peaks of the model in more detail.

### 1. Introduction

#### The Two-Parameter Portfolio Model

In Chapter 3, the two-parameter portfolio model was introduced. We now proceed in these steps, this chapter

**A. Framework**

| Normal Distribution | Risk Aversion and the Efficient Set | 213 |

---

**The Two-Parameter Portfolio Model**
The assumption that portfolio return distributions are normal causes the

B. The Simplications Obtained When Portfolio

return distribution to a stock and portfolio returns are normal.

C. The Simplifications Obtained When Inverses

...
The Two-Parameter Portfolio Model

The graph illustrates the relationship between the expected return and the variance (or standard deviation) of a portfolio. Each point on the graph represents a possible combination of expected returns and standard deviations, forming a surface in two dimensions. The surface is concave, indicating that diversification reduces risk, and it suggests the trade-off between expected return and risk.

In the context of the Capital Asset Pricing Model (CAPM), the efficient frontier represents the set of portfolios that offer the highest expected return for a given level of risk. Portfolios that lie above the efficient frontier are inefficient, as they offer lower expected returns than other portfolios with the same level of risk.

The diagram also illustrates the concept of the Capital Market Line (CML), which separates the region of attainable portfolios from the unattainable portfolios. The CML connects the risk-free rate to the tangent portfolio, which is the portfolio that maximizes the Sharpe ratio (the excess return per unit of deviation in an asset's return).

In summary, the graph helps investors understand the trade-offs between expected return and risk, and it guides them in constructing portfolios that align with their risk tolerance and investment goals.

---

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The portfolio selection problem is to determine how we take the portfolio set

\[
(01) \quad u(Ty) = (y_T y_0) + (e_T y_0)
\]

\[
(02) \quad u(Ty) = (y_T y_0) + (e_T y_0)
\]

The portfolio selection problem becomes

\[
(01) \quad u(Ty) = (y_T y_0) + (e_T y_0)
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(02) \quad u(Ty) = (y_T y_0) + (e_T y_0)
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(03) \quad u(Ty) = (y_T y_0) + (e_T y_0)
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(03) \quad u(Ty) = (y_T y_0) + (e_T y_0)
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\[
(03) \quad u(Ty) = (y_T y_0) + (e_T y_0)
\]

\[
(04) \quad u(Ty) = (y_T y_0) + (e_T y_0)
\]
The Two-Parameter Model
The Two-Parameter Portfolio Model

Proposition III

Let \( x \) and \( y \) be the proportions of two assets in a portfolio, and let \( b \) and \( c \) be the proportions of the other assets. Then the expected return of the portfolio is given by:

\[
E(R_p) = bx + cy
\]

where \( b \) and \( c \) are the proportions of the other assets in the portfolio. The variance of the portfolio is given by:

\[
\sigma_p^2 = \sigma_b^2 + \sigma_c^2 + 2\sigma_b\sigma_c \rho
\]

where \( \sigma_b^2 \) and \( \sigma_c^2 \) are the variances of the assets, and \( \rho \) is the correlation coefficient between the returns of the assets.

Proposition IV

The expected return of the portfolio is maximized when the correlation coefficient is zero, and the variances of the assets are equal. In this case, the portfolio is said to be diversified.

Proposition V

The expected return of the portfolio is minimized when the correlation coefficient is one, or when the variances of the assets are not equal. In this case, the portfolio is said to be concentrated.

Example

Suppose we have a portfolio consisting of two assets, A and B, with expected returns of 10% and 15%, respectively. The variances of the assets are 0.04 and 0.09, and the correlation coefficient between the assets is 0.5. The expected return of the portfolio is:

\[
E(R_p) = 0.5 \times 0.1 + 0.5 \times 0.15 = 0.125
\]

The variance of the portfolio is:

\[
\sigma_p^2 = 0.04 + 0.09 + 2 \times 0.5 \times 0.04 \times 0.09 = 0.052
\]

This example demonstrates the importance of diversification in portfolio management.
Let us simplify this short form of the compound interest model in further detail.

The Two-Parameter Portfolio Model

The analysis of short-setting
The two-Fund Portfolio Model
Suppose the curve be in Figure 7.2 represents the set of portfolios that

C. The Efficient Set with a Risk-Free Asset

some claim the effect of a risk-free asset on the investment opportunity set.
Markowitz equilibrium provided in the next chapter. Thus, we now discuss two
portfolio models proposed in the two-parametric model of capital
market equilibrium. The two-parametric model of capital

Figure 7.2 Portfolio Opportunity Frontier When There Is Short-Selling

Figure 7.3 Portfolio Opportunity Frontier When There Is No Short-Selling

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The Two-Parameter Portfolio Model

The expected return of any portfolio formed according to (1) is

\[ E_R = \frac{1}{1 + \frac{r}{p}} \]

where \( r \) is the risk-free rate and \( p \) is the portfolio's beta.

The variance of any portfolio formed according to (1) is

\[ \sigma^2 = \frac{1}{1 + \frac{r}{p}} \]

where \( \sigma^2 \) is the portfolio's variance.

For the two-parameter portfolio model, the optimal portfolio is

\[ \text{Optimal Portfolio} = \frac{1}{1 + \frac{r}{p}} \]

where \( r \) is the risk-free rate and \( p \) is the portfolio's beta.
In intuitive terms, the return on the return on a portfolio of $\frac{\gamma}{\gamma}x$ and $\frac{\gamma}{\gamma}(x-1)$ is:

$$\frac{(\gamma x) - (\gamma)(x-1)}{(\gamma)(x-1)} = \frac{\gamma}{\gamma}\frac{x}{x-1}$$

$$0 \leq x \leq \frac{\gamma}{\gamma}, \quad \frac{\gamma}{\gamma}(x-1) \neq 0$$

Then:

$$x \neq 0 \quad \frac{\gamma}{\gamma}(x-1) \neq 0$$

Consider two portfolios $\gamma$ and $\gamma(x-1)$.

1. Derive the expression for the return on the portfolio $\gamma(x-1)$ and $\gamma x$. If $\gamma$ is the return on the portfolio $\gamma(x-1)$, then the return on the portfolio $\gamma x$ is:

$$\frac{\gamma}{\gamma}(x-1) = \frac{\gamma}{\gamma}$$

2. If $\gamma = \gamma(x-1)$, then the return on the portfolio $\gamma x$ is:

$$\frac{\gamma}{\gamma}\frac{x}{x-1}$$

From (2)(1), if $\gamma = \gamma(x-1)$, then the return on the portfolio $\gamma x$ is:

$$\frac{\gamma}{\gamma}\frac{x}{x-1}$$

This is the intuitive expression for the return on the portfolio $\gamma x$. From (2)(1), if $\gamma = \gamma(x-1)$, then the return on the portfolio $\gamma x$ is:

$$\frac{\gamma}{\gamma}\frac{x}{x-1}$$

Equations (2) and (1) are called the two portfolio model.

The two portfolio model is a simple method for determining the expected return on a portfolio of assets. It assumes that the returns on the assets are independent and that the investor can choose any proportion of the assets in the portfolio. From (2)(1), if $\gamma = \gamma(x-1)$, then the return on the portfolio $\gamma x$ is:

$$\frac{\gamma}{\gamma}\frac{x}{x-1}$$

The two portfolio model is a useful tool for investors to determine the expected return on a portfolio of assets. It is especially useful for investors who are not able to diversify their portfolio, as it allows them to choose the proportion of assets in the portfolio that maximizes their return. From (2)(1), if $\gamma = \gamma(x-1)$, then the return on the portfolio $\gamma x$ is:

$$\frac{\gamma}{\gamma}\frac{x}{x-1}$$

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$$\frac{\gamma}{\gamma}\frac{x}{x-1}$$

The two portfolio model is a useful tool for investors to determine the expected return on a portfolio of assets. It is especially useful for investors who are not able to diversify their portfolio, as it allows them to choose the proportion of assets in the portfolio that maximizes their return.
The contribution of security i to the portfolio's risk is given by Equation (79):

\[
\rho_i = \sum_{j=1}^{n} \rho_{ij} \frac{1}{\sigma_i^2} \left( \frac{1}{\sigma_j^2} \right) x_j
\]

where \( \rho_{ij} \) is the covariance between the returns of securities i and j, and \( \sigma_i \) and \( \sigma_j \) are their respective standard deviations.

Chapter 7: Portfolio Choice

The Effect of Diversification

10. Portfolio Risk, Security Risk, and Covariance

When there is no risk in the portfolio, the portfolio variance is zero, which implies that the portfolio risk is also zero.

The different forms of portfolios are:

- Perfectly correlated portfolios
- Less than perfectly correlated portfolios
- More than perfectly correlated portfolios

The question is: How do these portfolios correlate to the risk of the individual securities?

Problem 10.1

The problem is to determine the risk of the portfolio given the correlation matrix and the variances of the individual securities.

The answer is: The risk of the portfolio is the sum of the risks of the individual securities weighted by their respective weights in the portfolio.

The Two-Factor Model

Dependence on factors

The two-factor model is a generalization of the capital asset pricing model (CAPM) that accounts for additional factors influencing asset returns. It is given by:

\[
E(R_i) = R_f + 
\]

\[
\beta_i (E(R_m) - R_f)
\]

where \( E(R_i) \) is the expected return on asset i, \( R_f \) is the risk-free rate, \( \beta_i \) is the beta of asset i, and \( E(R_m) \) is the expected market return.

The Two-Factor Portfolio Model

In a two-factor model, assets are classified into two categories, such as domestic and international, and each category is characterized by a different factor: the market factor and the country-specific factor.
The Two-Parameter Portfolio Model

The expected return makes a significant contribution to the expected return on the portfolio. If the expected return on the security is positive, then the expected return on the portfolio is also positive. If the expected return on the security is zero, then the expected return on the portfolio is also zero. If the expected return on the security is negative, then the expected return on the portfolio is also negative.

The standard deviation of the portfolio is greater than the standard deviation of the securities in the portfolio. The standard deviation of the portfolio is also greater than the standard deviation of the security. The standard deviation of the portfolio is also greater than the standard deviation of the security.

The correlation of the returns of the securities in the portfolio is less than the correlation of the securities in the portfolio. The correlation of the returns of the securities in the portfolio is also less than the correlation of the securities in the portfolio. The correlation of the returns of the securities in the portfolio is also less than the correlation of the securities in the portfolio.
\[
\left( \mu^d dX \right) \left( \int_0^T \frac{1}{N} \right) dX = \left( dY \right)_T
\]

so the contribution of security \( d \) to the sample variance of the return on

(38)

\[
\left( \int_0^T \frac{1}{N} \right) \left( \frac{\mu^d dX}{\mu^d} \right) = \left( dY \right)_T
\]

is

```
Rejection is

(37)

\[
\left( \int_0^T \frac{1}{N} \right) \left( \frac{\mu^d dX}{\mu^d} \right) = \left( dY \right)_T
\]

```

where

\[
\left( \mu^d dX \right) \left( \int_0^T \frac{1}{N} \right) dX = \left( dY \right)_T
\]

with equation \( (38) \) says the contribution of security \( d \) to the sample variance of the return on portfolio \( d \) is equal to the sample variance of the return on security \( d \) in the portfolio. The variance of the return on the portfolio is the sum of the variances of the individual securities and the covariances between them.

Rejection is

```
(37)

\[
\left( \frac{\mu^d dX}{\mu^d} \right) \left( \int_0^T \frac{1}{N} \right) = \left( dY \right)_T
\]

```

where

\[
\left( \mu^d dX \right) \left( \int_0^T \frac{1}{N} \right) dX = \left( dY \right)_T
\]

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B. Portfolio Risk and Security Risk: Empirical Examples

The Two-Factor Model

```
expected security returns

\[
\mu^d = \frac{1}{T} \int_0^T \mu^d dX = \frac{1}{T} \int_0^T \mu^d dX
\]

```

where \( \mu^d \) is the sample mean of the returns on security \( d \) in the portfolio, and \( \mu^d \) is the sample mean of the returns on all securities.

```
(32)

\[
\mu^d = \frac{1}{T} \int_0^T \mu^d dX = \frac{1}{T} \int_0^T \mu^d dX
\]

```

where \( \mu^d \) is the sample mean of the returns on security \( d \) in the portfolio, and \( \mu^d \) is the sample mean of the returns on all securities.

```
(32)

\[
\mu^d = \frac{1}{T} \int_0^T \mu^d dX = \frac{1}{T} \int_0^T \mu^d dX
\]

```

where \( \mu^d \) is the sample mean of the returns on security \( d \) in the portfolio, and \( \mu^d \) is the sample mean of the returns on all securities.

```
(32)

\[
\mu^d = \frac{1}{T} \int_0^T \mu^d dX = \frac{1}{T} \int_0^T \mu^d dX
\]

```

where \( \mu^d \) is the sample mean of the returns on security \( d \) in the portfolio, and \( \mu^d \) is the sample mean of the returns on all securities.

```
(32)

\[
\mu^d = \frac{1}{T} \int_0^T \mu^d dX = \frac{1}{T} \int_0^T \mu^d dX
\]

```

where \( \mu^d \) is the sample mean of the returns on security \( d \) in the portfolio, and \( \mu^d \) is the sample mean of the returns on all securities.
Although the estimated risk of security $i$ in the portfolio,
\[
\sum_{j=1}^{N} \left( \frac{N}{N_i} \right)^{1/2} \left( \frac{1}{N_i} \right)^{1/2} \frac{dX_i}{dX} x_i
\]

is the contribution of security $i$ to the risk $\frac{1}{N} \sum_{j=1}^{N} \frac{dX_j}{dX} x_j$ of the portfolio, this expression is useful in a portfolio optimization context, where the objective is to minimize the risk. However, in practice, the risk minimization problem is often formulated as a constrained optimization problem, where the portfolio weights $x_i$ are subject to certain constraints, such as the total weight constraint or the individual weight constraints. Therefore, the portfolio optimization problem is typically solved using numerical optimization techniques such as quadratic programming or constrained optimization methods.

### Table 1

<table>
<thead>
<tr>
<th>Security</th>
<th>$\frac{dX_i}{dX}$</th>
<th>$\left( \frac{N}{N_i} \right)^{1/2}$</th>
<th>$\left( \frac{1}{N_i} \right)^{1/2}$</th>
<th>$\frac{dX_i}{dX} x_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1</td>
<td>0.5</td>
<td>0.773</td>
<td>0.628</td>
<td>0.316</td>
</tr>
<tr>
<td>J2</td>
<td>0.3</td>
<td>0.773</td>
<td>0.628</td>
<td>0.204</td>
</tr>
<tr>
<td>J3</td>
<td>0.2</td>
<td>0.773</td>
<td>0.628</td>
<td>0.137</td>
</tr>
<tr>
<td>J4</td>
<td>0.1</td>
<td>0.773</td>
<td>0.628</td>
<td>0.069</td>
</tr>
</tbody>
</table>

If we call the weighted average of the estimates of the risk of the portfolio the risk of the portfolio, then from (38) the estimate of the risk of the portfolio is
\[
\left( \frac{N}{N_i} \right)^{1/2} \left( \frac{1}{N_i} \right)^{1/2} \sum_{j=1}^{N} \frac{dX_j}{dX} x_j
\]
<table>
<thead>
<tr>
<th>N</th>
<th>( \frac{1}{N} \sum_{i=1}^{N} x_i )</th>
<th>( \frac{1}{N} \sum_{i=1}^{N} x_i^2 )</th>
<th>( \frac{1}{N} \sum_{i=1}^{N} x_i^3 )</th>
<th>( \frac{1}{N} \sum_{i=1}^{N} x_i^4 )</th>
<th>( \frac{1}{N} \sum_{i=1}^{N} x_i^5 )</th>
</tr>
</thead>
<tbody>
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<tr>
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<tr>
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<td>3000000</td>
</tr>
<tr>
<td>40</td>
<td>4000000</td>
<td>4000000</td>
<td>4000000</td>
<td>4000000</td>
<td>4000000</td>
</tr>
<tr>
<td>50</td>
<td>5000000</td>
<td>5000000</td>
<td>5000000</td>
<td>5000000</td>
<td>5000000</td>
</tr>
</tbody>
</table>

**TABLE 1.**
\[
\left( \frac{\delta y}{y} \right)_x^N = \left( \frac{\delta y}{y} \right)_x \frac{N}{1} = \left( \frac{\delta y}{y} \right)_x \frac{d(x)}{d(x)} \sum_{x}^N
\]

7.1: where the values of the stocks in the portfolio are replaced by the total returns of the stocks in the portfolio. This is because the total return of the portfolio is the sum of the returns of the individual stocks. The total return of the portfolio is calculated as:

\[
\left( \frac{\delta P}{P} \right)_p = \sum_{i=1}^{N} \left( \frac{\delta P}{P} \right)_i
\]

where \( P \) is the total value of the portfolio.

The two-factor model for portfolio returns is given by:

\[
\left( \frac{\delta P}{P} \right)_p = \left( \frac{\delta P}{P} \right)_m + \left( \frac{\delta P}{P} \right)_s
\]

where \( \left( \frac{\delta P}{P} \right)_m \) is the market factor and \( \left( \frac{\delta P}{P} \right)_s \) is the size factor. The market factor represents the systematic risk of the portfolio, while the size factor represents the size effect.

The importance of the two-factor model in finance is that it helps to explain the variation in portfolio returns and can be used to construct diversified portfolios that are less sensitive to market fluctuations.
The Two-Parameter Portfolio Model

![Graph showing the standard deviation of portfolio return as a function of the number of securities in the portfolio.](image)

The standard deviation of a portfolio return as a function of the number of securities in the portfolio.

**Figure 7.9**

---

**The Effect of Diversification**

Diversification reduces the risk of a portfolio by spreading investments across a variety of assets. The more diversified a portfolio is, the lower its risk. This is illustrated in Figure 7.9, which shows how the standard deviation of portfolio return decreases as the number of securities increases.

**Equation 7.3**

The standard deviation of portfolio return can be calculated using the formula:

\[
\sigma_p = \sqrt{\sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}}
\]

**Equation 7.4**

The expected return of a diversified portfolio is given by:

\[
E(R_p) = \sum_{i=1}^{N} w_i E(R_i)
\]

where

- \( w_i \) is the weight of the \( i \)-th asset in the portfolio,
- \( E(R_i) \) is the expected return of the \( i \)-th asset.

**Example**

If a portfolio consists of 2 assets, with weights \( w_1 = 0.5 \) and \( w_2 = 0.5 \), and expected returns \( E(R_1) = 0.06 \) and \( E(R_2) = 0.04 \), the expected return of the portfolio is:

\[
E(R_p) = 0.5 \times 0.06 + 0.5 \times 0.04 = 0.05
\]

---

**The Two-Parameter Portfolio Model**

The two-parameter portfolio model is a framework for analyzing and selecting portfolios of investments. It is based on the assumptions that investors are risk-averse and that they can be described by a utility function that is linear in wealth and quadratic in the risk of the portfolio.

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The standard deviation of a portfolio is a function of the number of securities in the portfolio. This is important because the more securities in a portfolio, the lower the risk of the portfolio. The formula for the standard deviation of a portfolio is:

\[ \text{Portfolio Standard Deviation} = \sqrt{\sum_{i=1}^{n} \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \rho_{ij} \sigma_i \sigma_j} \]

where \( \sigma_i \) is the standard deviation of security \( i \), \( \rho_{ij} \) is the correlation coefficient between securities \( i \) and \( j \), and \( n \) is the number of securities in the portfolio.

The correlation coefficient is defined as:

\[ \rho_{ij} = \frac{\text{Cov}(R_i, R_j)}{\sigma_i \sigma_j} \]

where \( \text{Cov}(R_i, R_j) \) is the covariance between returns of securities \( i \) and \( j \).

The covariance is defined as:

\[ \text{Cov}(R_i, R_j) = \frac{1}{n} \sum_{t=1}^{n} (R_{it} - \overline{R}_i)(R_{jt} - \overline{R}_j) \]

where \( R_{it} \) is the return of security \( i \) at time \( t \), \( \overline{R}_i \) is the mean return of security \( i \), and \( n \) is the number of observations.

The standard deviation of the portfolio is equal to the square root of the portfolio variance, which is:

\[ \text{Portfolio Variance} = \sum_{i=1}^{n} \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \rho_{ij} \sigma_i \sigma_j \]

The correlation coefficient can take on values between -1 and 1. A correlation coefficient of 1 indicates a perfect positive linear relationship between the returns of two securities, while a coefficient of -1 indicates a perfect negative linear relationship. A coefficient of 0 indicates no linear relationship.

Problem 6C:

Do it! Answer

The Two-Parameter Portfolio Model

\[ \text{Portfolio Return} = \sum_{i=1}^{n} w_i R_i \]

where \( w_i \) is the weight of security \( i \) in the portfolio and \( R_i \) is the return of security \( i \).

Figure 7.10: The standard deviation of a portfolio as a function of the number of securities