Economics Letters 72 (2001) 325–333

economics letters

www.elsevier.com/locate/econbase

Buy prices in online auctions: irrationality on the internet?

Eric B. Budish, Lisa N. Takeyama*

Amherst College, Department of Economics, Campus Box 2201, Amherst, MA 01002-5000, USA

Received 15 October 2000; accepted 10 January 2001

Abstract

Buy prices are puzzling: it makes sense for a seller at auction to set a minimum bidding level (i.e., a reserve price), surely, but a maximum? We explore the use of maximum bidding levels (buy prices) in online auctions and provide a rational explanation for this seemingly irrational auction mechanism. We show that augmenting an English auction with a buy price can improve the seller's profits by partially insuring (and therefore increasing the expected payment from) some risk-averse bidders. Perhaps more surprising is that the English auction augmented with a buy price can also be superior even to the first-price sealed-bid and Dutch auctions when bidders are risk averse.

Keywords: Buy price; Auctions

JEL classification: D44

1. Introduction

Buy prices are puzzling: it makes sense for a seller at auction to set a minimum bidding level (i.e., a reserve price), surely, but a maximum? Should we understand the countless sellers electing to use buy prices in their auctions at Yahoo! and Amazon.com to be displaying symptoms of irrationality on the Internet? Possibly, but we suspect not.

Here's how a buy price works.[1] A seller at auction announces a buy price, the existence and amount of which are publicly known. As long as the bidding is less than the buy price, the auction continues until no bidder is willing to increase the current bid. However, if a bidder bids the buy price, that

*Tel.: +1-413-542-2271; fax: +1-413-542-2090.

E-mail address: lntakeyama@amherst.edu (L.N. Takeyama).

[1]The term 'buy price' comes from Yahoo! Auctions. Amazon.com calls the same mechanism a 'take-it price'.

0165-1765/01/$ – see front matter
PII: S0165-1765(01)00438-4

bidder immediately wins the auction.[2,3] In some sense, the buy price is the functional opposite of a reserve price. While a reserve price establishes a minimum bidding level below which the seller is unwilling to part with the item, a buy price establishes a maximum bidding level at which the seller is willing to part with the item immediately. In effect, the buy price creates an upper bound on the maximum possible bid the seller may receive. Are sellers acting irrationally by intentionally capping their maximum possible payoff? We answer this question in the negative and show that, on the contrary, a seller's use of a buy price can in fact be quite rational.

In an English auction without a buy price, bidders face risky distributions of prices: a winning bidder pays a relatively high price when at least one other bidder has a similarly high valuation, and pays a relatively lower price otherwise. Augmenting an English auction with a buy price serves to facilitate risk reduction by making nonrandom the price paid by a winning buy-price bidder. Effectively, the buy price provides bidder insurance against the risky prospect that other bidders have similarly high (or higher) valuations. Consequently, augmenting an English auction with a buy price can improve the seller's profits by partially insuring (and therefore increasing the expected payment from) some risk-averse bidders.[4]

We use a simple two-bidder, two-valuation framework to illustrate how sellers can augment standard English auctions with buy prices to increase expected revenue when bidders are risk averse.[5] Perhaps more surprising, we also find that the English auction with a buy price can be superior even to the first-price sealed-bid and Dutch auctions when bidders are risk averse. The optimal auction mechanism when bidders are risk averse is quite complicated, but among the simple 'standard' auction mechanisms, the first-price sealed-bid and Dutch auctions are generally thought to be best (see for example, McAfee and McMillan, 1987).

In the following section, we present a model similar to that in Maskin and Riley (1985) to show that the seller's expected profits from the English auction can be enhanced by the use of a buy price when bidders are risk averse. In Section 3 we show that the English auction with a buy price can produce greater profits than even the first-price sealed-bid auction when bidders are risk averse. Conclusions follow in Section 4.

2. The English auction with buy price

Consider an independent private values auction in which there is one profit-maximizing seller and two bidders. Each bidder may be one of two types: high-valuation with type parameter V^{H}, or

[2] The buy-price auction thus bears some resemblance to the '$100-or-best-offer' pricing mechanism often found in secondhand markets.

[3] Ebay recently introduced a variant of the buy-price mechanism: the 'buy-it-now' price, which differs from a buy price in that the option to bid the buy-it-now price is withdrawn from all bidders once a bid less than the buy-it-now price is placed. More work is needed to understand the circumstances under which a seller would prefer this mechanism to the standard buy-price mechanism.

[4] That the seller has an incentive to reduce risk for high-value bidders is widely understood. See Maskin and Riley (1984, 1985).

[5] Although highly simplified, the two-bidder, two-valuation model has been employed to demonstrate for example, the Revenue Equivalence Theorem as well as the effects of asymmetry and risk aversion. See Maskin and Riley (1985). Nevertheless, as Klemperer (1999) notes, the two-bidder, two-valuation framework is mainly illustrative and does not always capture well the effects of some auction characteristics (e.g., correlated values).

low-valuation with type parameter V^L, $V^H > V^L$. Let p represent the probability a bidder is of type V^L and therefore $(1-p)$ is the probability a bidder is of type V^H. The utility of a bidder of type V^i that wins the auction with bid b is $U(V^i - b)$, where $i = \mathrm{H}$, L. We assume that $U(V^i - b)$ is twice differentiable and that $U' > 0$. The utility of a bidder who loses the auction and pays nothing is given by $U(0)$, where $U(0)$ is normalized to zero.

In the absence of a buy price, it is well known that it is a dominant strategy for each bidder to remain in the auction until the bidding exceeds the bidder's valuation. In the context of the present model, this implies that the expected revenue to the seller is given by:

$$E\Pi^1 = p^2 V^L + 2p(1-p)V^L + (1-p)^2 V^H. \tag{1}$$

Now consider the English auction augmented with a buy price. Since it is readily apparent that the optimal buy price can never be less than V^L, a low-valuation type's strategy is unaffected by the introduction of an optimal buy price. With this in mind, we now look for a symmetric equilibrium in which high-valuation bidders bid the buy price immediately while low-valuation types drop out at their reservation values. Given a buy price of B, the equilibrium expected utility for a high-valuation type will be given by:

$$pU(V^H - B) + \frac{(1-p)}{2}U(V^H - B), \text{ or } \frac{1+p}{2}U(V^H - B) \tag{2}$$

where we make the standard assumption that in the event of a tie (i.e., both bidders bid the buy price), the winning bidder is determined by a coin toss. Consider next the incentive for a high-valuation type to deviate from an immediate bid of the buy price. The largest potential benefit from deferring a buy-price bid occurs when the other bidder is a low-valuation type and the deviating high-valuation bidder wins the auction for a bid of V^L. Since this event occurs with at most probability p, the largest possible expected payoff for a high-valuation type who deviates from an immediate buy-price bid is:

$$pU(V^H - V^L). \tag{3}$$

Therefore, a symmetric equilibrium in which high-valuation bidders bid the buy price immediately while low-valuation types drop out at their reservation values will exist for any buy price B satisfying $B \geq V^L$ and:[6]

$$pU(V^H - B) + \frac{(1-p)}{2}U(V^H - B) \geq pU(V^H - V^L) \tag{4}$$

$$\Leftrightarrow U(V^H - B) \geq \frac{2p}{(1+p)}U(V^H - V^L) \tag{5}$$

$$\Leftrightarrow B \leq V^H - U^{-1}\left(\left(\frac{2p}{1+p}\right)U(V^H - V^L)\right) \tag{6}$$

Furthermore, since the seller will wish to establish the largest buy price possible, subject to the

[6] In fact, given a buy price satisfying (6), the resulting symmetric equilibrium is unique.

constraint in (6), the optimal buy price is given by:[7,8]

$$B^* = V^{H} - U^{-1}\left(\left(\frac{2p}{1+p}\right)U(V^{H} - V^{L})\right), \tag{7}$$

which generates expected revenue of:

$$E\Pi^2 = p^2V^{L} + (1-p^2)\left[V^{H} - U^{-1}\left(\left(\frac{2p}{1+p}\right)U(V^{H} - V^{L})\right)\right]. \tag{8}$$

The seller is thus better off with the addition of a buy price if and only if $E\Pi^2 > E\Pi^1$ or:

$$p^2V^{L} + (1-p^2)\left[V^{H} - U^{-1}\left(\left(\frac{2p}{1+p}\right)U(V^{H} - V^{L})\right)\right] > p^2V^{L} + 2p(1-p)V^{L} + (1-p)^2V^{H}. \tag{9}$$

$$\Leftrightarrow \left(\frac{2p}{1+p}\right)(V^{H} - V^{L}) > U^{-1}\left(\left(\frac{2p}{1+p}\right)U(V^{H} - V^{L})\right) \tag{10}$$

$$\Leftrightarrow U\left(\left(\frac{2p}{1+p}\right)(V^{H} - V^{L})\right) > \left(\frac{2p}{1+p}\right)U(V^{H} - V^{L}). \tag{11}$$

Since $2p/(1+p) \leq 1$, it follows that the inequality in (11) holds if and only if $U'' < 0$, or bidders are risk averse. That is, when bidders are risk averse, the seller is strictly better off with the addition of a buy price. On the other hand, if $U'' = 0$, such that bidders are risk neutral, the expected revenues in the two auctions are equivalent.[9,10]

3. Comparison to the first-price sealed-bid auction

In the first-price sealed-bid auction with two valuations, the low-valuation type simply bids his valuation while the high-valuation type randomizes his bid (see for example, Maskin and Riley, 1985). Specifically (as shown in Appendix A), the high-valuation type randomizes his bid according to the distribution function, $F(b)$, where

---

[7] Strictly speaking, the optimal buy price should actually be understood to be some arbitrarily small amount, ϵ, less than that defined in (7).

[8] Observe that as $p \to 0$ or 1, the optimal buy price converges to V^{H} or V^{L}, respectively.

[9] Another way to see this is to consider the lottery faced by a high-valuation bidder in the absence of a buy price: [p: $V^{H} - V^{L}$; $(1-p)$: 0]. With the addition of a buy price, B, the lottery becomes: [$(1+p)/2$: $V^{H} - B$; $(1-p)/2$: 0]. If $B = V^{H} - (2p/(1+p))(V^{H} - V^{L})$, the buy price lottery second-order stochastic dominates the first. Therefore, if bidders are risk averse, the seller is able to raise B to B^*, which extracts a premium for risk reduction.

[10] Note however, that in a more general framework with N valuations, the optimal buy price may be less than the second-highest valuation, which admits the possibility of inefficient outcomes. In this case, revenue equivalence breaks down and the effectiveness of the buy price to enhance sellers' profits when bidders are risk averse may be diminished.

$$F(b)=\frac{pU(V^{H}-V^{L})}{(1-p)U(V^{H}-b)}-\frac{p}{(1-p)}, \tag{12}$$

and where $V^{L}\le b\le B^{\max}$, with

$$B^{\max}=V^{H}-U^{-1}(pU(V^{H}-V^{L})). \tag{13}$$

The expected revenue to the seller is given by:

$$E\Pi^{3}=p^{2}V^{L}+2p(1-p)\int_{V^{L}}^{B^{\max}} bf(b)\,\mathrm{d}b+(1-p)^{2}\int_{V^{L}}^{B^{\max}} 2bf(b)F(b)\,\mathrm{d}b \tag{14}$$

where the density function $f(b)$ is given by:

$$f(b)=\frac{pU(V^{H}-V^{L})U'(V^{H}-b)}{(1-p)(U(V^{H}-b))^{2}} \tag{15}$$

It is well established that, when bidders are risk averse, the expected revenue to the seller is higher in the first-price sealed-bid auction than in the English auction (see for example McAfee and McMillan, 1987). The main question we are interested in here is whether or not the seller's expected revenue can be higher in the English auction with a buy price than in the first-price sealed-bid auction. Notice first that $B^{\max}\ge B^{*}$, or the maximum (random) bid in the first-price sealed-bid auction is never less than the optimal buy price in the English auction. In fact, it is relatively straightforward to show that the buy price is actually the median bid of the randomizing high-value bidders in the first-price sealed-bid auction, a fact which will be useful shortly.[11]

Comparing the seller's expected revenue in (14) to that in (8), we see that the seller is better off using the English auction augmented with a buy price of B^{*} than the first-price sealed-bid auction if and only if:

$$(1-p^{2})\left[V^{H}-U^{-1}\left(\left(\frac{2p}{1+p}\right)U(V^{H}-V^{L})\right)\right]>2p(1-p)\int_{V^{L}}^{B^{\max}} bf(b)\,\mathrm{d}b$$
$$+(1-p)^{2}\int_{V^{L}}^{B^{\max}} 2bf(b)F(b)\,\mathrm{d}b \tag{16}$$

$$\Leftrightarrow(1-p^{2})B^{*}>2p(1-p)\int_{V^{L}}^{B^{\max}} bf(b)\,\mathrm{d}b+(1-p)^{2}\int_{V^{L}}^{B^{\max}} 2bf(b)F(b)\,\mathrm{d}b \tag{17}$$

[11] From (5) and the definition of the optimal buy price (which implies (5) holds with equality), we have: $U(V^{H}-B^{*})=2p/(1+p)U(V^{H}-V^{L})$. Substituting for $U(V^{H}-B^{*})$ in $F(B^{*})$ gives $F(B^{*})=[(1+p)pU(V^{H}-V^{L})]/[2p(1-p)U(V^{H}-V^{L})]-p/(1-p)=1/2$.

$$\Leftrightarrow \frac{(1+p)}{2} B^* > p \int_{V^L}^{B^{\max}} b f(b)\, \mathrm{d}b + (1-p) \int_{V^L}^{B^{\max}} b f(b) F(b)\, \mathrm{d}b \tag{18}$$

As demonstrated in Appendix A the condition in (18) is equivalent to:

$$\frac{B^*}{U(V^H - B^*)} > \int_{V^L}^{B^{\max}} f(b) \frac{b}{U(V^H - b)}\, \mathrm{d}b, \tag{19}$$

or

$$g(B^*) > \int_{V^L}^{B^{\max}} f(b) g(b)\, \mathrm{d}b, \tag{20}$$

where $g(b) = b/[U(V^H - b)]$, defined over $V^L \le b \le B^{\max}$. Since $g(b)$ is monotonically increasing, and B^* is the value of the median bid for randomizing high-value bidders in the first-price sealed-bid auction, it must be the case that $g(B^*)$ is the median value of $g(b)$. Also, since the right hand side of the inequality in (20) is the expected value of $g(b)$, the inequality can be established by demonstrating that the median of $g(b)$ exceeds its expectation. As shown in Appendix A, the density of $g(b)$ is given by:

$$h(b) = \frac{p}{(1-p)} U(V^H - V^L) \left[\frac{U'(V^H - b)}{U(V^H - b) + bU'(V^H - b)} \right]. \tag{21}$$

Furthermore,

$$h'(b) = \frac{-p}{(1-p)} U(V^H - V^L) \left[\frac{U(V^H - b) U''(V^H - b)}{(U(V^H - b) + bU'(V^H - b))^2} \right] \tag{22}$$

Notice that $h(b)$ is strictly increasing over the appropriate range of b provided $U'' < 0$, i.e., bidders are risk averse. Furthermore, given a strictly increasing density function, it must be the case that the median exceeds the mean, which proves the inequality in (18). Therefore, we conclude that, when bidders are risk averse, the expected revenue to the seller in the English auction with an optimal buy price can exceed even that generated by the first-price sealed-bid auction.[12]

4. Conclusions

This paper has proposed a rational explanation for a seemingly irrational auction mechanism: the buy price. We argue that, by reducing risk for some high-value bidders, the buy price enables the seller to extract a premium for risk reduction. Consequently, when bidders are risk averse, an

[12] Note that if $p = 0$ or 1, the seller's revenues in the two auctions are equivalent.

optimally set buy price can increase the expected payment to the seller. Of particular note is that the seller's expected revenue is potentially higher in an English auction with a buy price than in either a standard first-price sealed-bid or Dutch auction. While the optimal auction mechanism when bidders are risk averse is quite complicated, among the simple 'standard' auction mechanisms, the first-price and Dutch auctions have generally been thought to be best.

Clearly, the natural extension of this work is to consider a more general setting with N bidders, perhaps having continuously distributed valuations. As noted previously, this would admit the possibility of inefficient outcomes under certain conditions, in which case the effectiveness of the buy price to augment the seller's profits may be diminished. Another possibility is to consider the relative advantages and disadvantages of a buy price vis-à-vis the reserve price as well as the possibility that the two may coexist. It is worth noting that such an analysis would necessarily encompass take-it-or-leave-it mechanisms, since, in effect, a buy price set equal to the reserve price becomes a take-it-or-leave-it offer.[13]

Acknowledgements

We would like to thank Walter Nicholson and Geoffrey Woglom for their helpful comments and suggestions on an earlier draft of this paper. Of course, any remaining errors are entirely our own.

Appendix A

1. Derivation of the distribution function $F(b)$ and $b^{\max}$:

   Given a high-valuation bidder randomizes according to $F(b)$, $b \in [V^{L}, B^{\max}]$, the probability that the bidder wins the auction is given by $[p + (1-p)F(b)]$. In the optimal mixed strategy, the bidder will select $F(b)$ such that the bidder's expected utility is constant:

$$[p + (1-p)F(b)]U(V^{H} - b) = EU = \text{constant} \tag{A.1}$$

$$\Rightarrow F(b) = \frac{EU}{(1-p)U(V^{H} - \mathrm{b})} - \frac{p}{(1-p)} \tag{A.2}$$

   Since $F(V^{L}) = 0$, EU must be equal to $pU(V^{H} - V^{L})$. Therefore,

$$F(b) = \frac{pU(V^{H} - V^{L})}{(1-p)U(V^{H} - b)} - \frac{p}{(1-p)}. \tag{A.3}$$

   Finally, since $F(B^{\max}) = 1$, we may solve for $B^{\max} = V^{H} - U^{-1}(pU(V^{H} - V^{L}))$.

2. Demonstration that the condition

[13] For some preliminary work on extending the model to include bidder entry, correlated values, and reserve prices, see Budish (2000).

$$\frac{(1+p)}{2}B^* > p\int_{V^L}^{B^{max}} bf(b)\,db + (1-p)\int_{V^L}^{B^{max}} bf(b)F(b)\,db$$

is equivalent to the condition

$$\frac{B^*}{U(V^H - B^*)} > \int_{V^L}^{B^{max}} f(b)\frac{b}{U(V^H - b)}\,db:$$

From 1) above, we have that:

$$[p + (1-p)F(b)]U(V^H - b) = pU(V^H - V^L). \tag{A.4}$$

When $b = B^*$, (A.4) becomes:

$$U(V^H - B^*) = \frac{2p}{(1+p)}U(V^H - V^L), \tag{A.5}$$

since $F(B^*) = 1/2$, as discussed in the text and demonstrated in footnote 11. Multiplying both sides of (A.5) by $B^*$and rearranging gives: $(1+p)/2B^* = B^*pU(V^H - V^L)/U(V^H - B^*)$. Next, multiplying both sides of (A.4) by $f(b)b/U(V^H - b)$ then integrating, gives:

$$p\int_{V^L}^{B^{max}} bf(b)\,db + (1-p)\int_{V^L}^{B^{max}} bf(b)F(b)\,db = pU(V^H - V^L)\int_{V^L}^{B^{max}} \frac{bf(b)}{U(V^H - b)}\,db. \tag{A.6}$$

Making appropriate substitutions, the inequality given by

$$\frac{(1+p)}{2}B^* > p\int_{V^L}^{B^{max}} bf(b)\,db + (1-p)\int_{V^L}^{B^{max}} bf(b)F(b)\,db$$

can now be written:

$$\frac{B^*pU(V^H - V^L)}{U(V^H - B^*)} > pU(V^H - V^L)\int_{V^L}^{B^{max}} \frac{bf(b)}{U(V^H - b)}\,db \tag{A.7}$$

$$\Rightarrow \frac{B^*}{U(V^H - B^*)} > \int_{V^L}^{B^{max}} f(b)\frac{b}{U(V^H - b)}\,db \tag{A.8}$$

3. Demonstration that the density of $g(b)$ (where $g(b) = b/U(V^H - b)$) is equal to

$$h(b) = \frac{p}{(1-p)}U(V^H - V^L)\left[\frac{U'(V^H - b)}{U(V^H - b) + bU'(V^H - b)}\right]:$$

Since $g(b)$ is a monotonic function of b, the density of $g(b)$ is given by $f(b)\mathrm{d}b/\mathrm{d}g$, where $f(b)$ is the density of b, given by:

$$f(b) = F'(b) = \frac{pU(V^{\mathrm{H}} - V^{\mathrm{L}})U'(V^{\mathrm{H}} - b)}{(1-p)(U(V^{\mathrm{H}} - b))^2}. \tag{A.9}$$

It is straightforward to show

$$\frac{\mathrm{d}g}{\mathrm{d}b} = \frac{U(V^{\mathrm{H}} - b) + bU'(V^{\mathrm{H}} - b)}{(U(V^{\mathrm{H}} - b))^2}$$

and therefore, using monotonicity of

$$g(b), \frac{\mathrm{d}b}{\mathrm{d}g} = \left[\frac{\mathrm{d}g}{\mathrm{d}b}\right]^{-1} = \frac{(U(V^{\mathrm{H}} - b))^2}{U(V^{\mathrm{H}} - b) + bU'(V^{\mathrm{H}} - b)}.$$

Therefore,

$$f(b)\frac{\mathrm{d}b}{\mathrm{d}g} = \left[\frac{pU(V^{\mathrm{H}} - V^{\mathrm{L}})U'(V^{\mathrm{H}} - b)}{(1-p)(U(V^{\mathrm{H}} - b))^2}\right]\left[\frac{(U(V^{\mathrm{H}} - b))^2}{U(V^{\mathrm{H}} - b) + bU'(V^{\mathrm{H}} - b)}\right] \tag{A.10}$$

$$= \frac{p}{(1-p)}U(V^{\mathrm{H}} - V^{\mathrm{L}})[\frac{U'(V^{\mathrm{H}} - b)}{U(V^{\mathrm{H}} - b) + bU'(V^{\mathrm{H}} - b)}] \tag{A.11}$$

References

Maskin, E.S., Riley, J.G., 1984. Optimal auctions with risk averse buyers. Econometrica 52, 1473–1518.
Maskin, E.S., Riley, J.G., 1985. Auction theory with private values. American Economic Association Papers and Proceedings 75, 150–155.
McAfee, R. Preston, McMillan, J., 1987. Auctions and bidding. Journal of Economic Literature, 699–738.
Klemperer, P., 1999. Auction theory: a guide to the literature. Journal of Economic Surveys 13, 227–286.
Budish, E.B., 2000. English auctions with buy prices: irrationality on the internet? Senior honors thesis, Amherst College.