REALIZED VOLATILITY FORECASTING and OPTION PRICING

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Abstract

A growing literature advocates the use of microstructure noise-contaminated high-frequency data for the purpose of volatility estimation. This paper evaluates and compares the quality of several recently-proposed estimators in the context of a relevant economic metric, i.e., profits from option pricing and trading. Using forecasts obtained by virtue of alternative volatility estimates, agents price short-term options on the S&P 500 index before trading with each other at average prices. The agents' average profits and the Sharpe ratios of the profits constitute the metrics used to evaluate alternative volatility estimates and the corresponding forecasts. For our data, we find that estimators with superior finite sample mean-squared-error properties generate higher average profits and higher Sharpe ratios, in general. We confirm that, even from a forecasting standpoint, there is scope for optimizing the finite sample properties of alternative volatility estimators as advocated by Bandi and Russell (2005) in recent work.

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1 Introduction

The recent, stimulating work on nonparametric volatility estimation by virtue of high-frequency asset price data has largely focused on designing estimators with satisfactory statistical properties when realistic market microstructure contaminations play a role. Important theoretical emphasis is generally placed on the asymptotic features of the proposed estimators (see, e.g., Barndorff-Nielsen, Hansen, Lunde, and Shephard, 2005, 2006, Kalnina and Linton, 2006, Zhang, Mykland, and Aït-Sahalia, 2005, and Zhang, 2006). A related strand of this literature focuses on the estimators' finite sample performance and the importance of optimizing this performance explicitly (Bandi and Russell, 2003, 2005). Bandi and Russell (2006b), Barndorff-Nielsen and Shephard (2006), and McAleer and Medeiro (2006) provide thorough reviews of this growing literature.

Somewhat sparse work examines the forecasting ability of alternative high-frequency volatility estimators. This paper considers this issue and evaluates volatility forecasting from the vantage point of a relevant economic criterion. Specifically, we evaluate the profits/losses that option dealers would derive from trading on the basis of alternative volatility forecasts. To this extent, we employ a methodology proposed by Engle, Hong, and Kane (1990) and effectively operate in the context of an artificial ("hypothetical," in their terminology) option market. Consider two generic option traders: Trader A and Trader B. Trader A (B) estimates volatility using Method A (B). Alternative volatility estimates yield alternative volatility forecasts and, hence, different option prices. The pair-wise trades are conducted at the mid-point of the traders' prices. The trader with the highest volatility (option price) forecast will buy a straddle (a call and a put option) from his/her counterpart. All options are executed on the following day. The average dollar profits and Sharpe ratios obtained from repeated implementations of this strategy represent the economic metrics used to evaluate alternative variance estimates/forecasts. The logic is simple. If the high volatility forecast is accurate, the straddle (whose price is twice the mid-point of the call/put price forecasts in our framework) is underpriced. The trader who buys the straddle is expected to make a profit. Since a straddle is just a portfolio containing a call option and a put option, high volatility is expected to result in either option being substantially in the money. If the cost of this position is relatively low, as determined by the above-mentioned underpricing, then a profit will arise.

We work with a variety of variance estimates and forecasts (Methods). Each trader uses a Method and trades with every other trader. The pair-wise trades always occur at the mid-point of the price forecasts derived from the corresponding pair of Methods. For each trader, we report the mean of the average profits/losses and the corresponding Sharpe ratios across the pair-wise trades. We employ the following Methods:

1. Realized variance constructed using 5-minute returns.

2. Realized variance constructed using 15-minute returns.

3. Optimally-sampled realized variance as proposed by Bandi and Russell (2003).

4. The "near consistent" Bartlett kernel estimator in Barndorff-Nielsen, Hansen, Lunde and Shephard (2005) with an optimal (in a finite sample) choice for the number of autocovariances as suggested by Bandi and Russell (2005).

5. The two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) with an asymptotically optimal choice for the number of subsamples as suggested by Zhang, Mykland and Aït-Sahalia (2005).

6. The two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) with an optimal (in a finite sample) choice for the number of subsamples as suggested by Bandi and Russell (2005).

7. The bias-corrected two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) with an asymptotically optimal choice for the number of subsamples as suggested by Zhang, Mykland and Aït-Sahalia (2005).

8. The bias-corrected two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) with an optimal (in a finite sample) choice for the number of subsamples.

9. The flat-top *Bartlett* kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an asymptotically optimal choice for the number of autocovariances as suggested by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006).

10. The flat-top *Bartlett* kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an optimal (in a finite sample) choice for the number of autocovariances as suggested by Bandi and Russell (2005).

11. The flat-top *cubic* kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an asymptotically optimal choice for the number of autocovariances as suggested by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006).

12. The flat-top *cubic* kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an optimal (in a finite sample) choice for the number of autocovariances as suggested by Bandi and Russell (2005).

13. The flat-top *modified Tukey-Hanning* kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an asymptotically optimal choice for the number of autocovariances as suggested by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006).

14. The flat-top *modified Tukey-Hanning* kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an optimal (in a finite sample) choice for the number of autocovariances as suggested by Bandi and Russell (2005).

Agents price and trade short-term (at-the-money) options on the S&P 500 index and use high-frequency data on the Standard and Poor's depository receipts (SPIDERS) to construct the volatility estimates. Since these estimates rely on high-frequency data, they are derived for a 6-hour period. Thus, we consider three scenarios. The first scenario studies 6-hour option trading to avoid issues posed by the need to account for the overnight returns in volatility estimation/forecasting. The second and third scenario focus on 1-day options and evaluate two alternative methods for dealing with the overnights.

Our results suggest that explicit optimization of the finite sample mean-squared-error (MSE) properties of the proposed estimators, as advocated by Bandi and Russell (2003, 2005), results in important economic gains. We find that the optimized (in a finite sample) flat-top kernel estimators largely constitute the most favorable Methods both in terms of average profits and in terms of Sharpe ratios. The "near consistent" Bartlett kernel estimator and the (bias-corrected and unadjusted) two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) can perform very well, and sometimes as well as the flat-top kernel estimators, when the number of subsamples/autocovariances is carefully chosen using finite sample methods. Choices of subsamples/autocovariances based on asymptotic, rather than finite sample, criteria leads to unnecessarily suboptimal performance. This is particularly evident in the case of the unadjusted two-scale estimator due to a potentially large finite sample (downward) bias. Optimally-sampled realized variance almost always dominates 5- and 15-minute realized variance.

As mentioned, volatility forecasting in the presence of market microstructure noise is still a largely underexplored subject. Bandi and Russell (2006a) and Bandi, Russell, and Zhu (2006) use reduced-form models to show that optimally-sampled (for each trading day) realized variances (covariances) outperform realized variances (covariances) constructed using ad-hoc (5- or 15-minute, say) intervals in predicting variances (covariances) out-of-sample. Ghysels and Sinko (2006a) employ the MIDAS approach of Ghysels, Santa-Clara, and Valkanov (2006) to evaluate the relative performance of realized variance based on fixed intervals, bias-corrected realized variance, and power variation. Power variation is the preferred estimator in their framework. Large (2006) uses the HAR-RV model of Corsi (2003) to find that his "alternation estimator" can have better forecasting properties than realized variance constructed using ad-hoc, fixed intervals. Aït-Sahalia and Mancini (2006) study the forecasting performance of the two-scale estimator and realized variance using a variety of simulation set-ups for stochastic volatility and microstructure noise. An empirical comparison relying on coefficients of determination from Mincer-Zarnowitz-style regressions is also provided. In their framework, the two-scale estimator outperforms realized variance both in terms of MSE and in terms of forecasting ability. Conditional predictive densities and confidence intervals for alternative integrated variance estimators are studied in Corradi, Distaso, and Swanson (2006).

Two economic metrics have been proposed, thus far. Bandi and Russell (2006a) consider a *portfolio choice* problem and the long-run utility that a mean-variance representative investor derives from alternative variance forecasts as the relevant performance criterion. A similar portfolio-based approach has been recently implemented by Bandi, Russell, and Zhu (2006) and De Pooter, Martens, and Van Dijk (2006) in a multivariate context (see Fleming, Kirby, and Ostdiek, 2001, 2003, and West, Edison, and Cho, 1993, in the no noise case). Bandi and Russell (2005, 2006c) study volatility forecasting for the purpose of *option pricing* as in the current paper. However, their focus is only on optimally-sampled realized variance and fixed-interval realized variance (Bandi and Russell, 2006c) and on optimally-sample realized variance and the two-scale estimator (Bandi and Russell, 2005).

The extant approaches focus on subsets of the existing estimators. Their findings generally point to the preferability of optimally-sampled realized variance over realized variance constructed using fixed intervals as well as to the out-of-sample usefulness of the consistent estimators, such as the two-scale estimator, when optimized using finite sample methods. Stimulating recent studies towards a more comprehensive analysis of the forecasting performance of alternative volatility estimation methods have been conducted by Andersen, Bollerslev, and Meddahi (2006) and Ghysels and Sinko (2006b). These papers use empirically-relevant stochastic volatility models and analytic expressions to evaluate forecasting performance in linear regressions of integrated variance on alternative variance estimates. Mincer-Zarnowitz-style regression models (Andersen, Bollerslev, and Meddahi, 2006) and MIDAS regressions (Ghysels and Sinko, 2006b) are used to predict variance in practise. In Andersen, Bollerslev, and Meddahi (2006) and Ghysels and Sinko (2006b), the forecasting metric has a statistical nature. The current paper's goal is to provide a rich comparison between alternative estimation methods in the context of an important economic criterion. We show that estimators with superior finite sample MSE properties generally have superior predictive ability in our set-up.

The paper proceeds as follows. Section 2 describes the Methods. In Section 3 we discuss the pricing and trading mechanism. Section 4 is about the data. Section 5 contains the profit-based rankings. Tables and Figures are in the Appendix.

2 The Methods

Consider, for simplicity, a trading day of length $h = 1$. Assume availability of $M + 1$ equispaced logarithmic asset prices over $[0, 1]$ and write

$$p_{j\delta} \;=\; p^*_{j\delta} + \eta_{j\delta}$$

or, in terms of continuously-compounded returns,

$$\underbrace{p_{j\delta} - p_{(j-1)\delta}}_{r_{j\delta}} = \underbrace{p^*_{j\delta} - p^*_{(j-1)\delta}}_{r^*_{j\delta}} + \underbrace{\eta_{j\delta} - \eta_{(j-1)\delta}}_{\varepsilon_{j\delta}},$$

where p^* denotes the *unobservable* equilibrium price, η denotes *unobservable* market microstructure noise, and $\delta = \frac{1}{M}$ represents the time distance between adjacent price observations. Assume the equilibrium price process evolves in time as a stochastic volatility local martingale, i.e.,

$$p^*_t = \int_0^t \sigma_s dW_s,$$

where σ_t is a càdlàg stochastic volatility process and W_t is a standard Brownian motion.

The object of econometric interest is $V = \int_0^1 \sigma_s^2 ds$. The Methods attempt to identify V by mitigating the impact that the presence of the noise component η has on nonparametric volatility estimates constructed using intra-daily, noise-contaminated asset returns r. Because we use SPIDERS high-frequency mid-quotes in this study, the price formation mechanism should be understood as the SPIDERS' mid-quote formation mechanism.

In what follows, we use the symbol $\perp\!\!\!\perp$ to signify "independence." The symbol $\Rightarrow$ denotes "weak convergence."

- *Realized variance constructed using 5-minute returns,* $\widehat{V}^{5\,\mathrm{min}}$

The classical realized variance estimator of Andersen, Bollerslev, Diebold, and Labys (2003) and Barndorff-Nielsen and Shephard (2002) is simply defined as the sum of the squared returns over the trading day, i.e.,

$$\widehat{V} = \sum_{j=1}^{M} r^2_{j\delta}.$$

In the absence of noise, $\widehat{V}$ is consistent for V as the sampling frequency increases, i.e., as $M \to \infty$ over $[0,1]$ (see, e.g., Protter, 1995). In the presence of empirically-reasonable noise contaminations, $\widehat{V}$ is inconsistent as shown by Bandi and Russell (2003) and Zhang, Mykland and Aït-Sahalia (2005). To reduce the impact of the noise component, Andersen, Bollerslev, Diebold, and Labys (2003) and Andersen, Bollerslev, Diebold, and Ebens (2001), among others, suggest using 5-minute returns, rather than returns sampled at the highest frequencies, to estimate V in practise. Thus, $\widehat{V}^{5\,\min}$ is equal to $\widehat{V}$ with $M = 72$ $(= 6 \times 60/5)$.

- *Realized variance constructed using 15-minute returns,* $\widehat{V}^{15\,\min}$

Since, provided $p^* \perp\!\!\!\perp \eta$, the bias of the realized variance estimator is equal to $M\mathbf{E}_M(\varepsilon^2)$, this bias can be further reduced by selecting a lower number of intra-daily returns for the purpose of variance estimation. Relying on plots of realized variance versus alternative sampling frequencies (i.e., "volatility signature plots") and the levelling off of realized variance around the 15-minute frequency, Andersen, Bollerslev, Diebold, and Labys (1999, 2000) recommend using 15- or 20-minute frequencies for the purpose of constructing realized variance. Hence, $\widehat{V}^{15\,\min}$ is equal to $\widehat{V}$ with $M = 24$ $(= 6 \times 60/15)$.

- *Optimally-sampled realized variance,* $\widehat{V}^{Opt}$

While low sampling frequencies reduce the extent of the estimator's bias component, they increase its variance. An optimal sampling frequency can then be chosen by optimally trading-off bias and variance as suggested by Bandi and Russell (2003, 2006a). Bandi and Russell (2003) discuss selection of the optimal MSE-based frequency $\frac{1}{M^{opt}}$ in the presence of noise dependence but provide a particularly simple rule-of-thumb to select this frequency when $\eta \perp\!\!\!\perp p^*$ and η is, as an approximation at least, i.i.d. in discrete time. Leverage effects ($W \perp\!\!\!\perp \sigma$) are also ruled out.[1] In this case,

$$M^{opt} \approx \left(\frac{\int_0^1 \sigma_s^4 ds}{\left(\mathbf{E}(\varepsilon^2)\right)^2} \right)^{\frac{1}{3}}.$$

Clearly, M^{opt} depends on a signal-to-noise ratio. The larger the signal coming from the equilibrium price process $Q = \int_0^1 \sigma_s^4 ds$ relative to the noise component $\left(\mathbf{E}(\varepsilon^2)\right)^2$, the larger M^{opt}. Both quantities in M^{opt} can be evaluated. The quarticity Q can be estimated inconsistently, but with little bias, by calculating $\widehat{Q} = \frac{M}{3} \sum_{j=1}^{M} r_{j\delta}^4$ (realized quarticity) with low frequency returns, e.g., $M = 24$ or 15-minute intervals. Under the same assumptions leading to M^{opt}, the noise second moment

[1] Bandi and Russell (2003) discuss the validity of this assumption at length.

can be estimated consistently by computing $\widehat{\mathbf{E}}(\varepsilon^2) = \sum_{j=1}^{M} r_{j\delta}^2/M$ at the highest frequency at which the data arrives (see, e.g., Bandi and Russell, 2003, and Zhang, Mykland and Aït-Sahalia, 2005).[2] Hence, $\widehat{V}^{Opt} = \widehat{V}$ with $\widehat{M}^{opt} = \left(\widehat{Q}/\left(\widehat{\mathbf{E}}(\varepsilon^2)\right)^2\right)^{1/3}$.

- *The "near consistent" Bartlett kernel estimator in Barndorff-Nielsen, Hansen, Lunde and Shephard (2005) with an optimal (in a finite sample) choice for the number of autocovariances as suggested by Bandi and Russell (2005).*

Here, and below, we assume the data is not necessarily equispaced. Consider a HAC-type estimator defined as

$$\widehat{V}^{Bar} = \left(\frac{M-1}{M}\frac{q-1}{q}\right)\widehat{\gamma}_0 + 2\sum_{s=1}^{q}\left(\frac{q-s}{q}\right)\widehat{\gamma}_s,$$

where $\widehat{\gamma}_s = \sum_{j=1}^{M-s} r_j r_{j+s}$ is the s-th return (realized) autocovariance. Clearly, $\widehat{\gamma}_0 = \widehat{V}$, hence the estimator weighs the classical realized variance estimator (possibly applied to non-equispaced data) and the first q return autocovariances by virtue of Bartlett-type kernel weights. This estimator is in the spirit of the first-order bias-corrected estimator studied by Zhou (1996), Hansen and Lunde (2006), and Oomen (2005).

Under i.i.d. η's and $p^* \perp\!\!\!\perp \eta$, Barndorff-Nielsen, Hansen, Lunde, and Shephard (2005) show that, if $M, q \to \infty$ with $\frac{q}{M} \to 0$ and $\frac{q^2}{M} \to \infty$, even though inconsistent for V, $\widehat{V}^{Bar}$ has a small (empirically) limiting variance given by $4\left(\mathbf{E}(\eta^2)\right)^2$.

We implement $\widehat{V}^{Bar}$ by selecting the number of autocovariances on the basis of the MSE-based approximation suggested by Bandi and Russell (2005):[3]

$$q^{BR} \approx \left(\frac{3}{2}\frac{\frac{V^2}{M^2}}{Q}\right)^{1/3} M.$$

As earlier in the case of M^{opt}, q^{BR} is determined on the basis of a bias-variance trade-off. Leaving proportionality factors aside, the term $\frac{V^2}{M^2}$ represents the leading term of the estimator's (squared) bias, whereas Q represents the leading term of its variance. The larger the bias term relative to the variance term, the larger the number of autocovariances. The optimal number of autocovariances

[2] Only the i.i.d. noise assumption is important for consistent estimation of the second moment of the noise by virtue of the estimator described in the text. One can allow for dependence between the noise and the efficient price as well as leverage effects with no effect on asymptotic inference (Bandi and Russell, 2003).

[3] This approximation is valid under the same conditions yielding "near consistency" of the estimator as in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2005) and $W \perp\!\!\!\perp \sigma$. The approximation relies on the finite sample MSE of the estimator. The full MSE can be optimized directly. See Bandi and Russell (2005).

is selected as a fraction of the number of observations M for each trading day in the sample. As earlier, preliminary, roughly unbiased, estimates of V and Q can be obtained by employing realized variance, $\widehat{V}$, and realized quarticity, $\widehat{Q}$, with low frequency returns (for instance, 15-minute returns).

- *The two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) with an asymptotically optimal choice for the number of subsamples as suggested by Zhang, Mykland and Aït-Sahalia (2005),* $\widehat{V}^{ZMA}_{ZMA}$

Consider q non-overlapping sub-grids $\Psi^{(i)}$ of the original grid of M arrival times with $i = 1, ..., q$. The first sub-grid starts at t_0 and takes every q-th arrival time, i.e., $\Psi^{(1)} = (t_0, t_{0+q}, t_{0+2q}, ...,)$, the second sub-grid starts at t_1 and also takes every q-th arrival time, i.e., $\Psi^{(2)} = (t_1, t_{1+q}, t_{1+2q}, ...,)$, and so on. Given the generic i-th sub-grid of arrival times, the corresponding realized variance estimator is defined as

$$\widehat{V}^{(i)} = \sum_{t_j, t_{j+} \in \Psi^{(i)}} \left(p_{t_{j+}} - p_{t_j} \right)^2, \tag{1}$$

where t_j and t_{j+} denote adjacent elements in $\Psi^{(i)}$. Zhang, Mykland, and Aït-Sahalia's two-scale or subsampling estimator is constructed as

$$\widehat{V}^{ZMA} = \frac{\sum_{i=1}^{q} \widehat{V}^{(i)}}{q} - \overline{M}\widehat{\mathbf{E}}(\varepsilon^2), \tag{2}$$

where $\overline{M} = \frac{M-q+1}{q}$, $\widehat{\mathbf{E}}(\varepsilon^2) = \frac{\sum_{j=1}^{M} \left(p_{t_{j+}} - p_{t_j} \right)^2}{M}$ is a consistent estimate of the second moment of the noise return, as discussed earlier, and $\overline{M}\widehat{\mathbf{E}}(\varepsilon^2)$ is a bias-correction. Thus, the estimator averages the realized variance estimates constructed on the basis of subsamples and bias-corrects them.

Under i.i.d. $\eta^{'}$s and $p^* \perp\!\!\!\perp \eta$, Zhang, Mykland, and Aït-Sahalia (2005) show that, if $M, q \to \infty$ with $\frac{q}{M} \to 0$ and $\frac{q^2}{M} \to \infty$, the estimator is consistent. Specifically, if $q = cM^{2/3}$,

$$M^{1/6} \left(\widehat{V}^{ZMA} - V \right) \Rightarrow \left(\sqrt{8c^{-2} \left(\mathbf{E}(\eta^2) \right)^2 + c\frac{4}{3}Q} \right) N(0, 1). \tag{3}$$

The constant c can be selected optimally in order to minimize the estimator's limiting variance. This minimization leads to an asymptotically optimal number of subsamples given by

$$q^{ZMA} = c^{ZMA} M^{2/3} = \left(\frac{16 \left(\mathbf{E}(\eta^2) \right)^2}{\frac{4}{3}Q} \right)^{1/3} M^{2/3} = \left(\frac{3 \left(\mathbf{E}(\varepsilon^2) \right)^2}{Q} \right)^{1/3} M^{2/3} \tag{4}$$

(Zhang, Mykland, and Aït-Sahalia, 2005). Q and $\mathbf{E}(\varepsilon^2)$ can be estimated as in the case of M^{opt}. In sum, $\widehat{V}^{ZMA}_{ZMA} = \widehat{V}^{ZMA}$ with $q = q^{ZMA}$.

- *The two-scale estimator of Zhang, Mykland, and Aït-Sahalia (2005), with an optimal (in a finite sample) choice for the number of subsamples as suggested by Bandi and Russell (2005),* $\widehat{V}_{BR}^{ZMA}$

Barndorff-Nielsen, Hansen, Lunde, and Shephard (2005, 2007) have shown that the two-scale estimator can be rewritten as a modified Bartlett-type kernel estimator, i.e.,

$$\widehat{V}^{ZMA} = \left(1 - \frac{M-q+1}{Mq}\right)\widehat{\gamma}_0 + 2\sum_{s=1}^{q}\left(\frac{q-s}{q}\right)\widehat{\gamma}_s - \frac{1}{q}\vartheta_q, \tag{5}$$

where $\widehat{\gamma}_s = \sum_{j=1}^{M-s} r_j r_{j+s}$, $\vartheta_1 = 0$, and $\vartheta_q = \vartheta_{q-1} + (r_1 + ... + r_{q-1})^2 + (r_{M-q+2} + ... + r_M)^2$ for $q \geq 2$. The term $\frac{1}{q}\vartheta_q$, which mechanically derives from subsampling, is crucial for the estimator's consistency and differentiates $\widehat{V}^{ZMA}$ from the inconsistent Bartlett kernel estimator $\widehat{V}^{Bar}$.

Under i.i.d. η's, $p^* \perp\!\!\!\perp \eta$, and $W \perp\!\!\!\perp \sigma$, Bandi and Russell (2005) find that, despite the inconsistency of $\widehat{V}^{Bar}$, the finite sample MSEs of $\widehat{V}^{ZMA}$ and $\widehat{V}^{Bar}$ are very similar in practise.[4] They suggest using the same rule-of-thumb q^{BR} as for $\widehat{V}^{Bar}$ in constructing $\widehat{V}^{ZMA}$. Hence, $\widehat{V}_{BR}^{ZMA}$ is equal to $\widehat{V}^{ZMA}$ with $q = q^{BR}$.

- *The bias-corrected two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) with an asymptotically optimal choice for the number of subsamples as suggested by Zhang, Mykland and Aït-Sahalia (2005),* $\widehat{V}_{ZMA}^{ZMAadj}$

Albeit consistent, the two-scale estimator is (downward) biased in finite samples. However, under i.i.d. η's, $p^* \perp\!\!\!\perp \eta$, and $W \perp\!\!\!\perp \sigma$, the form of the bias has been provided (Bandi and Russell, 2005, and Barndorff-Nielsen, Hansen, Lunde, and Shephard, 2005).[5] Therefore, following a suggestion

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[4] When studying $\widehat{V}^{ZMA}$, their results also rely on the assumption $\sigma_j^2 = \int_{t_{j-1}}^{t_j} \sigma_s^2 ds = \frac{V}{M}\ \forall j$. This condition is exact when sampling occurs in business time (see, e.g., Oomen, 2005). It is an approximation when volatility does not vary drastically during the day. Importantly, the condition is solely introduced to facilitate empirical tractability of the resulting MSE expansion. In other words, the exact MSE expansion of $\widehat{V}^{ZMA}$ may be provided while dispensing with this assumption. As an example, below we report the general form of the estimator's bias.

[5] The exact bias is given by

$$\left(\frac{-M+q-1}{qM}\right)V - \frac{1}{q}\sum_{s=1}^{q-1}(q-s)(\sigma_s^2 + \sigma_{M+1-s}^2).$$

Under the additional assumption $\sigma_j^2 = \int_{t_{j-1}}^{t_j} \sigma_s^2 ds = \frac{V}{M}\ \forall j$ (discussed above), the bias reduces to

$$\left(\frac{-M+2q-q^2-1}{qM}\right)V.$$

contained in Zhang, Mykland and Aït-Sahalia (2005), the estimator can be bias-corrected. The bias-corrected estimator can be written as $\widehat{V}^{ZMAadj} = c(q,M)\widehat{V}^{ZMA}$, where

$$c(q,M) = \left(\frac{qM - 1 + 2q - q^2 - M}{qM}\right)^{-1}.$$

Note that, for reasonable values of q and M, the factor $c(q,M)$ will increase the magnitude of the estimates, thereby off-setting the downward bias of the original estimator.[6] Because $\widehat{V}^{ZMAadj}$ is asymptotically equivalent to $\widehat{V}^{ZMA}$, the asymptotically optimal number of subsamples is given by q^{ZMA}. This choice leads to $\widehat{V}^{ZMAadj}_{ZMA}$.

- *The bias-corrected two-scale estimator of Zhang, Mykland and Aït-Sahalia (2005) with an optimal (in a finite sample) choice for the number of subsamples,* $\widehat{V}^{ZMAadj}_{BRY}$.

The optimal (in a finite sample) number of subsamples of $\widehat{V}^{ZMAadj}$ is provided by optimizing the finite sample variance of the estimator. Under i.i.d. η's, $p^* \perp\!\!\!\perp \eta$, $W \perp\!\!\!\perp \sigma$, and $\sigma_j^2 = \int_{t_{j-1}}^{t_j} \sigma_s^2 ds = \frac{V}{M}$ $\forall j$, write

$$var\left(\widehat{V}^{ZMAadj}\right) = (c(q,M))^2\, var\left(\widehat{V}^{ZMA}\right),$$

where, if $\frac{q}{M} \leq 1/2$,

[6] The form of the bias-correction is slightly different from that contained in Zhang, Mykland and Aït-Sahalia (2005) where $\widehat{V}^{ZMAadj} = \left(\frac{qM-1+q-M}{qM}\right)^{-1}\widehat{V}^{ZMA}$. Specifically, under $\sigma_j^2 = \int_{t_{j-1}}^{t_j} \sigma_s^2 ds = \frac{V}{M}$ $\forall j$, our proposed bias-correction is exact. Hence, the estimator's finite sample MSE reduces to its finite sample variance in Eq. (6). Zhang, Mykland and Aït-Sahalia (2005) do not adjust for the term $\frac{1}{q}\sum_{s=1}^{q-1}(q-s)(\sigma_s^2 + \sigma_{M+1-s}^2)$ (or $\left(\frac{q^2-q}{qM}\right)V$ under $\sigma_j^2 = \int_{t_{j-1}}^{t_j} \sigma_s^2 ds = \frac{V}{M}$ $\forall j$) in the bias expansion. Since this term is of order $\frac{q}{M} - \frac{1}{M}$, the difference between the exact correction introduced here and the approximate correction in Zhang, Mykland and Aït-Sahalia (2005) is asymptotically immaterial and can be empirically small.

$$var\left(\widehat{V}^{ZMA}\right) = K^{ZMA} - \frac{1}{3}(Q+V^2)\left(\frac{q}{M}\right)^2 + \left(-\frac{1}{3}V^2\frac{1}{M} - 4V^2\frac{1}{M^2} + \frac{4}{3}Q\right)\frac{q}{M}$$
$$+[-\frac{4}{M^4}\left(Q+V^2\right) + \left(\frac{8\sigma_\eta^4 + 16\sigma_\eta^2 V - 8Q - \frac{56}{3}V^2}{M^3}\right) + \left(\frac{24\sigma_\eta^2 V - \frac{10}{3}Q + 8\sigma_\eta^4}{M^2}\right)$$
$$+\left(\frac{-8\sigma_\eta^4 + 8\sigma_\eta^2 V}{M}\right)]\frac{M}{q} + [\frac{2}{M^5}Q + \left(\frac{-4\sigma_\eta^4 - 8\sigma_\eta^2 V + 4Q - 8V^2}{M^4}\right) + \left(\frac{-4\sigma_\eta^4 - 16\sigma_\eta^2 V + 2Q}{M^3}\right)$$
$$+\left(\frac{8\sigma_\eta^4 - 8\sigma_\eta^2 V}{M^2}\right) + \frac{8}{M}\sigma_\eta^4]\frac{M^2}{q^2}, \tag{6}$$

and

$$K^{ZMA} = (-4\sigma_\eta^4 - 8V\sigma_\eta^2)\frac{1}{M} + \left(-4\sigma_\eta^4 - 8\sigma_\eta^2 V + \frac{13}{3}Q + \frac{79}{3}V^2\right)\frac{1}{M^2} + \frac{1}{M^3}\left(2Q + 8V^2\right)$$

with $\sigma_\eta^2 = \mathbf{E}(\eta^2)$ (see Bandi and Russell, 2005). The optimal q ($= q^{BRY}$) is now simply the minimizer of $var\left(\widehat{V}^{ZMAadj}\right)$. We call the resulting estimator $\widehat{V}_{BRY}^{ZMAadj}$.

- *The flat-top Bartlett kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an asymptotically optimal choice for the number of autocovariances as suggested by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006),* $\widehat{V}_{BNHLS}^{BNHLS(Bar)}$

In recent work, Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) have advocated using unbiased (under i.i.d. η's and $p^* \perp\!\!\!\perp \eta$) flat-top symmetric kernels of the type

$$\widehat{V}^{BNHLS} = \widehat{\gamma}_0 + \sum_{s=1}^{q} w_s(\widehat{\gamma}_s + \widehat{\gamma}_{-s}), \tag{7}$$

where $\widehat{\gamma}_s = \sum_{j=1}^{M} r_j r_{j-s}$ with $s = -q, ..., q$, $w_s = k\left(\frac{s-1}{q}\right)$ and k is a function on $[0,1]$ satisfying $k(0) = 1$ and $k(1) = 0$. If $q = cM^{2/3}$, this family of estimators is consistent (at rate $M^{1/6}$) and asymptotically mixed normal. When $k(x) = 1 - x$ (the Bartlett case), the limiting variance of $\widehat{V}^{BNHLS}$ is the same as that of the two-scale estimator. Hence, q can be chosen asymptotically as suggested by Zhang, Mykland, and Aït-Sahalia (2005).

- *The flat-top Bartlett kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an optimal (in a finite sample) choice for the number of autocovariances as suggested by Bandi and Russell (2005),* $\widehat{V}_{BR}^{BNHLS(Bar)}$

As earlier, Bandi and Russell (2005) provide an alternative way to choose the number of autocovariances in finite samples. Consistent with the case of $\widehat{V}^{ZMA}$ (or $\widehat{V}^{ZMAadj}$) and $\widehat{V}^{Bar}$, for a generic $k(x)$, they advocate choosing q as $\phi^{BR}M$ with $0 < \phi^{BR} \leq 1$, where ϕ^{BR} minimizes the finite sample MSE of the estimator, i.e.,[7]

$$MSE = (bias(\phi))^2 + var(\phi),$$

where

$$bias(\phi) = 0$$

and

$$var\,(\phi) = \frac{Q}{M} w^{\intercal}\Omega_1 w + 4\left(\mathbf{E}(\eta^2)\right)^2 M(w^{\intercal}\Omega_2 w) + 4\left(\mathbf{E}(\eta^2)\right)^2 (w^{\intercal}\Omega_3 w) + (2\mathbf{E}(\eta^2)V)4(w^{\intercal}\Omega_4 w),$$

with

$$w = \left(1, 1, k\left(\frac{1}{\phi M}\right), ..., k\left(\frac{\phi M - 1}{\phi M}\right)\right)^{\intercal},$$

and Ω_a $a = 1, ..., 4$ are $(\phi M + 1, \phi M + 1)$ square matrices. For $j \leq \phi M$, the matrices Ω_1 and Ω_4 are defined as follows:

$$\begin{aligned}
\Omega_1[1,1] &= 2,\ \Omega_1[1+j, 1+j] = 4, \\
\Omega_4[1,1] &= 1,\ \Omega_4[2,1] = -1,\ \Omega_4[1,2] = -1,\ \Omega_4[2,2] = 2,\ \Omega_4[1+j, 1+j] = 2, \\
\Omega_4[1+j, j] &= -1,\ \Omega_4[j, j+1] = -1.
\end{aligned}$$

For $j \leq \phi M - 1$, the matrices Ω_2 and Ω_3 are defined as follows:

[7] The MSE is obtained under i.i.d. $\eta^{'}$s and $p^* \perp\!\!\!\perp \eta$ (the same conditions used to prove the limiting properties of this class of estimators as well as the limiting properties of the estimators discussed above) as well as $W \perp\!\!\!\perp \sigma$. In addition, normality of the errors is assumed solely to dispense with the estimation of the fourth noise moment. This assumption can be easily relaxed (Barndorff-Nielsen, Hansen, Lunde, and Shephard, 2006, and Bandi and Russell, 2005).

$$
\begin{aligned}
\Omega_2[1,1] &= 3,\ \Omega_2[1,2]=-4,\ \Omega_2[2,1]=-4,\ \Omega_2[2,2]=7,\\
\Omega_2[2+j,2+j] &= 6,\ \Omega_2[2+j,1+j]=-4,\ \Omega_2[1+j,2+j]=-4,\\
\Omega_2[2+j,j] &= 1,\ \Omega_2[j,2+j]=1,\\
\Omega_3[1,1] &= -1,\ \Omega_3[1,2]=2,\ \Omega_3[2,1]=2,\ \Omega_3[2,2]=-4.5,\ \Omega_3[j+2,j+2]=-3(j+1)-1,\\
\Omega_3[2+j,1+j] &= 2(j+1),\ \Omega_3[1+j,2+j]=2(j+1),\\
\Omega_3[2+j,j] &= -(j+1)/2,\ \Omega_3[j,2+j]=-(j+1)/2.
\end{aligned}
$$

Hence, $\widehat{V}_{BR}^{BNHLS(Bar)}$ is equal to $\widehat{V}^{BNHLS}$ with $q=\phi^{BR}M$ and $k(x)=1-x$.

- *The flat-top cubic kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an asymptotically optimal choice for the number of autocovariances as suggested by Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006),* $\widehat{V}_{BNHLS}^{BNHLS(Cubic)}$

Consider again the class of estimators in Eq. (7). Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) have shown that, if $k(.)$ is also chosen in such a way as to guarantee that $k^{'}(0)=0$ and $k^{'}(1)=0$, the number of autocovariances can be selected as $q=cM^{1/2}$ and the estimator is consistent at rate $M^{1/4}$. Specifically,

$$
\begin{aligned}
&M^{1/4}\left(\widehat{V}^{BNHLS}-V\right)\\
\Rightarrow\ &\left(\sqrt{4ck_{\bullet}^{0,0}Q-8c^{-1}k_{\bullet}^{0,2}\mathbf{E}(\eta^2)\left(V+\frac{\mathbf{E}(\eta^2)}{2}\right)+4\left(\mathbf{E}(\eta^2)\right)^2c^{-3}\left\{k'''(0)+k_{\bullet}^{0,4}\right\}}\right)N(0,1), \quad (8)
\end{aligned}
$$

where $k_{\bullet}^{0,0}=\int_0^1 k^2(x)dx$, $k_{\bullet}^{0,2}=\int_0^1 k(x)k^{''}(x)dx$, and $k_{\bullet}^{0,4}=\int_0^1 k(x)k^{''''}(x)dx$. Importantly, when $k(x)=1-3x^2+2x^3$, the limiting distribution of $\widehat{V}^{BNHLS}$ is the same as that of the multi-scale estimator of Zhang (2006). We define $\widehat{V}_{BNHLS}^{BNHLS(Cubic)}$ as $\widehat{V}^{BNHLS(Cubic)}$ with $q=cM^{1/2}$ and c chosen as the minimizer of the limiting variance in Eq. (8).

- *The flat-top cubic kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an optimal (in a finite sample) choice for the number of autocovariances as suggested by Bandi and Russell (2005),* $\widehat{V}_{BR}^{BNHLS(Cubic)}$

We define $\widehat{V}_{BR}^{BNHLS(Cubic)}$ as $\widehat{V}^{BNHLS}$ with $q=\phi^{BR}M$ and $k(x)=1-3x^2+2x^3$.

- *The flat-top modified Tukey-Hanning kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an asymptotically optimal choice for the number of autocovariances as suggested in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006),* $\widehat{V}_{BNHLS}^{BNHLS(TH)}$

Finally, we consider the modified Tukey-Hanning flat-top kernel estimator since it is asymptotically more efficient than the cubic estimator, i.e., $k(x) = \left(1 - \cos\pi(1-x)^2\right)/2$ (Barndorff-Nielsen, Hansen, Lunde, and Shephard, 2006). We define $\widehat{V}_{BNHLS}^{BNHLS(TH)}$ as $\widehat{V}^{BNHLS(TH)}$ with $q = cM^{1/2}$ and c chosen as the minimizer of the limiting variance in Eq. (8).

- *The flat-top modified Tukey-Hanning kernel estimator of Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) with an optimal (in a finite sample) choice for the number of autocovariances as suggested in Bandi and Russell (2005),* $\widehat{V}_{BR}^{BNHLS(TH)}$

We define $\widehat{V}_{BR}^{BNHLS(TH)}$ as $\widehat{V}^{BNHLS}$ with $q = \phi^{BR}M$ and $k(x) = \left(1 - \cos\pi(1-x)^2\right)/2$.

2.1 Out-of-sample forecasting

All variance estimates are obtained using intra-daily high-frequency returns over a 6-hour (10am to 4pm) period. The choice variables, such as the optimal sampling frequency of the realized variance estimator, the number of subsamples of the two-scale estimators, and the number of autocovariances of the flat-top symmetric kernel estimators, for instance, should be interpreted as time-varying. Specifically, for each trading day, optimization of the (finite sample and asymptotic) MSE of the estimators translates into the selection of an optimal number of intra-daily returns in the case of realized variance and into the selection of an optimal number of autocovariances/subsamples, for a *given* number of intra-daily returns, in the case of the alternative kernel-based estimators.

The out-of-sample forecasts are derived using ARFIMA models.[8] For each Method, we use 3 sample lengths to estimate the ARMA parameters: (1) 1,000 days, (2) 1,500 days, and (3) the entire set of daily data with no less than 1,500 observations.[9] This effectively translates into a total of $3 \times 14 = 42$ Methods. As in Engle, Hong, and Kane (1990) we add three additional Methods, namely the average of the daily forecasts (Mean), and the daily minimum (Min) and maximum (Max) forecast, for a grand total of 45 Methods. As emphasized by Engle, Hong, and Kane (1990), the average of n forecasts that are conditionally independent and of similar quality will converge

[8] A previous version of the paper used ARMA specifications. The resulting ranking of the forecasts was virtually the same as that reported below.

[9] The fractional d parameter is estimated by virtue of the GPH estimator (Geweke and Porter-Hudak, 1983) using the full sample. Because this parameter can hardly be estimated efficiently, we opt for using the longest span of available data for its estimation. While this choice reduces the genuine out-of-sample nature of our empirical exercise, we find that alternative, reasonable choices of the d parameter do not affect our results in any meaningful way. The d estimates are traditional and range between 0.3 for $\widehat{V}^{5\,\min}$ and 0.49 for $\widehat{V}_{ZMA,BR}^{ZMAadj}$ (see Table 1).

to an accurate forecast at a fast rate. Failure to do so indicates large economic differences and/or dependence of the forecasts. The use of the minimum (maximum) forecast should highlight the effects of persistent upward (downward) biases in the forecasts. Specifically, if the distribution of the volatility forecasts is roughly symmetric (which appears to be realistic for our data) and the forecasts are largely downward biased, then the maximum forecast should have better performance than the minimum forecast and the forecasts that are considerably downward biased. Conversely, if the forecasts are largely upward biased, then the minimum forecast should have better performance than the maximum forecast and the forecasts that are very upward biased.

To assess the influence of the overnights in variance forecasting, we consider three scenarios. The first involves pricing 6-hour options. Since the intra-daily variance estimates are for a 6-hour period, straightforward one-step-ahead forecasting yields the needed variance prediction. The second and third scenario involve pricing 1-day options. We account for the overnights (i.e., price changes between 4pm and 10am of the following day) in two alternative ways. The first procedure multiplies each original 6-hour variance estimate $\widetilde{V}$ (before forecasting) by a constant factor ζ defined as

$$\zeta = \frac{\sum_{i=1}^{n} \left(R_i^{S\&P500}\right)^2}{\sum_{i=1}^{n} \widetilde{V}_i}, \tag{9}$$

where $R_i^{S\&P500}$ is the daily return on the S&P 500 index for day i and n is the total number of days in our sample. This procedure ensures that the average of the transformed variances, i.e., $\zeta\widetilde{V}$, is equal to the average of the squared daily returns. Hence, ζ will blow up the 6-hour variance estimates. The second procedure simply adds the square of the corresponding overnight returns to each variance estimate.[10]

3 Option pricing and option trading

We follow Engle, Hong, and Kane (1990) in designing an hypothetical option market. Every agent is associated with a Method. Given his/her Method the agent prices 6-hour or 1-day options on a \$1 share of the S&P 500 index. The exercise price of the options is taken to be \$1, the risk-free rate is taken to be zero. Agents use Black-Scholes to price. Under these assumptions, the predicted call price is given by

[10] Hansen and Lunde (2005) provide a theoretical justification for these traditional adjustments while studying the optimal combination of overnight squared returns and intra-daily realized variance for the purpose of daily integrated variance estimation.

$$P_t = 2\Phi\left(\frac{1}{2}\sigma_t\right) - 1,$$

where Φ is the cumulative normal distribution and σ_t is a specific volatility forecast, as delivered by a Method. By put/call parity, P_t is also the corresponding put price. We are now specific about the various stages of the pricing and trading process.

1. Given a Method, each agent computes his/her fair Black-Scholes option price for either a 6-hour (at-the-money) option or a 1-day (at-the-money) option on a $1 share of the S&P 500 index.

2. The pair-wise trades take place. The trades are conducted at the mid-point of the agents' prices. Agents with variance forecasts higher than the mid-point will perceive the options to be underpriced. Hence, they will buy from their counterpart. Importantly, the agents buy or sell straddles (one put and one call). Hence, agents with high variance forecasts effectively speculate on volatility in that, on the one hand, they perceive the straddle to be underpriced while, on the other hand, they count on either option to end up considerably in the money due to the expected high volatility.

   Finally, the positions are hedged. The delta, or hedge ratio, of the call option is given by $\Phi\left(\frac{1}{2}\sigma_t\right)$. Hence, a trader who buys a call should go short $\Phi\left(\frac{1}{2}\sigma_t\right)$ shares of the stock for a riskless (given small changes in the stock price) position in the option and in the stock. Similarly, a trader who buys a put should go long $1 - \Phi\left(\frac{1}{2}\sigma_t\right)$ for a riskless position. The hedge ratio of the straddle is the sum of the two hedge ratios, i.e., $1 - 2\Phi\left(\frac{1}{2}\sigma_t\right)$.

   The daily profit to a trader who buys the straddle is then given by

$$\left|R_t^{S\&P500}\right| - 2P_t + R_t^{S\&P500}\left(1 - 2\Phi\left(\frac{1}{2}\sigma_t\right)\right).$$

   The daily profit to a seller is given by

$$2P_t - \left|R_t^{S\&P500}\right| - R_t^{S\&P500}\left(1 - 2\Phi\left(\frac{1}{2}\sigma_t\right)\right).$$

3. For each trading day, the profits/losses are computed for each agent (Method). The total daily profit for each agent is just the sum of the pair-wise profits divided by $l - 1$, where l is the total number of agents (Methods), i.e., 45. The profits/losses are then averaged across days.

The goal of this paper is to use a dollar metric in comparing competing volatility forecasts. In doing so, we consider a market where all agents effectively "price" volatility using the same model, a classical Black-Scholes model. The use of Black-Scholes places agents on equal footing. While our metric is sensible, future work could consider the sensitivity of our rankings (as reported below) to the assumptions made regarding the pricing model and/or the option's horizon. Because we are pricing at-the-money options over a short horizon it is likely that alternative pricing models would deliver similar rankings.

We conclude this section with two observations. First, by design, traders with median forecasts (or forecasts near the median value) have the potential to make profits off of market making (by selling to those with higher price forecasts and buying from those with lower price forecasts) even if their volatility forecasts are inaccurate. By looking at the resulting traders' positions, we find that the upward (downward) biased forecasts lead to an overwhelming majority of buy (sell) orders, as expected. Importantly, the best performing Methods have positions that are nicely dispersed around the neutral (market-making) position, with no obvious emphasis on the neutral position. Second, comparing multiple forecasts in an hypothetical market is not an obvious task. While our current set-up provides a meaningful comparison, future work should consider the case of a single market price for all transactions. Consider the price associated with the trader with the median forecast, for instance. On the one hand, differently from our current framework, traders should be allowed to choose whether to trade with other traders or not. If this were not the case, then some agents above (or below) the median price would engage in trades with a negative expected profit.[11] On the other hand, agents on different sides of the market, as determined by the clearing (median) price, should trade with each other and/or with the trader associated with the median price forecast (the market maker).

4 The data

We employ high-frequency data on the Standard and Poor's depository receipts (SPIDERS) to construct the index's intra-daily volatility estimates. SPIDERS are shares in a trust which owns stocks in the same proportion as that found in the S&P 500 index. SPIDERS trade like a stock (with the ticker symbol SPY on the Amex) at approximately one-tenth of the level of the S&P 500 index. They are widely used by institutions and traders as bets on the overall direction of the market or as a means of passive management (see, e.g., Elton, Gruber, Comer, Li, 2002).

Our sample extends from January 2, 1998 to March 31, 2006. We use SPIDERS mid-quotes

[11] Consider two traders whose price forecasts are above the median price forecast, for example. The trader whose price forecast is relatively lower would sell even though he/she deems the straddle to be underpriced.

on the NYSE. We remove quotes whose associated price changes and/or spreads are larger than 10%. Table 1 contains descriptive statistics. In our sample, the average duration between quote updates is 11.53 seconds. The average spread and the average price level are 0.0015 and 117.27, respectively.

The second moment of the noise $\mathbf{E}(\varepsilon^2)$, V, and Q are necessary inputs in our sampling frequency and bandwidth selection procedures. We estimate $\mathbf{E}(\varepsilon^2)$ using quote-to-quote continuously-compounded returns. The variance and quarticity estimates are obtained by using $\widehat{V}$ (realized variance) and $\widehat{Q}$ (realized quarticity) with fixed, 15-minute, calendar-time intervals.[12] The prevailing quote method is used in the absence of a quote. The out-of-sample forecasts are derived by virtue of an ARFIMA$(1,1)$ model. This simple specification provides a reasonable and parsimonious model for all series. The standard errors used to evaluate the statistical significance of the profits are Newey-West standard errors. The correlation structures of the various profits is mild. We use 6 autocovariances for all profit series based on their inspection. The results are insensitive to the use of an alternative, reasonable number of autocovariances.

The average optimal sampling interval of the realized variance estimator is 5.7 minutes. The average optimal (in a finite sample) number of autocovariances of $\widehat{V}_{BRY}^{ZMAadj}$, $\widehat{V}_{BR}^{ZMA}$ (and $\widehat{V}^{Bar}$), $\widehat{V}_{BR}^{BNHLS(Bar)}$, $\widehat{V}_{BR}^{BNHLS(Cub)}$, and $\widehat{V}_{BR}^{BNHLS(TH)}$ is 7, 16, 7, 6, and 10, respectively. The average asymptotically optimal number of autocovariances of $\widehat{V}_{ZMA}^{ZMA}$ (and $\widehat{V}_{ZMA}^{ZMAadj}$), $\widehat{V}_{BNHLS}^{BNHLS(Bar)}$, $\widehat{V}_{BNHLS}^{BNHLS(Cub)}$, and $\widehat{V}_{BNHLS}^{BNHLS(TH)}$ is 5, 5, 6, and 8, respectively. These figures are very consistent with values obtained by Bandi and Russell (2005, 2006a) using alternative assets and sample periods. Finite sample criteria lead to a number of autocovariances which is larger on average (but generally less volatile) than asymptotic criteria. The difference is particularly evident for the unadjusted two-scale estimator. The "near consistent" Bartlett kernel estimator and the unadjusted two-scale estimator have favorable asymptotic properties but, as discussed in Bandi and Russell (2005), display a potentially large finite sample (downward) bias in general. The optimal (in a finite sample) choice of the number of autocovariances/subsamples is affected by this bias component. Specifically, effective bias reduction in a finite sample requires the selection of a fairly large number of autocovariances/subsamples (16, on average, in our case). The optimal (average) asymptotic choice (5) is likely to leave a substantial (time-varying) bias component in the resulting estimates. The class of flat-top symmetric kernels is, under conditions discussed in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2006) and in Section 2 above, unbiased in a finite sample. The bias-corrected two-scale estimator is also unbiased under the same conditions. Not surprisingly, the required op-

[12] Bandi and Russell (2006c) evaluate by simulation the empirical validity of this straightforward (from an applied standpoint), albeit admittedly inefficient, method to estimate the price moments of interest. As is well-known, efficient estimation of the integrated quarticity is an open research issue.

timal (in a finite sample) number of autocovariances is, on average, smaller than in the unadjusted two-scale case (7, 6, and 10 vs. 16 in the case of the flat-top kernel estimators and 7 vs. 16 in the case of the bias-corrected two-scale estimator). Also, again not surprisingly, the difference between finite sample and asymptotic choices is smaller in the case of estimators with no obvious biases. If the conditions under which Bandi and Russell (2005), Zhang, Mykland and Aït-Sahalia (2005) and Barndorff-Nielsen, Hansen, Lunde and Shephard (2005) prove the finite sample and limiting properties of these estimators are satisfied at least as an approximation ($p^* \perp\!\!\!\perp \eta$ and i.i.d. η's for asymptotic results, $p^* \perp\!\!\!\perp \eta$, i.i.d. η's, and $W \perp\!\!\!\perp \sigma$ for finite sample results[13]), then (1) the bias-corrected two-scale estimator and the class of flat-top symmetric kernels should have no systematic biases, (2) the optimized (using finite sample methods) bias-corrected two-scale estimator and the flat-top symmetric kernels should have a smaller variance than their asymptotically optimal counterparts, (3) the optimized (using finite sample methods) unadjusted two-scale estimator should have a smaller bias component than the asymptotically optimal unadjusted two-scale estimator.

The time variation in the optimal sampling intervals of the realized variance estimator and in the optimal (asymptotic and finite sample) number of autocovariances of the kernel estimators is substantial. Figs. 1 and 2 contain the corresponding daily time series. Since the optimal number of autocovariances is positively related to the number of trades in each day, in all cases we experience a clear upward trend reflecting the corresponding upward trend in the number of trades. Similarly, the downward trend in the optimal sampling intervals of the realized variance estimator are largely due to decreases in the noise second moment relative to the quarticity of the underlying.

Fig. 3 (levels) and Fig. 4 (differences) compare finite sample and asymptotic choices of the number of autocovariances. In the case of the unadjusted two-scale estimator, the optimal (in a finite sample) choice is almost always higher than the asymptotically optimal choice. The only exceptions are for values at the end of the sample (for which, of course, asymptotic approximations are expected to fare better in virtue of the corresponding higher number of observations). As pointed out earlier, in light of the absence of a large finite sample bias component, asymptotic and finite sample choices are expected to be more similar in the case of the flat-top symmetric kernels and in the case of the bias-corrected two-scale estimator. Figs. 3 and 4 confirm this intuition. In both cases, large deviations are generally associated with a lower, asymptotically optimal number of autocovariances.

[13]As stressed above, the additional assumption used by Bandi and Russell (2005) in the case of the two-scale estimator, namely $\sigma_j^2 = \int_{t_{j-1}}^{t_j} \sigma_s^2 ds = \frac{V}{M}\ \forall j$, is simply intended to facilitate the empirical tractability of the resulting finite sample MSE expansion.

Fig. 5 contains time-series plots of $\widehat{V}^{5\,\mathrm{min}}$, $\widehat{V}^{15\,\mathrm{min}}$, and $\widehat{V}^{Opt}$. Consistent with theory, $\widehat{V}^{5\,\mathrm{min}}$ and $\widehat{V}^{15\,\mathrm{min}}$ have slightly more erratic behavior then $\widehat{V}^{Opt}$. Fig. 6 presents plots of $\widehat{V}^{Bar}$, $\widehat{V}^{ZMA}_{ZMA}$, and $\widehat{V}^{ZMA}_{BR}$. Again, in agreement with theory, $\widehat{V}^{Bar}$ and $\widehat{V}^{ZMA}_{BR}$ (the unadjusted two-scale estimator with an optimal, in a finite sample, number of subsamples) are virtually identical. $\widehat{V}^{ZMA}_{ZMA}$, the unadjusted two-scale estimator with an asymptotically optimal number of subsamples, is less volatile and appears to be downward biased. Fig. 7 and Fig. 8 report the three flat-top kernel estimates for asymptotic and finite sample choices of the number of autocovariances. The dynamic evolution of the intra-daily volatility estimates is similar in this case. Fig. 9 contains plots of $\widehat{V}^{ZMAadj}_{ZMA}$ and $\widehat{V}^{ZMAadj}_{BRY}$. With the exception of a few spikes (also documented for other kernel estimates), $\widehat{V}^{ZMAadj}_{ZMA}$ is slightly lower, on average, then $\widehat{V}^{ZMA(adj)}_{BRY}$.

In Table 2 we report the average variance forecasts, the standard deviations of the forecasts, and the average forecast errors (defined here as the difference between the variance forecasts and the squared 6-hour returns) as a percentage of the sample variance of the 6-hour returns. The realized variance estimates tend to lead to higher forecasts than the flat-top symmetric kernel estimates and the bias-corrected two-scale estimates. Similarly, the asymptotically optimal unadjusted two-scale estimator leads to lower forecasts. This is indicative of a likely upward bias in the realized variance estimates and, again, a likely downward bias in the asymptotically optimal unadjusted two-scale estimator. We now turn to option pricing and the profits from trading.

5 Profit-based ranking

Table 3 reports the average option prices (in cents), the average profits from holding a straddle (in cents), as well as the standard deviations of the profits. The relevant horizon is 6 hours. In other words, the intra-daily variance estimates are not adjusted for lack of overnights.

The profit from holding a straddle is $\left|R_t^{S\&P500}\right| - 2P_t$, where P_t is the call/put price forecast induced by the corresponding Method. Here we are assuming that each agent goes long a straddle and pays a price for it given by the agent's volatility forecast. If the forecast is accurate, the corresponding profit should be *small* in absolute value. For the longest data lengths, the smallest profits are delivered by the finite sample optimal bias-corrected two-scale estimator $\widehat{V}^{ZMA(adj)}_{BR}$, ranking second, and the flat-top Bartlett kernel estimator $\widehat{V}^{BNHLS(Bar)}$ in its two versions (asymptotically optimal and finite sample optimal), ranking first and third. The largest profits are given by 5-minute realized variance, $\widehat{V}^{5\,\mathrm{min}}$. The magnitudes of the profits associated with Max, Min, and Mean suggest lack of systematic (across estimators) downward or upward biases. However, the upward-biased realized variance-based forecasts translate into upward-biased price estimates (and large losses), whereas the downward-biased forecasts induced by the asymptotically optimal unadjusted two-scale estimator lead to downward-biased price estimates (and large positive profits). The standard deviations of the profits do not vary much across alternative Methods.

We now turn to trading, as described in Section 3, and consider first the case of options with a 6-hour expiration time. We rank the Methods based on average profits (Table 4) and Sharpe ratios (Table 5). The Sharpe ratios are simply defined as the average profits divided by the standard deviations of the profits. In what follows, the symbols $\overline{V}1,000$, $\overline{V}1,500$, and $\overline{V}1,500M$, define the generic Method $\overline{V}$ with $ARMA$ parameters estimated using the three sample lengths described in Section 2, i.e., 1,000 observations, 1,500 observations, and the entire sample of data with at least 1,500 observations, respectively.

When ranked based on average profits, 3 Methods perform better than Mean, namely $V_{BR}^{BNHLS(Bart)}1500$, $V_{BR}^{BNHLS(Cubic)}1500$, and $V_{BR}^{BNHLS(Cubic)}1000$. The cross-sectional dispersion of the profits is substantial, thereby indicating economically significant differences between alternative variance forecasts. We confirm that, even though there are no systematic biases in the forecasts (Max and Min give profits equal to -16.85 and -5.3 with rankings equal to 45 and 42, respectively), upward biases are associated with the realized variance measures ($\widehat{V}^{Opt}$, $\widehat{V}^{5\,\mathrm{min}}$, and $\widehat{V}^{15\,\mathrm{min}}$), while downward biases are associated with $\widehat{V}_{ZMA}^{ZMA}$. The windows on the right of Table 4 break down the Methods based on their data lengths. In all cases the largest profits are given by the optimized (in a finite sample) flat-top symmetric kernels. The performance of the asymptotically optimal flat-top symmetric kernels is quite satisfactory, mainly in the Bartlett and cubic cases. The finite sample optimal (bias-corrected and unadjusted) two-scale estimators perform well and, sometimes, as well as the optimized (in a finite sample) flat-top symmetric kernels. Consistent with MSE-based arguments laid out in Section 2, when using an asymptotic bandwidth choice, bias-correcting the two-scale estimator is beneficial. For the longest data lenghts, $\widehat{V}^{Opt}$ performs better than $\widehat{V}^{5\,\mathrm{min}}$ and $\widehat{V}^{15\,\mathrm{min}}$.

Differently from the pricing exercise in Table 3, the standard deviation of the profits is quite variable across different Methods. The smallest standard deviation is associated with Mean, thus suggesting some degree of independence between alternative forecasts. Not surprisingly, ranking based on Sharpe ratios favors the Mean (see Table 5). The subsequent four Methods in the ranking are $\widehat{V}_{BR}^{BNHLS(Bar)}1,500$, $\widehat{V}_{BR}^{BNHLS(Cub)}1,500$, $\widehat{V}_{BR}^{BNHLS(Bar)}1,500M$, and $\widehat{V}_{BR}^{ZMA}1,500$. Importantly, when we break down the Methods on the basis of data length, we largely find the same ranking as earlier. The largest Sharpe ratios are generally associated with the optimized (in a finite sample) flat-top symmetric kernels. The optimized (in a finite sample) two-scale estimators performs well. Again, when choosing an asymptotically optimal bandwidth, bias-correcting the two-scale estimator is helpful. As earlier, $\widehat{V}^{Opt}$ outperforms $\widehat{V}^{5\,\text{min}}$ and $\widehat{V}^{15\,\text{min}}$ for the longest data lengths.

We now deal with the overnights explicitly and price 1-day options (Table 6 and Table 7). First, we multiply each variance estimate by ζ in Eq. (9). Naturally, this adjustment has the potential to partly reduce the economic significance of alternative high-frequency volatility estimates/forecasts. Despite the adjustment, the overall picture remains fairly unchanged. Mean is now the 5th best forecast. The optimized (in a finite sample) flat-top symmetric kernels largely dominate all other forecasts for every choice of sample length. The asymptotically optimal flat-top symmetric kernels, the optimized (in a finite sample) "near consistent" Bartlett kernel, and the optimized (in a finite sample) two-scale estimators continue to fare well. Similarly, $\widehat{V}^{Opt}$ continues to outperform $\widehat{V}^{5\,\text{min}}$ and $\widehat{V}^{15\,\text{min}}$. Interestingly, the asymptotically optimal unadjusted two-scale estimator is now outperformed by all realized variance estimators. This is consistent with findings in Bandi and Russell (2005) where the same adjustment for lack of overnights is employed. Bias-correcting the asymptotically optimal two-scale estimator is again useful.[14] Examining the Sharpe ratios, rather than the average profits, does not modify our results.

The overall picture does not change when explicitly using the overnight returns, rather than the adjustment ζ, to obtain daily variance forecasts for the purpose of pricing daily options (Table 8 and Table 9). If anything, notwithstanding the obvious contamination that this correction might entail, the resulting ranking appears even cleaner, mainly in the longest data length case.

Importantly, the differences between alternative average profits/Sharpe ratios can be quite large. This result is consistent across experiments and indicates that, from an economic standpoint, there is scope for employing finite sample adjustments even in situations where the performance of the asymptotically optimal estimators is, in terms of ranking, similar to the performance of the optimal (in finite samples) estimators.

Not only are the profits economically significant, they are also statistically significant. Table 4, 6, and 8 contain the corresponding tests. Joint Chi-squared tests of the null of zero profits reject the null overwhelmingly in all cases. The 5% critical value of a Chi-squared test with 45 degrees of freedom, as in our case, is about 61. The tests' values are 249.2, 85.5, and 398.2, respectively. Hence, at least one strategy earns profits statistically significantly different from zero. Pairwise t-tests of the null of equal profits between optimal (in a finite sample) kernel estimators and asymptotically optimal kernel estimators favor the former unequivocally. The profits are *always* larger in the case of the optimal (in a finite sample) kernel estimators and generally statistically so. Interestingly, even in situations where a certain asymptotically optimal kernel estimator performs as well as (or

[14] By construction, due to the adjustment, the unconditional mean of the variance estimates is the same across Methods. Hence, it is the *time-variation* of the bias of the unadjusted two-scale estimator which makes the bias-correction compelling.

better) than an alternative optimal (in a finite sample) kernel estimator, the corresponding optimal (in a finite sample) version of that estimator delivers profits that are, in general, statistically higher. Hence, it is valuable to optimize the performance of estimators that would fare satisfactorily even when implemented using asymptotic bandwidth selection criteria. Finally, the asymptotically optimal bias-corrected two-scale estimator always outperforms its unadjusted asymptotically optimal counterpart while yielding gains that are generally statistically significant.

To summarize, we find that:

1. Optimized (in a finite sample) flat-top symmetric kernels generally outperform other choices.

2. Within the class of flat-top symmetric kernels, the Bartlett and cubic kernel are the preferred choices in most of our experiments. The modified Tukey-Hanning kernel is virtually always inferior to these alternatives, despite its favorable theoretical properties.

3. The asymptotically optimal flat-top symmetric kernels, the optimized (in a finite sample) "near consistent" Bartlett kernel, and the finite sample optimal (adjusted or unadjusted) two-scale estimators can perform very well.

4. When using asymptotically optimal bandwidth choices, bias-correcting the two-scale estimator is always beneficial for our data and metric.

5. Importantly, *regardless of the estimator*, optimal (in a finite sample) bandwidth choices always yield higher profits than the corresponding asymptotically optimal choices for our data and metric. These higher profits are generally statistically significant.

6. Optimally-sampled realized variance generally dominates realized variance based on ad-hoc intervals.

We find it interesting, and of course not obvious, that our reported rankings almost perfectly mirror that found by Bandi and Russell (2005) using finite sample MSE expansions as the relevant metric. Bandi and Russell (2005) emphasize that evaluating volatility estimators solely based on their limiting properties is, in general, not the right criterion. Certain "near consistent" and consistent estimators, such as the Bartlett kernel estimator and the unadjusted two-scale estimator, have favorable asymptotic properties but can display large finite sample biases. Equivalently, estimators with the same asymptotic properties, such as the unadjusted two-scale estimator and the unbiased flat-top Bartlett kernel estimator, can have drastically different finite sample features, largely due to bias considerations. For a variety of relevant metrics, the (time-varying) finite sample bias needs to be reduced for certain estimators to perform at their full potential. Our option pricing metrics, for example, penalize forecasts which drastically overstate or understate the level of volatility. Finite sample bandwidth selection methods, such as the one proposed by Bandi and Russell (2005), can yield bias reduction while optimally trading-off bias and variance. Because some consistent or "near consistent" estimators are asymptotically unbiased, while being biased in finite samples, asymptotic bandwidth selection methods can be detrimental in practise. In general, these methods do not capture the finite sample bias of these estimators. We also show that, even in the case of estimators with no obvious biases, such as the bias-corrected two-scale estimator and the class of flat-top kernel estimators, there is scope for optimizing their finite sample variance properties by appropriately selecting the corresponding number of autocovariances.

If we were to consider reduced-form linear forecasting models as in Andersen, Bollerslev, and Meddahi (2006), Bandi and Russell (2006a), and Ghysels and Sinko (2006a, 2006b), among others, we would expect the finite sample bias of alternative Methods for the regressor, *when constant across days*, to play no role. If the regressand is roughly unbiased, then using regressors with smaller variance would lead to (roughly) unbiased forecasts and forecasting gains from an R^2 perspective (Andersen, Bollerslev, and Meddahi, 2006, and Ghysels and Sinko, 2006b). This said, an unbiased regressand is necessary in a linear regression context for the forecast to be unbiased, i.e., for the level of volatility to be correct. While emphasis is placed on variance considerations in Andersen, Bollerslev, and Meddahi (2006) and Ghysels and Sinko (2006b), important bias considerations lead to their choice of unbiased regressands for the purpose of volatility prediction. Future research should evaluate how the effective separation between bias issues (leading to the level of the volatility forecast) and variance issues (leading to the accuracy of the forecast) in a linear regression forecasting model would fare in the context of the option pricing metric used in this paper.

Here we employ classical linear time-series models to predict out-of-sample. Bias and variance considerations are dealt with by directly evaluating the finite sample MSEs of the individual estimators (for each trading day). We find it important, from an applied standpoint, that explicit optimization of the finite sample MSE properties of these estimators, as suggested by Bandi and Russell (2003, 2005), leads to substantial economic gains (for our metrics) and a somewhat expected, but not obvious, ranking between alternative forecasts.

6 The VIX

Although the goal of this paper is to compare alternative high-frequency variance forecasts, it is tempting to evaluate how these forecasts would fare against a readily-available market-based forecast, such as the VIX, in the context of our metrics. We therefore consider today's VIX as a forecast for today's volatility and associate it with an additional trader. We let all dealers trade daily options and document findings for both Scenario 2 and 3.

The unconditional mean of the daily VIX is larger than the unconditional mean of the alternative daily variance measures, thereby suggesting the potential presence of a volatility risk premium. In order to put all variance measures on equal footing, we therefore multiply the daily VIX by the deflator $\zeta_{VIX} = \frac{\sum_{i=1}^{n} \left(R_i^{S\&P500}\right)^2}{\sum_{i=1}^{n} VIX_i}$. This adjustment guarantees that the average daily VIX value is the same as the average of the squared daily returns (and the average of the daily volatility estimates when using Scenario 2). We find that the reported rankings are hardly affected by the inclusion of this additional market-based Method. In addition, the VIX ranks 38 out of 45 in Scenario 2 and 43 out of 45 in Scenario 3 (Table 10 through 13).

7 Conclusions

The relative success of alternative high-frequency variance measures depends on their out-of-sample forecasting performance. Yet, a variety of loss functions can be proposed in practise. Either statistical or economic metrics could be entertained. Within each broad category, several alternative criteria can be invoked. It is legitimate to argue that well-posed economic loss functions represent crucial measures of the success of different forecasts. We take this position in this paper. Specifically, we attempt to provide a rich comparison between recently-proposed high-frequency measures of variance in the context of important, nonlinear economic criteria.

In the context of our proposed metrics, we do not find a significant difference between the "ex-ante" ranking that one would derive from careful evaluation (and optimization) of the finite sample MSE properties of alternative estimators and the "ex-post" ranking of the estimators' out-of-sample forecasts.

Among other things, future work should evaluate the pricing and trading of longer-maturity options, and the resulting ranking of long-term volatility estimates/forecasts. It is also of interest to examine situations where pricing and trading decisions are affected by forecast precision, rather than solely by point forecasts. More importantly for our purposes, while we empirically show that MSE-based forecasts largely preserve their "ex-ante" ranking in the context of our metrics, the optimal forecasts are in general not MSE-based optimal forecasts. Direct optimization of certain economic criteria could then be very beneficial and is left as a crucial topic for future work.

References

[1] Aït-Sahalia, Y., and L. Mancini (2006). Out-of-sample forecasts of quadratic variation. *Working paper.*

[2] Andersen, T.G., T. Bollerslev, and N. Meddahi (2006). Realized volatility forecasting and market microstructure noise. *Working paper.*

[3] Andersen, T.G., T. Bollerslev, F.X. Diebold, and H. Ebens (2001). The distribution of realized stock return volatility. *Journal of Financial Economics 61,* 43-76.

[4] Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys (1999). (Understanding, optimizing, using, and forecasting) Realized volatility and correlation. *Working paper.*

[5] Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys (2000). Great Realizations. *Risk Magazine 13,* 105-108.

[6] Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys (2003). Modeling and forecasting realized volatility. *Econometrica 71*, 579-625.

[7] Bandi, F.M., and J.R. Russell (2003). Microstructure noise, realized volatility, and optimal sampling. *Working paper.*

[8] Bandi, F.M., and J.R. Russell (2005). Market microstructure noise, integrated variance estimators, and the accuracy of asymptotic approximations. *Working paper.*

[9] Bandi, F.M., and J.R. Russell (2006a). Separating microstructure noise from volatility. *Journal of Financial Economics 79,* 655-692.

[10] Bandi, F.M., and J.R. Russell (2006b). Volatility. In J. Birge and V. Linetski (Eds.) *Handbook of financial engineering. Elsevier, North Holland,* forthcoming.

[11] Bandi, F.M., and J.R. Russell (2006c). Microstructure noise, realized variance, and optimal sampling. *Review of Economic Studies, forthcoming.*

[12] Bandi, F.M., J.R. Russell, and Yinghua Zhu (2006). Using high-frequency data in dynamic portfolio choice. *Econometric Reviews, forthcoming.*

[13] Barndorff-Nielsen, O.E., and N. Shephard (2002). Econometric analysis of realized volatility and its use in estimating stochastic volatility models. *Journal of the Royal Statistical Society, Series B 64*, 253-280.

[14] Barndorff-Nielsen, O.E., and N. Shephard (2006). Variation, jumps, market frictions and high-frequency data in financial econometrics. In R. Blundell, P. Torsten and W. K. Newey (Eds.) *Advances in Economics and Econometrics. Theory and Applications.* Ninth World Congress. Econometric Society Monographs, Cambridge University Press, forthcoming.

[15] Barndorff-Nielsen, O.E., P. Hansen, A. Lunde and N. Shephard (2005). Regular and modified kernel-based estimators of integrated variance: the case with independent noise. *Working paper.*

[16] Barndorff-Nielsen, O.E., P. Hansen, A. Lunde and N. Shephard (2006). Designing realized kernels to measure ex-post variation of equity prices in the presence of noise. *Working paper.*

[17] Barndorff-Nielsen, O.E., P. Hansen, A. Lunde and N. Shephard (2007). Subsampling realized kernels. *Working paper.*

[18] Corradi, V., W. Distaso, and N. Swanson (2006). Predictive inference for integrated volatility. *Working paper.*

[19] Corsi (2003). A simple long-memory model of realized volatility. *Working paper.*

[20] De Pooter, M., M. Martens, and D. Van Dijk (2006). Predicting the daily covariance matrix for S&P100 stocks using intraday data: but which frequency to use? *Econometric Reviews, forthcoming.*

[21] Elton, E.J., M. Gruber, G. Comer, K. Li (2002). Spiders: Where are the bugs? *Journal of Business 75*, 453-472

[22] Engle, R.F., C.H. Hong, and A. Kane (1990). Valuation of variance forecasts with simulated option markets. NBER Working paper No: 3350.

[23] Fleming, J., C. Kirby, and B. Ostdiek (2001). The economic value of volatility timing. *Journal of Finance 56*, 329-352.

[24] Fleming, J., C. Kirby, and B. Ostdiek (2003). The economic value of volatility timing using “realized volatility.” *Journal of Financial Economics 67*, 473-509.

[25] Geweke, J. and S. Porter-Hudak (1983). The estimation and application of long memory time series models. *Journal of Time Series Analysis 4*, 221-238.

[26] Ghysels, E. and A. Sinko (2006a). Comment on Hansen and Lunde. *Journal of Business and Economic Statistics 24,* 192-194.

[27] Ghysels, E. and A. Sinko (2006b). Volatility forecasting and microstructure noise. *Working paper.*

[28] Ghysels, E., P. Santa-Clara, and R. Valkanov (2006). Predicting volatility: getting the most out of return data sampled at different frequencies. *Journal of Econometrics, forthcoming.*

[29] Hansen, P.R., and A. Lunde (2005). A realized variance for the whole day based on intermittent high-frequency data. *Journal of Financial Econometrics 4, 525-554.*

[30] Hansen, P.R., and A. Lunde (2006). Realized variance and market microstructure noise (with discussions). *Journal of Business and Economic Statistics 24,* 127-161.

[31] Kalnina, I. and O. Linton (2006). Estimating quadratic variation consistently in the presence of correlated measurement error. *Working paper.*

[32] Large (2006). Estimating quadratic variation when quoted prices change by constant increments. *Working paper.*

[33] McAleer, M., and M. Medeiros (2006). Realized volatility: A review. *Working paper.*

[34] Oomen, R.C.A. (2005). Properties of bias-corrected realized variance under alternative sampling schemes. *Journal of Financial Econometrics 3,* 555–577.

[35] Protter, P. (1995). *Stochastic integration and differential equations.* A new approach. Springer.

[36] West, K.D., H.J. Edison, and D. Cho (1993). A utility-based comparison of some models of exchange rate volatility. *Journal of International Economics 35*, 23-45.

[37] Zhang, L., P. Mykland, and Y. Aït-Sahalia (2005). A tale of two time scales: determining integrated volatility with noisy high-frequency data. *Journal of the American Statistical Association 100,* 1394-1411.

[38] Zhang, L. (2006a). Efficient estimation of stochastic volatility using noisy observations: a multi-scale approach. *Bernoulli, forthcoming.*

[39] Zhou, B. (1996). High-frequency data and volatility in foreign-exchange rates. *Journal of Business and Economic Statistics 14*, 45-52.

Appendix

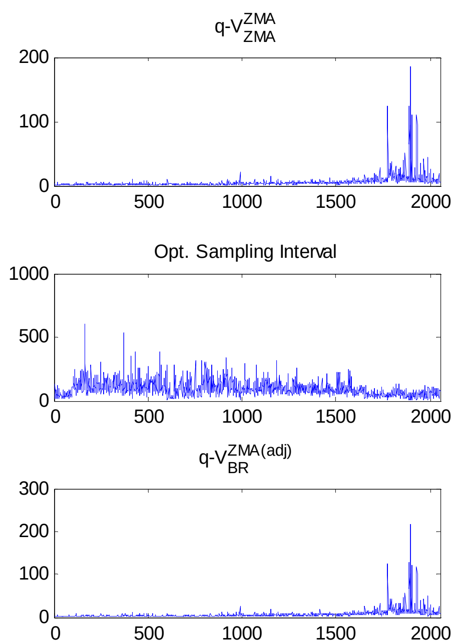

Figure 1. The asymptotic and finite sample optimal number of autocovariances of the (bias-corrected and unadjusted) two-scale estimator, the optimal number of observations of the realized variance estimator, and the number of daily trades.

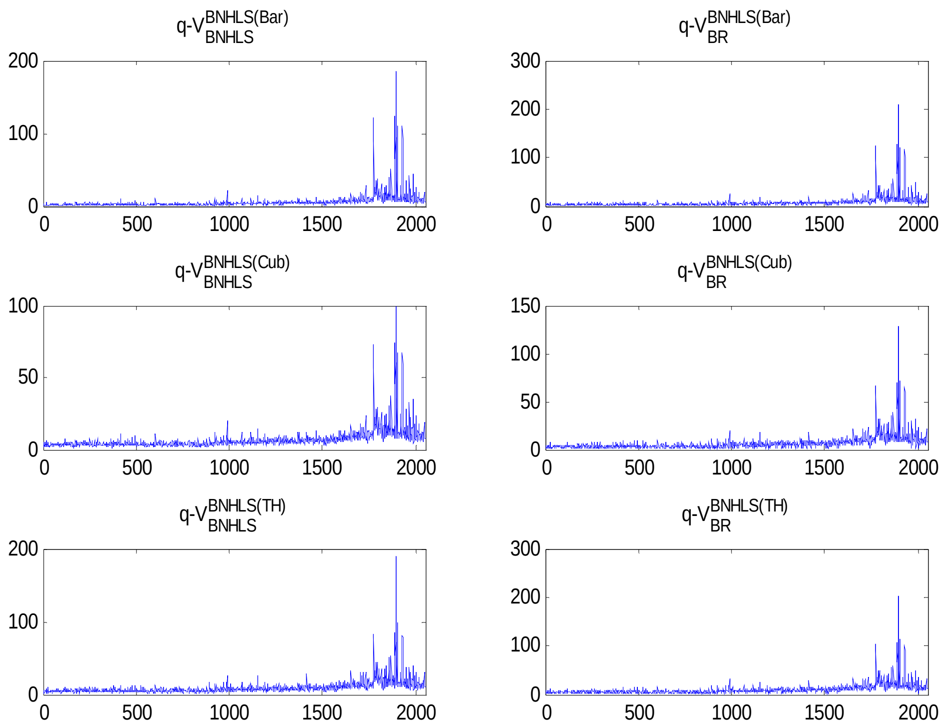

Figure 2. The asymptotic and finite sample optimal number of autocovariances of the flat-top symmetric kernel estimators (Bartlett, cubic, and modified Tukey-Hanning).

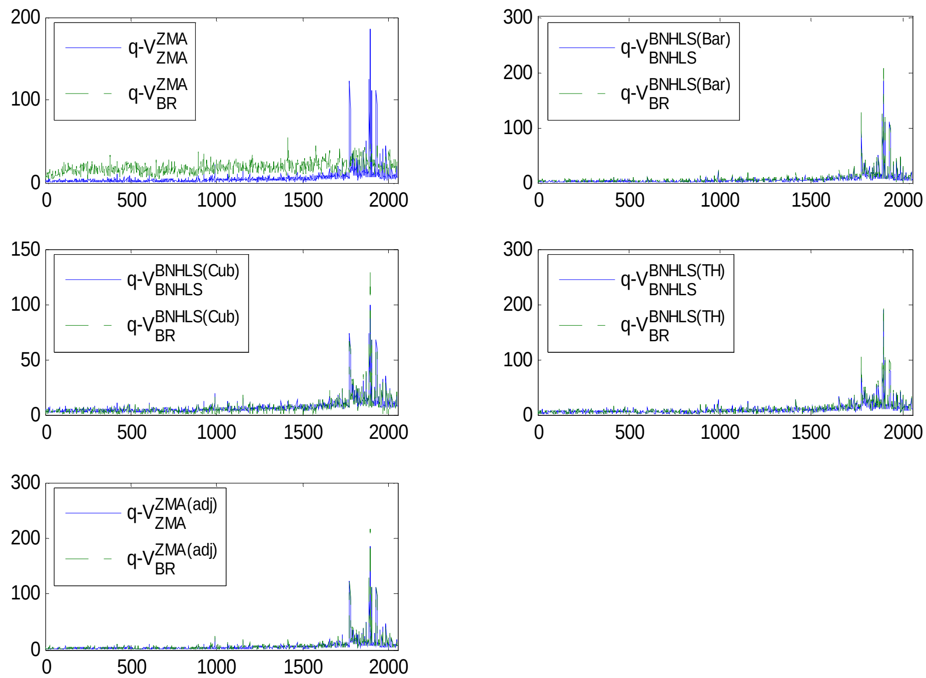

Figure 3. The asymptotic and finite sample optimal number of autocovariances of the (bias-corrected and unadjusted) two scale estimator and of the flat-top symmetric kernel estimators (Bartlett, cubic, and modified Tukey-Hanning).

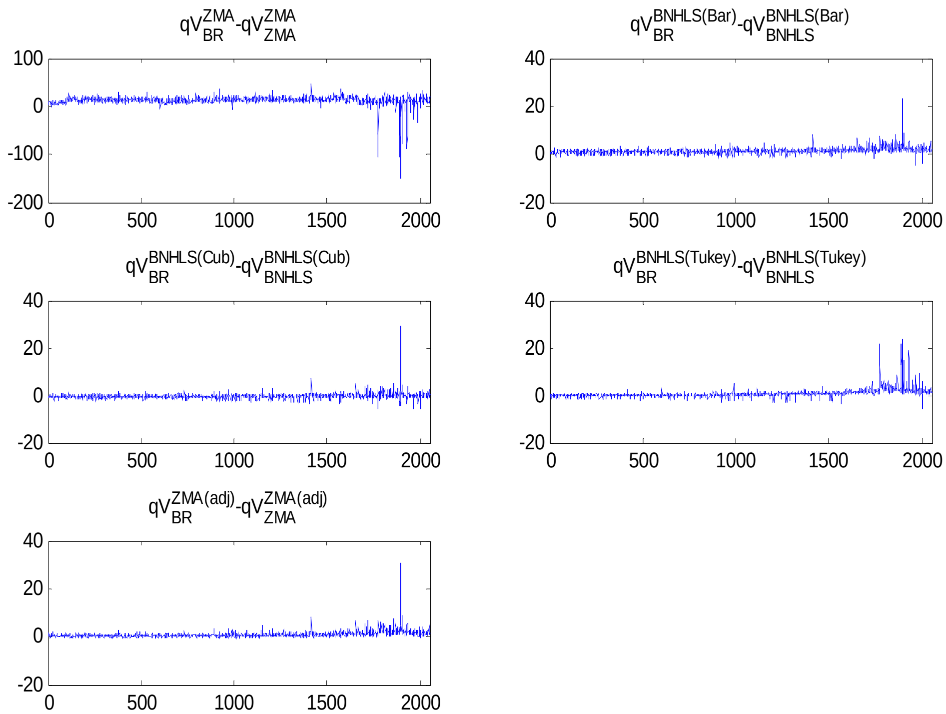

Figure 4. The difference between the asymptotic and finite sample optimal number of autocovariances of the (bias-corrected and unadjusted) two-scale estimator and of the flat-top symmetric kernel estimators (Bartlett, cubic, and modified Tukey-Hanning).

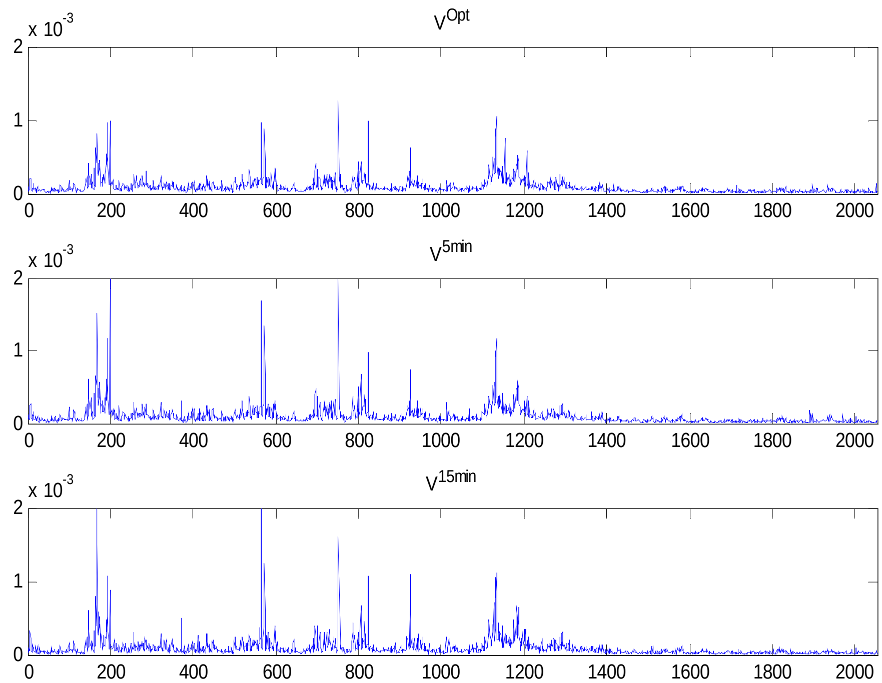

Figure 5. The realized variance estimates (optimally-sampled, 5-minute, and 15-minute).

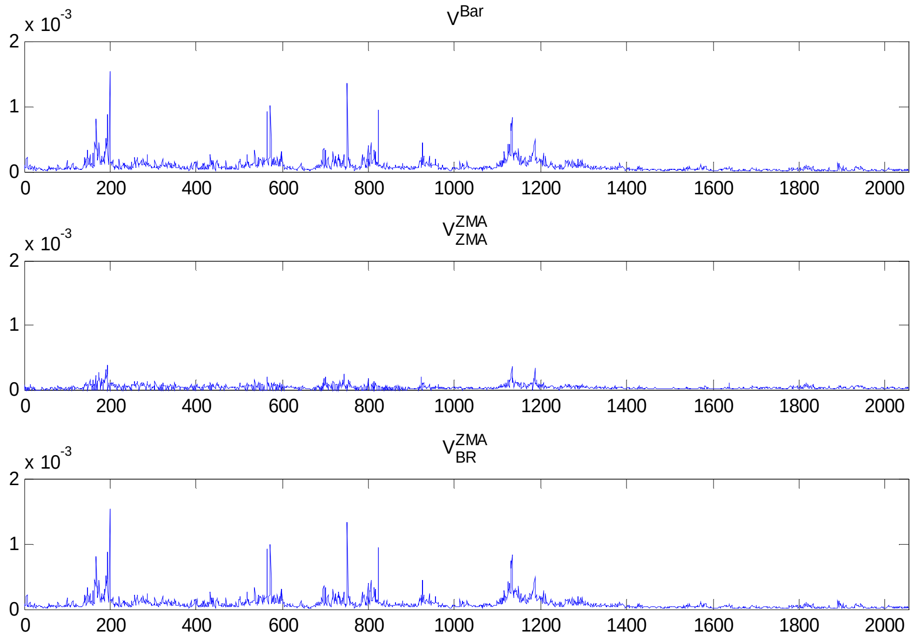

Figure 6. Variance estimates obtained by virtue of the "near consistent" Bartlett kernel estimator and the unadjusted two-scale estimator (asymptotically optimal and optimal in a finite sample).

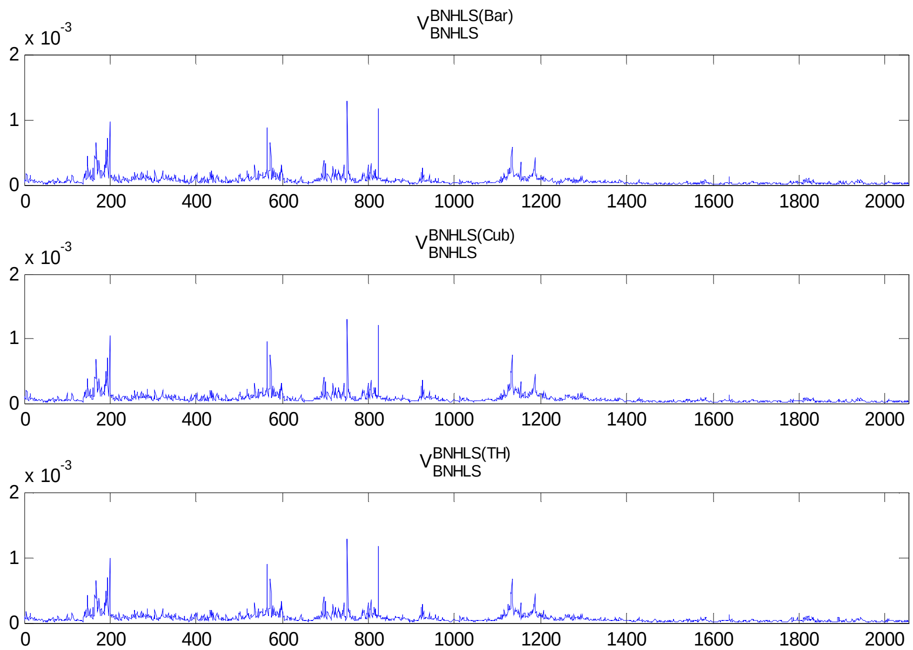

Figure 7. Variance estimates obtained by virtue of the flat-top symmetric kernel estimators (Bartlett, cubic, and modified Tukey-Hanning) with an asymptotically optimal number of autocovariances.

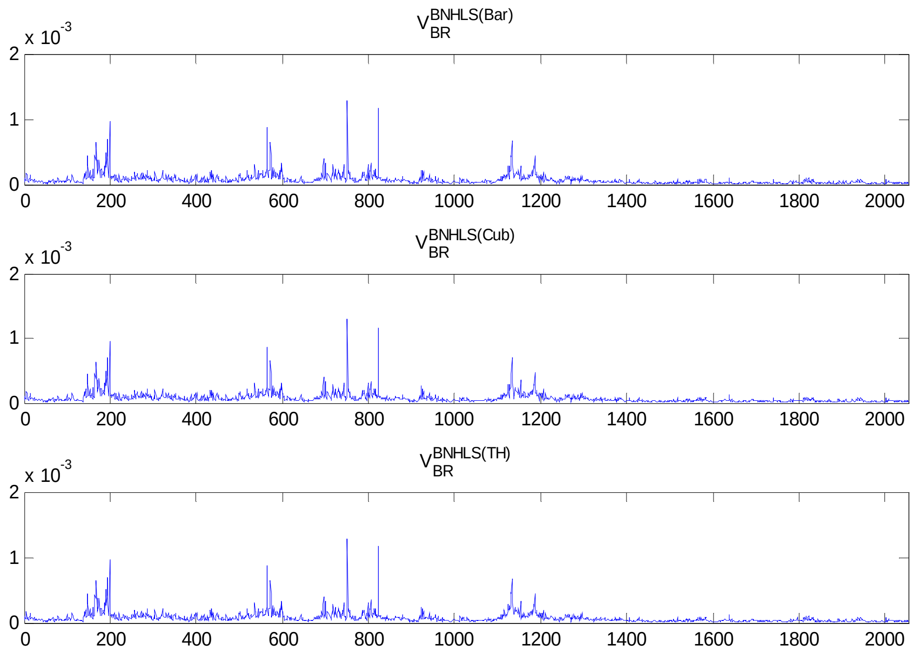

Figure 8. Variance estimates obtained by virtue of the flat-top symmetric kernel estimators (Bartlett, cubic, and modified Tukey-Hanning) with an optimal (in finite sample) number of autocovariances.

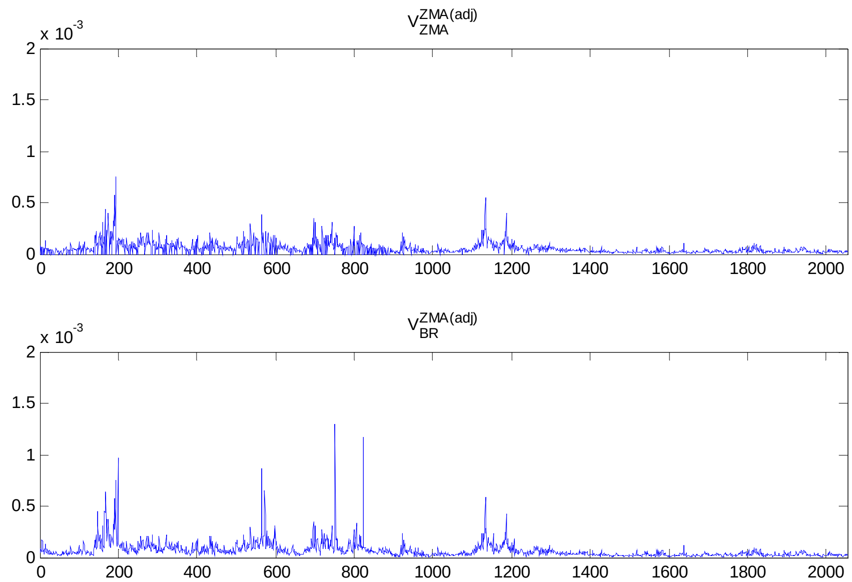

Figure 9. Variance estimates obtained by virtue of the bias-corrected two-scale estimator with an asymptotically optimal and a finite sample optimal number of autocovariances.

Table 1: Summary Statistics

Avg. Dur.	Avg. Sprd.	Avg. Price	M	Opt. Inter.
11.53	0.0015	117.27	3648	5.72

Asy. q		ZMA/ZMAadj	FlatBart	FlatCubic	FlatTukey
Mean		5	5	6	8
Max		185	208	129	202
Min		1	1	1	1
Std		10.22	10.99	6.99	10.85
Finite q	ZMAadj	ZMA/Bart	FlatBart	FlatCubic	FlatTukey
Mean	7	16	7	6	10
Max	216	57	185	99	189
Min	1	3	1	1	1
Std	11.18	6.45	10.18	6.53	9.19

We report summary statistics about the data (mid-quotes on SPIDERS between January 2, 1998 and March 31, 2006): average duration between mid-quote updates, average spread, average price, and average number of daily observations. Opt. Inter. denotes the average optimal sampling interval (in minutes) of the realized variance estimator. The remaining statistics summarize the empirical distribution of the optimal (finite sample and asymptotic) number of auto-covariances of the "near consistent" Bartlett kernel estimator (Bart), of the bias-corrected and unadjusted two-scale estimators (ZMAadj and ZMA), and of the flat-top kernel estimators in their Bartlett, cubic, and modified Tukey-Hanning versions (FlatBart, FlatCubic, and FlatTukey).

The d estimates
(Table 1 continued)

	d
Vopt	0.4964
VBart	0.4268
VFinZMA	0.4290
VFlatBart	0.4799
VFlatCubic	0.4335
VFlatTukey	0.4474
VZMA	0.4891
V5min	0.2927
V15min	0.3613
VFinFlatBart	0.4618
VFinFlatCubic	0.4619
VFinFlatTukey	0.4528
VZMAadj	0.4931
VFinZMAadj	0.4913

We report estimates of the fractional d parameter obtained by virtue of the GPH estimator (Geweke and Porter-Hudak, 1983).

Table 2: Average Forecasts, Std.s Forecasts, and Forecast Errors

	Avg. Vol. %	Std. Vol. %	MSE/AV*	Std.
OptV1000	9.4719	1.5145	0.1186	1.5970
OptV1500	9.6889	1.3334	0.1628	1.5918
OptV1500M	9.7547	1.3360	0.1785	1.5935
BartV1000	8.8737	1.6383	-0.0100	1.5935
BartV1500	9.2153	1.3381	0.0542	1.5851
BartV1500M	9.4110	1.2352	0.0953	1.5849
FinZMAV1000	8.8489	1.6444	-0.0151	1.5933
FinZMAV1500	9.2083	1.3234	0.0522	1.5847
FinZMAV1500M	9.3747	1.2413	0.0872	1.5855
FlatBart1000	8.3975	1.3465	-0.1204	1.5861
FlatBart1500	8.7207	1.0898	-0.0608	1.5830
FlatBart1500M	8.8236	1.1090	-0.0384	1.5849
FlatCubic1000	8.8232	1.3887	-0.0300	1.5838
FlatCubic1500	9.1745	1.0939	0.0379	1.5801
FlatCubic1500M	9.2681	1.0867	0.0587	1.5815
FlatTukey1000	8.7674	1.4222	-0.0408	1.5904
FlatTukey1500	9.1027	1.1369	0.0231	1.5847
FlatTukey1500M	9.2060	1.1412	0.0462	1.5865
ZMAV1000	7.5928	1.3061	-0.2780	1.5944
ZMAV1500	7.6441	1.2375	-0.2707	1.5923
ZMAV1500M	7.7036	1.2416	-0.2595	1.5935
V5min1000	10.0263	1.7488	0.2592	1.6096
V5min1500	10.8950	1.0626	0.4565	1.5842
V5min1500M	11.1393	0.9844	0.5200	1.5858
V15min1000	9.5219	1.3767	0.1253	1.5853
V15min1500	9.9901	1.0749	0.2273	1.5802
V15min1500M	10.0865	1.0704	0.2507	1.5840
FinFlatBart1000	8.5915	1.3367	-0.0807	1.5777
FinFlatBart1500	8.9250	1.0762	-0.0174	1.5766
FinFlatBart1500M	9.0299	1.0875	0.0058	1.5781
FinFlatCubic1000	8.7978	1.3211	-0.0377	1.5759
FinFlatCubic1500	9.1094	1.0821	0.0232	1.5750
FinFlatCubic1500M	9.2054	1.0932	0.0448	1.5763
FinFlatTukey1000	8.7762	1.3807	-0.0403	1.5844
FinFlatTukey1500	9.1123	1.1067	0.0245	1.5808
FinFlatTukey1500M	9.2062	1.1163	0.0456	1.5825
ZMAadjV1000	8.0983	1.4115	-0.1782	1.5947
ZMAadjV1500	8.2542	1.2187	-0.1534	1.5891
ZMAadjV1500M	8.3405	1.2413	-0.1353	1.5921
FinZMAadjV1000	8.2609	1.2743	-0.1504	1.5860
FinZMAadjV1500	8.5163	1.1009	-0.1033	1.5834
FinZMAadjV1500M	8.6127	1.1447	-0.0821	1.5854
Max	11.2371	1.1482	0.5508	1.5944
Min	7.4888	1.2109	-0.3000	1.5896
Mean	9.1982	1.1374	0.0444	1.5799

*: The ratio of the average forecast error (i.e., the variance forecast minus the 6-hour squared return) divided by the variance of the 6-hour returns.

Table 3: Hypothetical Option Prices and Average Profits from Straddles (in cents)

	Avg. Option Price	Avg. Prof. Straddle	Profits HAC Std.
OptV1000	0.23804	-0.03721	0.01557
OptV1500	0.24349	-0.04812	0.01562
OptV1500M	0.24514	-0.05143	0.01563
BartV1000	0.22301	-0.00715	0.01571
BartV1500	0.23159	-0.02432	0.01576
BartV1500M	0.23651	-0.03416	0.01582
FinZMAV1000	0.22238	-0.00590	0.01573
FinZMAV1500	0.23141	-0.02397	0.01577
FinZMAV1500M	0.23559	-0.03233	0.01585
FlatBart1000	0.21104	0.01679	0.01563
FlatBart1500	0.21916	0.00054	0.01583
FlatBart1500M	0.22175	-0.00463	0.01586
FlatCubic1000	0.22174	-0.00461	0.01540
FlatCubic1500	0.23056	-0.02227	0.01561
FlatCubic1500M	0.23292	-0.02697	0.01564
FlatTukey1000	0.22033	-0.00180	0.01553
FlatTukey1500	0.22876	-0.01866	0.01575
FlatTukey1500M	0.23136	-0.02385	0.01577
ZMAV1000	0.19081	0.05723	0.01623
ZMAV1500	0.19210	0.05466	0.01626
ZMAV1500M	0.19360	0.05166	0.01629
V5min1000	0.25197	-0.06508	0.01556
V5min1500	0.27380	-0.10874	0.01554
V5min1500M	0.27994	-0.12102	0.01561
V15min1000	0.23929	-0.03973	0.01534
V15min1500	0.25106	-0.06326	0.01545
V15min1500M	0.25348	-0.06810	0.01545
FinFlatBart1000	0.21591	0.00704	0.01545
FinFlatBart1500	0.22429	-0.00973	0.01562
FinFlatBart1500M	0.22693	-0.01500	0.01564
FinFlatCubic1000	0.22110	-0.00333	0.01533
FinFlatCubic1500	0.22893	-0.01900	0.01551
FinFlatCubic1500M	0.23134	-0.02382	0.01552
FinFlatTukey1000	0.22055	-0.00225	0.01546
FinFlatTukey1500	0.22900	-0.01914	0.01562
FinFlatTukey1500M	0.23136	-0.02386	0.01564
ZMAadjV1000	0.20352	0.03182	0.01602
ZMAadjV1500	0.20744	0.02399	0.01611
ZMAadjV1500M	0.20961	0.01965	0.01617
FinZMAadjV1000	0.20760	0.02365	0.01582
FinZMAadjV1500	0.21402	0.01081	0.01598
FinZMAadjV1500M	0.21645	0.00597	0.01598
Max	0.28240	-0.12594	0.01557
Min	0.18820	0.06246	0.01608
Mean	0.23116	-0.02346	0.01547

Rank Breakdown	Avg. Abs. Profits
Forecast Length 1000	
FlatTukey1000	0.0018
FinFlatTukey1000	0.0022
FinFlatCubic1000	0.0033
FlatCubic1000	0.0046
FinZMAV1000	0.0059
FinFlatBart1000	0.0070
BartV1000	0.0071
FlatBart1000	0.0168
FinZMAadjV1000	0.0237
ZMAadjV1000	0.0318
OptV1000	0.0372
V15min1000	0.0397
ZMAV1000	0.0572
V5min1000	0.0651
Forecast Length 1500	
FlatBart1500	0.0005
FinFlatBart1500	0.0097
FinZMAadjV1500	0.0108
FlatTukey1500	0.0187
FinFlatCubic1500	0.0190
FinFlatTukey1500	0.0191
FlatCubic1500	0.0223
FinZMAV1500	0.0240
ZMAadjV1500	0.0240
BartV1500	0.0243
OptV1500	0.0481
ZMAV1500	0.0547
V15min1500	0.0633
V5min1500	0.1087
Forecast Length >1500	
FlatBart1500M	0.0046
FinZMAadjV1500M	0.0060
FinFlatBart1500M	0.0150
ZMAadjV1500M	0.0197
FinFlatCubic1500M	0.0238
FlatTukey1500M	0.0238
FinFlatTukey1500M	0.0239
FlatCubic1500M	0.0270
FinZMAV1500M	0.0323
BartV1500M	0.0342
OptV1500M	0.0514
ZMAV1500M	0.0517
V15min1500M	0.0681
V5min1500M	0.1210

Scenario I: 6-hour Options

Table 4: Rank by Annualized Daily Profits (in cents)

	Avg. Prof.	HAC Std.	Rank
OptV1000	-0.1149	2.4576	30
OptV1500	-0.6360	2.5044	33
OptV1500M	-2.1653	2.6680	34
BartV1000	2.5347	2.2001	17
BartV1500	3.6490	1.5335	9
BartV1500M	1.1831	1.9742	25
FinZMAV1000	2.2769	2.3014	19
FinZMAV1500	4.1112	1.4375	6
FinZMAV1500M	1.7384	1.8121	21
FlatBart1000	1.9480	2.3277	20
FlatBart1500	3.9282	1.5659	7
FlatBart1500M	2.2990	1.4695	18
FlatCubic1000	2.8112	1.6944	16
FlatCubic1500	3.1022	1.4250	11
FlatCubic1500M	0.9878	1.8519	26
FlatTukey1000	1.7262	1.8137	22
FlatTukey1500	1.5302	1.6028	23
FlatTukey1500M	-0.2343	1.8894	31
ZMAV1000	-4.7069	3.6191	39
ZMAV1500	-3.2457	3.4138	37
ZMAV1500M	-2.6136	3.2316	36
V5min1000	-5.1638	2.8213	41
V5min1500	-12.2026	3.4791	43
V5min1500M	-15.6579	3.7250	44
V15min1000	0.7428	2.4936	27
V15min1500	-3.4578	2.7453	38
V15min1500M	-5.0107	2.8004	40
FinFlatBart1000	4.1716	2.0844	5
FinFlatBart1500	5.7373	1.1920	1
FinFlatBart1500M	3.8587	1.2782	8
FinFlatCubic1000	4.6568	1.6310	3
FinFlatCubic1500	5.0170	1.2324	2
FinFlatCubic1500M	3.0646	1.5726	13
FinFlatTukey1000	2.9372	1.6889	15
FinFlatTukey1500	2.9717	1.2922	14
FinFlatTukey1500M	1.3474	1.5591	24
ZMAadjV1000	-2.2520	2.8441	35
ZMAadjV1500	0.5084	2.5018	29
ZMAadjV1500M	-0.4525	2.3944	32
FinZMAadjV1000	0.5479	2.5795	28
FinZMAadjV1500	3.0726	1.9956	12
FinZMAadjV1500M	3.2501	1.7353	10
Max	-16.8509	3.8173	45
Min	-5.3163	3.8745	42
Mean	4.4162	0.7528	4

	Avg. Prof.
Forecast Length 1000	
FinFlatCubic1000	4.6568
FinFlatBart1000	4.1716
FinFlatTukey1000	2.9372
FlatCubic1000	2.8112
BartV1000	2.5347
FinZMAV1000	2.2769
FlatBart1000	1.9480
FlatTukey1000	1.7262
V15min1000	0.7428
FinZMAadjV1000	0.5479
OptV1000	-0.1149
ZMAadjV1000	-2.2520
ZMAV1000	-4.7069
V5min1000	-5.1638
Forecast Length 1500	
FinFlatBart1500	5.7373
FinFlatCubic1500	5.0170
FinZMAV1500	4.1112
FlatBart1500	3.9282
BartV1500	3.6490
FlatCubic1500	3.1022
FinZMAadjV1500	3.0726
FinFlatTukey1500	2.9717
FlatTukey1500	1.5302
ZMAadjV1500	0.5084
OptV1500	-0.6360
ZMAV1500	-3.2457
V15min1500	-3.4578
V5min1500	-12.2026
Forecast Length >1500	
FinFlatBart1500M	3.8587
FinZMAadjV1500M	3.2501
FinFlatCubic1500M	3.0646
FlatBart1500M	2.2990
FinZMAV1500M	1.7384
FinFlatTukey1500M	1.3474
BartV1500M	1.1831
FlatCubic1500M	0.9878
FlatTukey1500M	-0.2343
ZMAadjV1500M	-0.4525
OptV1500M	-2.1653
ZMAV1500M	-2.6136
V15min1500M	-5.0107
V5min1500M	-15.6579

Table 5: Rank by Sharpe Ratios

	Avg. Prof./HAC Std.	Rank
OptV1000	-0.0468	30
OptV1500	-0.2540	33
OptV1500M	-0.8116	36
BartV1000	1.1521	18
BartV1500	2.3796	8
BartV1500M	0.5993	25
FinZMAV1000	0.9893	19
FinZMAV1500	2.8600	5
FinZMAV1500M	0.9593	20
FlatBart1000	0.8369	24
FlatBart1500	2.5085	7
FlatBart1500M	1.5645	16
FlatCubic1000	1.6591	15
FlatCubic1500	2.1770	10
FlatCubic1500M	0.5334	26
FlatTukey1000	0.9518	22
FlatTukey1500	0.9547	21
FlatTukey1500M	-0.1240	31
ZMAV1000	-1.3006	39
ZMAV1500	-0.9508	37
ZMAV1500M	-0.8088	35
V5min1000	-1.8303	42
V5min1500	-3.5074	43
V5min1500M	-4.2035	44
V15min1000	0.2979	27
V15min1500	-1.2595	38
V15min1500M	-1.7893	41
FinFlatBart1000	2.0014	11
FinFlatBart1500	4.8134	2
FinFlatBart1500M	3.0188	4
FinFlatCubic1000	2.8552	6
FinFlatCubic1500	4.0708	3
FinFlatCubic1500M	1.9488	12
FinFlatTukey1000	1.7391	14
FinFlatTukey1500	2.2996	9
FinFlatTukey1500M	0.8642	23
ZMAadjV1000	-0.7918	34
ZMAadjV1500	0.2032	29
ZMAadjV1500M	-0.1890	32
FinZMAadjV1000	0.2124	28
FinZMAadjV1500	1.5397	17
FinZMAadjV1500M	1.8729	13
Max	-4.4144	45
Min	-1.3721	40
Mean	5.8663	1

	Avg. Prof./HAC Std.
Forecast Length 1000	
FinFlatCubic1000	2.8552
FinFlatBart1000	2.0014
FinFlatTukey1000	1.7391
FlatCubic1000	1.6591
BartV1000	1.1521
FinZMAV1000	0.9893
FlatTukey1000	0.9518
FlatBart1000	0.8369
V15min1000	0.2979
FinZMAadjV1000	0.2124
OptV1000	-0.0468
ZMAadjV1000	-0.7918
ZMAV1000	-1.3006
V5min1000	-1.8303
Forecast Length 1500	
FinFlatBart1500	4.8134
FinFlatCubic1500	4.0708
FinZMAV1500	2.8600
FlatBart1500	2.5085
BartV1500	2.3796
FinFlatTukey1500	2.2996
FlatCubic1500	2.1770
FinZMAadjV1500	1.5397
FlatTukey1500	0.9547
ZMAadjV1500	0.2032
OptV1500	-0.2540
ZMAV1500	-0.9508
V15min1500	-1.2595
V5min1500	-3.5074
Forecast Length >1500	
FinFlatBart1500M	3.0188
FinFlatCubic1500M	1.9488
FinZMAadjV1500M	1.8729
FlatBart1500M	1.5645
FinZMAV1500M	0.9593
FinFlatTukey1500M	0.8642
BartV1500M	0.5993
FlatCubic1500M	0.5334
FlatTukey1500M	-0.1240
ZMAadjV1500M	-0.1890
ZMAV1500M	-0.8088
OptV1500M	-0.8116
V15min1500M	-1.7893
V5min1500M	-4.2035

Tests
(Table 4 continued)

Joint test all b's (profits) are equal to zero:
b'*inv(var(b))*b=249.2

Pair-wise tests between finite sample profits and asymptotic profits:

	1000	1500	>1500
t(FinBart-AsyBart)	1.73	1.72	1.42
t(FinCubic-AsyCubic)	1.75	2.82	2.80
t(FinTukey-AsyTukey)	2.49	2.16	2.45
t(FinZMA-AsyZMA)	2.19	2.23	1.08
t(FinZMAadj-ZMAadj)	3.11	3.29	2.70
t(ZMAadj-ZMA)	1.90	3.01	1.64

Scenario II: 1-day Options (first adjustment)

Table 6: Rank by Annualized Daily Profits (in cents)

	Avg. Prof.	HAC Std.	Rank
OptV1000	0.2282	2.8843	25
OptV1500	1.4741	2.5052	19
OptV1500M	1.0151	2.4773	20
BartV1000	0.2227	2.6974	26
BartV1500	1.4768	2.2765	18
BartV1500M	1.6577	1.9858	14
FinZMAV1000	0.0239	2.6797	28
FinZMAV1500	1.5893	2.2472	15
FinZMAV1500M	1.4842	2.0402	17
FlatBart1000	0.1877	2.1275	27
FlatBart1500	-0.5808	1.9014	32
FlatBart1500M	-1.1129	2.0240	33
FlatCubic1000	2.8895	2.2533	11
FlatCubic1500	4.2241	1.5021	4
FlatCubic1500M	3.0021	1.3820	9
FlatTukey1000	0.7762	2.1234	23
FlatTukey1500	1.5229	1.6521	16
FlatTukey1500M	-0.2860	1.7227	30
ZMAV1000	-7.6278	3.2403	43
ZMAV1500	-7.4413	3.1052	42
ZMAV1500M	-7.7843	3.1828	44
V5min1000	-1.6036	3.2693	34
V5min1500	0.8966	2.8442	21
V5min1500M	-0.1504	2.8772	29
V15min1000	-0.5118	2.9695	31
V15min1500	0.7834	2.8291	22
V15min1500M	0.5232	2.6352	24
FinFlatBart1000	4.9687	1.8925	1
FinFlatBart1500	3.2974	1.2197	8
FinFlatBart1500M	2.8223	1.4189	13
FinFlatCubic1000	4.9313	2.1086	2
FinFlatCubic1500	4.5489	1.4320	3
FinFlatCubic1500M	3.7632	1.3679	6
FinFlatTukey1000	2.9988	2.1593	10
FinFlatTukey1500	3.3046	1.3640	7
FinFlatTukey1500M	2.8380	1.3147	12
ZMAadjV1000	-4.8756	2.7887	39
ZMAadjV1500	-4.7779	2.5778	38
ZMAadjV1500M	-5.0953	2.7134	40
FinZMAadjV1000	-2.5947	2.3755	37
FinZMAadjV1500	-1.8392	2.2005	35
FinZMAadjV1500M	-2.3207	2.3607	36
Max	-7.9026	3.8924	45
Min	-6.4832	3.9416	41
Mean	3.9504	0.8150	5

	Avg. Prof.
Forecast Length 1000	
FinFlatBart1000	4.9687
FinFlatCubic1000	4.9313
FinFlatTukey1000	2.9988
FlatCubic1000	2.8895
FlatTukey1000	0.7762
OptV1000	0.2282
BartV1000	0.2227
FlatBart1000	0.1877
FinZMAV1000	0.0239
V15min1000	-0.5118
V5min1000	-1.6036
FinZMAadjV1000	-2.5947
ZMAadjV1000	-4.8756
ZMAV1000	-7.6278
Forecast Length 1500	
FinFlatCubic1500	4.5489
FlatCubic1500	4.2241
FinFlatTukey1500	3.3046
FinFlatBart1500	3.2974
FinZMAV1500	1.5893
FlatTukey1500	1.5229
BartV1500	1.4768
OptV1500	1.4741
V5min1500	0.8966
V15min1500	0.7834
FlatBart1500	-0.5808
FinZMAadjV1500	-1.8392
ZMAadjV1500	-4.7779
ZMAV1500	-7.4413
Forecast Length >1500	
FinFlatCubic1500M	3.7632
FlatCubic1500M	3.0021
FinFlatTukey1500M	2.8380
FinFlatBart1500M	2.8223
BartV1500M	1.6577
FinZMAV1500M	1.4842
OptV1500M	1.0151
V15min1500M	0.5232
V5min1500M	-0.1504
FlatTukey1500M	-0.2860
FlatBart1500M	-1.1129
FinZMAadjV1500M	-2.3207
ZMAadjV1500M	-5.0953
ZMAV1500M	-7.7843

Table 7: Rank by Sharpe Ratios

	Avg. Prof./HAC Std.	Rank
OptV1000	0.0791	27
OptV1500	0.5884	19
OptV1500M	0.4098	20
BartV1000	0.0826	26
BartV1500	0.6487	18
BartV1500M	0.8348	15
FinZMAV1000	0.0089	28
FinZMAV1500	0.7072	17
FinZMAV1500M	0.7275	16
FlatBart1000	0.0882	25
FlatBart1500	-0.3055	32
FlatBart1500M	-0.5498	34
FlatCubic1000	1.2824	13
FlatCubic1500	2.8121	3
FlatCubic1500M	2.1723	9
FlatTukey1000	0.3656	21
FlatTukey1500	0.9218	14
FlatTukey1500M	-0.1660	30
ZMAV1000	-2.3540	43
ZMAV1500	-2.3964	44
ZMAV1500M	-2.4457	45
V5min1000	-0.4905	33
V5min1500	0.3153	22
V5min1500M	-0.0523	29
V15min1000	-0.1723	31
V15min1500	0.2769	23
V15min1500M	0.1985	24
FinFlatBart1000	2.6255	6
FinFlatBart1500	2.7034	5
FinFlatBart1500M	1.9891	11
FinFlatCubic1000	2.3387	8
FinFlatCubic1500	3.1766	2
FinFlatCubic1500M	2.7511	4
FinFlatTukey1000	1.3888	12
FinFlatTukey1500	2.4228	7
FinFlatTukey1500M	2.1587	10
ZMAadjV1000	-1.7484	39
ZMAadjV1500	-1.8535	40
ZMAadjV1500M	-1.8778	41
FinZMAadjV1000	-1.0923	37
FinZMAadjV1500	-0.8358	35
FinZMAadjV1500M	-0.9830	36
Max	-2.0303	42
Min	-1.6448	38
Mean	4.8473	1

	Avg. Prof./HAC Std.
Forecast Length 1000	
FinFlatBart1000	2.6255
FinFlatCubic1000	2.3387
FinFlatTukey1000	1.3888
FlatCubic1000	1.2824
FlatTukey1000	0.3656
FlatBart1000	0.0882
BartV1000	0.0826
OptV1000	0.0791
FinZMAV1000	0.0089
V15min1000	-0.1723
V5min1000	-0.4905
FinZMAadjV1000	-1.0923
ZMAadjV1000	-1.7484
ZMAV1000	-2.3540
Forecast Length 1500	
FinFlatCubic1500	3.1766
FlatCubic1500	2.8121
FinFlatBart1500	2.7034
FinFlatTukey1500	2.4228
FlatTukey1500	0.9218
FinZMAV1500	0.7072
BartV1500	0.6487
OptV1500	0.5884
V5min1500	0.3153
V15min1500	0.2769
FlatBart1500	-0.3055
FinZMAadjV1500	-0.8358
ZMAadjV1500	-1.8535
ZMAV1500	-2.3964
Forecast Length >1500	
FinFlatCubic1500M	2.7511
FlatCubic1500M	2.1723
FinFlatTukey1500M	2.1587
FinFlatBart1500M	1.9891
BartV1500M	0.8348
FinZMAV1500M	0.7275
OptV1500M	0.4098
V15min1500M	0.1985
V5min1500M	-0.0523
FlatTukey1500M	-0.1660
FlatBart1500M	-0.5498
FinZMAadjV1500M	-0.9830
ZMAadjV1500M	-1.8778
ZMAV1500M	-2.4457

Tests
(Table 6 continued)

Joint test all b's (profits) are equal to zero
b'*inv(var(b))*b=85.5

Pair-wise tests between finite sample profits and asymptotic profits:

	1000	1500	>1500
t(FinBart-AsyBart)	2.52	2.51	2.31
t(FinCubic-AsyCubic)	1.79	0.35	0.85
t(FinTukey-AsyTukey)	2.44	2.25	2.56
t(FinZMA-AsyZMA)	1.79	2.35	2.36
t(FinZMAadj-ZMAadj)	1.98	2.74	2.77
t(ZMAadj-ZMA)	2.10	3.31	3.38

Scenario III: 1-day Options (second adjustment)

Table 8: Rank by Annualized Daily Profits (in cents)

	Avg. Prof.	HAC Std.	Rank
OptV1000	1.4076	2.5370	23
OptV1500	0.5264	2.5543	28
OptV1500M	-0.3525	2.7280	30
BartV1000	1.2990	2.2287	25
BartV1500	2.0689	1.6783	18
BartV1500M	0.1787	1.8703	29
FinZMAV1000	0.8541	2.2989	26
FinZMAV1500	2.1193	1.6677	17
FinZMAV1500M	0.5302	1.7678	27
FlatBart1000	1.7326	2.2167	21
FlatBart1500	2.8598	1.5014	12
FlatBart1500M	1.9536	1.4623	20
FlatCubic1000	3.4300	1.7250	11
FlatCubic1500	3.7406	1.3513	8
FlatCubic1500M	2.4769	1.7202	15
FlatTukey1000	2.2242	1.9027	16
FlatTukey1500	2.5142	1.4608	14
FlatTukey1500M	1.5339	1.8011	22
ZMAV1000	-6.7700	3.5534	41
ZMAV1500	-5.5774	3.3871	40
ZMAV1500M	-4.8736	3.2202	39
V5min1000	-4.3665	2.6956	38
V5min1500	-8.9750	3.3812	43
V5min1500M	-11.5596	3.6134	44
V15min1000	-0.5003	2.4909	31
V15min1500	-2.8123	2.6699	35
V15min1500M	-3.3766	2.6718	37
FinFlatBart1000	3.8728	1.9850	7
FinFlatBart1500	4.7183	1.1070	3
FinFlatBart1500M	4.1549	1.1808	5
FinFlatCubic1000	4.7270	1.6137	2
FinFlatCubic1500	5.0566	1.1566	1
FinFlatCubic1500M	4.5133	1.4334	4
FinFlatTukey1000	3.4439	1.7315	10
FinFlatTukey1500	3.5900	1.1692	9
FinFlatTukey1500M	2.7823	1.5091	13
ZMAadjV1000	-2.9695	2.6927	36
ZMAadjV1500	-1.1581	2.4610	33
ZMAadjV1500M	-1.5666	2.2879	34
FinZMAadjV1000	-0.8493	2.4498	32
FinZMAadjV1500	1.3619	2.0950	24
FinZMAadjV1500M	2.0156	1.7424	19
Max	-12.0798	3.7689	45
Min	-7.9341	3.7769	42
Mean	4.0408	0.7947	6

	Avg. Prof.
Forecast Length 1000	
FinFlatCubic1000	4.7270
FinFlatBart1000	3.8728
FinFlatTukey1000	3.4439
FlatCubic1000	3.4300
FlatTukey1000	2.2242
FlatBart1000	1.7326
OptV1000	1.4076
BartV1000	1.2990
FinZMAV1000	0.8541
V15min1000	-0.5003
FinZMAadjV1000	-0.8493
ZMAadjV1000	-2.9695
V5min1000	-4.3665
ZMAV1000	-6.7700
Forecast Length 1500	
FinFlatCubic1500	5.0566
FinFlatBart1500	4.7183
FlatCubic1500	3.7406
FinFlatTukey1500	3.5900
FlatBart1500	2.8598
FlatTukey1500	2.5142
FinZMAV1500	2.1193
BartV1500	2.0689
FinZMAadjV1500	1.3619
OptV1500	0.5264
ZMAadjV1500	-1.1581
V15min1500	-2.8123
ZMAV1500	-5.5774
V5min1500	-8.9750
Forecast Length >1500	
FinFlatCubic1500M	4.5133
FinFlatBart1500M	4.1549
FinFlatTukey1500M	2.7823
FlatCubic1500M	2.4769
FinZMAadjV1500M	2.0156
FlatBart1500M	1.9536
FlatTukey1500M	1.5339
FinZMAV1500M	0.5302
BartV1500M	0.1787
OptV1500M	-0.3525
ZMAadjV1500M	-1.5666
V15min1500M	-3.3766
ZMAV1500M	-4.8736
V5min1500M	-11.5596

Table 9: Rank by Sharpe Ratios

	Avg. Prof./HAC Std.	Rank
OptV1000	0.5548	25
OptV1500	0.2061	28
OptV1500M	-0.1292	30
BartV1000	0.5828	24
BartV1500	1.2327	18
BartV1500M	0.0955	29
FinZMAV1000	0.3715	26
FinZMAV1500	1.2708	17
FinZMAV1500M	0.2999	27
FlatBart1000	0.7816	22
FlatBart1500	1.9048	12
FlatBart1500M	1.3360	16
FlatCubic1000	1.9884	10
FlatCubic1500	2.7681	8
FlatCubic1500M	1.4399	15
FlatTukey1000	1.1689	19
FlatTukey1500	1.7212	14
FlatTukey1500M	0.8517	21
ZMAV1000	-1.9052	41
ZMAV1500	-1.6467	40
ZMAV1500M	-1.5135	38
V5min1000	-1.6199	39
V5min1500	-2.6544	43
V5min1500M	-3.1991	44
V15min1000	-0.2009	31
V15min1500	-1.0534	35
V15min1500M	-1.2638	37
FinFlatBart1000	1.9511	11
FinFlatBart1500	4.2621	3
FinFlatBart1500M	3.5188	4
FinFlatCubic1000	2.9293	7
FinFlatCubic1500	4.3719	2
FinFlatCubic1500M	3.1487	5
FinFlatTukey1000	1.9889	9
FinFlatTukey1500	3.0704	6
FinFlatTukey1500M	1.8437	13
ZMAadjV1000	-1.1028	36
ZMAadjV1500	-0.4706	33
ZMAadjV1500M	-0.6847	34
FinZMAadjV1000	-0.3467	32
FinZMAadjV1500	0.6501	23
FinZMAadjV1500M	1.1568	20
Max	-3.2052	45
Min	-2.1007	42
Mean	5.0850	1

	Avg. Prof./HAC Std.
Forecast Length 1000	
FinFlatCubic1000	2.9293
FinFlatTukey1000	1.9889
FlatCubic1000	1.9884
FinFlatBart1000	1.9511
FlatTukey1000	1.1689
FlatBart1000	0.7816
BartV1000	0.5828
OptV1000	0.5548
FinZMAV1000	0.3715
V15min1000	-0.2009
FinZMAadjV1000	-0.3467
ZMAadjV1000	-1.1028
V5min1000	-1.6199
ZMAV1000	-1.9052
Forecast Length 1500	
FinFlatCubic1500	4.3719
FinFlatBart1500	4.2621
FinFlatTukey1500	3.0704
FlatCubic1500	2.7681
FlatBart1500	1.9048
FlatTukey1500	1.7212
FinZMAV1500	1.2708
BartV1500	1.2327
FinZMAadjV1500	0.6501
OptV1500	0.2061
ZMAadjV1500	-0.4706
V15min1500	-1.0534
ZMAV1500	-1.6467
V5min1500	-2.6544
Forecast Length >1500	
FinFlatBart1500M	3.5188
FinFlatCubic1500M	3.1487
FinFlatTukey1500M	1.8437
FlatCubic1500M	1.4399
FlatBart1500M	1.3360
FinZMAadjV1500M	1.1568
FlatTukey1500M	0.8517
FinZMAV1500M	0.2999
BartV1500M	0.0955
OptV1500M	-0.1292
ZMAadjV1500M	-0.6847
V15min1500M	-1.2638
ZMAV1500M	-1.5135
V5min1500M	-3.1991

Tests
(Table 8 continued)

Joint test all b's (profits) are equal to zero:
b'*inv(var(b))*b=398.2

Pair-wise tests between finite sample profits and asymptotic profits:

	1000	1500	>1500
t(FinBart-AsyBart)	1.58	1.49	1.68
t(FinCubic-AsyCubic)	1.12	1.90	2.87
t(FinTukey-AsyTukey)	2.29	1.68	2.02
t(FinZMA-AsyZMA)	2.15	2.05	1.30
t(FinZMAadj-ZMAadj)	1.88	2.35	2.48
t(ZMAadj-ZMA)	2.80	4.05	2.72

Table 10: Rank by Ann. Daily Profits (in cents). Scenario II

	Avg. Prof.	HAC Std.	Rank		Avg. Prof.
OptV1000	0.4648	2.8071	26	Forecast Length 1000	
OptV1500	1.6036	2.4473	18	FinFlatBart1000	5.0986
OptV1500M	1.1373	2.4219	20	FinFlatCubic1000	5.0557
BartV1000	0.4761	2.6442	25	FinFlatTukey1000	3.1989
BartV1500	1.6536	2.2311	17	FlatCubic1000	2.9072
BartV1500M	1.8446	1.9445	14	FlatTukey1000	0.9478
FinZMAV1000	0.2934	2.6263	28	BartV1000	0.4761
FinZMAV1500	1.7787	2.2096	15	OptV1000	0.4648
FinZMAV1500M	1.6596	2.0017	16	FlatBart1000	0.3616
FlatBart1000	0.3616	2.1357	27	FinZMAV1000	0.2934
FlatBart1500	-0.4620	1.9078	31	V15min1000	-0.7477
FlatBart1500M	-0.9562	2.0320	33	V5min1000	-1.3390
FlatCubic1000	2.9072	2.1933	13	FinZMAadjV1000	-2.4011
FlatCubic1500	4.3331	1.4568	4	ZMAadjV1000	-4.6757
FlatCubic1500M	3.1675	1.3465	10	ZMAV1000	-7.4141
FlatTukey1000	0.9478	2.1168	23	Forecast Length 1500	
FlatTukey1500	1.5658	1.6633	19	FinFlatCubic1500	4.7061
FlatTukey1500M	-0.3741	1.7323	30	FlatCubic1500	4.3331
ZMAV1000	-7.4141	3.2295	44	FinFlatBart1500	3.4163
ZMAV1500	-7.2617	3.0982	43	FinFlatTukey1500	3.3819
ZMAV1500M	-7.5892	3.1753	45	FinZMAV1500	1.7787
V5min1000	-1.3390	3.1750	34	BartV1500	1.6536
V5min1500	1.0310	2.7843	22	OptV1500	1.6036
V5min1500M	-0.0064	2.8203	29	FlatTukey1500	1.5658
V15min1000	-0.7477	2.8971	32	V15min1500	1.0974
V15min1500	1.0974	2.7685	21	V5min1500	1.0310
V15min1500M	0.7854	2.5867	24	FlatBart1500	-0.4620
FinFlatBart1000	5.0986	1.8787	1	FinZMAadjV1500	-1.5951
FinFlatBart1500	3.4163	1.2199	7	ZMAadjV1500	-4.5952
FinFlatBart1500M	2.9406	1.4220	12	ZMAV1500	-7.2617
FinFlatCubic1000	5.0557	2.0760	2	Forecast Length >1500	
FinFlatCubic1500	4.7061	1.4003	3	FinFlatCubic1500M	3.8832
FinFlatCubic1500M	3.8832	1.3518	6	FlatCubic1500M	3.1675
FinFlatTukey1000	3.1989	2.1348	9	FinFlatTukey1500M	2.9731
FinFlatTukey1500	3.3819	1.3633	8	FinFlatBart1500M	2.9406
FinFlatTukey1500M	2.9731	1.3246	11	BartV1500M	1.8446
ZMAadjV1000	-4.6757	2.7836	40	FinZMAV1500M	1.6596
ZMAadjV1500	-4.5952	2.5767	39	OptV1500M	1.1373
ZMAadjV1500M	-4.9433	2.7092	41	V15min1500M	0.7854
FinZMAadjV1000	-2.4011	2.3811	37	V5min1500M	-0.0064
FinZMAadjV1500	-1.5951	2.1965	35	FlatTukey1500M	-0.3741
FinZMAadjV1500M	-2.1779	2.3561	36	FlatBart1500M	-0.9562
VIX	-3.1641	3.2149	38	FinZMAadjV1500M	-2.1779
Max	-12.1265	3.7964	46	VIX	-3.1641
Min	-6.2812	3.9384	42	ZMAadjV1500M	-4.9433
Mean	4.1100	0.8236	5	ZMAV1500M	-7.5892

Table 11: Rank by Sharpe Ratios. Scenario II

	Avg. Prof./HAC Std.	Rank
OptV1000	0.1656	27
OptV1500	0.6552	19
OptV1500M	0.4696	20
BartV1000	0.1801	25
BartV1500	0.7412	18
BartV1500M	0.9486	14
FinZMAV1000	0.1117	28
FinZMAV1500	0.8050	17
FinZMAV1500M	0.8291	16
FlatBart1000	0.1693	26
FlatBart1500	-0.2422	31
FlatBart1500M	-0.4706	34
FlatCubic1000	1.3255	13
FlatCubic1500	2.9743	3
FlatCubic1500M	2.3524	9
FlatTukey1000	0.4478	21
FlatTukey1500	0.9414	15
FlatTukey1500M	-0.2160	30
ZMAV1000	-2.2958	43
ZMAV1500	-2.3438	44
ZMAV1500M	-2.3901	45
V5min1000	-0.4217	33
V5min1500	0.3703	23
V5min1500M	-0.0023	29
V15min1000	-0.2581	32
V15min1500	0.3964	22
V15min1500M	0.3036	24
FinFlatBart1000	2.7139	6
FinFlatBart1500	2.8004	5
FinFlatBart1500M	2.0679	11
FinFlatCubic1000	2.4353	8
FinFlatCubic1500	3.3608	2
FinFlatCubic1500M	2.8727	4
FinFlatTukey1000	1.4985	12
FinFlatTukey1500	2.4808	7
FinFlatTukey1500M	2.2444	10
ZMAadjV1000	-1.6798	40
ZMAadjV1500	-1.7833	41
ZMAadjV1500M	-1.8247	42
FinZMAadjV1000	-1.0084	38
FinZMAadjV1500	-0.7262	35
FinZMAadjV1500M	-0.9244	36
VIX	-0.9842	37
Max	-3.1942	46
Min	-1.5949	39
Mean	4.9905	1

Table 12: Rank by Ann. Daily Profits (in cents). Scenario III

	Avg. Prof.	HAC Std.	Rank		Avg. Prof.
OptV1000	1.7316	2.4844	23	Forecast Length 1000	
OptV1500	0.8262	2.4833	27	FinFlatCubic1000	4.8997
OptV1500M	-0.0115	2.6563	30	FinFlatBart1000	4.0371
BartV1000	1.5711	2.1945	25	FinFlatTukey1000	3.7092
BartV1500	2.2817	1.6349	19	FlatCubic1000	3.6482
BartV1500M	0.5307	1.8004	29	FlatTukey1000	2.5003
FinZMAV1000	1.1529	2.2669	26	FlatBart1000	1.9399
FinZMAV1500	2.3899	1.6246	17	OptV1000	1.7316
FinZMAV1500M	0.8097	1.7042	28	BartV1000	1.5711
FlatBart1000	1.9399	2.2234	21	FinZMAV1000	1.1529
FlatBart1500	3.1065	1.5049	12	V15min1000	-0.1155
FlatBart1500M	2.3050	1.4636	18	FinZMAadjV1000	-0.6229
FlatCubic1000	3.6482	1.7062	11	ZMAadjV1000	-2.7163
FlatCubic1500	4.0984	1.2786	6	V5min1000	-3.9079
FlatCubic1500M	2.7993	1.6517	14	ZMAV1000	-6.4943
FlatTukey1000	2.5003	1.9154	16	Forecast Length 1500	
FlatTukey1500	2.7499	1.4411	15	FinFlatCubic1500	5.3257
FlatTukey1500M	1.7914	1.7615	22	FinFlatBart1500	4.9327
ZMAV1000	-6.4943	3.5324	41	FlatCubic1500	4.0984
ZMAV1500	-5.2575	3.3636	40	FinFlatTukey1500	3.8921
ZMAV1500M	-4.5884	3.2030	39	FlatBart1500	3.1065
V5min1000	-3.9079	2.6236	38	FlatTukey1500	2.7499
V5min1500	-8.5582	3.3090	42	FinZMAV1500	2.3899
V5min1500M	-11.1063	3.5511	45	BartV1500	2.2817
V15min1000	-0.1155	2.4111	31	FinZMAadjV1500	1.5901
V15min1500	-2.3662	2.5987	35	OptV1500	0.8262
V15min1500M	-2.9819	2.6013	37	ZMAadjV1500	-0.9410
FinFlatBart1000	4.0371	1.9921	7	V15min1500	-2.3662
FinFlatBart1500	4.9327	1.1015	2	ZMAV1500	-5.2575
FinFlatBart1500M	4.4276	1.1671	5	V5min1500	-8.5582
FinFlatCubic1000	4.8997	1.6064	3	Forecast Length >1500	
FinFlatCubic1500	5.3257	1.1066	1	FinFlatCubic1500M	4.8729
FinFlatCubic1500M	4.8729	1.3726	4	FinFlatBart1500M	4.4276
FinFlatTukey1000	3.7092	1.7394	10	FinFlatTukey1500M	3.0386
FinFlatTukey1500	3.8921	1.1412	9	FlatCubic1500M	2.7993
FinFlatTukey1500M	3.0386	1.4636	13	FlatBart1500M	2.3050
ZMAadjV1000	-2.7163	2.6949	36	FinZMAadjV1500M	2.2689
ZMAadjV1500	-0.9410	2.4586	33	FlatTukey1500M	1.7914
ZMAadjV1500M	-1.3770	2.2937	34	FinZMAV1500M	0.8097
FinZMAadjV1000	-0.6229	2.4507	32	BartV1500M	0.5307
FinZMAadjV1500	1.5901	2.0879	24	OptV1500M	-0.0115
FinZMAadjV1500M	2.2689	1.7443	20	ZMAadjV1500M	-1.3770
VIX	-8.7703	3.3217	43	V15min1500M	-2.9819
Max	-14.4358	3.7449	46	ZMAV1500M	-4.5884
Min	-8.8752	3.7805	44	VIX	-8.7703
Mean	3.8973	0.8456	8	V5min1500M	-11.1063

Table 13: Rank by Sharpe Ratios. Scenario III

	Avg. Prof./HAC Std.	Rank
OptV1000	0.6970	25
OptV1500	0.3327	28
OptV1500M	-0.0043	30
BartV1000	0.7159	24
BartV1500	1.3956	18
BartV1500M	0.2948	29
FinZMAV1000	0.5086	26
FinZMAV1500	1.4711	17
FinZMAV1500M	0.4751	27
FlatBart1000	0.8725	22
FlatBart1500	2.0643	12
FlatBart1500M	1.5750	16
FlatCubic1000	2.1382	9
FlatCubic1500	3.2054	7
FlatCubic1500M	1.6947	15
FlatTukey1000	1.3054	19
FlatTukey1500	1.9082	14
FlatTukey1500M	1.0170	21
ZMAV1000	-1.8385	41
ZMAV1500	-1.5631	40
ZMAV1500M	-1.4326	38
V5min1000	-1.4895	39
V5min1500	-2.5863	43
V5min1500M	-3.1276	45
V15min1000	-0.0479	31
V15min1500	-0.9105	35
V15min1500M	-1.1463	37
FinFlatBart1000	2.0265	13
FinFlatBart1500	4.4782	3
FinFlatBart1500M	3.7937	4
FinFlatCubic1000	3.0501	8
FinFlatCubic1500	4.8127	1
FinFlatCubic1500M	3.5501	5
FinFlatTukey1000	2.1325	10
FinFlatTukey1500	3.4105	6
FinFlatTukey1500M	2.0762	11
ZMAadjV1000	-1.0080	36
ZMAadjV1500	-0.3827	33
ZMAadjV1500M	-0.6004	34
FinZMAadjV1000	-0.2542	32
FinZMAadjV1500	0.7616	23
FinZMAadjV1500M	1.3007	20
VIX	-2.6403	44
Max	-3.8548	46
Min	-2.3476	42
Mean	4.6087	2