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Management\***  
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Activist shareholders have lately been attempting to assert themselves in a struggle with management and regulators over control of corporate decisions. These efforts have met with mixed success. Meanwhile, shareholders have been pressing for changes in the rules governing access to the corporate proxy process, especially in regard to nominating directors. The key issue which these events have brought to light is whether, in fact, shareholders will be better off with enhanced control over corporate decisions. Proponents of increased shareholder participation argue that such participation is needed to counter the agency problems associated with management decisions. In this view, boards of directors do not exercise sufficient control over self-interested managers because management insiders typically hand-pick directors through their control of the proxy process. Opponents offer several arguments such as that shareholders lack the requisite knowledge and expertise to make effective decisions or that shareholders may have incentives to make value-reducing decisions. In this paper, we investigate what determines the optimality of shareholder control, taking account of some of the above arguments, both pro and con. Our main contribution is to use formal modeling to uncover some factors overlooked in these arguments. For example, we show that the claims that shareholders should not have control over important decisions because they lack sufficient information to make an informed decision or because they have a non-value-maximizing agenda are flawed. On the other hand, it has been argued that, since shareholders have the “correct” objective (value maximization) and can always delegate the decision to insiders when they believe insiders will make a better decision, shareholders should control all major decisions. We show that this argument is also flawed.

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# Control of Corporate Decisions: Shareholders vs. Management

by Milton Harris and Artur Raviv

Activist shareholders have lately been attempting to assert themselves in a struggle with management and regulators over control of corporate decisions. These efforts have met with mixed success. Carl Icahn, along with some hedge funds, attempted, without success, to force a breakup of Time Warner, even going so far as to commission a study by Lazard Frères that touted the benefits of such a breakup. Kirk Kerkorian succeeded in placing his representative on the board of General Motors in a bid to get GM to enter into an alliance with Nissan and Renault. Again, this attempt at wresting control of some corporate decisions failed. On the other hand, Nelson Peltz succeeded in getting himself and an ally elected to the board of H.J. Heinz Co. and in getting management to implement accelerated cost cuts and restructuring. Meanwhile, Jay Sidhu, longtime CEO of Sovereign Bancorp resigned under pressure from Ralph Whitworth of Relational Investors who also succeeded in getting himself and another independent director elected to the board of Sovereign. These examples illustrate the recent trend toward shareholder activism and, despite some successes, the difficulty that these shareholders face in affecting corporate decisions under the current rules.

On another front, shareholders have been pressing for changes in the rules governing access to the corporate proxy process, especially in regard to nominating directors. For example, the American Federation of State, County and Municipal Employees (AFSCME), as a stockholder in American International Group (AIG), proposed that AIG shareholders be allowed to vote on a measure to give them a greater voice in the selection of directors. When AIG succeeded in obtaining permission from the SEC to keep the measure off the proxy, AFSCME sued, eventually winning in an appeal. The SEC plans to discuss a possible rule change as a result of the court ruling.

The key issue which these events have brought to light is whether, in fact, shareholders will be better off with enhanced control over corporate decisions. Proponents of increased shareholder participation argue that such participation is needed to counter the agency problems associated with management decisions.<sup>1</sup> In this view, boards of directors do not exercise sufficient control over self-interested managers because management insiders typically hand-pick directors through their control of the proxy process. Opponents offer several arguments such as that shareholders lack the requisite knowledge and expertise to make effective decisions or that shareholders may have incentives to make value-reducing decisions. They argue, for example that large institutional shareholders, who will drive the decisions if shareholders are given more power, may try to inflate the firm's stock price with short-term measures that actually reduce firm value. Another example is that shareholders with social, political or environmental agendas may dominate the decision-making process.<sup>2</sup> In this paper, we investigate what determines the optimality of shareholder control, taking account of some of the above arguments, both pro and con. Our main contribution is to use formal modeling to uncover some factors overlooked in these arguments. For example, we show that the claims that shareholders should not have control over important decisions because they lack sufficient information to make an informed decision or because they have a non-value-maximizing agenda are flawed. On the other hand, it has been argued that, since shareholders have the "correct" objective (value maximization) and can always delegate the decision to

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<sup>1</sup> The leading proponent, at least in the academy, is Lucian Bebchuk. See Bebchuk (2005).

<sup>2</sup> See, for examples, Bainbridge (2006), Lipton (2002), Stout (2006), and Strine (2006).

insiders when they believe insiders will make a better decision, shareholders should control all major decisions.<sup>3</sup> We show that this argument is also flawed.

In general, our analysis highlights the complicated interaction among control rights, who actually makes the decision, and the extent of communication between the parties. In particular, we emphasize that control over a decision by a given party does not require that that party actually make the decision but does determine who chooses which party makes the decision. This delegation choice determines not only the decision maker but also, in part, how much information will be used to make the decision. In particular, the decision maker's own private information will be fully utilized, but the other party's information will be only partially communicated to the decision maker. The extent of this communication is limited by the importance of the managerial agency problem, as well as by the extent of non-value-maximizing behavior on the part of shareholders. The result is that whether shareholder control is optimal depends on such characteristics of the decision being controlled as the extent of private information on both sides and the extent of agency problems (potentially on both sides).

Our analysis results in several surprising conclusions:

- Shareholders should *always* control decisions about which they have no private information.
- Shareholders should *not* control some decisions about which they have private information.
- Misinformed shareholders *should* control some decisions.
- Shareholders with social/political/environmental agendas *should* control some decisions.

The model we use focuses on managerial agency problems and private information available both to management and shareholders. We begin by assuming that shareholders' objective is to maximize the value of their shares and that they understand fully the limitations of their information. In this case we obtain a result that, at first glance, may seem somewhat counterintuitive, namely that shareholders should always control decisions about which they have no private information (we explain the intuition for this result below). When shareholders do have private information, whether they should control a given decision depends on how important their private information is relative to that of managers and relative to the agency costs. In general, when shareholders have private information, we show that shareholders should be in control of decisions whenever insiders' information is less important than agency costs. When insiders' information is more important than agency costs, insiders should be in control when their information is sufficiently important relative to that of shareholders.

To gain some intuition for these results, we begin with the case in which shareholders have no private information. A naïve approach, often taken by opponents to greater shareholder participation, suggests that, in such cases, shareholders should not be in control. This approach, however, ignores the possibility that shareholders will delegate the decision to better-informed managers. We show that, in such situations, shareholders, recognizing that they have no information not already known to management, will delegate the decision to management if and only if management's private information is sufficiently valuable that it outweighs the cost due to the managerial agency problem. That is,

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<sup>3</sup> For example, Bebchuk (2005, pp. 881-882), arguing that shareholder ignorance is no excuse for denying shareholders control, states, "After balancing the considerations for and against deference [to management], rational shareholders might often conclude that deference would be best on an expected-value basis. Other times, however, they might reach the opposite conclusion. Although shareholders cannot be expected to get it right in every case, it is their money that is on the line, and they thus naturally have incentives to reach decisions that would best serve their interests."

shareholders delegate the decision to managers in precisely the correct situations to maximize share value. This raises the question of why, given that shareholders are fully aware of their information limitations and their preferences are perfectly aligned with our criterion for optimality, they should not control all important corporate decisions as argued by some proponents of greater shareholder participation. The reason that shareholder control is not always optimal is that, when shareholders have private information, they will fail to delegate the decision to managers in some situations in which such delegation would increase share value. This stems from a commitment problem that we discuss in detail below.

Having considered shareholders who are fully aware of their limitations, we next consider the case when shareholders believe they have more information than they, in fact, do. This analysis addresses the criticisms that not only are shareholders ill-informed relative to insiders but also are overconfident in their ability to understand the issues involved in some decisions. We consider mainly the extreme case in which shareholders believe they have substantial private information but in reality have no private information at all. We show that even in this case, some decisions are better controlled by shareholders.

Finally, we examine the case for shareholder control when some shareholders have agendas other than firm value maximization. It is often claimed that some shareholders want to use corporate resources to further a social or political agenda at the expense of profits. For example, some shareholders may want the firm to pursue environmentally friendly production techniques even though these are not legally required and are more costly than other techniques. Other examples include wealth redistribution (*e.g.*, to workers), support for certain political candidates, boycotts of products of certain firms or countries, etc. Thus, similar to management, decisions made by non-value maximizing shareholders entail an agency cost. Again, we assume a worst case for shareholder control, namely that the non-value-maximizing shareholders control any decision assigned to the shareholders. And again we find that shareholders should control some decisions, even though we maintain the criterion of share value maximization. In particular, it is optimal for shareholders to control decisions for which non-value-maximizing shareholders' bias is smaller than that of management or in the opposite direction and management's private information is sufficiently unimportant relative to the net bias (management's bias net of shareholders' bias). If shareholders' bias is in the same direction as and greater than that of management, insider control is optimal. Of course, when management's information is sufficiently important that shareholders delegate the decision to them regardless of shareholders' information, control is irrelevant.

The next section presents our model. In section 2, we analyze the "base case" in which shareholders want to maximize the value of their shares and accurately assess the extent of their private information. The case in which shareholders believe they have better information than they actually have is analyzed in section 3. We consider shareholders with non-value-maximizing objectives in section 3. Section 5 concludes.

## **1 The Model**

We consider a firm whose value depends on the value of a strategic decision denoted  $s$ . Some examples of the kinds of decisions we have in mind are the reservation price for sale of the firm or some of its assets, the reservation price for acquiring other firms, the size of a major investment, etc. The issue we analyze is who should optimally control this decision, shareholders or insiders. Control of the decision allows the controlling group to make this decision themselves or to delegate it to the other group. We assume that the board of directors is controlled by management insiders and hence always acts in their interests which diverge from the interests of shareholders, as will be explained below.<sup>4</sup> We refer to this group equivalently as insiders and management.

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<sup>4</sup> Obviously, if the interests of shareholders are perfectly represented by independent directors on the board whose information includes any private information of shareholders, then the issue becomes who should control the board or various decisions made by the board, not whether shareholders should directly control these decisions. This is the topic addressed in Harris and Raviv (2007a). Here, we make the opposite assumption that the board does

We do not consider how conflicts among the members of either group, insiders or shareholders, are resolved, nor do we model the sharing of information among the members of a group. Instead, we model insiders as if they behave as a single agent with the preferences and information described below. Similarly we model shareholders in the same way. In particular, we do not analyze voting by shareholders or other schemes for aggregating their preferences and information into decisions. Of course, this is an important consideration in deciding whether shareholder control is optimal. Our aim is simply to understand, assuming the difficult issue of preference and information aggregation can be successfully resolved, which decisions it makes sense for shareholders to control.

The optimal (value-maximizing)<sup>5</sup> decision depends on private information of management (“agents”),  $\tilde{a}$ , and private information of shareholders (“principals”),  $\tilde{p}$ . To ensure that the private information of the two parties is complementary for the decision, we assume that the optimal decision is given by the sum of  $\tilde{a}$  and  $\tilde{p}$ . To the extent that the actual decision,  $s$ , differs from this optimum, there is a loss in firm value given by the quadratic function<sup>6</sup>

$$(s - (\tilde{a} + \tilde{p}))^2. \quad (1)$$

For most of the paper, we assume that shareholders seek to maximize expected firm value, or, equivalently, to minimize the expected loss derived from (1), given whatever information they have.<sup>7</sup> It follows that shareholders’ optimal decision is given by  $s = E(\tilde{a} + \tilde{p})$ , where the expectation is conditional on whatever information shareholders have about  $\tilde{a}$  and  $\tilde{p}$ .

Insiders, on the other hand, are biased relative to value maximization. In particular, we assume that insiders seek to minimize, given whatever information they have, the expectation of the loss function

$$(s - (\tilde{a} + \tilde{p} + b))^2. \quad (2)$$

where *insiders’ bias*,  $b$ , is a positive parameter that measures the extent of the agency problem between shareholders and management.<sup>8</sup> Insiders’ optimal decision is given by  $s = E(\tilde{a} + \tilde{p}) + b$ , where the expectation is conditional on whatever information insiders have about  $\tilde{a}$  and  $\tilde{p}$ .

Because of the quadratic loss functions in (1) and (2), the difference between the expected loss that results when the insiders make the decision and the expected-loss-minimizing decision, for any given information, is  $b^2$ . We therefore refer to  $b^2$  (and sometimes  $b$ ) as the *agency cost*.

The shareholders’ information cannot be obtained by insiders, but may be communicated to them by shareholders. Similarly, shareholders cannot obtain the insiders’ information directly, but insiders

not effectively represent the interests of shareholders. For evidence on the extent to which CEO involvement in the selection of new board members, results in appointments of less independent directors, see Shivadasani and Yermack (1999).

<sup>5</sup> We will use the word “optimal” to mean value-maximizing throughout the paper, even in cases where some shareholders have other objectives. We also assume that maximizing share value and maximizing firm value are equivalent.

<sup>6</sup> The specific assumptions that the optimal decision is the sum of  $\tilde{a}$  and  $\tilde{p}$  and that the loss is quadratic can be generalized somewhat for some of our results, but these assumptions greatly simplify the analysis. We believe that the insights derived from this model do not depend on these assumptions.

<sup>7</sup> In section 3, we consider non-value-maximizing behavior on the part of shareholders.

<sup>8</sup> It is not essential that  $b$  be positive. This is assumed only for expositional convenience.

may communicate it to them. The extent to which one party will communicate its information to the other is limited by the agency problem described above.

We make the following assumptions regarding the distributions of  $\tilde{a}$  and  $\tilde{p}$ :

*Assumption 1.* The variables  $\tilde{a}$  and  $\tilde{p}$  are independent with  $\tilde{a}$  uniformly distributed on  $[0, A]$  and  $\tilde{p}$  uniformly distributed on  $[0, P]$ .<sup>9</sup>

In some cases, it will be more convenient to work with the standard deviations of the random variables  $\tilde{a}$  and  $\tilde{p}$ , as well as those of other uniformly distributed random variables, instead of the parameters  $A$  and  $P$ . Consequently, for any  $x \geq 0$ , denote by  $\sigma(x)$  the standard deviation of a random variable uniformly distributed on an interval of width  $x$ , i.e.,  $\sigma(x) = x/\sqrt{12}$ . We will use  $\sigma_a$  to denote  $\sigma(A)$  and  $\sigma_p$  to denote  $\sigma(P)$ .

Because of the quadratic cost function in (1), it turns out that if an unbiased decision-maker chooses  $s$ , the difference in firm value between knowing  $\tilde{p}$  (respectively,  $\tilde{a}$ ) and having no information about  $\tilde{p}$  (respectively,  $\tilde{a}$ ) is exactly  $\sigma_p^2$  (respectively,  $\sigma_a^2$ ). We will therefore refer to  $\sigma_p^2$  and  $\sigma_a^2$  ( $\sigma_a^2$  and  $\sigma_p^2$ ) as the *importance of the shareholders' (insiders') information*. We focus on the case in which the agency problem (as measured by  $b$ ) is severe relative to the importance of shareholders' information.<sup>10</sup> Formally, we assume

*Assumption 2:*  $\sigma_p \leq b$ .

Recall that control of a decision empowers the controlling party to make the decision or to delegate it to the other party. If the controlling party does not delegate, it will make the decision based on its own information and any information communicated to it by the other party.<sup>11</sup> The sequence of events is assumed to be the following. After observing its private information, the controlling party decides whether to delegate to the other party or not. The party not making the decision may communicate some or all of its private information to the decision maker. Finally, the decision maker chooses  $s$  and firm value is realized.

## 2 Base Case Analysis

In this section, we analyze the case in which shareholders are value-maximizers and understand perfectly the extent of their private information as well as all other parameters of the model. Essentially, the current model is a special case of the model in Harris and Raviv (2007a).<sup>12</sup> Consequently, we will

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<sup>9</sup> Our results depend only on the widths of the supports of  $\tilde{a}$  and  $\tilde{p}$ , not on their locations.

<sup>10</sup> It will be shown below that this assumption guarantees that management will not want to delegate to shareholders and that shareholders will never reveal any of their private information to management.

<sup>11</sup> We do not consider the possibility of constrained delegation, i.e., that one party allows the other party to choose  $s$  subject to a constraint. One can show, for example, that if the shareholders have no private information, it is optimal for them, when they delegate to insiders, to put an upper bound on insiders' choice that is sometimes binding. When shareholders have private information, the problem is more complicated, since their choice of a constraint may convey information. Since this model is sufficiently complicated already, we leave this possibility for future work.

<sup>12</sup> Shareholders in the current model are like the outsiders in Harris and Raviv (2007a), while management/insiders in the current model correspond to insiders in Harris and Raviv (2007a). In the current model, we drop the assumption that outsiders (shareholders) must pay a cost to acquire their private information.

borrow liberally from the results in that paper. We first analyze separately the two cases in which shareholders are assumed to be in control of the decision and insiders are assumed to be in control. We then determine optimal control by comparing the equilibrium firm values for the two cases.

## 2.1 Shareholder Control

First, assume that shareholders do not delegate the decision to insiders. In this case, shareholders will choose  $s$  based on their own information and any information communicated to them by insiders. Denote by  $r(a)$  the report of insiders to shareholders if insiders observe  $\tilde{a} = a$ . Because of the agency problem, the report will not fully communicate the reporting party's information, as will be seen below.

As noted above, if shareholders observe  $\tilde{p} = p$ , their optimal decision is given by

$$s(p, r) = \bar{a}(r) + p, \quad (3)$$

where  $\bar{a}(r) = E(\tilde{a}|r)$  is the mean of shareholders' posterior belief about  $\tilde{a}$ , given the insiders' report,  $r$ .

Because of the agency problem, insiders will not fully reveal their information. More precisely, in the Pareto-best Bayes equilibrium of the game in which shareholders choose  $s$ , insiders will partition the support of  $\tilde{a}$ ,  $[0, A]$ , into cells  $[a_i, a_{i+1}]$  (of unequal widths) and report a value that is uniformly distributed on the cell in which the true realization of  $\tilde{a}$  lies.<sup>13</sup> Thus shareholders learn only the cell in which the true value of insiders' information lies, and their posterior belief is that  $\tilde{a}$  is uniformly distributed on that cell. It follows that if the report  $r$  is in  $[a_i, a_{i+1}]$ ,  $\bar{a}(r) = \frac{a_i + a_{i+1}}{2}$ . The number of cells in the partition is denoted by  $N(b, A)$  (see Harris and Raviv (2007b) for an explicit formula). Note that the number of cells is a measure of the extent to which insiders communicate their information to shareholders. For example, if there is only one cell,  $N(b, A) = 1$ , insiders communicate nothing shareholders do not already know. This will be the case, for example, if insiders' information is less important than agency cost, i.e.,  $\sigma_a \leq b$ . On the other hand as  $N(b, A)$  gets very large (as will be the case if agency cost,  $b$ , approaches zero), the information communicated approaches perfect information about  $\tilde{a}$ . We state the following result about the function  $N(b, x)$  for future reference (the proof can be found in Harris and Raviv (2005)).

**Lemma 1.**  $N(b, x) = 1$  if and only if  $x \leq 4b$  or, equivalently,  $\sigma(x) \leq 2b/\sqrt{3}$ .

Since  $2 \geq \sqrt{3}$ , the lemma implies that  $N(b, A) = 1$  if  $\sigma_a \leq b$ , as mentioned above.

Since insiders do not fully communicate their private information, there is a consequent loss in firm value. Let  $L(b, A)$  denote this information cost, i.e., the expected loss in firm value due to having only information about  $\tilde{a}$  that is transmitted in equilibrium (as opposed to full information). That is,

$$L(b, A) = E[\bar{a}(r(\tilde{a})) + p - (\tilde{a} + p)]^2 = E[\bar{a}(r(\tilde{a})) - \tilde{a}]^2. \quad (4)$$

<sup>13</sup> The game in which shareholders choose  $s$  is analyzed formally in Harris and Raviv (2005). Here we simply summarize the results and provide intuition.

The loss  $L(b, A)$  depends on how much information is transmitted in the report, as measured by  $N(b, A)$ .<sup>14</sup> In particular, if  $N(b, A) = 1$ , i.e., no information is transmitted to shareholders, then  $L(b, A)$  is the entire variance,  $\sigma_a^2$ , of  $\tilde{a}$ , as is obvious from (4). If some information is transmitted ( $N(b, A) > 1$ ), the expected cost is smaller than the variance.

Now suppose that shareholders do delegate the decision to insiders. Then insiders will choose  $s$  based on their own information and any information communicated to them by shareholders. Lemma 1, together with Assumption 2, implies that  $N(b, P) = 1$ . Thus, shareholders' report will convey no information about  $\tilde{p}$  to insiders. Although shareholders do not directly share any information about  $\tilde{p}$ , they may reveal some information through their delegation decision, as will be seen presently. If insiders observe  $\tilde{a} = a$ , they choose

$$s(a) = a + \hat{p} + b, \quad (5)$$

where  $\hat{p} = E(\tilde{p} | \text{delegation})$  is the mean of the insiders' posterior belief about  $\tilde{p}$  given the fact that the decision has been delegated. Therefore, to analyze the equilibrium in this case, one must first understand in which circumstances, i.e., for which values of  $\tilde{p}$ , shareholders will delegate.

To understand when shareholders will delegate, consider the loss in firm value from delegating and the loss from not delegating. There are two components of the loss from delegating, namely the direct agency cost,  $b^2$ , and the loss due to imperfect communication of shareholders' private information. The loss from not delegating is the loss due to imperfect communication of insiders' private information, i.e.,  $L(b, A)$ .

If shareholders have no private information ( $\sigma_p = 0$ ), there is no loss due to imperfect communication about shareholders' private information, so the relevant comparison is between  $L(b, A)$  and the direct agency cost,  $b^2$ . Consequently, shareholders delegate in this case if and only if  $L(b, A) \geq b^2$ . It is shown in Lemma 1 of Harris and Raviv (2007b), however, that  $L(b, A) \geq b^2$  if and only if  $\sigma_a \geq b$ . Therefore, when shareholders have no private information, they will delegate if and only if insiders' information is more important than agency costs. This result is quite intuitive.

Now assume that shareholders have private information ( $\sigma_p > 0$ ). Now there is a cost of delegating due to loss of information about  $\tilde{p}$ . Consequently, the delegation region, i.e., the set of values of  $\tilde{p}$  for which shareholders will delegate, must satisfy the following, equilibrium property. Suppose insiders believe that delegation occurs if and only if  $\tilde{p}$  is in some region  $D \subset [0, P]$ . Then it must be optimal for the shareholders to delegate if and only if  $\tilde{p}$  is in the region  $D$ , i.e., the information cost from delegating plus the agency cost must be smaller than the information cost of not delegating,  $L(b, A)$ , if and only if  $\tilde{p}$  is in  $D$ .

We argue that the equilibrium delegation region is of the form  $[p^*, P]$  for some threshold  $p^* \in [0, P]$ , i.e., that shareholders will delegate if and only if  $\tilde{p}$  exceeds some threshold  $p^*$ . To see this,

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<sup>14</sup> An explicit formula for  $L$  is given in Harris and Raviv (2007b) where it is shown that this expected loss depends only on the agency cost,  $b$ , and the width  $A$  of the support of  $\tilde{a}$ .

suppose the interval  $D = [d_1, d_2] \subset [0, P]$  is the equilibrium delegation region.<sup>15</sup> In this case, because  $\tilde{p}$  is uniformly distributed, insiders' expectation of  $\tilde{p}$  given delegation,  $\hat{p}$ , is the midpoint of  $D$ . Since insiders choose  $s = a + \hat{p} + b$ , shareholders' loss from delegating if they observe  $\tilde{p} = p$  is  $[a + \hat{p} + b - (a + p)]^2 = (\hat{p} + b - p)^2$ , i.e., the squared distance between  $\hat{p} + b$  and  $p$ . But since  $b > 0$  and  $\hat{p}$  is the midpoint of  $D$ ,  $\hat{p} + b$  is to the right of the midpoint of  $D$ . If  $d_2 < P$ , then for some values of  $p$  strictly between  $d_2$  and  $P$ ,  $p$  is closer to  $\hat{p} + b$  than some values of  $p$  in the delegation region  $D$ . But this means that there are values of  $\tilde{p}$  not in the delegation region for which shareholders would prefer to delegate. Hence, if  $d_2 < P$ ,  $D$  cannot be an equilibrium delegation region, i.e., any equilibrium delegation region must have  $P$  as its right endpoint. This shows that delegation by shareholders, if it occurs, will occur for all values of  $\tilde{p}$  above some threshold,  $p^*$ .

It is convenient to denote by  $d$  the width of the interval of values of  $\tilde{p}$  over which shareholders delegate, i.e.,  $d = P - p^*$ . We characterize  $p^*$  and  $d$  more precisely in the following proposition.<sup>16</sup>

**Proposition 1:** If shareholders are in control of a decision, they will delegate the decision if and only if the realized value of their private information exceeds a threshold  $p^* \in [0, P]$  satisfying the following three conditions:

$$\text{If } \sigma_a \leq b, \text{ shareholders never delegate, i.e., } p^* = P \text{ and } d = 0; \quad (6)$$

$$\text{if } P \leq 2 \left[ \sqrt{L(b, A)} - b \right], \text{ shareholders always delegate, i.e., } p^* = 0 \text{ and } d = P; \quad (7)$$

$$\text{otherwise, } p^* \in (0, P), \text{ } d \in (0, P) \text{ and satisfy } P - p^* = d = 2 \left[ \sqrt{L(b, A)} - b \right]. \quad (8)$$

Note that for  $d \in (0, P)$ ,  $d$  is independent of  $P$ . We will refer to the case in condition (7) in which shareholders always delegate as one in which *insiders' information is critical*.

It follows from Proposition 1 that, when shareholders are in control, the expected loss in firm value is given by

$$L_s = \frac{d}{P} (b^2 + \sigma(d)^2) + \left(1 - \frac{d}{P}\right) L(b, A). \quad (9)$$

To understand this expression, first recall that if shareholders delegate, they do not share any information about  $\tilde{p}$  with insiders other than the fact that  $\tilde{p} \geq p^*$ , due to Assumption 2. Consequently, the expected loss in firm value if shareholders delegate is the agency cost,  $b^2$ , plus the loss from knowing only that  $\tilde{p} \in [p^*, P]$ ,  $\sigma(P - p^*)^2 = \sigma(d)^2$ . Thus the first term on the right hand side of (9) is the probability that shareholders delegate,  $d/P$ , times the expected loss if they do,  $b^2 + \sigma(d)^2$ . The second term on the right

<sup>15</sup> This is an intuitive argument that assumes the delegation region is an interval. See Harris and Raviv (2005) for a formal proof that there is an equilibrium in which shareholders delegate if and only if  $\tilde{p}$  exceeds a threshold.

<sup>16</sup> As mentioned above, if shareholders have no private information ( $\sigma_p = 0$ ), they will delegate if and only if insiders' information is more important than agency costs ( $\sigma_a \geq b$ ). This is a special case of conditions (6)–(8).

hand side of (9) is the probability that shareholders do not delegate,  $1 - d/P$ , times the expected loss if shareholders make the decision,  $L(b, A)$ .

## 2.2 Management Control

When insiders are in control, their delegation decision is the mirror image of shareholders' delegation decision. That is, management will delegate if and only if  $\tilde{a}$  is *below* a threshold. This is because, from the point of view of management, shareholders are biased toward smaller choices of  $s$  by  $-b$ . The analysis of the delegation decision of the previous subsection goes through with obvious modification, except that, in the present case, if insiders delegate, they may convey additional information. The counterpart of condition (6) holds, however, so Assumption 2 implies that insiders will never delegate the decision to shareholders. As before, Assumption 2 also implies that shareholders will refuse to share any information about  $\tilde{p}$  with insiders. Consequently, the expected loss in firm value under management control is the agency cost,  $b^2$ , plus the loss from knowing only that  $\tilde{p} \in [0, P]$ ,  $\sigma_p^2$ , i.e., the expected loss in firm value under management control is

$$L_M = b^2 + \sigma_p^2. \quad (10)$$

## 2.3 Optimal Control

Our goal in this subsection is to characterize the values of the parameters  $b$ ,  $\sigma_p$ , and  $\sigma_a$  that lead to control by each of the two parties. Obviously, shareholder control of the decision is optimal if and only if the net gain to shareholder control,  $\Delta \equiv L_M - L_S$ , is non-negative, i.e., if and only if

$$\Delta \equiv L_M - L_S = b^2 + \sigma_p^2 - \left[ \frac{d}{P} (b^2 + \sigma(d)^2) + \left( 1 - \frac{d}{P} \right) L(b, A) \right] \geq 0. \quad (11)$$

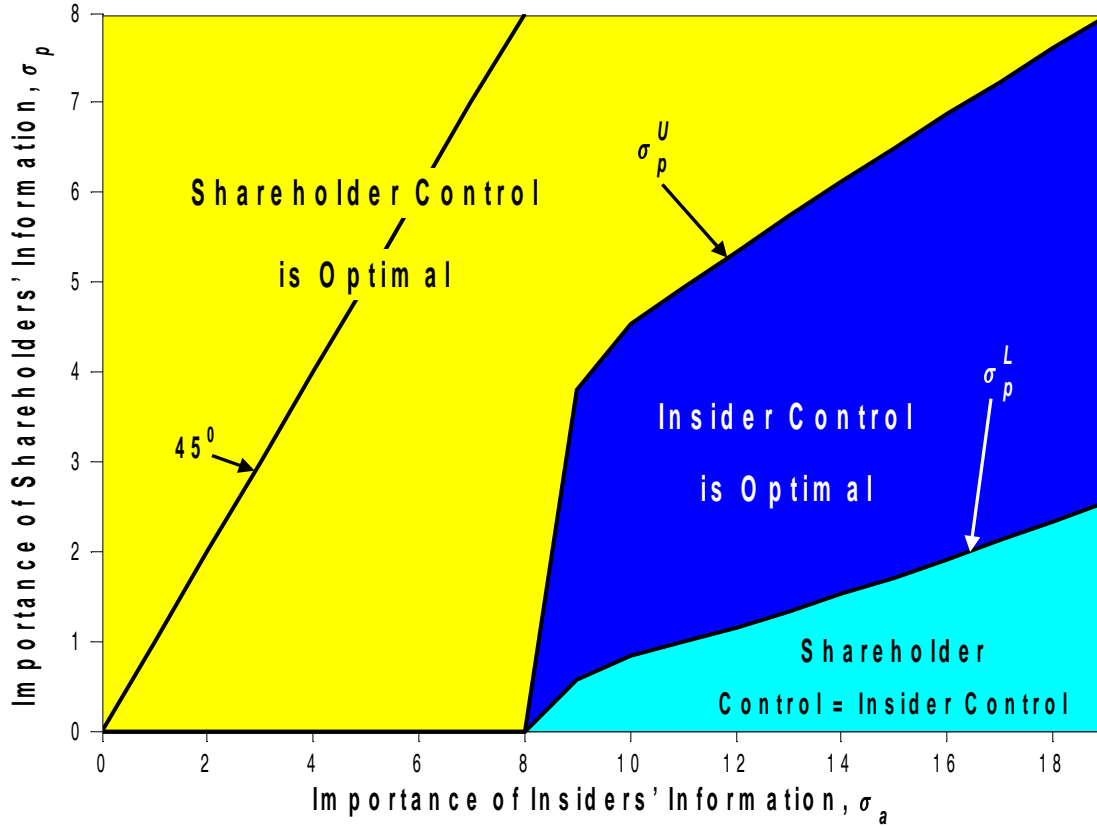
The main result of this section is stated in the following proposition (proved in the appendix).

### Proposition 2.

- (i) If the information of insiders is less important than agency costs ( $\sigma_a < b$ ), then for any  $\sigma_p \in [0, b]$ , shareholder control is optimal ( $\Delta > 0$ ).
- (ii) For every  $\sigma_a \geq b$ , there are two boundaries for  $\sigma_p$ ,  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$ , such that when the importance of the shareholders' information  $\sigma_p \in (\sigma_p^L(\sigma_a), \sigma_p^U(\sigma_a))$ , insider control is strictly optimal ( $\Delta < 0$ ). When the importance of the shareholders' information  $\sigma_p > \sigma_p^U(\sigma_a)$ , shareholder control is strictly optimal ( $\Delta > 0$ ). When  $\sigma_p \leq \sigma_p^L(\sigma_a)$  or  $\sigma_p = \sigma_p^U(\sigma_a)$ , control is irrelevant ( $\Delta = 0$ ).
- (iii) The functions  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$  satisfy the following properties:  $\sigma_p^L(b) = \sigma_p^U(b) = 0$ , and for every  $\sigma_a > b$ ,  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$  are strictly increasing in  $\sigma_a$ , and  $\sigma_a > \sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a) > 0$ .

Proposition 2 shows that the parameter space  $(\sigma_a, \sigma_p)$  can be divided into three regions as shown in Figure 1. It is optimal for insiders to control decisions for which the importance of the information of the two parties lies between the upward sloping curves defined by  $\sigma_p = \sigma_p^U(\sigma_a)$  and  $\sigma_p = \sigma_p^L(\sigma_a)$ . For decisions for which management's information is less important than agency cost ( $\sigma_a < b$ ) or for which

the importance of the information of the two parties is above  $\sigma_p = \sigma_p^U(\sigma_a)$  for  $\sigma_a \geq b$ , it is optimal for shareholders to control the decision. Finally, control is irrelevant for decisions for which the importance of the information of the two parties is below  $\sigma_p = \sigma_p^L(\sigma_a)$ .



**Figure 1**

This graph shows how optimal control varies with the two information parameters,  $\sigma_a$ , the importance of insiders' information, and  $\sigma_p$ , the importance of shareholders' information. For this figure,  $b = 8$ .

Given that shareholders' objective is to maximize firm value and given that, if shareholders believe that insiders' choice of  $s$  will result in higher value than their own, they may delegate the decision to insiders, one might ask why it would ever be optimal for insiders to control the decision. Indeed, from the point of view of assigning control, which is done *ex ante*, if shareholders were able, *ex ante*, to commit to a delegation threshold, it would always be optimal to assign them control.

For suppose shareholders could announce and commit to any delegation cutoff,  $P - x$  with  $x \in [0, P]$ , before observing their private information. Since shareholders are committed to delegate if and only if  $\tilde{p} \geq P - x$ , if shareholders delegate, insiders correctly infer that  $\tilde{p} \geq P - x$ . In this case, the *ex ante* expected loss from shareholder control is given by equation (9) with  $d$  replaced by  $x$ :

$$\frac{x}{P} (b^2 + \sigma(x)^2) + \left(1 - \frac{x}{P}\right) L(b, A).$$

The *ex-ante*-optimal threshold is found by choosing  $x \in [0, P]$  to minimize the above expression.<sup>17</sup> Clearly, a feasible solution to this problem is  $x = P$  (always delegate), which results in an *ex ante* expected loss from shareholder control of  $b^2 + \sigma_p^2$ . But this is equal to the expected loss from management control,  $L_M$  (see equation (10)). Therefore, the *ex-ante*-optimal delegation threshold results in an expected loss from shareholder control that is no greater than the expected loss from insider control. Consequently, if the threshold can be chosen *ex ante* to minimize the expected loss from shareholder control, and shareholders can commit to the chosen threshold, it is always optimal for shareholders to control any decision.

In some situations, however, the *ex-ante*-optimal delegation threshold does indeed require commitment on the part of shareholders. That is, for some parameter values, the *ex-ante*-optimal threshold is different from the equilibrium threshold,  $p^*$ . Indeed, we argue that, when the equilibrium delegation threshold  $p^* \in (0, P)$ , shareholders can do better if they can commit to a delegation threshold that requires them to delegate for some realizations of  $\tilde{p} < p^*$ . Moreover, for some of these realizations, *ex post*, the cost of delegating exceeds the cost of not delegating, so that shareholders must indeed be able to commit to delegating for those realizations. Thus, the equilibrium delegation threshold  $p^*$  results in too little delegation compared to an *ex-ante*-optimal threshold with commitment. The reason for this is that the delegation decision conveys information about  $\tilde{p}$  to insiders in addition to determining who actually makes the decision. If the delegation threshold is chosen *ex ante*, shareholders take into account the effect of the threshold on the information content of the information conveyed by delegating. In the *ex post* equilibrium, however, shareholders take the threshold as given.

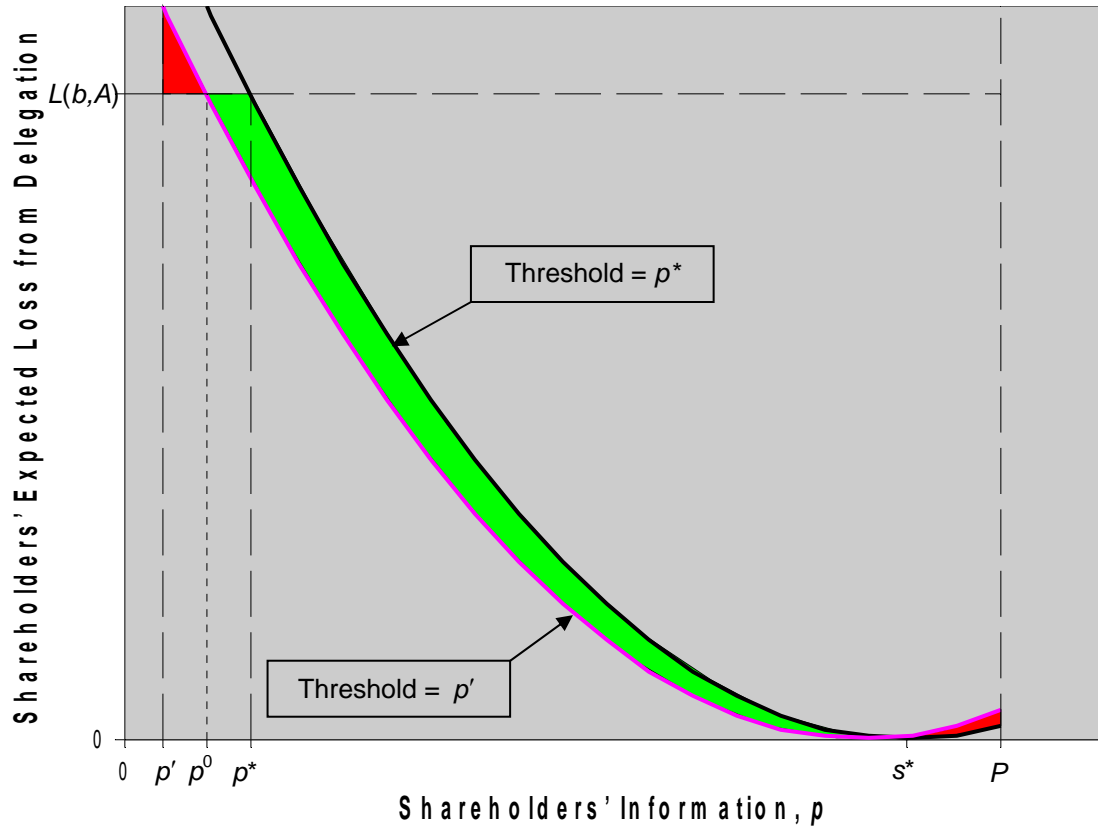
To understand this result, suppose the equilibrium delegation threshold  $p^* \in (0, P)$ . The dark blue (or black) curve in Figure 2 represents the expected loss from delegating as a function of the realization  $p$  of  $\tilde{p}$ , given that insiders believe the delegation threshold is  $p^*$ . Note that this cost is exactly equal to the expected loss from not delegating,  $L(b, A)$ , at  $\tilde{p} = p^*$ . Consider the *ex ante* expected costs and benefits of reducing the delegation threshold below  $p^*$  to say  $p'$ , assuming that the insiders adjust their beliefs accordingly and shareholders can commit to the resulting delegation decisions. The pink (or gray) curve in Figure 2 represents the expected loss from delegating, given that insiders believe the delegation threshold is  $\hat{p}$ . Since, when  $\tilde{p} = p^*$ , the loss from delegating exactly equals the loss from not delegating, reducing the threshold to  $p'$  results in some realizations of  $\tilde{p}$ , those between  $p'$  and  $p^0$  in Figure 2, for which shareholders are supposed to delegate but, *ex post*, prefer not to. Given their commitment, however, shareholders delegate anyway. This is the cost of reducing the threshold to  $p'$ , represented by the triangular red (or dark) area between  $p'$  and  $p^0$  in Figure 2. The benefit is, that by committing to delegate for lower realizations of  $\tilde{p}$ , shareholders change the inference of insiders about  $\tilde{p}$  causing them to choose a smaller value of  $s$ . Is this really a benefit? For realizations of  $\tilde{p}$  between  $p^0$  and the point labeled  $s^*$  in Figure 2,<sup>18</sup> insiders' new (lower) choice of  $s$  is closer to the optimal value of  $s$ , and there is a corresponding reduction in expected loss represented by the green (or gray) area in the figure. For realizations of  $\tilde{p}$  that are above  $s^*$ , however, the new choice is farther from the optimal

<sup>17</sup> Note that we are not claiming the solution to this problem is an optimal delegation *policy*. We claim only that, among policies that are characterized by a lower threshold for delegating, the solution is optimal. Nevertheless, we show that, even if one is restricted to such threshold policies, shareholder control is always optimal.

<sup>18</sup>  $s^* + a$  is insiders' choice of  $s$  when the threshold is  $p^*$ , but since  $a$  cancels in calculating shareholders' expected loss from delegating,  $s^*$  is the relevant comparison point.

value of  $s$ , resulting in an increase in expected loss as shown by the red (or dark) area to the right of  $s^*$ . Since  $s^*$  is above the midpoint of the (original) delegation region, however,  $\tilde{p}$  is less likely to be above  $s^*$  than below it. That is, inducing the insiders to lower their choice of  $s$  is, on balance a benefit (the green or gray area exceeds the red or dark area to the right of  $s^*$ ). In fact, it turns out that, for some values of  $p'$ , this benefit exceeds the additional cost of lowering the threshold, i.e., the additional cost of delegating for some realizations of  $\tilde{p}$  for which, *ex post*, shareholders would prefer not to. In terms of the figure, the green (or gray) area exceeds the total of the two red (or dark) areas. In the appendix, we compute the optimal such threshold, denoted  $p^{**}$ , and show that it is indeed smaller than  $p^*$ .

The above argument shows that the equilibrium delegation threshold for shareholders,  $p^*$ , is not *ex-ante* optimal and that the *ex-ante*-optimal such threshold requires commitment on the part of shareholders to delegate for some realizations of  $\tilde{p}$  for which, *ex post*, they would prefer not to. By assumption, however, shareholders cannot so commit, i.e., the delegation threshold must be the equilibrium threshold, not the *ex-ante*-optimal threshold. Because shareholders sometimes do not delegate optimally from an *ex ante* perspective, it can be better for shareholders in those situations to have insiders in control. Indeed, putting insiders in control is like a commitment to delegate. This accounts for the non-empty region in Figure 1 in which it is optimal for insiders to control the decision.



**Figure 2**

This figure shows the loss to shareholders from delegating as a function of the realization of their private information. The loss is shown for two values of the delegation threshold, the equilibrium value,  $p^*$  (dark blue or black curve), and a lower value  $p' < p^*$  (pink or gray curve). The value of  $p$  that minimizes shareholders' expected loss from delegating, given that insiders believe shareholders delegate for  $\tilde{p} \geq p^*$ , is shown as  $s^*$ . The red (or dark) triangular region on the left represents the extra expected cost of reducing the threshold from  $p^*$  to  $p'$  due to having to delegate for some values of  $\tilde{p}$  for which, *ex post*, not delegating is less costly. The green (or gray) region minus the small red (or dark) region on the right represents the expected benefit of reducing the insiders' choice of  $s$  by delegating for smaller realizations of  $\tilde{p}$ . For this graph,  $b = 10$ ,  $\sigma_p = 9$ , and  $\sigma_a = 50$ .

Proposition 2 has several interesting and important implications. First, when shareholders have no private information ( $\sigma_p = 0$ ), it is strictly optimal for them to be in control when management's information is less important than agency cost ( $\sigma_a < b$ ) and weakly optimal when management's information is more important than agency cost ( $\sigma_a \geq b$ ). This is because, when shareholders have no private information, they make an *ex-ante*-optimal delegation decision, namely to delegate if management's information is more important than agency cost and not otherwise. On the other hand, when shareholders do have private information, it is sometimes optimal for them not to be in control as shown in Proposition 2. Thus we obtain the somewhat counterintuitive result that shareholders should

control any decision about which they are ignorant. Indeed, many commentators have argued just the opposite.<sup>19</sup>

Second, the region of the  $(\sigma_a, \sigma_p)$ -plane in which it is strictly optimal for shareholders to be in control includes the diagonal. Thus, for decisions in which shareholders and insiders have information of approximately equal importance, shareholders should be in control, and insiders should control a decision only if their information is sufficiently more important than shareholders'. Shareholders should also control all decisions for which insiders' information is less important than the agency cost, since for such decisions, shareholders, if in control, delegate optimally to insiders.

Third, in the region below the curve  $\sigma_p = \sigma_p^L(\sigma_a)$ , control doesn't matter, because for decisions in this region, insiders' information is of critical importance, so shareholders will always delegate and convey no information. Since insiders never delegate, in this region, insiders actually make the decision with no information from shareholders, regardless of who controls the decision.

Finally, there is a non-empty region between the lower and upper boundaries,  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$ , for which insider control is strictly optimal. In this region, insiders' information is sufficiently more important than shareholders' information that firm value is higher if insiders make the decision but not sufficiently more important that shareholders will delegate to them for every realization of  $\tilde{p}$ .

Proposition 2 immediately yields the following comparative statics result.

**Proposition 3.** An increase in the importance of shareholders' information can result in a shift from insider control being strictly optimal to shareholder control being strictly optimal but not the reverse. Similarly, an increase in the importance of management's information can result in a shift from shareholder control being strictly optimal to management control being strictly optimal but not the reverse.

### 3 Shareholders Are Misinformed but Don't Realize It

As we have seen in the previous section, if shareholders fully understand their private information (or lack thereof), the case for shareholder control is actually stronger when shareholders are poorly informed because, in this case, they make better delegation decisions. Critics of shareholder control whose opposition is based on shareholder ignorance may argue, however, that not only are shareholders poorly informed but also overestimate the extent of their information. Consequently, in this section, we assume that shareholders misperceive their private information. Indeed, we rig the game against shareholder control by assuming that while shareholders believe they observe  $\tilde{p}$ , in fact they observe a constant independent of  $\tilde{p}$ . Even in this case, we find that it is optimal for shareholders to control some decisions.

Assume that, instead of observing  $\tilde{p}$ , shareholders actually observe a constant,  $q \in (0, P)$ , but believe they have observed  $\tilde{p}$ . Both shareholders and insiders believe  $\tilde{p}$  is uniform on  $[0, P]$ , but insiders realize that shareholders actually observe a constant (it doesn't matter whether they know what the constant is). All other assumptions are the same as in section 2.

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<sup>19</sup> See, e.g., Bainbridge (2006).

### 3.1 Shareholder Control

First, suppose shareholders are in control. Since shareholders believe they are in the situation modeled in section 2, they will delegate to management if and only if  $q \geq p^*$ , where  $p^*$  is the equilibrium delegation threshold defined in section 2.1.

If, in fact,  $q \geq p^*$ , so shareholders do delegate to insiders, insiders, knowing that shareholders have no information about  $\tilde{p}$ , do not change their beliefs based on the fact that shareholders chose to delegate. Insiders choose  $s = \bar{p} + a + b$ , where  $a$  is the realization of  $\tilde{a}$ . Shareholders' actual *ex ante* expected loss from being in control is

$$E(\bar{p} + \tilde{a} + b - (\tilde{p} + \tilde{a}))^2 = \sigma_p^2 + b^2. \quad (12)$$

Since shareholders always delegate in this case, their *ex ante* expected loss consists of the loss due to having no information about  $\tilde{p}$  and the agency cost.

Now suppose that  $q < p^*$ , so that shareholders do not delegate, and insiders observe  $\tilde{a} = a$ . From the point of view of insiders, shareholders are biased for two reasons. First, shareholders are biased by  $-b$ , i.e., toward *smaller s* relative to insiders' optimum. Second, shareholders are also biased by their misperception of  $\tilde{p}$ . This misperception bias is given by  $\tilde{e} = q - \tilde{p}$ . Since insiders do not observe  $\tilde{p}$ , they are interested in shareholders' average misperception bias,  $\bar{e} = E(\tilde{e}) = q - \bar{p}$ . Therefore, when shareholders misperceive  $\tilde{p}$  and do not delegate, insiders view them as biased by  $-B$ , where

$$-B = -b + \bar{e}. \quad (13)$$

We refer to  $-B$  as shareholders' *effective bias*.

In the equilibrium of the communication game when shareholders do not delegate, insiders will behave exactly as in section 2, except that they will replace  $-b$  by the shareholders' effective bias  $-B$ . Shareholders' average misperception bias may offset or reinforce the original bias, depending on whether shareholders' misperception bias is positive or negative on average. If the shareholders perceive  $\tilde{p}$  correctly on average, i.e.,  $\bar{e} = 0$ , then  $B = b$ , and the communication game in this case is the same as in section 2.1. If shareholders' perception of  $\tilde{p}$  is larger than its average value ( $\bar{e} > 0$ ), this reduces their bias toward smaller  $s$  from the point of view of insiders and, interestingly, *improves* communication from insiders to shareholders. Just the opposite is true if shareholders' perception of  $\tilde{p}$  is smaller than its average value.<sup>21</sup>

When shareholders do not delegate, shareholders' actual expected loss is

<sup>20</sup> Note that this is not how shareholders perceive their expected loss. We are interested only in their actual expected loss, since it is this loss that determines whether it is optimal for shareholders to control the decision.

<sup>21</sup> A formal proof of the argument in this paragraph is given in the appendix. Note that the maximum number of elements in the equilibrium partition of  $[0, A]$  is now  $N(B, A)$  instead of  $N(b, A)$ . Since  $B$  may be negative, to allow for negative biases, throughout the remainder of this paper the functions  $N(x, y)$  and  $f(x, y)$  will be redefined as the original functions evaluated at  $(|x|, y)$ . There is no need to redefine  $L(x, y)$ , since  $L$  depends on  $x$  only through  $x^2$ .

$$E[\bar{a}(r(\tilde{a})) - \tilde{a} + q - \tilde{p}]^2 = L(B, A) + E(\tilde{e}^2) = L(B, A) + \sigma_p^2 + \bar{e}^2. \quad (14)$$

This expression differs from shareholders' expected loss when they do not delegate in the base case of section 2.1,  $L(b, A)$ , in three respects. First, the loss due to imperfect communication from insiders about  $\tilde{a}$ ,  $L(B, A)$ , reflects the shareholders' effective bias (from the point of view of insiders) rather than the actual bias. Second, there is an additional loss of  $\sigma_p^2$  due to the fact that shareholders do not observe  $\tilde{p}$ . Third, there is an additional loss of  $\bar{e}^2$  due to the fact that shareholders misperceive  $\tilde{p}$ .

To summarize, if shareholders are in control, their actual expected loss is given by (12) if they always delegate ( $q \geq p^*$ ) and is given by (14) if they never delegate ( $q < p^*$ ).

### 3.2 Management Control

Now suppose insiders are in control. Insiders will never delegate to shareholders. This is because the only reason for insiders to delegate to shareholders is to take advantage of shareholders' information about  $\tilde{p}$ . In this case, however, shareholders have no information about  $\tilde{p}$ .<sup>23</sup> Consequently, shareholders' expected loss if management controls the decision is the same as in section 2.2, namely  $L_M = \sigma_p^2 + b^2$ .

### 3.3 Optimal Control

In this subsection we show that, even though shareholders are misinformed, there are nevertheless decisions for which shareholder control is optimal. When shareholders are in control, their misperception introduces three effects in addition to those already considered. First, shareholders' delegation decision is affected. Second, if shareholders do not delegate, their misperception bias affects the extent of communication from insiders to shareholders. Third, if shareholders do not delegate, their decision is biased relative to the value-maximizing decision. Opponents of shareholder control focus on this third effect which clearly weakens the case for shareholder control. Since shareholder control is strictly optimal for some decisions when shareholders are not misinformed, it will still be optimal in some cases if shareholders misperception bias is small. Moreover, the second effect in some circumstances can

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<sup>22</sup> The function  $r(\tilde{a})$  is the equilibrium reporting function from section 2.1, except that  $b$  is replaced by  $B$ . The equality follows from the fact that  $E[\bar{a}(r(\tilde{a}))] = E(\tilde{a})$  and the independence of  $\tilde{a}$  and  $\tilde{p}$ .

<sup>23</sup> Formally, insiders' expected loss from delegating is the loss due to imperfect communication of insiders' information plus  $Var(\tilde{e}) + B^2$ . Since  $Var(\tilde{e}) = \sigma_p^2$ , insiders' expected loss from delegating is at least  $\sigma_p^2$  and greater than that if  $B \neq 0$ . The insiders' loss from not delegating is simply  $\sigma_p^2$ . Consequently, insiders will never delegate in this situation. Interestingly, if shareholders have some private information, insiders may strictly prefer to delegate. The reason for this is that shareholders' misperception may make their effective bias very small resulting in communication from insiders about  $\tilde{a}$  that is very precise. In this case, by delegating, insiders can take advantage of shareholders' information while losing little due to shareholders' effective bias and imprecise communication about  $\tilde{a}$ . As an extreme example, suppose shareholders observe  $\alpha\tilde{p} + (1-\alpha)\bar{p} + b$  (but believe they observe  $\tilde{p}$ ), where  $\alpha \in (0,1)$  and  $b < (1-\alpha)\bar{p}$  (the last condition guarantees that shareholders' observation is in  $[0, P]$ ). In this case,  $\tilde{e} = (1-\alpha)(\bar{p} - \tilde{p}) + b$ ,  $\bar{e} = b$ , and  $B = 0$ . Consequently, by delegating, insiders lose nothing from imprecise communication of their private information, nor from shareholders' effective bias, i.e., their expected loss is  $Var(\tilde{e}) = (1-\alpha)^2 \sigma_p^2$ . Since their expected loss from not delegating is  $\sigma_p^2$ , we see that insiders prefer to delegate regardless of the realization of  $\tilde{a}$ .

enhance communication from insiders by reducing shareholders' effective bias from the point of view of insiders. This may overcome the third effect.

The main result of this section is stated in the following proposition (see the Appendix for a proof).

**Proposition 4.**

(i) If insiders' information is less important than their agency cost ( $\sigma_a < b$ ) and shareholders' average misperception bias,  $\bar{e}$ , is sufficiently small, shareholder control is optimal.

Now suppose shareholders believe  $\tilde{p}$  is smaller than the equilibrium delegation threshold ( $q < p^*$ ) and insiders' information is more important than their agency cost ( $\sigma_a \geq b$ ). Then

(ii) If shareholders believe  $\tilde{p}$  is smaller than the average value of  $\tilde{p}$  ( $q \leq \bar{p}$ ), insider control is optimal.

(iii) If shareholders believe  $\tilde{p}$  is greater than the average value of  $\tilde{p}$  ( $q > \bar{p}$ ), for some values of shareholders' average misperception bias  $\bar{e}$  between 0 and  $b$ , shareholder control is optimal. If, however, shareholders' average misperception bias exceeds  $b$ , it is optimal for insiders to control.

Finally, suppose shareholders believe  $\tilde{p}$  is larger than the equilibrium delegation threshold ( $q \geq p^*$ ). Then control is irrelevant.

To understand this result, first consider the case in which shareholders always delegate (the last claim in the proposition). Since management, if in control, never delegates, in this case insiders make the decision with no information about  $\tilde{p}$ , regardless of who is in control.

Next note that, regardless of who is in control, the expected loss includes the loss from not observing  $\tilde{p}$ ,  $\sigma_p^2$  (recall that neither party observes  $\tilde{p}$  in this section). Consequently, this part of the expected loss does not affect optimal control. When shareholders delegate if in control, therefore, the comparison of expected losses from shareholder control with those from insider control reduces to a comparison of the agency cost saving from shareholder control,  $b^2$ , with the cost of the misperception bias,  $\bar{e}^2$ , plus the loss due to imperfect communication of  $\tilde{a}$  by insiders,  $L(B, A)$ . Thus it is optimal for shareholders to control if and only if

$$b^2 > \bar{e}^2 + L(B, A).$$

Now consider the case in which  $\sigma_a < b$ . In this case, the equilibrium and optimal delegation decision for shareholders from section 2.1 is not to delegate regardless of the realization of  $\tilde{p}$ , i.e.,  $p^* = P$ . Consequently,  $q < p^*$ , so shareholders also prefer not to delegate in the present situation in which they are misinformed, and the optimality of shareholder control is determined by the above comparison. The loss due to imperfect information about  $\tilde{a}$  cannot exceed the loss from knowing nothing about  $\tilde{a}$ ,  $\sigma_a^2$ , however. Thus the relevant part of the expected loss if shareholders are in control in this case is not greater than  $\bar{e}^2 + \sigma_a^2$ . Since  $\sigma_a < b$ , if  $\bar{e}$  is sufficiently small, it is optimal for shareholders to be in control, even though they are misinformed and never delegate.<sup>24</sup> Essentially, in this

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<sup>24</sup> For example, if  $b = 1$ ,  $\bar{e} = 0.7575$  (so  $B = 0.2425$ ), and  $\sigma_a = 0.28141147 < b$ ,  $\bar{e}^2 + L(B, A) = 0.6524106 < 1 = b^2$ .

case insiders' information is less important than their agency cost and shareholders' choice of  $s$  is not too far from what they would choose if they realized they did not observe  $\tilde{p}$ . Therefore, even though shareholders do not delegate, shareholder control is optimal.

Next consider the case in which  $\sigma_a \geq b$  and  $q < p^*$ , so that, if shareholders are in control, they never delegate. If  $\bar{e} = q - \bar{p} \leq 0$ , shareholders' misperception bias reinforces their bias toward smaller  $s$  (from the point of view of insiders), so shareholders' effective bias is even larger than their actual bias, i.e.,  $B \geq b$ . This reduces communication from insiders. Given that insiders' information is more important than their agency cost in this case, the expected loss from poor communication exceeds the saving in agency cost if shareholders control. Consequently, insider control is optimal in this case.

Finally, suppose shareholders' average misperception bias is positive but less than the agency cost of insiders ( $\bar{e} \in (0, b)$ ). In this case, there are two opposing effects. The misperception bias reduces the value of shareholder control, but overestimating  $\tilde{p}$  on average results in better communication from insiders if shareholders are in control. Indeed, as  $\bar{e}$  increases from zero to  $b$ , communication from insiders improves and approaches full revelation of  $\tilde{a}$  as  $\bar{e}$  approaches  $b$ . As is shown by some numerical examples (see the appendix), the latter effect can dominate, leading to the counterintuitive result that an increase in average misperception bias on the part of shareholders can increase the advantage of putting shareholders in control. Clearly, however, if shareholders' average misperception bias is greater than insiders' bias, it is optimal for insiders to control, since they have valuable information, but shareholders do not.

A counterintuitive implication of Proposition 4 is that *decreases* in the importance of insiders' information can result in optimal control switching from shareholders to insiders. If insiders' information is critical, for example, then  $p^* = 0$ , so, regardless of  $q$ , shareholders will always delegate. In such situations control is irrelevant, so one might observe shareholders in control. If insiders' information decreases in importance to the point that shareholders never delegate ( $q < p^*$ ), it may then be optimal for insiders to be in control. Thus declining importance of insider information can cause control to shift from shareholders to insiders.

#### **4 Some Shareholders Have a “Social Agenda”**

In this section we consider whether shareholder control may still maximize firm value even when some shareholders have goals other than value maximization. In particular, we have in mind a situation in which some shareholders prefer that the firm sacrifice some profits to further another goal, e.g., preservation of the environment, support of a political agenda, etc. As in the previous section, we rig the game against shareholder control by assuming that, although such shareholders hold only a small proportion of the firm's stock (so that firm-value maximization is still an appropriate goal), they somehow hijack shareholder decisions. The question then is under what circumstances, if any, the value-maximizing (VM) shareholders are better off with their non-value-maximizing (NVM) co-investors in control than with insiders in control.

We model NVM shareholders as minimizing the loss function  $E(s - (\tilde{p} + \tilde{a} + \beta))^2$ . NVM shareholders' optimal decision is the same as that of insiders', except that  $b$  is replaced by  $\beta$ . That is, NVM shareholders will choose  $s = E(\tilde{a} + \tilde{p}) + \beta$ , where the expectation is conditional on whatever information they have about  $\tilde{a}$  and  $\tilde{p}$ . Here, the parameter  $\beta$  measures the extent to which these shareholders will deviate from the optimal decision to further their social agenda.<sup>25</sup> Note that  $\beta$  could be

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<sup>25</sup> We use the term “optimal” in this section to mean firm-value maximizing.

either positive or negative. For example, suppose the decision,  $s$ , is a minimum acceptable bid for selling the firm and that NVM shareholders anticipate losing control if the firm is sold. In this case, they may prefer a higher-than-optimal minimum acceptable bid, i.e., have a positive  $\beta$ . On the other hand, suppose the decision is the size of a new plant and that NVM shareholders are willing to sacrifice profits to reduce emissions from the plant. In this case, these shareholders may prefer a smaller-than-optimal plant if this will reduce emissions, i.e., have a negative  $\beta$ . Since it affects NVM shareholders in exactly the same way as  $b$  affects insiders, we refer to  $\beta$  as the *NVM shareholders' bias*.

Insiders minimize  $E\left(s - (\tilde{p} + \tilde{a} + b)\right)^2$ , given their information, as before. Since  $b - \beta$  plays a similar role to  $b - \bar{e}$  in the previous section, we redefine  $B = b - \beta$  and refer to it as the *net bias*. We also assume that the parameter values are such that insiders, if in control, never delegate to shareholders.<sup>26</sup> Note that  $B$  may be positive or negative.

If  $B > 0$ , all the results of section 2 apply, except that  $B$  replaces  $b$  in all calculations. In particular,  $p^*$  and  $d = P - p^*$  satisfy (6)–(8) with  $b$  replaced by  $B$ .

If  $B < 0$ , then NVM shareholders delegate when  $\tilde{p} \in [0, p^*]$ , and  $p^*$  and  $d$  are calculated as in section 2.1 except that  $b$  is replaced by  $|B|$  and  $p^* = d$ .<sup>27</sup>

Also, for  $B < 0$ , insiders never delegate to shareholders if and only if  $\sigma_p \leq |B|$ . Thus the counterpart of Assumption 2 in this case is  $\sigma_p \leq |B|$ , which also implies that (NVM) shareholders do not communicate any private information to insiders (other than what may be communicated by their delegation decision).

Since we assume that  $\sigma_p \leq |B|$ ,  $L(B, P) = \sigma_p^2$  and  $L(B, d) = \sigma(d)^2$ . Assuming that  $\sigma_p > 0$ , if (NVM) shareholders are in control, the expected loss in firm value is

$$\frac{d}{P} \left( b^2 + \sigma(d)^2 \right) + \left( 1 - \frac{d}{P} \right) \left( \beta^2 + L(B, A) \right). \quad (15)$$

The expression in (15) is the same as in the base case (equation (9)), except for three effects that are similar to those discussed in section 3.3. First, the size of the delegation region,  $d$ , is determined by the net bias  $B$  rather than insiders' bias  $b$ . Second, the loss when shareholders do not delegate is increased by the cost of the NVM shareholders' bias,  $\beta^2$ . This effect obviously reduces the attractiveness of shareholder control and is the effect on which opponents of shareholder control focus. Third, the loss due to imperfect communication from insiders is determined by the net bias instead of insiders' bias. Since the net bias can be smaller than the insiders' bias, communication of insiders' information can be more precise than in the base case, resulting in smaller loss. As is shown in Proposition 5 below, this effect causes shareholder control to be optimal in some cases.

<sup>26</sup> Exactly what this requires will be made explicit below.

<sup>27</sup> The argument for delegation when  $\tilde{p}$  is below a threshold if  $B < 0$  is essentially the same as the argument for delegation when  $\tilde{p}$  is above a threshold in section 2.1, except that, when  $B < 0$ , insiders' choice of  $s$  results in shareholders' expected loss from delegation being minimized at a value of  $\tilde{p}$  that is below the midpoint of the delegation region, instead of above it.

If insiders are in control, the expected loss in firm value is  $b^2 + \sigma_p^2$ , as before. Therefore, if the objective is to maximize firm value, shareholders should control if and only if (note the similarity with (11))

$$\frac{d}{P}(b^2 + \sigma(d)^2) + \left(1 - \frac{d}{P}\right)(\beta^2 + L(B, A)) \leq b^2 + \sigma_p^2. \quad (16)$$

Note that if  $\sigma_a \leq |B|$ , neither party will delegate to the other nor will they communicate any of their private information. Therefore, control is equivalent to making the decision with no information from the other party. The expected loss in firm value resulting from control by a given party is simply that party's agency cost plus the loss from not observing the other party's information. Condition (16) becomes

$$\beta^2 + \sigma_a^2 \leq b^2 + \sigma_p^2 \text{ or } \beta^2 - b^2 \leq \sigma_p^2 - \sigma_a^2.$$

The second inequality in the above condition makes it clear that optimality of shareholder control in this case hinges on a comparison between the difference in the importance of the parties' information and the difference in their agency costs. It will be convenient to restate the condition in terms of the net bias,  $B$ :

$$\sigma_a^2 \leq \sigma_p^2 - (B^2 - 2bB). \quad (17)$$

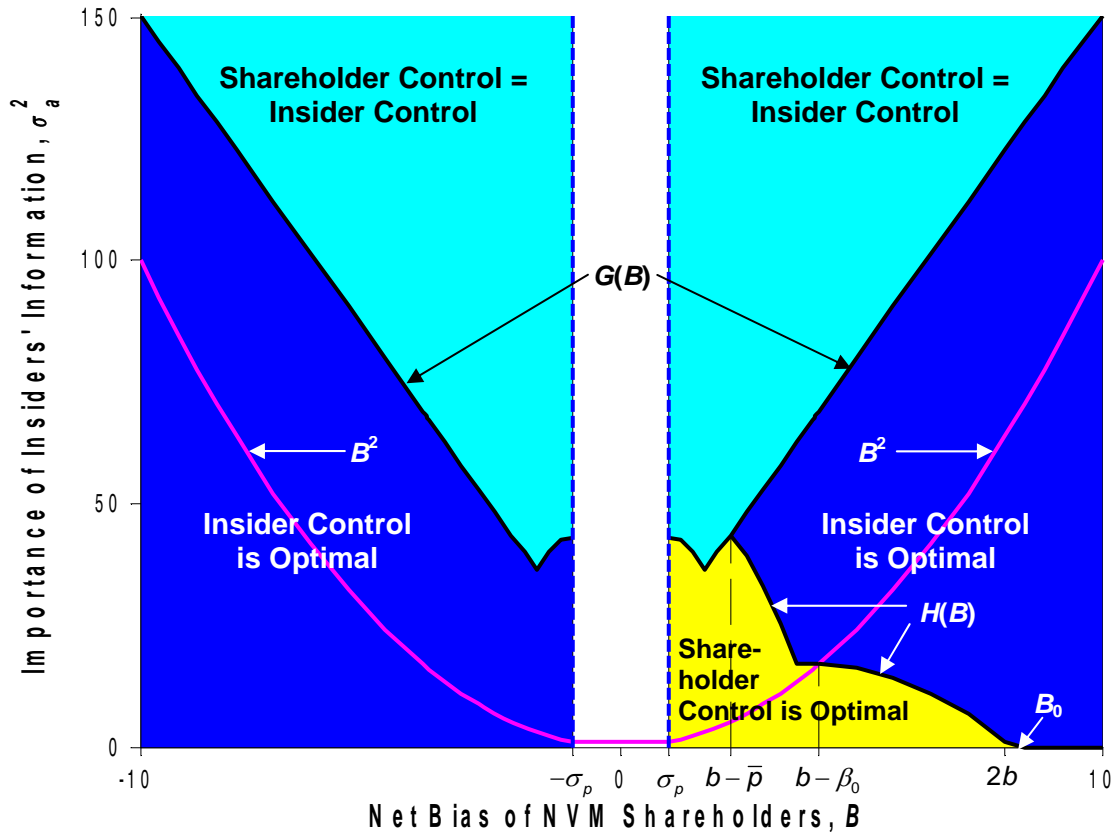
The main result of this section (proved in the appendix) is to characterize optimal control of decisions for various values of the net bias  $B$  and the importance of insiders' information  $\sigma_a^2$ , for fixed values of insiders' bias  $b$  and the importance of shareholders' information  $\sigma_p$ .

**Proposition 5.** Assume  $b - \bar{p} > \sigma_p$ .<sup>28</sup> Define  $B_0 = b + \sqrt{b^2 + \sigma_p^2}$ . Then there exist continuous functions,  $G(B) > B^2$ , with  $G$  symmetric with respect to  $B = 0$ , and  $H(B)$  such that  $H(b - \bar{p}) = G(b - \bar{p})$ ,  $H(B_0) = 0$ ,  $H(B)$  is decreasing in  $B$  for  $B \geq b/2$ , and

- for decisions for which  $B < -\sigma_p$ , insider-control is strictly optimal if  $\sigma_a^2 < G(B)$ , and otherwise insider-control and outsider-control are equivalent;
- for decisions for which  $\sigma_p < B < b - \bar{p}$ , shareholder-control is strictly optimal if  $\sigma_a^2 < G(B)$ , and otherwise, insider-control and outsider-control are equivalent;
- for decisions for which  $b - \bar{p} < B < B_0$ , shareholder-control is strictly optimal if  $\sigma_a^2 < H(B)$ , insider-control is strictly optimal if  $H(B) < \sigma_a^2 < G(B)$ , and otherwise, insider-control and outsider-control are equivalent;
- for decisions for which  $B_0 < B$ , insider-control is strictly optimal if  $\sigma_a^2 < G(B)$ , and otherwise, insider-control and outsider-control are equivalent.

The proposition is depicted in Figure 3.

<sup>28</sup> This is the richest case. When this inequality fails, the result is qualitatively similar.



**Figure 3**

This figure shows, for various combinations of values of the net bias,  $B$ , and the importance of insiders' information,  $\sigma_a^2$ , which party optimally controls the decision.

The values of other parameters are  $b = 4$ ,  $\sigma_p = 1$ . These values imply that  $\bar{p} = \sqrt{3}$  and  $B_0 = 4 + \sqrt{17} \approx 8.123$ .

When  $\sigma_a^2 > G(B)$ , insiders' information is sufficiently important that shareholders, if in control of the decision, will always delegate it to the insiders. Consequently, the delegation decision conveys no information to insiders. Since shareholders convey no information directly, it doesn't matter whether shareholders or insiders control such decisions: insiders will always actually make the decision with no input from shareholders.

For decisions for which the net bias  $B < 0$ , NVM shareholders are even more biased than insiders and in the same direction. Therefore, it is not surprising that insiders should control such decisions (unless  $\sigma_a^2 > G(B)$ , in which case control is irrelevant as just mentioned).

For decisions for which the net bias is positive, NVM shareholders are either less biased in the same direction as insiders ( $0 < \beta < b$ ) or are biased in the opposite direction ( $\beta < 0$ ). In this case, there is a tradeoff. When the net bias,  $B = b - \beta$ , is small, but still more important than shareholders' information ( $B > \sigma_p$ ), NVM shareholders' bias,  $\beta$ , is similar to that of insiders but smaller ( $0 < \beta < b$ ). Since the net bias is small, insiders are willing to communicate much of their information to shareholders if shareholders are in control and decide not to delegate. Since NVM shareholders' bias is smaller than that

of insiders, and little of insiders' information is lost if shareholders make the decision, the cost of letting shareholders decide is small, so even if shareholders do not delegate, it is optimal for them to control such decisions. Of course, when shareholders always delegate, i.e.,  $\sigma_a^2 > G(B)$ , shareholder control is weakly optimal. As the net bias increases, NVM shareholders' bias must decrease, since we are holding insiders' bias,  $b$ , fixed. This reduces communication from insiders, but, for  $\beta > 0$ , also reduces the inefficiency of the shareholders' decision, *cet. par.* The net effect on control could go either way. If  $\beta < 0$ , however, further reductions in  $\beta$  increase NVM shareholders' (absolute) bias. Now, both the communication effect and the direct effect on the decision of shareholders of the increase in net bias work against shareholder control of the decision. Thus for  $\beta < 0$ , i.e.,  $B > b$ , as the net bias increases, the importance of insiders' information must decrease in order for shareholder control to be optimal. That is, in Figure 3,  $H$  must be downward sloping for  $B > b$ .<sup>29</sup> For sufficiently large net bias,  $B \geq B_0$ , NVM shareholders' bias is so large that it is optimal for insiders to control even if they have no private information.

This section shows that, even when shareholders have biases that prevent them from choosing the expected profit-maximizing decision, it may still be optimal for them to control some decisions. In particular, they should control decisions for which both the net bias and the importance of insiders' information are small. The larger is the net bias, the less important must be insiders' information for shareholder control to be optimal.

## 5 Conclusions

In this paper, we address the issue of when direct shareholder control of decisions is appropriate. Using a model that accounts for private information, delegation, communication and agency considerations, we show that popular arguments both for and against direct shareholder control are flawed. For example, a strong intuitive argument has been advanced by several commentators that shareholders should not control major corporate decisions because, unlike management, they do not possess the relevant information. We show, however, that shareholders should control decisions for which they have none of the information possessed by management and have no private information of their own, provided these shareholders are aware of their ignorance and the extent of insiders' private information. This result follows, in part, from the failure of the simple argument to take account of the fact that shareholders can delegate the decision to management. On the other hand, others have argued that, because shareholders can delegate and want to maximize value (i.e., have no agency problem), they should control every major decision. We show that this argument is incorrect, because, if shareholders have private information, they will fail to delegate optimally. The reason for this is that shareholders take account of the inference made by management about shareholders' private information based on shareholders' decision to delegate. Indeed, we show that insiders best control decisions for which their private information is much more important than that of shareholders.

Others have argued against shareholder control on the grounds that either shareholders overestimate the extent of their information or that shareholders have agendas other than value maximization. We show that when shareholders think they have private information but, in fact, do not, there are still some decisions for which shareholder control is optimal. In particular, this will be the case if shareholders' average misperception is small. Finally, we show that, even when shareholders are themselves biased, it is firm-value maximizing for them to control decisions for which shareholders' bias is smaller than insiders' bias but not too much smaller.

Obviously, we have neglected a number of important issues regarding the optimality of direct shareholder control of decisions. The most glaring of these omissions is our assumption that there are no

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<sup>29</sup> Actually, it is shown in Proposition 5 that  $H$  is downward sloping for  $B \geq b/2$ .

differences of opinion, information, or preferences among shareholders (or at least among the controlling group of shareholders). When such differences exist, the issue arises as to how they are resolved in making decisions (both delegation decisions and “substantive” decisions). Obviously, this involves voting in some form or another. The same can be said about differences among insiders. Finally, we have assumed that all the parameters of the information structure and preferences are common knowledge. Relaxing these assumptions will, we believe, lead to interesting results. This, of course, is left for future work.

Finally, consider the question of who should control the decision of who controls substantive decisions. This is what Bebchuk (2005) refers to as controlling the “rules of the game.” Suppose that, when such rules-of-the-game decisions are made, the parameters  $b$ ,  $\sigma_a$ , and  $\sigma_p$  describing the substantive decisions are common knowledge and shareholders want to maximize firm value. If shareholders want to maximize firm value and are not misinformed, then they should control rules-of-the-game decisions (and allocate them as described in Proposition 2). If there is uncertainty about the relevant parameters when a rules-of-the-game decision is made, especially if there is *private* information about their likely values, it is not clear who should control the rules of the game, even if shareholders are not misinformed and do not have agendas other than value maximization. Matters become even more complicated if shareholders are misinformed or have other agendas. This topic is also left for future work.

## 6 Appendix

### 6.1 Ex-Post Delegation by Shareholders is Suboptimal

**Theorem.** Suppose shareholders are in control and  $p^* \in (0, P)$ . Let  $x^*$  solve

$$\min_{x \in [0, P]} \frac{x}{P} \left( \sigma(x)^2 + b^2 \right) + \left( 1 - \frac{x}{P} \right) L(b, A), \quad (18)$$

and let  $p^{**} = P - x^*$ . Then  $p^{**}$  is an *ex-ante*-optimal delegation threshold, and  $p^{**} < p^*$ .<sup>30</sup>

**Proof.** The problem in (18) has a solution, since the objective function is continuous and the constraint set is compact. Obviously, the objective function in (18) is the *ex ante* expected loss if shareholders delegate if and only if  $\tilde{p} \geq P - x$ . Consequently,  $p^{**}$  is an *ex-ante*-optimal delegation threshold.

Define, for any  $x \in [0, P]$ ,

$$g(b, x) \equiv \frac{x^2}{4} + b^2. \quad (19)$$

Then the derivative of the objective function in (18) with respect to  $x$  is

$$\frac{g(b, x) - L(b, A)}{P}. \quad (20)$$

Clearly,  $g$  is increasing in its second argument, so the objective function in (18) is convex. Also, for any  $x \in (0, P)$ ,

$$f(b, x) = \left( \frac{x}{2} + b \right)^2 > g(b, x). \quad (21)$$

Since  $p^* \in (0, P)$ , so  $d = P - p^* \in (0, P)$ , from (7) and (21), we have

$$f(b, d) = L(b, A) > g(b, d).$$

Therefore, from (20), the objective function in (18) is strictly decreasing in  $x$  at  $x = d$ . Since  $d \in (0, P)$ , we must have  $x^* > d$ . Consequently,  $p^{**} = P - x^* < P - d = p^*$ . Q.E.D.

### 6.2 Proof of Proposition 1

**Proposition 1:** If shareholders are in control of a decision, they will delegate the decision if and only if the realized value of their private information exceeds a threshold  $p^* \in [0, P]$  satisfying the following three conditions:

$$\text{If } \sigma_a \leq b, \text{ shareholders never delegate, i.e., } p^* = P \text{ and } d = 0; \quad (22)$$

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<sup>30</sup> Note that we do not claim that delegating when  $p \in [p^{**}, P]$  is an optimal *ex ante* delegation policy for shareholders. We claim only that this policy is optimal in the class of policies in which shareholders delegate when  $p$  is above some cutoff value and that it is better than the equilibrium *ex post* delegation policy of delegating when  $p \in [p^*, P]$ .

$$\text{if } P \leq 2\left[\sqrt{L(b,A)} - b\right], \text{ shareholders always delegate, i.e., } p^* = 0 \text{ and } d = P; \quad (23)$$

$$\text{otherwise, } p^* \in (0, P), \text{ } d \in (0, P) \text{ and satisfy } P - p^* = d = 2\left[\sqrt{L(b,A)} - b\right]. \quad (24)$$

**Proof.** Define a function  $f$  on  $(0, P)$  as follows: for any  $x \in (0, P)$ ,  $f(b, P - x)$  is the loss to shareholders of delegating, given that insiders believe the threshold is  $x$ , and given that the actual realization of  $\tilde{p}$  is exactly  $x$ . If insiders believe that shareholders delegate if and only if  $\tilde{p} \in [x, P]$ ,  $\hat{p} = \frac{P+x}{2}$ , so insiders who observe  $\tilde{a} = a$  choose  $s = a + \frac{P+x}{2} + b$  (recall that shareholders communicate no information about  $\tilde{p}$  other than what can be inferred from the fact of delegation). Thus

$$f(b, P - x) = \left(a + \frac{P+x}{2} + b - (a+x)\right)^2 = \left(\frac{P-x}{2} + b\right)^2. \quad (25)$$

Since  $x$  is the farthest point in the delegation region from the insiders' choice of  $s$  (remember,  $b > 0$ , so  $\hat{p}$  is more than halfway between  $x$  and  $P$ ),  $f(b, P - x)$  represents the worst-case loss from delegating. In order for  $x$  to be an equilibrium threshold for delegating, this worst-case loss from delegating must be just equal to the loss from not delegating,  $L(b, A)$ , provided that  $x \in (0, P)$ . If  $L(b, A) < f(b, P - x)$ , the loss from delegating will be greater than the loss from not delegating for some values of  $\tilde{p} > p^*$ . If  $L(b, A) > f(b, P - x)$ , the loss from delegating will be less than the loss from not delegating for some values of  $\tilde{p} < p^*$ . Thus, if  $p^* \in (0, P)$ ,  $p^*$  must satisfy

$$f(b, P - p^*) = \left(\frac{P-p^*}{2} + b\right)^2 = \left(\frac{d}{2} + b\right)^2 = L(b, A). \quad (26)$$

Solving (26) for  $d$  gives

$$d = 2\left[\sqrt{L(b,A)} - b\right]. \quad (27)$$

Clearly, the formula for  $d$  in (27) is valid only if it results in a value between 0 and  $P$ . If  $L(b, A) \leq b^2$ , then (27) results in  $d \leq 0$ . In this case, insiders do not have sufficient information to warrant delegating to them regardless of the realization of  $\tilde{p}$ , so  $d = 0$  and  $p^* = P$ . As mentioned above, however,  $L(b, A) \leq b^2$  if and only if  $\sigma_a \leq b$ , so we have that  $d = 0$  and  $p^* = P$  if and only if  $\sigma_a \leq b$ . If  $L(b, A) \geq f(b, P)$ , or, using (25),  $P \leq 2\left[\sqrt{L(b,A)} - b\right]$ , then (27) results in  $d \geq P$ . In this case, insiders have sufficient information to warrant delegating to them regardless of the realization of  $\tilde{p}$ , so  $d = P$  and  $p^* = 0$ . Q.E.D.

### 6.3 Proof of Proposition 2

#### Proposition 2.

- (i) If the information of insiders is less important than agency costs ( $\sigma_a < b$ ), then for any  $\sigma_p \in [0, b]$ , shareholder control is optimal ( $\Delta > 0$ ).

- (ii) For every  $\sigma_a \geq b$ , there are two boundaries for  $\sigma_p$ ,  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$ , such that when the importance of the shareholders' information  $\sigma_p \in (\sigma_p^L(\sigma_a), \sigma_p^U(\sigma_a))$ , insider control is strictly optimal ( $\Delta < 0$ ). When the importance of the shareholders' information  $\sigma_p > \sigma_p^U(\sigma_a)$ , shareholder control is strictly optimal ( $\Delta > 0$ ). When  $\sigma_p \leq \sigma_p^L(\sigma_a)$  or  $\sigma_p = \sigma_p^U(\sigma_a)$ , control is irrelevant ( $\Delta = 0$ ).
- (iii) The functions  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$  satisfy the following properties:  $\sigma_p^L(b) = \sigma_p^U(b) = 0$ , and for every  $\sigma_a > b$ ,  $\sigma_p^L(\sigma_a)$  and  $\sigma_p^U(\sigma_a)$  are strictly increasing in  $\sigma_a$ , and  $\sigma_a > \sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a) > 0$ .

**Proof.** First suppose  $\sigma_a < b$ . In this case, as shown in Proposition 1,  $d = 0$ , so from (11),

$$\Delta = b^2 + \sigma_p^2 - L(b, A) > \sigma_p^2 > 0,$$

since, from Lemma 1 of Harris and Raviv (2007b),  $\sigma_a < b$  implies that  $L(b, A) < b^2$ . This proves part (i). Henceforth, we assume  $\sigma_a \geq b$ .

Define  $\sigma_p^L$  by

$$f(b, \sigma_p^L \sqrt{12}) = \left( \frac{\sigma_p^L \sqrt{12}}{2} + b \right)^2 = L(b, \sigma_a \sqrt{12}), \quad (28)$$

or

$$\sigma_p^L(\sigma_a) = \frac{2}{\sqrt{12}} \left( \sqrt{L(b, \sigma_a \sqrt{12})} - b \right). \quad (29)$$

From Lemma 1 of Harris and Raviv (2007b),  $\sigma_a \geq b$  implies that  $\sigma_p^L$  as given in (29) is non-negative.

Since  $\sigma_a \geq b$  implies that  $L(b, A) = L(b, \sigma_a \sqrt{12}) = \sigma_a^2$ , it is obvious from (29) that  $\sigma_p^L(b) = 0$ . Since  $L$  is strictly increasing in its second argument (Lemma 1 of Harris and Raviv (2007b)), it is also obvious from (29) that  $\sigma_p^L(\sigma_a)$  is increasing in  $\sigma_a$ ,  $\forall \sigma_a > b$ , as claimed in part (iii).

Since  $f$  is clearly increasing in its second argument, (28) implies that  $f(b, \sigma_p \sqrt{12}) \leq L(b, \sigma_a \sqrt{12})$  if and only if  $\sigma_p \leq \sigma_p^L(\sigma_a)$ . Hence, from (6)–(8) and (28),  $d = \min\{\sigma_p^L(\sigma_a) \sqrt{12}, P\}$ . It follows immediately from (11) that, for  $\sigma_p \leq \sigma_p^L(\sigma_a)$ ,  $d = P$  and  $\Delta = 0$ , as claimed in part (ii).

The next step is to develop a formula for  $\sigma_p^U(\sigma_a)$ . Assuming for the time being that  $\sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a)$  and using  $d = \min\{\sigma_p^L(\sigma_a) \sqrt{12}, P\}$ , we can write the condition defining  $\sigma_p^U(\sigma_a)$  as

$$\sigma_p^U \left( R - (\sigma_p^U)^2 \right) = \sigma_p^L \left( R - (\sigma_p^L)^2 \right), \quad (30)$$

where  $R = L(b, \sigma_a \sqrt{12}) - b^2$ . Rewrite (30) as

$$(\sigma_p^U)^3 - (\sigma_p^L)^3 - R(\sigma_p^U - \sigma_p^L) = 0. \quad (31)$$

Since we are assuming that  $\sigma_p^U > \sigma_p^L$ , we can divide (31) by  $\sigma_p^U - \sigma_p^L$  to obtain

$$(\sigma_p^U)^2 + \sigma_p^U \sigma_p^L + (\sigma_p^L)^2 - R = 0.$$

The solution of this equation of interest to us is given by

$$\sigma_p^U = \frac{-\sigma_p^L + \sqrt{4R - 3(\sigma_p^L)^2}}{2}. \quad (32)$$

For  $\sigma_p^U$  to be given by equation (32), we need only show that this value exceeds  $\sigma_p^L$ . For this, it suffices to show that  $R > 3(\sigma_p^L)^2$ . But from (29) and the definition of  $R$ , we have that  $R > 3(\sigma_p^L)^2$  if and only if  $\sqrt{L(b, \sigma_a \sqrt{12})} + b > \sqrt{L(b, \sigma_a \sqrt{12})} - b$ , which is clearly true, since  $b > 0$ . Consequently, we have shown that  $\sigma_p^U$  is indeed given by equation (32) and that  $\sigma_p^U(\sigma_a) > \sigma_p^L(\sigma_a)$ , as claimed in (iii). For  $\sigma_a = b$ ,  $R = \sigma_a^2 - b^2 = 0$ , and, as shown previously,  $\sigma_p^L = 0$ . Consequently,  $\sigma_p^U(b) = 0$ , as claimed in (iii).

To show that  $\sigma_p^U(\sigma_a) < \sigma_a$ , from (32), it suffices to show that

$$R = L(b, \sigma_a \sqrt{12}) - b^2 < \sigma_a^2 + \sigma_a \sigma_p^L + (\sigma_p^L)^2. \quad (33)$$

It is easy to check that  $L(b, \sigma_a \sqrt{12}) \leq \sigma_a^2$ .<sup>31</sup> Therefore (33) is clearly satisfied since  $b$ ,  $\sigma_a$ , and  $\sigma_p^L$  are all positive.

To complete the proof of part (iii), it remains to show that  $\sigma_p^U(\sigma_a)$  is increasing in  $\sigma_a$ . For this, it suffices to show that  $\sigma_p^U(\sigma_a)$  is increasing in  $\sqrt{L(b, \sigma_a \sqrt{12})}$ . To make the formulas easier to read, let  $z = \sqrt{L(b, \sigma_a \sqrt{12})}$ . Then, substituting for  $R$  and  $\sigma_p^L$  in (32), we have

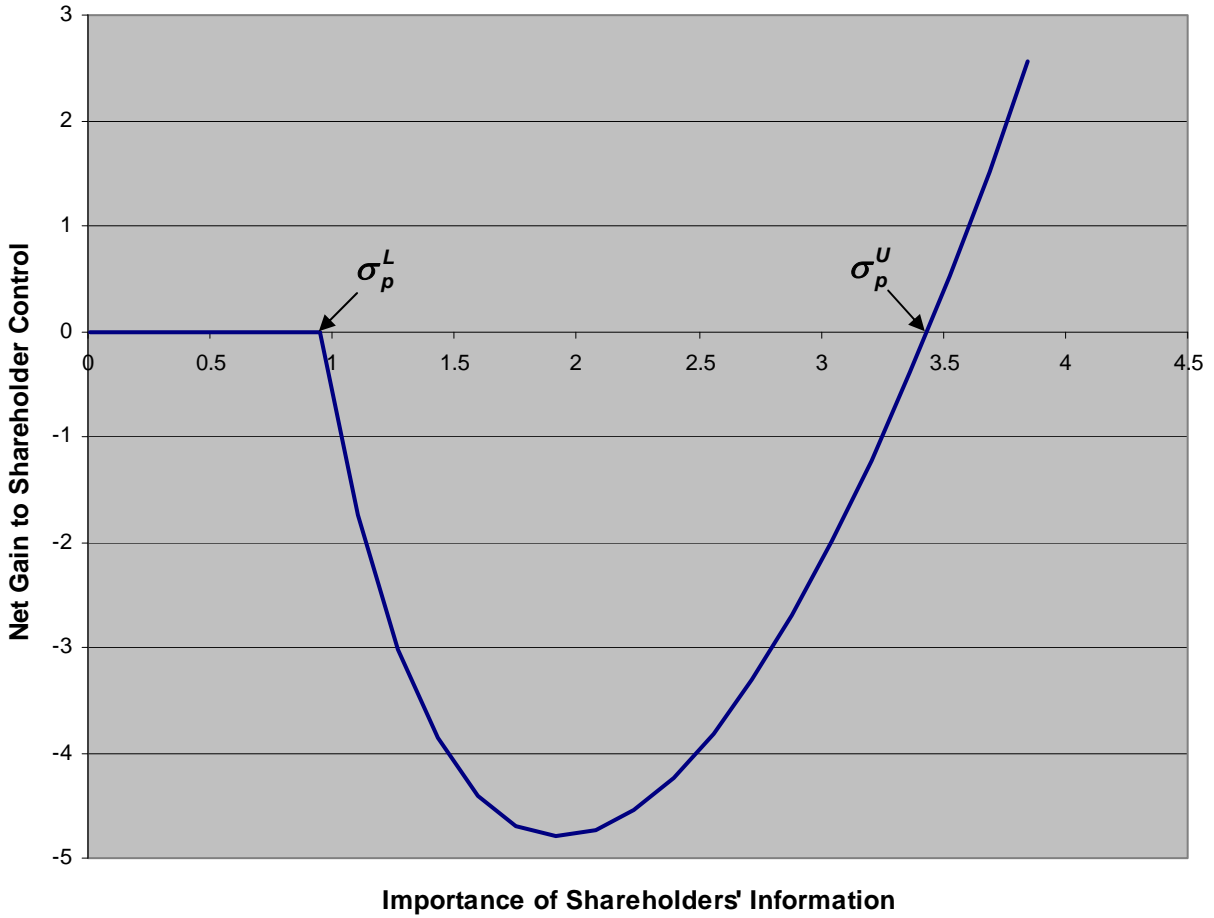
$$\sigma_p^U = \frac{1}{2} \left[ -\frac{2}{\sqrt{12}}(z - b) + \sqrt{3(z^2 - b^2) + 2b(z - b)} \right]. \quad (34)$$

It is easy to check that the derivative of the right hand side of (34) with respect to  $z$  is positive if and only if  $3z^2 + 2bz + b^2 > 0$ , which is clearly true. This completes the proof of part (iii).

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<sup>31</sup> If  $N(b, \sigma_a \sqrt{12}) = 1$ , then  $L(b, \sigma_a \sqrt{12}) = \sigma_a^2$ , and we are done. Suppose  $N(b, \sigma_a \sqrt{12}) = n \geq 2$ . Then  $L(b, \sigma_a \sqrt{12}) = \frac{\sigma_a^2}{n^2} + \frac{b^2(n^2 - 1)}{3} < \sigma_a^2$  if and only if  $\sigma_a^2 > \frac{(bn)^2}{3}$ . But, as shown in Lemma 1 of Harris and Raviv (2007b),  $N(b, \sigma_a \sqrt{12}) = n$  implies that  $A > 2bn(n - 1)$  or  $\sigma_a^2 > \frac{(bn(n - 1))^2}{3} \geq \frac{(bn)^2}{3}$ , for  $n \geq 2$ .

To complete the proof of part (ii), we must show that  $\Delta < 0$  for  $\sigma_p^L(\sigma_a) < \sigma_p < \sigma_p^U(\sigma_a)$ , and  $\Delta > 0$  for  $\sigma_p > \sigma_p^U(\sigma_a)$ . It is easy to check that  $\Delta$  is convex in  $\sigma_p$  for  $\sigma_p \geq \sigma_p^L$ . Since  $\Delta \equiv 0$  for  $\sigma_p \leq \sigma_p^L$ ,  $\Delta$  can cross zero at most once at some  $\sigma_p > \sigma_p^L$  and only from below (see Figure 4). Consequently, this must occur at  $\sigma_p^U$ , and  $\Delta < 0$  for  $\sigma_p^L(\sigma_a) < \sigma_p < \sigma_p^U(\sigma_a)$ ,  $\Delta > 0$  for  $\sigma_p > \sigma_p^U(\sigma_a)$ , and  $\Delta = 0$  for  $\sigma_p = \sigma_p^U(\sigma_a)$  as claimed. Q.E.D.



**Figure 4**

This graph shows the net gain to shareholder control,  $\Delta$ , as a function of the importance of shareholders' information. Shareholder control is optimal whenever the importance of shareholders' information,  $\sigma_p$ , exceeds  $\sigma_p^U \approx 3.45$ . For this figure,  $b = 4$ ,  $\sigma_a = 8$ , and  $\sigma_p^L = 0.95658525 < b$ .

#### 6.4 Communication from insiders when shareholders misperceive $\tilde{p}$

When shareholders misperceive  $\tilde{p}$  and do not delegate, insiders draw their report from a uniform distribution on the element that contains  $a$  of the equilibrium partition of  $[0, A]$  for the game in which both parties believe  $\tilde{a}$  is uniform on  $[0, A]$ . This partition will be somewhat different than that described in section 2. Specifically, the sequence  $\{a_i\}$  of endpoints of the elements of the equilibrium partition of  $[0, A]$  is calculated using the fact that insiders must be indifferent between reporting  $r \in [a_{i-1}, a_i]$  and reporting  $r \in [a_i, a_{i+1}]$ , when  $\tilde{a} = a_i$ . Insiders' expected loss for a report  $r \in [a_{i-1}, a_i]$  is

$$E \left[ \left( \frac{a_{i-1} + a_i}{2} + \tilde{e} - a - b \right)^2 \right] = \left( \frac{a_{i-1} + a_i}{2} - a - (b - \bar{e}) \right)^2 + \sigma_p^2. \quad (35)$$

The right hand side of equation (35) is exactly the same expected loss as in section 2, except that the bias  $b$  in section 2 is replaced by  $B$ .

## 6.5 Proof of Proposition 4

### Proposition 4.

(i) If insiders' information is less important than their agency cost ( $\sigma_a < b$ ) and shareholders' average misperception bias,  $\bar{e}$ , is sufficiently small, shareholder control is optimal.

Now suppose shareholders believe  $\tilde{p}$  is smaller than the equilibrium delegation threshold ( $q < p^*$ ) and insiders' information is more important than their agency cost ( $\sigma_a \geq b$ ). Then

(ii) If shareholders believe  $\tilde{p}$  is smaller than the average value of  $\tilde{p}$  ( $q \leq \bar{p}$ ), insider control is optimal.

(iii) If shareholders believe  $\tilde{p}$  is greater than the average value of  $\tilde{p}$  ( $q > \bar{p}$ ), for some values of shareholders' average misperception bias  $\bar{e}$  between 0 and  $b$ , shareholder control is optimal. If, however, shareholders' average misperception bias exceeds  $b$ , it is optimal for insiders to control.

Finally, suppose shareholders believe  $\tilde{p}$  is larger than the equilibrium delegation threshold ( $q \geq p^*$ ). Then control is irrelevant.

**Proof.** Optimal control is obtained by comparing  $L_M$  with shareholders' expected loss from either (12) or (14). Comparison with (12) shows that, when  $q \geq p^*$ , control doesn't matter. This proves the last part of the proposition.

If  $q < p^*$ , comparison of  $L_M$  with (14) shows that it is optimal for shareholders to be in control if and only if

$$b^2 > \bar{e}^2 + L(B, A). \quad (36)$$

First consider the case in which  $\sigma_a < b$ . In this case,  $p^* = P$  (see Proposition 1). Consequently,  $q < p^*$ . Since  $\sigma_a < b$ , we have  $L(B, A) \leq \sigma_a^2 < b^2$ , so if  $\bar{e}$  is sufficiently small, it is optimal for shareholders to be in control.<sup>32</sup> This establishes the claim in item (i).

Now assume  $\sigma_a \geq b$  and  $q < p^*$ . If shareholders are in control, they never delegate. Suppose  $\bar{e} \leq 0$ . Then, from (13),  $B \geq b$ , and, using Lemma 1 of Harris and Raviv (2007b),

$$L(B, A) \geq L(b, A) \geq b^2,$$

which implies that (36) is false. This proves item (ii).

Finally, suppose  $\bar{e} \in (0, b)$ . All that is required for the first claim in item (iii) is a numerical example. One such example involves  $\sigma_a = 5 > 4 = b$ . As  $q$  increases from  $\bar{p} + 0.8$  to  $\bar{p} + 2$ , the net advantage of shareholder control over insider control increases from  $-1.13$  to  $+1.75$ . It remains only to show that there is a value of  $\sigma_p < 4$  such that, for  $\sigma_a = 5$ , and  $b = 4$ ,  $p^* > \bar{p} + 2 = \frac{P}{2} + 2$ . One such value

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<sup>32</sup> For example, if  $b = 1$ ,  $\bar{e} = 0.7575$  (so  $B = 0.2425$ ), and  $\sigma_a = 0.28141147 < b$ ,  $\bar{e}^2 + L(B, A) = 0.6524106 < 1 = b^2$ .

is  $\sigma_p = 3.5$  ( $P = 12.124356$ ). For  $\sigma_p = 3.5$ ,  $\sigma_a = 5$ , and  $b = 4$ ,  $\bar{p} + 2 = 8.0621778 < p^* = 10.6903745$ .

Finally, if  $\bar{e} \geq b$ , it is obvious from (36) and the fact that  $L(B, A) \geq 0$ , that insider control is optimal.

This completes the proof of item (iii) and of the proposition. Q.E.D.

## 6.6 Proof of Proposition 5

Define

$$\beta_0 = \frac{1}{2} \left( b - \sqrt{b^2 + 2\sigma_p^2} \right). \quad (37)$$

**Lemma 2.**  $\beta_0 < 0$  and if  $b > \sigma_p/2$ , then  $b - \beta_0 < 2b$ .

**Proof.** From (37), it is obvious that  $\beta_0 < 0$  and that  $-b < \beta_0$  if and only if  $b > \sigma_p/2$ . Clearly,  $b - \beta_0 < 2b$  if and only if  $-b < \beta_0$ . Q.E.D.

**Proposition 5.** Assume  $b - \bar{p} > \sigma_p$ . Define  $B_0 = b + \sqrt{b^2 + \sigma_p^2}$ . Then there exist continuous functions,  $G(B) > B^2$ , with  $G$  symmetric with respect to  $B = 0$ , and  $H(B)$  such that  $H(b - \bar{p}) = G(b - \bar{p})$ ,  $H(B_0) = 0$ ,  $H(B)$  is decreasing in  $B$  for  $B \geq b/2$ , and

- for decisions for which  $B < -\sigma_p$ , insider-control is strictly optimal if  $\sigma_a^2 < G(B)$ , and otherwise, insider-control and outsider-control are equivalent;
- for decisions for which  $\sigma_p < B < b - \bar{p}$ , shareholder-control is strictly optimal if  $\sigma_a^2 < G(B)$ , and otherwise, insider-control and outsider-control are equivalent;
- for decisions for which  $b - \bar{p} < B < B_0$ , shareholder-control is strictly optimal if  $\sigma_a^2 < H(B)$ , insider-control is strictly optimal if  $H(B) < \sigma_a^2 < G(B)$ , and otherwise, insider-control and outsider-control are equivalent;
- for decisions for which  $B_0 < B$ , insider-control is strictly optimal if  $\sigma_a^2 < G(B)$ , and otherwise, insider-control and outsider-control are equivalent.

**Proof.** Define  $G(B)$  as the value of  $\sigma_a^2$  such that  $L(B, \sigma_a \sqrt{12}) = \left( \frac{P}{2} + |B| \right)^2$ . It is easy to check, using the facts that  $L(B, \sigma_a \sqrt{12}) = \sigma_a^2$  for  $\sigma_a^2 \leq B^2$  and  $L(B, \sigma_a \sqrt{12}) \rightarrow \infty$  as  $\sigma_a \rightarrow \infty$ , and  $L$  is continuous in its second argument (see Lemma 1 in Harris and Raviv (2007)), that such a value of  $\sigma_a^2$  exists and is larger than  $B^2$ . Thus  $G(B) > B^2$  for all  $B$ . Since  $L$  depends on  $B$  only through  $B^2$ , and  $\left( \frac{P}{2} + |B| \right)^2$  depends on  $B$  only through  $|B|$ ,  $G$  is symmetric with respect to  $B = 0$ . Finally, since  $L$  and  $f$  are continuous in  $B$  (for  $B \neq 0$ ), so is  $G$ . From the definition of  $G$ , for  $\sigma_a^2 \geq G(B)$ ,  $d = P$ , i.e., shareholders, if in control, always delegate to insiders. In this case, control is irrelevant, since insiders always make the decision with no information from shareholders, regardless of control, i.e., (16) is satisfied as an equality. For the remainder of the proof, we consider only the case in which  $\sigma_a^2 < G(B)$ .

First suppose  $\sigma_a^2 \leq B^2$  so that shareholder-control is optimal if and only if (17) holds. Define  $H_1(B) = \sigma_p^2 - (B^2 - 2bB)$ . Clearly, (17) is satisfied if and only if  $\sigma_a^2 \leq H_1(B)$ . If  $B < -\sigma_p$ , then  $B^2 - 2bB > B^2 \geq \sigma_p^2$ , since  $B < 0$ , so  $H_1(B) < 0$  for  $B$  in this range. Consequently, (17) cannot be satisfied, and insider-control is optimal. If  $B > \sigma_p$ , it is easy to check that  $H_1(B) \leq B^2$  if and only if  $B \geq b - \beta_0$  with equality if and only if  $B = b - \beta_0$ . Note that the assumption that  $b - \bar{p} > \sigma_p$  implies that  $b > \sigma_p/2$  which implies that  $0 < b - \beta_0 < 2b$  by Lemma 2. Moreover,  $H_1$  is decreasing in  $B$  for  $B \geq b - \beta_0$  and  $H_1(B_0) = 0$ . Thus, for  $\sigma_a^2 \leq B^2$ , insider-control is strictly optimal for  $B < -\sigma_p$ , for  $b - \beta_0 < B < B_0$  if and only if  $G(B) > \sigma_a^2 > H_1(B)$ , and for  $B \geq B_0$  for all  $\sigma_a^2 \leq B^2$ . For  $\sigma_p \leq B < b - \beta_0$ , shareholder control is optimal for all  $\sigma_a^2 \leq B^2$ . This completes the characterization of optimal control for  $\sigma_a^2 \leq B^2$ .

Now suppose  $G(B) > \sigma_a^2 > B^2$ . In this case  $d \in (0, P)$ , so we have

$L(B, A) = f(B, d) = \left(\frac{d}{2} + |B|\right)^2$ . Consequently we can write the left hand side of (16) as

$$F(d) \equiv \frac{d}{P}b^2 + \left(1 - \frac{d}{P}\right)\beta^2 + \frac{d}{P}\sigma(d)^2 + \left(1 - \frac{d}{P}\right)\left(\frac{d}{2} + |B|\right)^2.$$

We claim that the function  $F$  is strictly concave in  $d$  on  $(0, P)$ . To see this, note that

$$F''(d) = \frac{1}{2} - \frac{2}{P}\left(\frac{d}{2} + |B|\right) < \frac{1}{2} - \frac{2|B|}{P}.$$

But  $P = \sqrt{12}\sigma_p \leq \sqrt{12}|B| < 4|B|$ . Consequently,  $F''(d) < 0$  and  $F$  is strictly concave as claimed. Also  $F(0) = B^2 + \beta^2 = b^2 + 2B(B - b)$ , and  $F(P) = \sigma_p^2 + b^2 =$  right hand side of (16).

Since  $F$  is strictly concave and  $F(P) = \sigma_p^2 + b^2$ , if  $F(0) \geq \sigma_p^2 + b^2$ , then  $F(d) > \sigma_p^2 + b^2$  for all  $d \in (0, P)$ . Therefore, in this case (16) is false, i.e., insider-control is strictly optimal, for all  $\sigma_a^2 < G(B)$ .

Now suppose  $B < -\sigma_p$ . Then,  $2B(B - b) > B^2 \geq \sigma_p^2$ , so  $F(0) > \sigma_p^2 + b^2$ . Consequently, insider-control is strictly optimal for all  $B^2 < \sigma_a^2 < G(B)$ . Together with the previous result that insider-control is strictly optimal when  $B < -\sigma_p$  for all  $\sigma_a^2 \leq B^2$ , we have completed the proof of the first bullet.

Next, suppose  $B > \sigma_p$ . Then it is easy to check that  $F(0) < \sigma_p^2 + b^2$  if and only if  $B < b - \beta_0$ . Consequently, if  $B \geq b - \beta_0$ ,  $F(0) \geq \sigma_p^2 + b^2$ , so insider control is strictly optimal. Now consider  $\sigma_p < B < b - \beta_0$ , so that  $F(0) < \sigma_p^2 + b^2$ . There are two possible cases. Since  $F$  is strictly concave, if  $F'(P) \geq 0$ , then  $F(d) < \sigma_p^2 + b^2$  for all  $d \in (0, P)$  which implies that it is strictly optimal for shareholders to control regardless of the value of  $B^2 < \sigma_a^2 < G(B)$ . If  $F'(P) < 0$ , then there exists a unique  $d_0 \in (0, P)$  such that  $F(d) \leq \sigma_p^2 + b^2$  for all  $d \leq d_0$ ,  $F(d) > \sigma_p^2 + b^2$  for all  $d > d_0$ , and  $F'(d_0) > 0$ . That is, it is optimal for shareholders to control if and only if  $d \leq d_0$ . But  $d$  is a continuous,

increasing function of  $\sigma_a^2$ ,  $d = 0$  for  $\sigma_a^2 \leq B^2$ , and  $d = P$  for  $\sigma_a^2 \geq G(B)$ , so, for each  $B$  such that  $F'(P) < 0$ , there is a unique value of  $\sigma_a^2 \in (B^2, G(B))$  such that  $d = d_0$  for that value of  $\sigma_a^2$ . Define  $H_0(B)$  to be the value of  $\sigma_a^2$  for which  $d = d_0$  for  $B$  such that  $F'(P) < 0$ . Then,  $G(B) > H_0(B) > B^2$ . Since  $F$ ,  $L$ , and  $f$  are continuous in  $B$ , so is  $H_0$ .

It is easy to check that

$$F'(P) = \frac{1}{\bar{p}}(b - \bar{p} - B)B.$$

Consequently,  $F'(P) < 0$  for  $b - \bar{p} < B < b - \beta_0$ , and  $F'(P) \geq 0$  for  $\sigma_p \leq B \leq b - \bar{p}$ . It follows that shareholder-control is strictly optimal for  $\sigma_p \leq B \leq b - \bar{p}$  and  $B^2 < \sigma_a^2 < G(B)$  and for  $b - \bar{p} < B < b - \beta_0$  with  $B^2 < \sigma_a^2 < H_0(B)$ , while insider-control is strictly optimal for  $b - \bar{p} < B < b - \beta_0$  for  $H_0(B) < \sigma_a^2 < G(B)$ .

Now, since  $F'$  is continuous and  $F'(P) = 0$  for  $B = b - \bar{p}$ ,  $d_0 \uparrow P$  as  $B \downarrow b - \bar{p}$ . But, for any  $B$ ,  $G(B)$  is the smallest value of  $\sigma_a^2$  such that  $d = P$ . Consequently,  $H_0(B) \rightarrow G(b - \bar{p})$  as  $B \downarrow b - \bar{p}$ , and we can define  $H_0(b - \bar{p}) = G(b - \bar{p})$ . Moreover, as  $B \uparrow b - \beta_0$ ,  $d_0 \downarrow 0$ , so  $H_0(B) \rightarrow B^2 = H_1(b - \beta_0)$ . Therefore, define

$$H(B) = \begin{cases} H_0(B) & \text{for } B \in [b - \bar{p}, b - \beta_0], \\ H_1(B) & \text{for } B \in [b - \beta_0, B_0]. \end{cases}$$

Then  $H$  is continuous in  $B$ ,  $H(b - \bar{p}) = G(b - \bar{p})$ , and  $H(B_0) = 0$ .

Finally, note that, for  $B > 0$ ,

$$\frac{\partial F(d)}{\partial B} = 2 \left(1 - \frac{d}{P}\right) \left(2B - b + \frac{d}{2}\right).$$

Consequently,  $\frac{\partial F(d)}{\partial B} > 0$  for all  $d \in (0, P)$  if  $B \geq b/2$ . Since  $F'(d_0) > 0$ , it follows that, for  $B \geq b/2$ ,  $d_0$  is decreasing in  $B$ . Therefore, so is  $H_0(B)$ . Since we have already shown that  $H_1(B)$  is decreasing in  $B$  for  $B \geq b - \beta_0$ ,  $H(B)$  is decreasing in  $B$  for  $B \geq b/2$ .

Q.E.D.

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