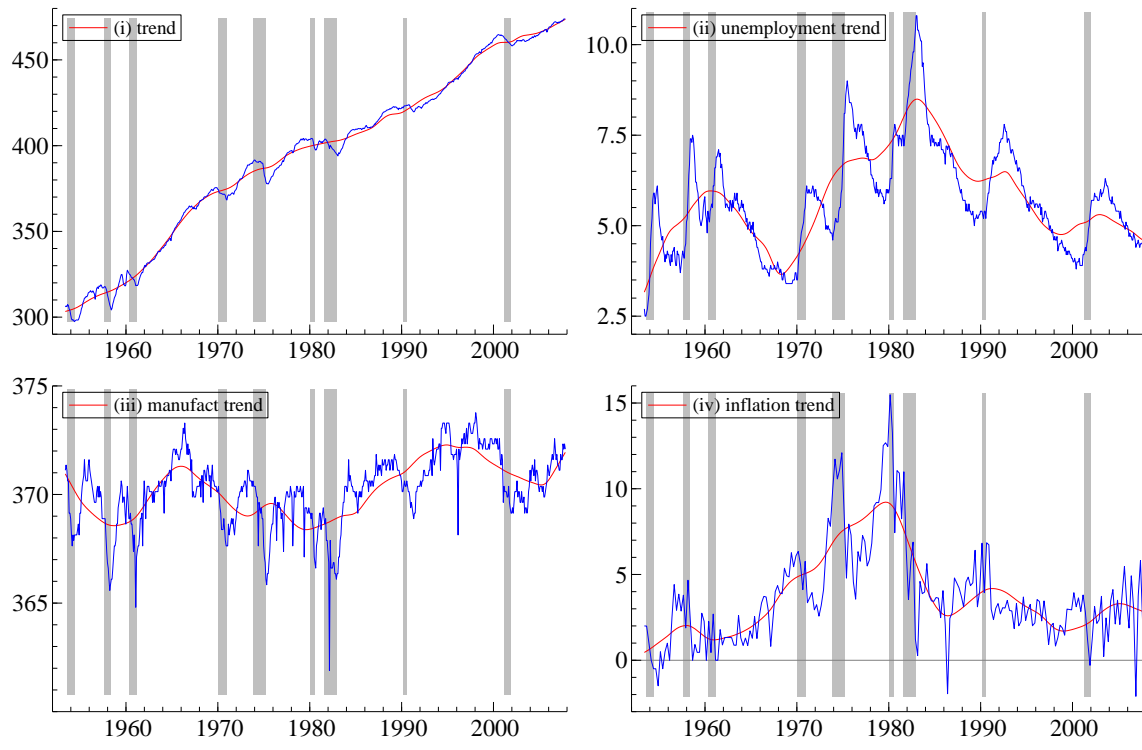


Figure 27: *Smoothed estimates of the trends. (i) industrial production; (ii) unemployment; (iii) manufacturing; (iv) inflation. The trends are plotted with each respective series. This is for the model with a known break in 1984 Q1.*

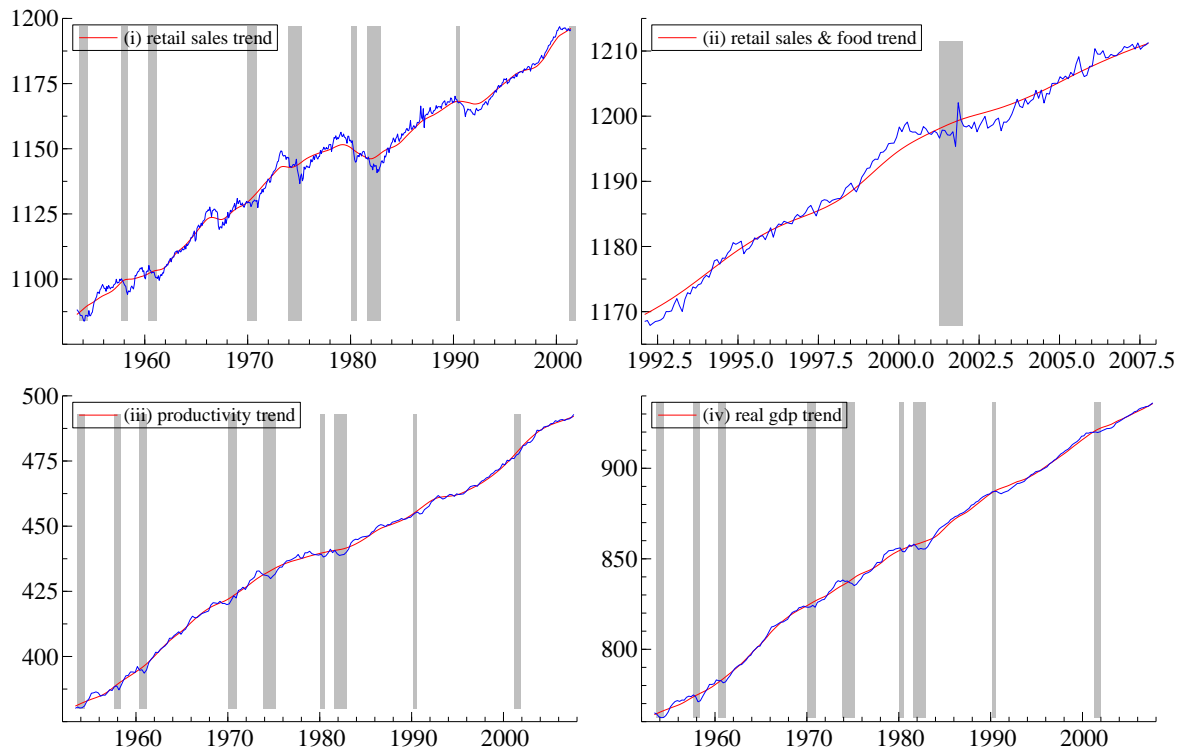


Figure 28: *Smoothed estimates of the trends. (i) retail sales; (ii) retail sales and food services; (iii) productivity of NFBS; (iv) real gdp. The trends are plotted with each respective series. This is for the model with a known break in 1984 Q1.*

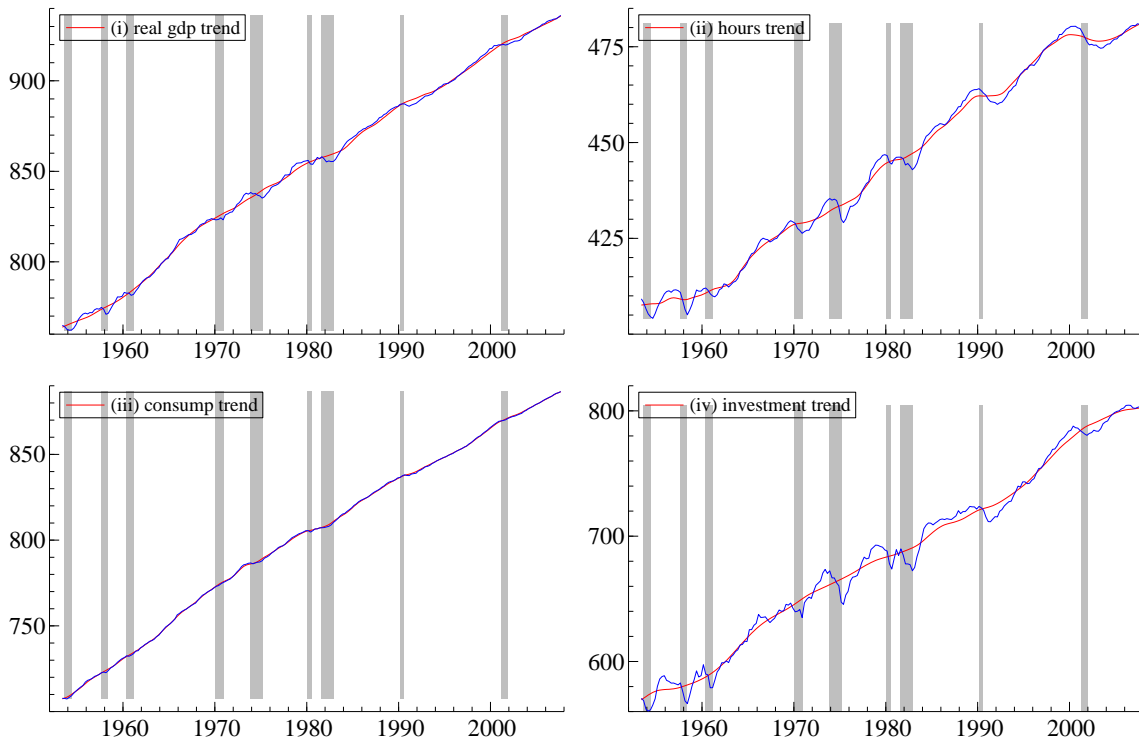


Figure 29: *Smoothed estimates of the trends. (i) real gdp; (ii) hours of NFBS; (iii) real consumption of nondurables + services; (iv) real consumption of durables + real fixed investment. The trends are plotted with each respective series. This is for the model with a known break in 1984 Q1.*

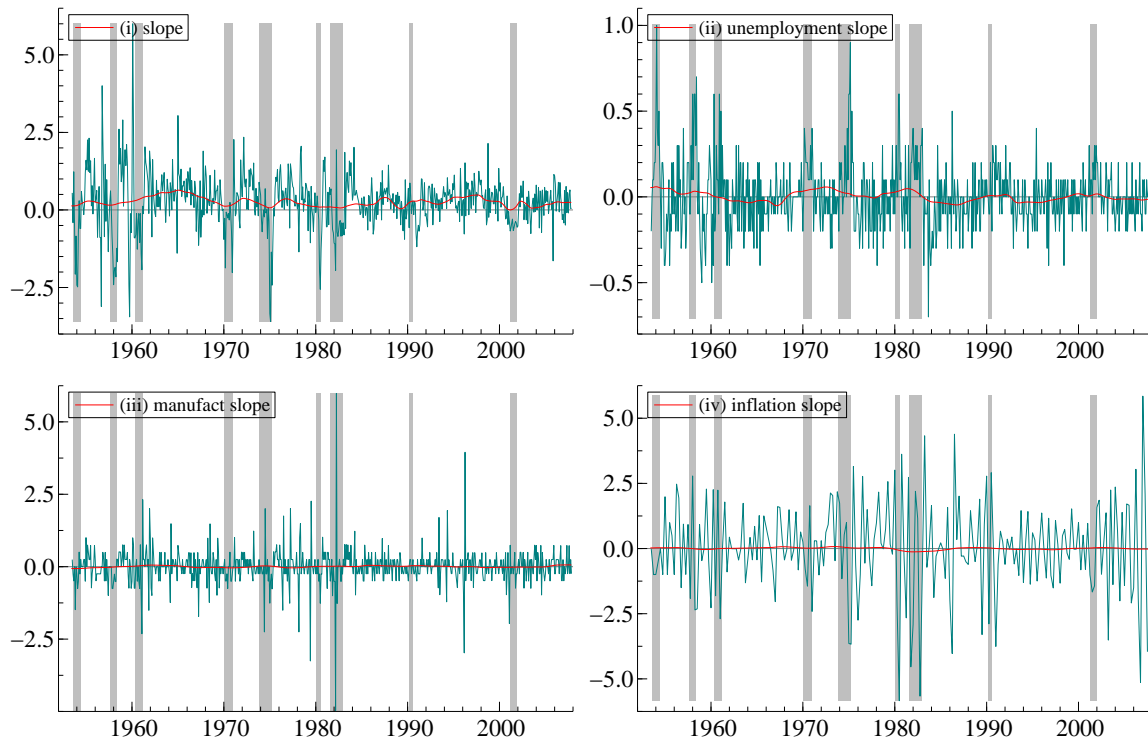


Figure 30: *Smoothed estimates of the slopes. (i) industrial production; (ii) unemployment; (iii) inflation; (iv) manufacturing. The slopes are plotted with the growth rates of the respective series. This is for the model with a known break in 1984 Q1.*

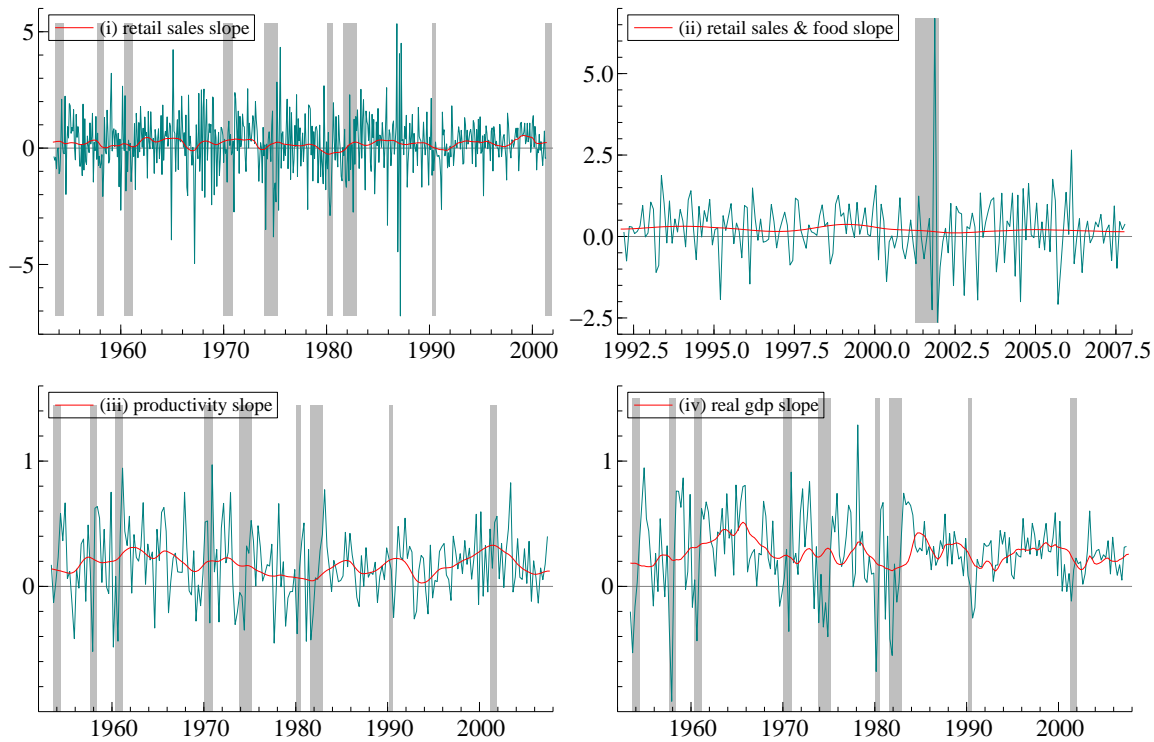


Figure 31: *Smoothed estimates of the slopes. (i) retail sales; (ii) retail sales and food services, (iii) real gdp; (iv) productivity of NFBS. The slopes are plotted with the growth rates of the respective series. This is for the model with a known break in 1984 Q1.*

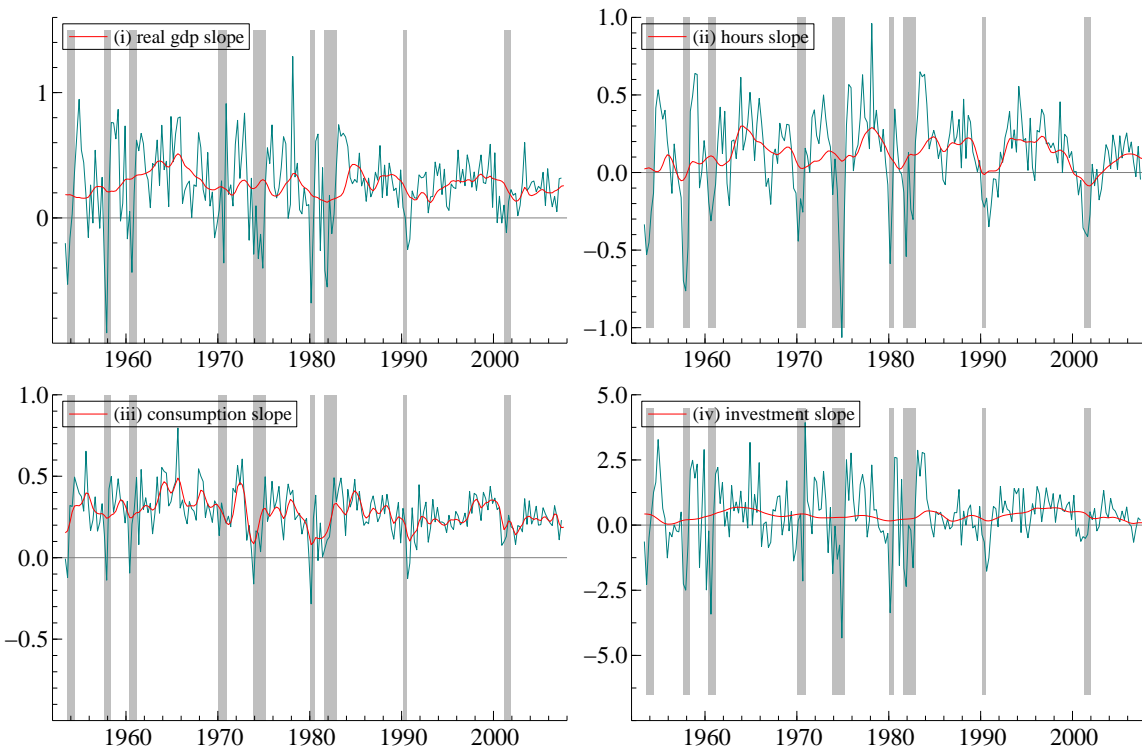


Figure 32: *Smoothed estimates of the slopes. (i) real gdp; (ii) hours of NFBS; (iii) real consumption of nondurables + services; (iv) real consumption of durables + real fixed investment. The slopes are plotted with the growth rates of the respective series. This is for the model with a known break in 1984 Q1.*