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ORGANIZATION DESIGN

by

Milton Harris

and

Artur Raviv[1]

ABSTRACT

This paper attempts to explain organization structure based on optimal coordination of interactions among activities. The main idea is that each manager is capable of detecting and coordinating interactions only within his limited area of expertise. Only the CEO can coordinate company-wide interactions. The optimal design of the organization trades off the costs and benefits of various configurations of managers. Our results consist of classifying the characteristics of activities and managerial costs that lead to the matrix organization, the functional hierarchy, the divisional hierarchy, or a flat hierarchy. We also investigate the effect of changing the fixed and variable costs of managers on the nature of the optimal organization, including the extent of centralization.

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[1]Harris is the Chicago Board of Trade Professor of Finance and Business Economics, Graduate School of Business, University of Chicago. Raviv is the Alan E. Peterson Distinguished Professor of Finance, Kellogg Graduate School of Management, Northwestern University. We thank Franklin Allen, Sel Becker, David Besanko, Ronen Israel, Ed Zajac and participants in seminars at the Stockholm School of Economics, Michigan, CMU, Berkeley, Stanford and the 2000 Utah Winter Finance Conference for helpful comments. Address correspondence to Professor Milton Harris, Graduate School of Business, 1101 East 58th Street, Chicago, IL 60637; telephone: 773-702-2549; E-mail: milt@uchicago.edu; Web: http://gsbwww.uchicago.edu/fac/milton.harris/more/.

Organization Design

by Milton Harris and Artur Raviv

Organizations are observed to exist with various structures. Many organizations are designed as hierarchies with each manager reporting to one and only one manager at the next higher level. Other organizations employ a matrix structure in which each low level manager reports to two or more superiors.

Within the hierarchical structure, there is considerable variation in the number of levels and in the set of activities grouped together. The two main groupings are "divisional" and "functional." In a divisional hierarchy, all the activities pertaining to a single product (or perhaps set of products) are grouped together into a division. For example, until the late 1980s Procter and Gamble (P & G) had a relatively flat hierarchical structure with only two levels. At the lower level were the brand managers. "Each brand – such as Tide, Crisco, Head and Shoulders and Scope – had its own brand manager who was singularly accountable for his or her brand's performance." [Robbins (1990, p. 295)]. The only other layer was headquarters. P & G then introduced a layer in between the brand managers and headquarters. This layer contains division managers for product categories, such as laundry detergents, each responsible for advertising, sales, production, research, etc. for the brands in his or her category. In a functional hierarchy, by contrast, activities pertaining to a particular function are organized into departments. For example, at Maytag, these departments include R & D, manufacturing, marketing, corporate planning, personnel, finance, labor relations, and legal [see Robbins (1990, pp. 286-87)]. In a functional hierarchy, the marketing department would, for example, coordinate marketing activities for all products.

Matrix structure, which involves "dual-authority relations" [Jennergren (1981, p. 43)] can combine divisional and functional structures. For example, the manager in charge of design for project A reports both to the Project A division manager and to the head of the design engineering group. In another example of the matrix form, the president of a unit producing power transformers in Norway for Asea Brown Boveri (ABB) reports to the president of ABB Norway and to the head of the Power Transformer Business Area [see Baron and Besanko (1997, p. 2)].

To clarify the various ways firms are typically organized, consider the following hypothetical example. ABC Company produces and sells two versions of a product in two countries, Norway and the U.S. Each version of the product requires occasional country-specific design adaptations, and, of course, each version must be marketed in each country. There are thus four basic tasks, design and marketing for each version of the product. These four activities may be organized in one of four commonly observed structures, depicted in **Figure 1**-**Figure 4**. In **Figure 1**, the structure is flat with the manager in charge of each activity reporting directly to the CEO. **Figure 2** depicts a divisional hierarchy in which there are two mid-level managers, each coordinating the two functions for a given country (product). In **Figure 3**, the hierarchy is organized along functional lines, i.e., each of the two middle managers is in charge of a function for both countries (products). Finally, **Figure 4** shows a matrix organization in which each bottom level manager reports to two middle managers, e.g., the marketing manager for Norway reports both to the middle manager in charge of Norway and the middle manager in charge of global marketing.

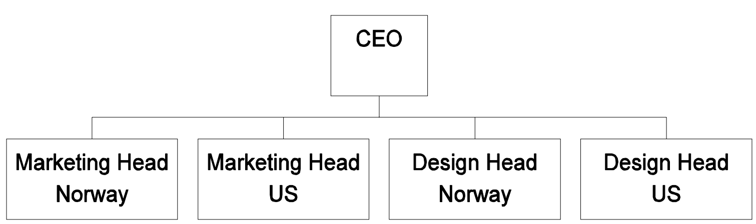

Figure 1: Flat Structure

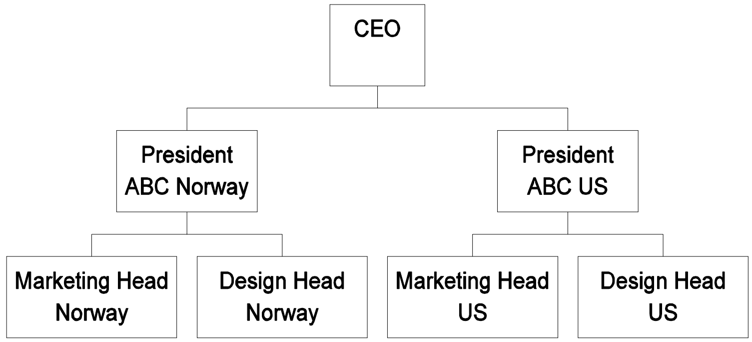

Figure 2: Divisional Hierarchy

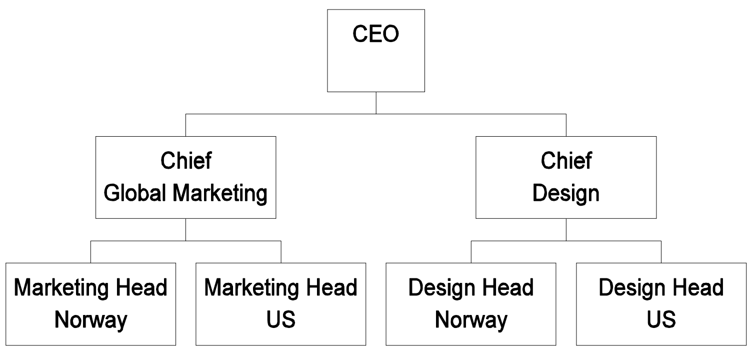

Figure 3: Functional Hierarchy

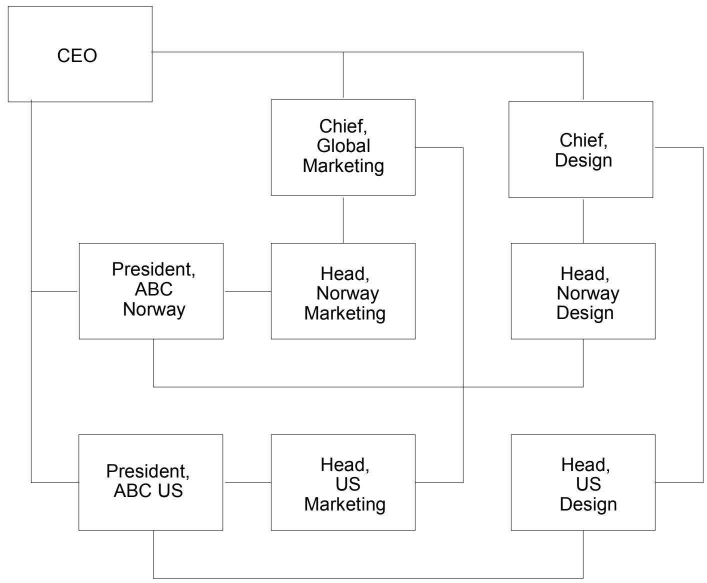

Figure 4: Matrix Organization

An interesting topic in the theory of the firm, relatively under explored in the economics literature, is what determines whether an organization adopts a matrix or hierarchical structure, how many levels are involved and how activities are grouped. Several authors in the organization behavior literature have argued that the choice between divisional and functional structures is driven by the relative importance of coordination of functional activities within a product line and economies of scale from combining similar functions across product lines.[2] The advantage of a divisional structure is that it allows better coordination among the various functions, such as manufacturing, product design, personnel, and marketing, required to produce and sell a product. Segregating these functions by product divisions, however, results in the failure to exploit economies of scale available if, for example, marketing for all products is handled by a central marketing department. Trading off these advantages, it

[2]See the survey by Jennergren (1981) for a summary of this literature.

is argued, determines whether one adopts a divisional or a functional hierarchy.

We address the issues relating to organization design mentioned above using a model based on coordinating interactions across various activities. In our view, coordinating interactions requires costly expertise embodied in managers. The optimal organizational structure trades off the benefits of coordination against the cost of the necessary expertise. In this sense it is similar to the arguments of organization theorists summarized in the previous paragraph. We provide a formal model that extends the ideas beyond the functional vs. divisional choice. The model endogenizes the choice of organization structure allowing us to make predictions regarding the use of matrix vs. hierarchical structures and the extent of decentralization. If a hierarchy is used, we rationalize the choice of functional vs. divisional grouping and vertical span.

We model a firm as consisting of activities such as producing products or components, designing products, marketing products, etc. Each activity originates with a "project manager" who is assumed to be essential to generating the activity and to have no function other than generating and possibly managing his activity. If a set of activities interacts, there are benefits to coordinating these activities. Reaping these benefits requires coordination by a manager with the correct expertise (project managers have no coordination expertise). The territory between the project managers and the CEO, may be populated by various "middle managers." Each middle manager is capable of detecting and coordinating a specific pair of interactions. In addition to the benefits of coordinating pairs of activities, there are incremental benefits to coordinating a set of activities on a company-wide basis. Only the CEO is capable of this company-wide coordination (the CEO can also coordinate pairs of activities and may engage in other, unmodeled activities). It is important to note that we abstract entirely from incentive problems. That is, in our model, managers act in the best interest of shareholders and have no incentive to hide or distort information that they discover. We return to this issue briefly in the concluding section.

These managers may have both fixed and variable costs. The fixed cost of a manager must be paid if the manager is available to coordinate activities for the firm, regardless of whether that manager is actually used. A manager's variable cost is paid only if the manager's expertise is actually used to coordinate activities. This is an opportunity cost of the manager's time that is related to his value in other activities not modeled here. For example, the variable cost of a CEO might be related to her value in strategic planning.

The organization design problem is to choose a set of middle managers who are available for coordinating activities and a set of instructions for using these managers, the project managers and the CEO, given the costs of having and using these managers and the expected coordination benefits. Our results consist of classifying the characteristics of activities and managerial costs that lead to various structures. When the fixed cost of middle managers is high, no middle managers are employed, and the resulting structure is a flat organization consisting only of the CEO and the project managers. When the middle managers' fixed cost is low, the resulting structure resembles a matrix form rich in middle managers. For intermediate fixed costs of the middle managers, a hierarchy with some middle managers is optimal. Increases in the variable cost of the CEO may also result in employing more middle managers as well as reducing the involvement of the CEO in coordination activities, i.e., reducing the centralization of decision making. Also, increases in the synergy gains from coordinating company-wide interactions increase centralization.

It is not surprising that increases in the fixed cost of middle managers lead to reduction in their employment or that increases in synergy gains or reductions in CEO variable cost lead to greater centralization. To understand the intuition for the result that increases in the variable cost of the CEO lead to more middle-management-intensive structures, it is helpful first to realize that the middle managers have two functions in our model. One is to coordinate pairs of projects when they interact. The other is to generate information that allows more efficient use of the CEO, i.e., to protect the CEO. In particular, middle managers allow a more accurate assessment of whether a company-wide interaction is present. This information enables the firm to reap the benefits of company-wide coordination in some situations when it would otherwise be suboptimal. It also allows the firm to avoid wasting the CEO's time when the company-wide interaction is unlikely to be present. Consequently, as the cost of the CEO's time in coordination activities increases, middle managers become more valuable in their function as protectors of the CEO.

A number of empirical implications follow from these results. Under certain additional assumptions, we show that organization structure will exhibit a sort of "life cycle" as the organization grows in complexity and size. In particular, we show that the structure will progress from a flat, but highly centralized structure, to a divisional hierarchy to a functional hierarchy and then either to a matrix structure or to a flat, highly decentralized structure. We also show that conglomerates that are organized as hierarchies may be expected to exhibit divisional, as opposed to functional, hierarchies. Finally, we show that firms that do not face tight resource constraints, highly regulated firms, and firms in stable environments will tend to have decentralized organizational structures.

The remainder of the paper is organized as follows. A brief review of the literature is contained in the next section. The model is presented formally in Section 2. We then solve the problem of how to use a given set of middle managers in Section 3. This results in an optimal value (i.e., coordination benefits net of costs) for each set of middle managers. These values are compared to obtain an overall best design in Section 4. Comparative statics results are presented in Section 5, empirical implications are considered in Section 6, and conclusions are presented in Section 7.

1 Literature Review

The economics literature on organization design is, as mentioned above, somewhat sparse. One approach, adopted by Radner (1993), is to assume that processing information takes valuable time. To reduce the delay, one can use "parallel processing" involving several people processing part of the information at the same time. Delay reduction can be traded off against the cost of more "processors." Generally, this does not result in the types of organization structures we usually observe. Bolton and Dewatripont (1994) have a similar approach but emphasize the tradeoff between specialization and communication. They show that in most cases, the optimal organization structure combines a hierarchy with a "conveyer belt" type of structure. Sah and Stiglitz (1986) also focus on sequential vs. parallel processing of information but investigate the tradeoff between type I and type II errors to determine when sequential processing is better than parallel processing and vice versa.

Garicano (1997) provides an elegant theory of hierarchies, based on expertise, that is similar in some respects to ours. He postulates the presence of experts who can be ranked according to the difficulty of the problems they can solve. Experts in a given cohort can solve all problems that can be solved by those in lower cohorts plus some more difficult problems. Experts who can solve more problems are correspondingly more expensive. More difficult problems occur less frequently than less difficult ones, however. This results in a pyramidical hierarchy with more workers at lower levels and fewer at higher levels. In analyzing hierarchies, we more-or-less assume a pyramidical structure but allow contingent referral of projects. We also consider experts with non-nested expertise allowing for the optimality of matrix forms.

Hart and Moore (1999) provide a model of hierarchies based on authority over the implementation of ideas for using assets. In their model, if individual i has authority over individual j with respect to ideas for a set of assets, then j's idea for those assets will be implemented if and only if i has no idea. The issue is how best to assign identical individuals to sets of assets, i.e., to which assets to assign each individual and in which order (where order indicates authority). Hart and Moore distinguish between coordination activities, in which an individual is assigned to more than one asset, and specialization activities, in which an individual is assigned to only one asset. To implement one's idea for a set of assets, one must have the highest authority of those who have an idea for any asset in the set.

Hart and Moore show that if the probability of having an idea for a set of assets decreases as the size of the set increases and if sets of assets for which synergies exist are disjoint, then the optimal structure will involve a pyramidical hierarchy with individuals whose tasks cover a larger set of assets appearing higher in the chain of command. Although this model shares with ours the idea that coordination of activities is an important determinant of organization design, the approaches are quite different. The Hart-Moore model focuses on authority, whereas we focus on information. Their results explain authority relations (e.g., coordinators should be senior to specialists) but do not explain hierarchical groupings (functional vs. divisional) or matrix forms as we do. Hart and Moore relate the extent of centralization to the size of coordination benefits, whereas we focus on costs of expertise as the main determinants of centralization.

Vayanos (1997) stresses the interaction of information, i.e., the idea that the best project in a subset may depend on the nature of projects outside that subset. This feature is absent from other models in the economics literature, e.g., Radner (1993), Bolton and Dewatripont (1994), and Garicano (1997), but is one we emphasize. The application Vayanos models is portfolio selection. He assumes a hierarchy in which managers at each level examine a set of portfolios suggested by subordinates and an exogenously determined set of assets (except the lowest level managers who examine only assets). Each manager chooses weights for combining these portfolios and assets into a larger portfolio (without changing the weights of the items in the submitted portfolios). Managers ignore assets outside their purview when choosing weights. The main result is that each agent in the hierarchy (except the lowest) has exactly one subordinate and also examines some assets directly.[3]

The organization behavior literature has investigated the empirical relationships between decentralization of decisions and such variables as size (measured by employment) and vertical integration. Blau and Schoenherr (1971), Child (1973), Donaldson and Warner (1974), Hinings and Lee

[3]Calvo and Wellisz (1979) also assume a hierarchical structure. Their main focus is explaining wage differentials across levels of the hierarchy.

(1971), Khandwalla (1974) and Pugh *et al.* (1969) all find a positive relationship between size and extent of decentralization. Kandwalla (1974) also documents a positive relationship between vertical integration and decentralization. Child (1973) finds that the vertical span (number of levels) of hierarchy is positively related to size.

2 Model

We model a firm that, for tractability, is assumed to engage in only four projects, labeled A, B, C, and D over a single period. Various subsets of these four projects may or may not interact. We denote an interaction between two projects by juxtaposing their labels, e.g., AB denotes an interaction between projects A and B. The set of feasible pairwise interactions is denoted by $\Omega = \{AB,CD,AC,BD\}$. Note that we have excluded two interactions, i.e., AD and BC. This greatly simplifies the analysis and reflects the idea that some interactions are *a priori* extremely unlikely. For example, the design of a product intended for sale in Norway and marketing of the U.S. version of the product are not likely to interact directly. Which of the feasible interactions occurs is given by an elementary event $e \subset \Omega$. That is, e is interpreted as the event that exactly those pairs of projects $\omega \in e$ interact and no others. Thus, for example, the event $e = \{AB,CD\}$ indicates that projects A and B interact, projects C and D interact, and no other projects interact. The event $e = \Omega$ indicates that all four possible interactions occurred. We refer to this event as a "company-wide" interaction. The event $e = \varnothing$ indicates that no interaction has occurred. The set of elementary events is denoted by $E = 2^{\Omega}$ (the set of all subsets of Ω).

To simplify the analysis and to capture the notion that some interactions tend to be similar to each other, we divide the set of feasible interactions, Ω, into two groups, $P = \{AB,CD\}$ and $R = \{AC,BD\}$. For example, suppose A is production of Tide, B is marketing of Tide, C is production of Cheer, and D is marketing of Cheer. Then the above grouping reflects the assumption that interactions within a product line are similar to each other. We take similarity to the extreme by assuming the interactions in a given group are identical in terms of probability of occurrence (later we assume that the benefits of coordinating these interactions are also the same). Formally, assume that the probability of either interaction in P is p, and the probability of either interaction in R is r. We also assume that interactions are independent, and these probabilities are observed by everyone in the firm.[4] We assume $p > r$, i.e, interactions between A and B and between C and D are the more likely interactions, while interactions between A and C and B and D are less likely. In terms of the above example, the assumption is that interactions within a product line are more likely than those across product lines. If, on the other hand, the economies of scale from combining production activities and those from combining marketing activities are more likely than interactions across functions within product lines, then we would simply relabel the activities so that A is production of Tide, B is production of Cheer, C is marketing of Tide, and D is marketing of Cheer.

There are three types of potential managers, project managers (one for each project), middle managers, and a CEO. Project managers are required to generate the projects and participate in managing them but cannot coordinate interactions between projects. They do, however, refer projects to middle management or the CEO for investigation and coordination of possible interactions.

If a set of projects does interact, there are benefits to coordinating them. Discovering and reaping these benefits requires investigation and coordination by a middle manager with the correct expertise (or by the CEO). For each interaction $\omega \in \Omega$, a middle manager, denoted m_ω, may be hired who can discover and coordinate this interaction and only this interaction. The set of potentially available middle managers is denoted by $M = \{m_\omega | \omega \in \Omega\}$. We can think of the middle managers in M as product division managers, managers of functional areas (e.g., marketing manager), country managers, etc., depending on the interpretation of the activities A, B, C, and D. Let M_P (M_R) denote the set of middle

[4]As was pointed out in the Introduction, we abstract from all private information and incentive problems. While these may also be important determinants of organization structure [see Qian (1994), Singh (1985)], we limit the scope of the current paper to a consideration of coordination issues.

managers who can discover and coordinate interactions in P (respectively, R), i.e.,

$$M_P = \{m_\omega | \omega \in P\} \text{ and } M_R = \{m_\omega | \omega \in R\}.$$

Incremental benefits from coordinating the pairwise interactions are assumed to be the same for all pairs and are normalized to one. Thus, the probabilities p and r also play the role of expected benefits of the potential interactions.

The CEO, denoted m*, is assumed to be able to discover and exploit any interaction, but only the CEO can exploit the company-wide interaction which is assumed to be present if all four pairwise interactions occur. Incremental benefits from coordinating the company-wide interaction are given by s.[5] Incremental benefits produced by the CEO depend on which benefits, if any, middle managers have already exploited. In particular, m* generates benefits of 1 for each interaction present but not exploited by a middle manager plus s if all four pairwise interactions are present.

This formalism admits many possible interpretations. For example, A and C can be electrical devices in Chevies and Cadillacs, respectively, and B and D can be chassis of Chevies and Cadillacs, respectively. If project A turns out to be headlight improvement and B crash resistance, then A and B are likely to interact in that both improve the safety of Chevies. If they do interact, exploiting this interaction through a coordinated marketing effort emphasizing safety will produce incremental benefits. If, however, B is roominess, then A and B are unlikely to interact. One can interpret m_{AB} (m_{CD}) as the manager of the Chevy (Cadillac) Division and m_{AC} (m_{BD}) as the head of electronics (chassis).

An alternative interpretation is that A and B are innovations in products 1 and 2, respectively, while C and D are new marketing techniques for the two products, respectively. The innovations may have common components calling for coordinated production. The new marketing techniques may call for a common ad campaign for the two products. One can interpret m_{AB} (m_{CD}) as the manager of the

[5]One role of the CEO in an organization may be to resolve conflicts in coordinating between two or more interactions involving the same project, e.g., between AB and AC. One can interpret s in our model as the benefit to such conflict resolution.

Production (Marketing) Division and m_{AC} (m_{BD}) as the manager of the Product 1 (2) Division.

As mentioned in the Introduction, managers have both fixed and variable costs. The fixed cost is a cost associated with employing the manager whether that manager is actually used to coordinate any projects. Empirically, we identify the manager's fixed cost with his salary. A manager's variable cost is an opportunity cost of the manager's time that is related to his value in other activities not modeled here.

Since project managers are assumed to be essential to generating projects and to have no function other than generating and possibly managing projects, the cost (both fixed and variable) of project managers can be ignored. For simplicity, we assume that all middle managers have the same fixed cost, denoted F and that the middle managers have no function other than discovering and coordinating interactions between projects. Therefore, the variable cost of the middle managers is zero. The CEO, however, is assumed to have other duties such as strategic planning. Consequently the CEO's variable cost is positive and is denoted by Q. These other duties of the CEO are sufficiently valuable that the CEO is required. Consequently, her fixed cost can be ignored. We further simplify the problem (and eliminate some uninteresting cases) by assuming that the value added by the CEO in coordinating the company-wide interaction exceeds her variable cost, i.e.,

Assumption 1. $s > Q$.

3 Optimal Use of a Given Set of Middle Managers

We find an optimal organization design in two stages. In this section, for each possible subset of available middle managers, we optimize their use and calculate the expected net benefits associated with this program. The overall organization design problem can then be solved by comparing these expected net benefits. This is done in Section 4.

Given the available middle managers and the project characteristics, p and r, the problem is to decide which managers should be used to check for and coordinate possible interactions and in what order and in which contingencies they should be used. The problem is vastly simplified, however, by the assumption that the middle managers have no variable costs. It follows that it is optimal to use any middle managers that are available.

In the next three subsections, we analyze in turn the cases of no middle managers, two middle managers, and all four middle managers, respectively.

3.1 No Middle Managers: Centralized and Decentralized Flat Structure

When no middle managers are employed (i.e., the structure is flat), the problem is simply to decide whether to refer all four projects to the CEO or to give up any coordination benefits and let the project managers run the projects. We refer to the flat structure when all projects are referred to the CEO as the *centralized flat structure*. The expected benefit of this structure is the expected benefit from coordinating the four pairwise interactions, p+p+r+r, plus the synergy gain, s, from the company-wide interaction times the probability of the company-wide interaction, p^2r^2. The cost of referring projects to the CEO is her opportunity cost, Q. Therefore, expected net profit for the centralized flat structure is $2(p+r)+p^2r^2s-Q$. We refer to the flat structure when no projects are referred to the CEO as the *decentralized flat structure*. In this case, there are no coordination benefits and also no costs, so the expected net profit is zero. Therefore, the net value of the flat structure is

$$V_F = \max\{2(p+r)+p^2r^2s-Q,0\}. \tag{1}$$

3.2 Two Middle Managers: Hierarchies

In this subsection we consider the problem assuming that only two middle managers are available. Since interactions are identical within a group, either P or R, there are only two relevant cases: only managers in M_R are available or only those in M_P are available.[6] We consider, for each case, the

[6]There is one other possibility, namely employing one manager from each of M_P and M_R. This structure does not, however, resemble a hierarchy (since one project will be referred to two middle managers) or any other commonly observed organizational forms. Consequently, we rule out this

optimal use of the given two managers.

When only managers from M_R or only those from M_P are available, the structure resembles a hierarchy in which each project is referred (at most) to one and only one manager at the next level. For example, if only managers from M_P are available, projects A and B are referred to m_{AB} and projects C and D are referred to m_{CD}. Consequently, we refer to these two situations as the R-hierarchy and the P-hierarchy, respectively.

First suppose only managers in M_R are present. To calculate the value of an optimal strategy in this case, we use backward induction. Suppose projects have been referred to the two managers in M_R. At this point one may either stop or refer all projects to the CEO. In the former case, the middle managers coordinate their interactions, and the CEO is not involved. Consequently, we refer to this case as the *decentralized R-hierarchy*. In the latter case, the middle managers coordinate their interactions, while the CEO coordinates the other pairwise interactions and the company-wide interaction. Accordingly, we refer to this case as the *centralized R-hierarchy*. Stopping results in a net additional expected benefit of zero. Referring projects to the CEO results in a net additional expected benefit that depends on whether the two managers in M_R both found interactions. If both interactions are found (this happens with probability r^2), the additional expected benefit of referring all projects to the CEO consists of the expected coordination benefits for the two projects in P, 2p, and the expected company-wide synergy gain, p^2s. The cost of referring to the CEO is Q, so the expected net benefit is $2p+p^2s-Q$. If at least one of the interactions from the R group failed to occur (this happens with probability $1-r^2$), the additional net benefit of referring all projects to the CEO is only $2p-Q$, since the company-wide interaction is not present in this case. Thus the expected value of an optimal continuation strategy, given that both managers from M_R have been consulted, is $r^2\max\{2p+p^2s-Q,0\}+(1-r^2)\max\{2p-Q,0\}$. The expected benefit from referring projects to the two middle managers is simply 2r. Therefore, the optimal

possibility.

value of the R-hierarchy (net of fixed costs) is

$$V_R = 2r + r^2\max\{2p+p^2s-Q,0\} + (1-r^2)\max\{2p-Q,0\} - 2F. \tag{2}$$

By symmetry we have the following analogous value for a P-hierarchy:[7]

$$V_P = 2p + p^2\max\{2r+r^2s-Q,0\} + (1-p^2)\max\{2r-Q,0\} - 2F. \tag{3}$$

3.3 Four Middle Managers: Matrix Structure

As mentioned above, given that all four middle managers are present, it is optimal to have them investigate the four possible interactions first, before referring any decisions to the CEO. This strategy allows the firm to reap any benefits from interactions that are present and involve the CEO only if it is known that a company-wide interaction requiring her special expertise exists. The strategy corresponds to the matrix organization described in the Introduction. That is, each project manager refers his project to two upper level managers: project A is referred both to m_{AB} and m_{AC}, project B is referred both to m_{AB} and m_{BD}, etc.

Using Assumption 1, the value of the matrix organization net of fixed costs can easily be computed as

$$V_M = 2(p+r) + p^2r^2(s-Q) - 4F. \tag{4}$$

The intuition for this expression is as follows. Given that all four middle managers will be used and that, if the company-wide interaction occurs, projects are referred to the CEO, the expected benefit is the expected benefit from each single interaction, p+p+r+r, plus the expected value added of the CEO net of her variable cost, $p^2r^2(s-Q)$. The expected cost is the fixed cost of the four middle managers, 4F. The difference between the expected benefit and the expected cost gives the value of the matrix form in (4).

[7]Note that, for the R-hierarchy, if Q < 2p, then all projects will eventually be referred to the CEO no matter what is discovered by the middle managers. A similar statement holds for the P-hierarchy when Q < 2r. For either hierarchy, this is equivalent simply to referring all projects directly to the CEO, skipping the middle managers. In this case, hierarchies would clearly be suboptimal, since a hierarchy would provide the same benefit as the centralized flat structure but would cost 2F more. Consequently, in an optimal design, if a centralized hierarchy is used, it will be one in which the CEO is involved only if both middle managers find interactions.

We summarize the results of this section in **Table 1** which shows, for various values of the opportunity cost Q of the CEO, the optimal value of each organization design. In constructing the table, we have made the following simplifying assumption.

Assumption 2. $r^2(1-p^2)s > 2p$.

This assumption allows us to define six ranges for Q such that in each range, the max operator that appears in the formulas for the values of each design can be resolved. Assumption 2 is consistent with the spirit of Assumption 1, namely that the value added of the CEO in coordinating activities is large.

Table 1: Value of Each Design as a Function of Q

<table>
<tr><th>For Q ∈</th><th>V_F</th><th>V_R</th><th>V_P</th><th>V_M</th></tr>
<tr><td>[0,2r)</td><td rowspan="3">G−Q</td><td rowspan="2">G−Q−2F</td><td>G−Q−2F</td><td rowspan="6">D(Q)−4F</td></tr>
<tr><td>[2r,2p)</td><td rowspan="3">$-2F+K_{pr}(Q)/2$</td></tr>
<tr><td>[2p,G)</td><td rowspan="3">$-2F+K_{rp}(Q)/2$</td></tr>
<tr><td>$[G,2r+r^2s)$</td><td rowspan="3">0</td></tr>
<tr><td>$[2r+r^2s,2p+p^2s)$</td><td rowspan="2">2p−2F</td></tr>
<tr><td>$[2p+p^2s,\infty)$</td><td>2r−2F</td></tr>
</table>

In the above table, the symbols D, G, and K are defined by

$$\begin{aligned} D(Q) &= 2(p+r)+p^2r^2(s-Q), \\ G &= 2(p+r)+p^2r^2s, \\ K_{xy}(Q) &= 4x+2x^2(2y+y^2s-Q), \ for \ (x,y) \in \{(p,r),(r,p)\}. \end{aligned}$$

4 Optimal Organization Design

The optimal organization design is found by comparing the values of the various designs. This comparison is most easily performed by row in **Table 1**.

For very low opportunity cost of the CEO, Q < 2r, even if there is no company-wide interaction, it is still optimal to use the CEO to discover and coordinate pairwise interactions. This fact has two important implications. First, it implies that, when using the flat structure, it is optimal to refer all projects to the CEO, i.e, use the centralized flat structure. Second, it also implies that, for either hierarchy, it is optimal to refer all projects to the CEO, *regardless of what is found by the two middle managers*. In this case, however, the cost, 2F, of the two middle managers is wasted, because the same result can be obtained from the flat structure with no middle managers by referring all projects directly to the CEO. Therefore, for $Q < 2r$, the optimal design is either the centralized flat structure or the matrix organization. In this range of Q, both designs exploit all interactions that are present. The advantage of the matrix is that the opportunity cost of the CEO is incurred only when it is known for sure that the company-wide interaction is present. That is, the matrix saves $A(Q) \equiv Q(1-p^2r^2)$ on average relative to the flat structure. The disadvantage is, of course, the fixed cost of the four middle managers, 4F. Thus when this fixed cost is large relative to the CEO's opportunity cost, the flat structure is preferred, i.e., the optimal design in this range of Q is the centralized flat structure when $4F > A(Q)$ and the matrix organization otherwise.

For $Q \in [2r,2p)$, the same argument as in the previous paragraph applies to the R-hierarchy, i.e., in the R-hierarchy, it is optimal to refer all projects to the CEO, regardless of what is found by the two middle managers. This is not true for the P-hierarchy, because for that hierarchy the expected payoff of referring to the CEO in the absence of a company-wide interaction is only $2r < Q$. Thus in this range of Q, the optimal design is either the centralized flat structure, the P-hierarchy, or the matrix organization. As before, the centralized flat structure dominates the matrix when $4F > A(Q)$. The advantages of the flat structure relative to the P-hierarchy are that it saves 2F in fixed costs of middle managers and it exploits all interactions that are present. The P-hierarchy refers projects to the CEO (it is centralized) but only when both of the high probability interactions are present. As a result, the centralized P-hierarchy saves $Q(1-p^2)$ on average in CEO costs but forgoes benefits of $2r(1-p^2)$ on average relative to the flat structure. Thus, the flat structure is better than the centralized P-hierarchy when $2F > (1-p^2)(Q-2r)$ or $4F > B_{pr}(Q) \equiv 2(1-p^2)(Q-2r)$. Consequently, for $Q \in [2r,2p)$, the flat structure is optimal when $4F > A(Q)$ and $4F > B_{pr}(Q)$.

Obviously, the matrix organization is better than the flat structure in this range of Q when $4F < A(Q)$. Relative to the centralized P-hierarchy, the matrix saves the opportunity cost of the CEO when the two high-probability interactions are present, but the company-wide interaction is not, i.e., the matrix saves $Q(p^2-p^2r^2)$. The matrix also always exploits the low-probability interactions when present, while the centralized P-hierarchy exploits them only when the high-probability interactions are present. Thus the matrix produces additional expected benefits of $2r(1-p^2)$ relative to the centralized P-hierarchy. On the other hand, the matrix costs 2F more in middle manager fixed costs. Thus the matrix is better than the centralized P-hierarchy when $2F < Qp^2(1-r^2)+2r(1-p^2)$ or $4F < C_{pr}(Q) \equiv 2Qp^2(1-r^2)+4r(1-p^2)$. Consequently, for $Q \in [2r,2p)$, the matrix organization is optimal when $4F < A(Q)$ and $4F < C_{pr}(Q)$.

When neither of the conditions in the previous two paragraphs is satisfied, i.e., when $A(Q) < 4F < C_{pr}(Q)$, the centralized P-hierarchy is optimal.

For $Q \in [2p,G)$, the comparisons between the flat structure, the matrix, and the P-hierarchy are the same as for $Q \in [2r,2p)$. In this range of Q, however, the R-hierarchy becomes a viable alternative. The comparisons between the R-hierarchy and the flat and matrix structures are, however, the same as those between the P-hierarchy and the other two organizations except that the roles of p and r are reversed. Thus, the centralized flat structure is optimal when $4F > \max\{A(Q),B_{pr}(Q),B_{rp}(Q)\}$, the matrix organization is optimal when $4F < \min\{A(Q),C_{pr}(Q),C_{rp}(Q)\}$, and otherwise one of the hierarchies is optimal. To compare the two hierarchies, first, recall that the difference between them is which two middle managers are available for discovering and coordinating interactions. In the R-hierarchy, the managers who can analyze the low-probability interactions are available. In the P-hierarchy, it is the managers who can analyze the high-probability interactions that are present. The advantage of the R-hierarchy is that the firm need pay the opportunity cost, Q, of the CEO only if both the low-probability interactions are discovered rather than when both high-probability interactions are discovered. Thus the R-hierarchy saves $Q(p^2-r^2)$ on average relative to the P-hierarchy. The advantage of the P-hierarchy is that, by starting with the high-probability interactions, one obtains a larger expected benefit from the two middle managers (2p vs. 2r) and has a higher probability of obtaining the benefits of the other interactions than if one had started with the low-probability interactions ($p^2(2r+r^2s)$ vs. $r^2(2p+p^2s)$). Thus the net advantage of the P-hierarchy over the R-hierarchy for $Q \in [2p,G)$ is $2p+p^2(2r+r^2s)-[2r+r^2(2p+p^2s)]-Q(p^2-r^2) = (p-r)[2(1+pr)-Q(p+r)]$. This implies that the centralized P-hierarchy is better than the centralized R-hierarchy for $Q \in [2p,G)$ when $Q < J \equiv 2(1+pr)/(p+r)$.

For $Q \in [G,2r+r^2s)$, it is no longer optimal in the flat structure to refer projects to the CEO, so the flat structure is decentralized and its value is zero. Now, the decentralized flat structure is optimal whenever the other designs generate net losses, i.e., for the matrix, when $4F > 2(p+r)+p^2r^2(s-Q) \equiv D(Q)$, for the (centralized) P-hierarchy when $4F > K_{pr}(Q)$, and for the (centralized) R-hierarchy when $4F > K_{rp}(Q)$ [see **Table 1**]. The comparison between the matrix organization and the two (centralized) hierarchies, as well as that between the two hierarchies themselves, is the same as for $Q \in [2p,G)$.

For $Q \in [2r+r^2s,2p+p^2s)$, the comparisons, except for the P-hierarchy, are the same as in the previous paragraph. In particular, the opportunity cost of the CEO is now sufficiently high that it is optimal not to refer projects to the CEO in the P-hierarchy even if both middle managers find interactions, i.e., the P-hierarchy is decentralized. Now the flat structure is better than the decentralized P-hierarchy when $2F > 2p$ or $4F > 4p$, and the matrix is better than the decentralized P-hierarchy when $2F > D(Q)-2p$ or $4F > 2D(Q)-4p \equiv E(Q)$. Finally, the decentralized P hierarchy is better than the (centralized) R-hierarchy if $2p > K_{rp}(Q)/2$ or $Q > T$, where $T \equiv 2p+p^2s-2(p-r)/r^2$ is the Q at which $K_{rp}(Q) = 4p$.

For $Q \geq 2p+p^2s$, the opportunity cost of the CEO is sufficiently high that it is optimal not to refer projects to the CEO in either hierarchy even if both middle managers find interactions, i.e., both hierarchies are decentralized. In this range of Q, the decentralized P-hierarchy is clearly better than the decentralized R-hierarchy, since it produces greater expected benefits at the same cost. The comparison among the remaining three organizations is similar to that of the previous paragraph, i.e., the decentralized flat structure is optimal when $4F > \max\{D(Q),4p\}$, the matrix is optimal when $4F < \min\{D(Q),E(Q)\}$, and otherwise the decentralized P-hierarchy is optimal.

The above discussion is summarized in the following proposition.

Proposition 1. The optimal organization design as a function of the opportunity cost of the CEO, Q, and the fixed cost of middle managers, F, is given by **Table 2**:

Table 2: Optimal Design as a Function of Q and F

For $Q \in$	Optimal Design
[0,2r)	Centralized Flat if $4F > A(Q)$ Matrix otherwise
[2r,2p)	Centralized Flat if $4F > \max\{A(Q),B_{pr}(Q)\}$ Matrix if $4F < \min\{A(Q),C_{pr}(Q)\}$ Centralized P-hierarchy otherwise
[2p,G)	Centralized Flat if $4F > \max\{A(Q),B_{pr}(Q),B_{rp}(Q)\}$ Matrix if $4F < \min\{A(Q),C_{pr}(Q),C_{rp}(Q)\}$ Centralized P-hierarchy if $C_{pr}(Q) < 4F < B_{pr}(Q)$ & $Q < J$ Centralized R-hierarchy otherwise
[G,2r+r²s)	Decentralized Flat if $4F > \max\{D(Q),K_{pr}(Q),K_{rp}(Q)\}$ Matrix if $4F < \min\{D(Q),C_{pr}(Q),C_{rp}(Q)\}$ Centralized P-hierarchy if $C_{pr}(Q) < 4F < K_{pr}(Q)$ & $Q < J$ Centralized R-hierarchy otherwise
[2r+r²s,2p+p²s)	Decentralized Flat if $4F > \max\{D(Q),4p,K_{rp}(Q)\}$ Matrix if $4F < \min\{D(Q),E(Q),C_{rp}(Q)\}$ Decentralized P-hierarchy if $E(Q) < 4F < 4p$ & $Q > T$ Centralized R-hierarchy otherwise
[2p+p²s,∞)	Decentralized Flat if $4F > \max\{D(Q),4p\}$ Matrix if $4F < \min\{D(Q),E(Q)\}$ Decentralized P-hierarchy otherwise

In this table,

$A(Q) = Q(1-p^2r^2) = D(Q)-G+Q$
$B_{xy}(Q) = 2(1-x^2)(Q-2y) = K_{xy}(Q)-2(G-Q)$
$C_{xy}(Q) = 4y(1-x^2)+2x^2(1-y^2)Q = 2D(Q)-K_{xy}(Q)$
$D(Q) = 2(p+r)+p^2r^2(s-Q)$
$E(Q) = 2[2r+p^2r^2(s-Q)] = 2D(Q)-4p$
$G = 2(p+r)+p^2r^2s$
$J = 2(1+pr)/(p+r)$
$T = 2p+p^2s-2(p-r)/r^2$
$K_{xy}(Q) = 4x+2x^2(2y+y^2s-Q)$

5 Comparative Statics

In this section, we analyze how the optimal design varies with the opportunity cost of the CEO, Q, and the fixed cost of the middle managers, F. Although **Table 2** defines the optimal design for each pair (Q,F), it is difficult to see from the table the effect of changing Q or F on the optimal design.

Consequently, here we first present the results in **Table 2** in the form of graphs in which, for each pair (Q,F), we show which organization design is optimal.

Notice from **Table 2** that the optimal design involves the relationship between 4F and the maximum or minimum among several linear functions of Q. Which of these linear functions is maximal or minimal turns out to depend on the relationships among several parameters that themselves depend on p, r, and s. Some of these, namely G, J, and T, have already been introduced. In particular, G is the expected gain to coordinating all interactions that are present. The parameter J is the value of Q at which the centralized P- and R-hierarchies are equivalent when Q is between G and $2r+r^2s$. The parameter T is the value of Q at which the function $K_{rp}(Q) = 4p$. In addition to these three parameters, we will also make use of the following:

$Q_B(x,y)$ = the Q for which $B_{xy}(Q) = A(Q)$ for $(x,y) \in \{(p,r),(r,p)\}$;

$Q_K(x,y)$ = the Q for which $K_{xy}(Q) = D(Q)$ for $(x,y) \in \{(p,r),(r,p)\}$;

Q_{4p} = the Q for which $D(Q) = 4p$.

Formulas for G, J, and T are given above; formulas for the other parameters are given in the appendix.

In the appendix, we show that there are six possible graphs. The six graphs are similar from the point of view of comparative statics results, the main differences being that for some configurations, one or more organization designs are suboptimal for all combinations of Q and F. In particular, of the six possible structures listed in **Table 2**, both flat structures, the matrix organization, and the decentralized P-hierarchy appear in all graphs. In some configurations, however, one or both of the centralized P- and R-hierarchies are suboptimal for all combinations of Q and F. In the text, we present and discuss in detail only one figure in which all six designs appear, i.e., each of the six is optimal for some region of the Q-F parameter space.[8] The other cases are included in the appendix.

[8]This graph corresponds to Case 3 in the appendix.

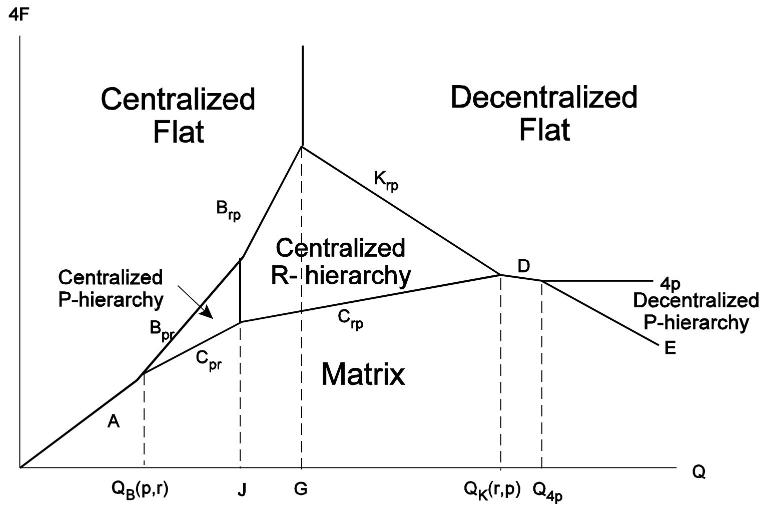

Figure 5: Optimal Organization Design for G > J ≥ $Q_B(r,p)$

Generally speaking, Proposition 1 implies that when the middle managers' fixed cost is high, the flat structure is optimal. It is not surprising that, when middle managers are expensive, it is optimal to do without them. One employs the flat structure with high CEO involvement (centralized) when the opportunity cost of the CEO's time is low and the flat structure with low CEO involvement (decentralized) when her opportunity cost is high. When the middle managers' fixed cost is low, the matrix organization is optimal (except for very low CEO cost). If the middle managers are sufficiently cheap, it is optimal to hire all four (even if the CEO is fairly expensive). This guarantees all pairwise interactions are exploited and that one never uses the CEO to coordinate projects unless the company-wide interaction is present. For intermediate fixed costs of the middle managers, one of the hierarchies is optimal (which one depends on the opportunity cost of the CEO).

In what follows, we discuss the effect of increasing the fixed costs of the middle managers on the optimal design for various values of the CEO's opportunity cost using **Figure 5** (results are qualitatively similar for the other figures).

- For $Q < Q_B(p,r)$, except for very low values of F, the firm will exhibit the centralized flat structure in which there are no middle managers and all projects are referred to the CEO. For very low F, however, one obtains the same result (i.e., all interactions are exploited) more cheaply by hiring all four middle managers (i.e., the matrix organization) and incurring the cost of the CEO only when the company-wide interaction is present. Obviously, the range of 4F for which this strategy is optimal, $[0,A(Q))$, increases with Q. One might wonder why, as F increases, it is not optimal to move from the matrix organization to one of the hierarchies. The reason is that, for such low values of Q, it is never optimal to forego coordination benefits (as sometimes happens when using a hierarchy) to save on CEO costs.
- For $Q_B(p,r) \leq Q < J$, as F increases, the optimal design changes from the matrix organization to the centralized P-hierarchy to the centralized flat structure. The intuition is similar to the previous situation, except that now Q is sufficiently large that it is optimal, for an intermediate range of fixed costs, to hire only two middle managers and forego some coordination benefits if at least one of the two middle managers fails to find an interaction. The advantage of the P-hierarchy is that, by starting with the high-probability interactions, one obtains a larger expected benefit from the two middle managers (2p vs. 2r) and has a higher probability of obtaining the benefits of the other interactions than if one had started with the low-probability interactions. The disadvantage of the P-hierarchy is that there is a greater chance of wasting the CEO's time because the low probability interactions are less likely to be present. Since the CEO's opportunity cost is still relatively low, the advantages of the P-hierarchy over the R-hierarchy outweigh the disadvantage.
- For $J \leq Q < G$, the situation is similar to that of the previous paragraph, except that now, for

intermediate values of F, the centralized R-hierarchy is optimal (instead of the centralized P-hierarchy). The intuition is the same as above, except that, since the opportunity cost of the CEO is larger, the disadvantage of the P-hierarchy, relative to the R-hierarchy, outweighs its advantages.

- For $G \leq Q < Q_K(r,p)$, the situation is similar to that of the previous paragraph, except that now, for large values of F, the decentralized flat structure is optimal (instead of the centralized flat structure). The intuition is simply that the CEO's opportunity cost is sufficiently high that it is no longer worthwhile to refer all projects directly to the CEO.
- For $Q_K(r,p) \leq Q < Q_{4p}$, as F increases, the optimal design changes directly from the matrix organization to the decentralized flat structure. For this range of Q, the CEO is so expensive that, even if both high-probability interactions are known to be present, it is not worth taking a chance on incurring the cost Q to obtain the remaining coordination benefits. Thus if a hierarchy were used, it would be used in its decentralized form. Clearly, the decentralized P-hierarchy dominates the decentralized R-hierarchy (both obtain the benefits of only two pairwise interactions, but the P-hierarchy obtains these benefits with higher probability). The P-hierarchy is better than the decentralized flat structure only if $p > F$. For F below p, however, the matrix organization is better than the P-hierarchy for Q in this range ($Q < Q_{4p}$), i.e., the expected net benefit of the two additional pairwise interactions, 2r, and the company-wide interaction, $p^2r^2(s-Q)$, exceeds the cost of the two additional middle managers, 2F, required to obtain them.
- For $Q \geq Q_{4p}$, the same argument as in the previous paragraph that the decentralized P-hierarchy is the best hierarchy applies. In this case, however, Q is sufficiently large ($Q \geq Q_{4p}$) and the expected net benefits of exploiting the company-wide interaction, $p^2r^2(s-Q)$, sufficiently small, that there is a range of $F < p$ such that it is not worth paying the two additional managers. Thus for $Q \geq Q_{4p}$, there is a range of F such that the optimal design is the decentralized P-hierarchy.

Note that, as Q increases, holding F constant, the optimal design moves from the centralized flat structure, in which projects are always referred to the CEO, toward either the decentralized flat structure or the decentralized P-hierarchy. In either of the latter two cases, projects are never referred to the CEO. For in-between values of Q, the optimal design may move from the centralized P-hierarchy to the centralized R-hierarchy to the matrix organization. The probability with which projects are referred to the CEO in these designs changes from p^2 to r^2 to p^2r^2 . Although one or more of these intermediate structures may be missing from the progression (see the appendix), in all cases the probability of CEO involvement decreases as the optimal design changes with increases in CEO opportunity cost. We summarize this result in the following proposition.

Proposition 2. As Q increases, *ceteris paribus*, the probability that projects are referred to the CEO decreases.

This completes our comparative statics on Q and F. Comparative statics results on p and r are difficult to prove in general, because all the boundaries of the various regions in **Figure 5** (and the figures in the appendix as well) shift in complicated ways with p and r. One result we can obtain, however, concerns the special case in which r is very small. It is easy to see from equations (1)-(4) that the R-hierarchy, the centralized P-hierarchy and the matrix organization are strictly suboptimal when $r = 0$. By continuity, this statement also holds for $r > 0$ but sufficiently small (as long as $F > 0$). The result is quite intuitive: when the probability of low-probability interactions is sufficiently small, it makes no sense to pay the fixed costs of middle managers who are experts in detecting and coordinating these interactions or to waste the time of the CEO in such activities. We summarize this result in the following proposition.

Proposition 3. For r sufficiently small, the R-hierarchy, the centralized P-hierarchy and the matrix organization are strictly suboptimal.

Finally, we consider comparative statics results for s. The following result is proved in the appendix.

Proposition 4. Increases in the synergy gain, s, from exploiting the company-wide interaction may cause the optimal design to change from a decentralized to a centralized structure but not the reverse. In particular, if the organization design was originally any of the centralized structures (centralized flat, centralized P- or R-hierarchy, or matrix), the design will not change. If the *new* design is decentralized (either decentralized flat or decentralized P-hierarchy), the original design was identical. Moreover, some firms that were originally decentralized will move to a centralized structure when s increases.

We now turn to the empirical implications of the results.

6 Empirical Implications

First, consider the optimal organization design of conglomerates. For such highly diversified firms, it seems reasonable to suppose that the most likely interactions are those across functions within a given product, and interactions across products are extremely unlikely. In terms of our model, conglomerates are firms in which the interactions with probability p are those within products, and r is very small. Consequently, Proposition 3 predicts that highly diversified conglomerates will not exhibit the matrix form, and those organized as hierarchies will be organized as *divisional* hierarchies, i.e., along product lines, as opposed to functional hierarchies.

Second, consider the result of changes in the opportunity cost of the CEO's time in coordinating projects. If we identify the CEO's variable cost with the size or complexity of the firm, the model makes a prediction regarding the "life cycle" of the firm's organization structure.[9] In particular, it suggests that young firms will have a centralized flat structure in which the CEO is highly involved in coordinating activities. Moreover, Proposition 2 implies that, as the firm matures, the frequency with which projects are referred to the CEO will decrease. This result is consistent with the findings of the organization

[9]In discussing the life cycle implications of the model, we are holding constant the fixed cost of middle managers as well as other parameters.

behavior literature cited in the Introduction which documents a positive relationship between size and extent of decentralization.

Firms for which the cost of middle managers is high will become highly decentralized with no middle managers and little involvement of the CEO as they mature. Firms for whom middle managers are inexpensive, will first switch from the centralized flat structure to the matrix organization then to a decentralized hierarchy oriented toward exploiting the most likely interactions. Finally, firms for whom middle managers are neither very cheap nor very expensive will switch from the centralized flat structure to a centralized hierarchy. This centralized hierarchy may be designed to exploit either the more likely or the less likely interactions or may switch from the more likely to the less likely as the CEO's opportunity cost increases. As these firms become even larger and/or more complex, the structure will shift in one of three directions. For firms in this group with the most expensive middle managers, the structure will become flat with no CEO involvement. For firms in this group with the least expensive middle managers, the structure will shift to the matrix organization, then to a decentralized hierarchy oriented toward exploiting the most likely interactions. For firms in this group with in-between middle manager costs, the matrix phase will disappear. Such firms will move directly to a decentralized hierarchy that exploits the most likely interactions.[10]

Further implications are available if we identify more specifically which interactions are most likely and which are least likely. Suppose interactions between functions relating to a given product are more likely than economies of scale from combining a function across products. In this case, if, as the size/complexity of the firm increases, the firm's organization structure changes from one type of centralized hierarchy to the other, the progression will be from a divisional (P-)hierarchy to a functional (R-)hierarchy. Moreover, if the firm exhibits a decentralized hierarchy, it will always be a decentralized

[10]The statements regarding firms with intermediate middle manager costs are true in all cases except one. In that case, the firm never exhibits a centralized hierarchy (see Case 1 in the appendix).

divisional hierarchy.

Next we examine the effects of changes in the incremental benefit of coordinating company-wide interactions, s. Recall from Proposition 4 that increases in s shrink the set of combinations of Q and F that result in decentralized structures. Possible empirical proxies for s include tightness of resource constraints, the extent to which incentive schemes focus on unit performance, the extent of regulation, and the stability of the environment. When units must compete for scarce corporate resources, the gains to company-wide coordination of the allocation are likely to be large. Similarly, when compensation schemes do not give unit managers an incentive to take account of the effects of their choices on the company as a whole, there should be greater benefits to coordination by the CEO. On the other hand, severe regulation may allow little scope for the CEO to improve performance through coordination of activities. Likewise, stable environments do not require frequent intervention by the CEO to reap coordination benefits. Thus, firms with weak resource constraints, compensation schemes that reward company-wide performance, strong regulatory constraints, and/or stable environments are more likely to have highly decentralized organization structure.[11]

Finally, consider the impact of changes in the fixed cost (salaries) of the middle managers, i.e., their productivity in the next best alternative employment, holding the variable cost of the CEO, Q, fixed. From the discussion in Section 5, as salaries increase, perhaps because of increased demand for middle managers, one expects firms to move toward flatter structures. This might involve changing from a matrix form to a hierarchy or to a flat structure. For firms whose CEOs have relatively low opportunity cost of coordinating projects, as middle management salaries increase, the organization design will change from the matrix form to a centralized flat structure, possibly passing through a centralized hierarchical structure. For firms whose CEOs have relatively high opportunity cost of coordinating

[11]Obviously we are assuming that firms with different values of s share the same distribution of Q and F.

projects, as middle management salaries increase, the organization design will change from the matrix form to a decentralized flat structure, possibly passing through a hierarchical structure. Note, however, in testing such implications, it is important to control for changes in the other parameters. In particular, it is likely that when salaries increase so do the benefits provided by middle managers, presenting a difficult identification problem.

7 Conclusions

This paper attempts to explain organization structure based on optimal coordination of interactions among activities. The main idea is that each middle manager is capable of detecting and coordinating interactions only within his limited area of expertise. Only the CEO can coordinate company-wide interactions. The optimal design of the organization trades off the costs and benefits of various configurations of managers.

The model provides a number of empirical predictions regarding firms' organization design. In obtaining these results, we made a number of simplifying assumptions. Perhaps the most important of these is that middle managers have no opportunity cost of coordinating interactions. This assumption allows us to ignore a large number of solutions that would be optimal for various levels of this opportunity cost. Since these solutions are rarely observed in practice, we believe that ignoring the opportunity cost of middle managers is justified.

A more important abstraction embedded in the model is the absence of incentive problems. These introduce a large set of considerations revolving around providing incentives to transfer information truthfully across managers within the organization structure. In particular, centralization of decisions will, no doubt, be more costly in such situations. This will bias the organization design toward flatter structures.

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Appendix

Recall

$$A(Q) = Q(1-p^2r^2) = D-G+Q,$$

$$B_{xy}(Q) = 2(1-x^2)(Q-2y) = K_{xy}(Q)-2(G-Q),$$

$$C_{xy}(Q) = 4y(1-x^2)+2x^2(1-y^2)Q = 2D(Q)-K_{xy}(Q),$$

$$D(Q) = 2(p+r)+p^2r^2(s-Q),$$

$$E(Q) = 2[2r+p^2r^2(s-Q)] = 2D(Q)-4p,$$

$$G = 2(p+r)+p^2r^2s,$$

$$J = \frac{2(1+pr)}{p+r},$$

$$T = 2p+p^2s-\frac{2(p-r)}{r^2},$$

$$K_{xy}(Q) = 4x+2x^2(2y+y^2s-Q).$$

The following equalities are easy to check:

$$B_{xy}-A = A-C_{xy}, \tag{5}$$

$$B_{rp}-B_{pr} = C_{pr}-C_{rp} = K_{rp}-K_{pr} = 2(p^2-r^2)(Q-J), \tag{6}$$

$$A-D = Q-G, \tag{7}$$

$$B_{xy}-K_{xy} = 2(A-D) = 2(Q-G), \tag{8}$$

$$C_{xy}-D = D-K_{xy}, \tag{9}$$

$$D-4p = E-D, \tag{10}$$

$$K_{xy}-4p = E-C_{xy}, \tag{11}$$

Note that $B_{xy}(Q)$ is upward sloping in Q and negative at Q = 0, whereas A(Q) is upward sloping in Q and A(Q) = 0 at Q = 0. Therefore $B_{xy}(Q)$ crosses A(Q) at most once. Let $Q_B(x,y)$ be such that $B_{xy}(Q) = A(Q)$ (if it exists), i.e.,

$$Q_B(x,y) = \frac{4(1-x^2)y}{2(1-x^2)+p^2r^2-1}. \tag{12}$$

For our purposes, $Q_B(x,y) < 0$ is the same as $Q_B(x,y)$ failing to exist. Henceforth, we will use the phrase $Q_B(x,y) < 0$ to mean either situation. It is clear from (12) that $Q_B(x,y) < 0$ if and only if $2(1-x^2) \leq 1-p^2r^2$. It is also easy to check that, since $p > r$, $Q_B(r,p) > 0$. Also, if $Q_B(x,y) > 0$, $J \geq Q_B(x,y)$ if and only if

$$2(1-p^2)(1-r^2) \geq (1+pr)^2(1-pr). \tag{13}$$

Note also that $K_{xy}(Q)$ and $D(Q)$ are downward sloping in Q and $K_{xy}(Q)$ is steeper than $D(Q)$. Moreover, $K_{pr}(Q)$ is steeper than $K_{rp}(Q)$ and $D(Q) = G$ at $Q = 0$. Consequently, $K_{xy}(Q)$ intersects $D(Q)$ exactly once at

$$Q_K(x,y) = \frac{2(x-y)+4x^2y+p^2r^2s}{x^2(2-y^2)}. \tag{14}$$

$Q_K(p,r) > 0$, but $Q_K(r,p)$ may be positive, negative or zero. Also, $D(Q)$ intersects $4p$ exactly once at

$$Q_{4p} = s-\frac{2(p-r)}{p^2r^2}. \tag{15}$$

Again, Q_{4p} can be positive, negative or zero depending on whether $G > 4p$, $G < 4p$, or $G = 4p$.

Let

$$H_1(Q) = \max\,\{A(Q),B_{pr}(Q),B_{rp}(Q)\};$$

$$L_1(Q) = \min\,\{A(Q),C_{pr}(Q),C_{rp}(Q)\};$$

$$H_2(Q) = \max\,\{D(Q),K_{pr}(Q),K_{rp}(Q),4p\};$$

$$L_2(Q) = \min\,\{D(Q),C_{pr}(Q),C_{rp}(Q),E(Q)\}.$$

For $Q < 2r$, $B_{pr}(Q) < 0 < A(Q)$, and, for $Q < 2p$, $B_{rp}(Q) < 0 < A(Q)$. Therefore,

$$H_1(Q) = \begin{cases} A(Q), \; for \;\; Q < 2r, \\ \max\,\{A(Q),B_{pr}(Q)\}, \; for \;\; 2r \leq Q < 2p. \end{cases}$$

Similarly, for $Q < 2r+r^2s$, $K_{pr}(Q) > 4p$, for $Q \geq 2r+r^2s$, $K_{pr}(Q) \leq 4p$, and for $Q \geq 2p+p^2s$, $K_{rp}(Q) \leq 4r < 4p$. Therefore,

$$H_2(Q) = \begin{cases} \max\{D(Q), K_{pr}(Q), K_{rp}(Q)\}, \text{ for } Q < 2r+r^2s, \\ \max\{D(Q), 4p, K_{rp}(Q)\}, \text{ for } 2r+r^2s \leq Q < 2p+p^2s, \\ \max\{D(Q), 4p\}, \text{ for } 2p+p^2s \leq Q. \end{cases}$$

Consequently, **Table 2** can be rewritten as

Table 3: Optimal Design as a Function of Q and F (revised)

For Q ∈	Optimal Design
[0,G)	Centralized Flat if $4F > H_1(Q)$ Matrix if $4F < L_1(Q)$ Centralized P-hierarchy if $L_1(Q) < 4F < H_1(Q)$ & $Q < J$ Centralized R-hierarchy otherwise
[G,∞)	Decentralized Flat if $4F > H_2(Q)$ Matrix if $4F < L_2(Q)$ Centralized P-hierarchy if $L_2(Q) < 4F < H_2(Q)$ & $Q < \min\{J, 2r+r^2s\}$ Decentralized P-hierarchy if $L_2(Q) < 4F < H_2(Q)$ & $Q > \max\{T, 2r+r^2s\}$ Centralized R-hierarchy otherwise

Now, it is easy to check that

$$L_1(Q) = 2A(Q) - H_1(Q), \tag{16}$$

and

$$L_2(Q) = 2D(Q) - H_2(Q). \tag{17}$$

Note that $L_1(Q) \leq H_1(Q)$, since, if $H_1(Q) = A(Q)$, $L_1(Q) = A(Q)$, and if $H_1(Q) > A(Q)$, then $L_1(Q) < A(Q) < H_1(Q)$. Similarly $L_2(Q) \leq H_2(Q)$.

Lemma 1. The behavior of the functions $H_1(Q)$ and $L_1(Q)$ is as follows:

- For $Q_B(r,p) \geq J$,

$$H_1(Q) = \begin{cases} A(Q), \ for \ Q < Q_B(r,p), \\ B_{rp}(Q), \ for \ Q_B(r,p) \le Q, \end{cases}$$

$$L_1(Q) = \begin{cases} A(Q), \ for \ Q < Q_B(r,p), \\ C_{rp}(Q), \ for \ Q_B(r,p) \le Q. \end{cases}$$

- For $J > Q_B(r,p)$,

$$H_1(Q) = \begin{cases} A(Q), \ for \ Q < Q_B(p,r), \\ B_{pr}(Q), \ for \ Q_B(p,r) \le Q < J, \\ B_{rp}(Q), \ for \ J \le Q, \end{cases}$$

$$L_1(Q) = \begin{cases} A(Q), \ for \ Q < Q_B(p,r), \\ C_{pr}(Q), \ for \ Q_B(p,r) \le Q < J, \\ C_{rp}(Q), \ for \ J \le Q, \end{cases}$$

Proof. First suppose $Q_B(r,p) \geq J$. It is easy to check, using (12) and (13), that if $Q_B(p,r) > 0$, then $Q_B(p,r) > Q_B(r,p) > 0$ if and only if $J < Q_B(r,p)$. Therefore, for this case, either $Q_B(p,r) < 0$, or $Q_B(p,r) > Q_B(r,p)$. For $Q < Q_B(r,p)$, $B_{rp}(Q) < A(Q)$ since $B_{rp}(Q) < 0$ at $Q = 0$. If $Q_B(p,r) < 0$, $B_{pr}(Q) < A(Q)$ for all Q. If $Q_B(p,r) > Q_B(r,p)$, $Q < Q_B(r,p)$ implies that $Q < Q_B(p,r)$, so $B_{pr}(Q) < A(Q)$ since $B_{pr}(Q) < 0$ at $Q = 0$. Therefore $H_1(Q) = A(Q)$ and (16) implies $L_1(Q) = A(Q)$.

For $Q \geq Q_B(r,p)$, $B_{rp}(Q) \geq A(Q)$ [$B_{xy}(Q)$ and $A(Q)$ can cross at most once and $B_{xy}(Q) < A(Q)$ for $Q < Q_B(x,y)$]. Also, (6) implies that for $Q > J$, $B_{rp}(Q) > B_{pr}(Q)$. Hence, $Q_B(r,p) > J$ implies that $B_{rp}(Q) > B_{pr}(Q)$ for all $Q > Q_B(r,p)$. Therefore $H_1(Q) = B_{rp}(Q)$ and (16) and (5) imply $L_1(Q) = C_{rp}(Q)$.

Now suppose $Q_B(r,p) < J$. We first show that, for this case, $Q_B(p,r) > 0$. Suppose not. Then, since $J > Q_B(r,p)$, $Q_B(r,p) > 0$ implies (13). But $Q_B(p,r) < 0$ and (13) then imply that $1-r^2 \geq 1+pr$, which is impossible. Therefore, $Q_B(p,r) > 0$.

For $Q < Q_B(p,r) < Q_B(r,p)$, $B_{pr}(Q) < A(Q)$ and $B_{rp}(Q) < A(Q)$ as argued for the previous case. Therefore, $H_1(Q) = L_1(Q) = A(Q)$ as before.

For $J > Q \geq Q_B(p,r)$, $B_{pr}(Q) > B_{rp}(Q)$ and $B_{pr}(Q) > A(Q)$. Therefore, $H_1(Q) = B_{pr}(Q)$ and (16) and (5) imply $L_1(Q) = C_{pr}(Q)$.

For $Q \geq J \geq Q_B(r,p)$, $B_{rp}(Q) \geq B_{pr}(Q)$ and $B_{rp}(Q) > A(Q)$. Therefore, $H_1(Q) = B_{rp}(Q)$ and (16) and (5) imply $L_1(Q) = C_{rp}(Q)$.

Q.E.D.

Now, let

$$H(Q) = \begin{cases} H_1(Q), \ for \ Q \leq G, \\ H_2(Q), \ for \ Q > G. \end{cases}$$

Define L(Q) similarly. We now use Lemma 1 to construct H and L as functions of the feasible relationships among the various parameters. Note that in all cases, the optimal organization design involves the centralized flat structure for $4F > H(Q)$ and $Q < G$, the decentralized flat structure for $4F > H(Q)$ and $Q \geq G$, and the matrix organization for $4F < L(Q)$. The optimal design for $L(Q) \leq 4F \leq H(Q)$ differs by case and will be developed for each case in what follows. Some cases will be ruled out by the following lemma.

Lemma 2. Under Assumption 2, $Q_K(r,p) \geq T \geq 2r+r^2s$ is impossible for $r > 0$. Also, if $2r+r^2s \geq J$, then $T \geq 2r+r^2s$. In particular, $G \geq J$ implies $T > 2r+r^2s$.

Proof. It is easy to check that $Q_K(r,p) \geq T$ if and only if $2p/(1-p^2) \geq T$. But Assumption 2 implies that $r^2s > 2p/(1-p^2)$. Therefore $Q_K(r,p) \geq T \geq 2r+r^2s$ implies that $r^2s > 2r+r^2s$ which is impossible.

For the second part, since K_{rp} is flatter than K_{pr}, (6) implies that $K_{rp}(Q) > K_{pr}(Q)$, for all $Q \geq J$. In particular, if $2r+r^2s \geq J$, $K_{rp}(2r+r^2s) \geq K_{pr}(2r+r^2s) = 4p$. Consequently, $T \geq 2r+r^2s$. Moreover, by

Assumption 2, $2r+r^2s > G$, so $G \geq J$ implies $T > 2r+r^2s$.

Q.E.D.

Lemma 2 implies that $Q_K(r,p) \geq T$ and either $T \geq 2r+r^2s$ or $G > J$ or $J \leq 2r+r^2s$ (or any of these) is impossible.

Case 1: $Q_B(r,p) \geq \max\{J,G\}$

Since $J \leq Q_B(r,p)$, if $Q_B(p,r) > 0$, $Q_B(r,p) \leq Q_B(p,r)$. Hence, either $Q_B(p,r) < 0$ or $G \leq Q_B(r,p) \leq Q_B(p,r)$. In either case, $B_{pr}(G) \leq A(G)$. Therefore, $K_{pr}(G) = B_{pr}(G) \leq A(G) = D(G)$, using (8). Also, $G \leq Q_B(r,p)$ implies that $B_{rp}(G) \leq A(G)$, so $K_{rp}(G) = B_{rp}(G) \leq A(G) = D(G)$ as before. Therefore, since $D(Q)$ is less steep than $K_{xy}(Q)$, $D(Q) \geq K_{xy}(Q)$ for all $Q \geq G$. In particular, since $G < 2r+r^2s$, $D(2r+r^2s) > K_{pr}(2r+r^2s) = 4p$. Therefore, $Q_{4p} > 2r+r^2s$, and, for this case, we have that

$$H(Q) = \begin{cases} A(Q), \; for \;\; Q \leq G, \\ D(Q), \; for \;\; Q_{4p} > Q > G, \\ \;\; 4p, \; for \;\; Q \geq Q_{4p}, \end{cases} \tag{18}$$

and

$$L(Q) = \begin{cases} A(Q), \; for \;\; Q \leq G, \\ D(Q), \; for \;\; Q_{4p} > Q > G, \\ E(Q), \; for \;\; Q \geq Q_{4p}. \end{cases} \tag{19}$$

Moreover, if $T > G$, then $D(T) > K_{rp}(T) = 4p$. Therefore, if $T > G$, then $Q_{4p} > T$. Consequently, the optimal organization design in this case for 4F between L(Q) and H(Q) is the decentralized P-hierarchy. The optimal design is pictured in **Figure 6**.

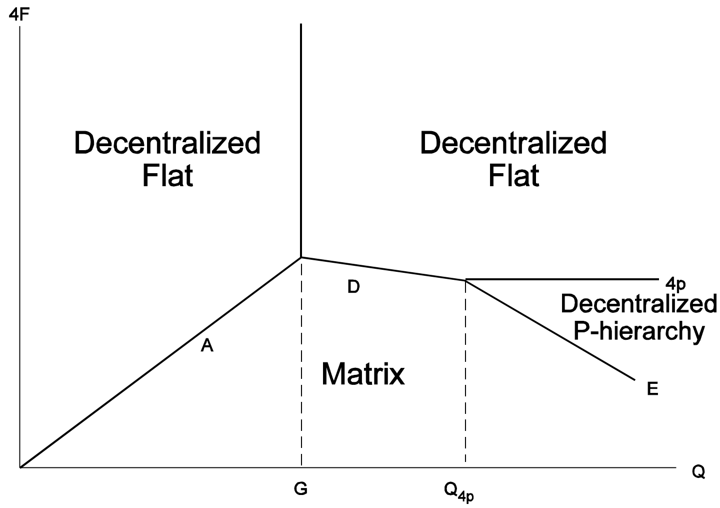

Figure 6: Optimal Organization Design for $0 < \max\{G,J\} \leq Q_B(r,p)$

Case 2: $G > Q_B(r,p) \geq J$

Note that, by Lemma 2, since $G > J$, we must have $T \geq Q_K(r,p)$. In this case,

$$H(Q) = \begin{cases} A(Q), \text{ for } Q \leq Q_B(r,p), \\ B_{rp}(Q), \; Q_B(r,p) < Q < G, \\ K_{rp}(Q), \text{ for } G \leq Q < Q_K(r,p), \\ D(Q), \text{ for } Q_K(r,p) \leq Q < Q_{4p}, \\ 4p, \text{ for } Q \geq Q_{4p}, \end{cases} \tag{20}$$

and

$$L(Q) = \begin{cases} A(Q), \; for \; Q \le Q_B(r,p), \\ C_{rp}(Q), \; Q_B(r,p) < Q < Q_K(r,p), \\ D(Q), \; for \; Q_K(r,p) \le Q < Q_{4p}, \\ E(Q), \; for \; Q \ge Q_{4p}. \end{cases} \tag{21}$$

It is easy to check that $Q_K(r,p) < T < Q_{4p}$. Consequently, since $J < Q_B(r,p)$ in this case, the optimal design for $L(Q) < 4F < H(Q)$ is the centralized R-hierarchy for $Q_B(r,p) \le Q < Q_K(r,p)$, and the decentralized P-hierarchy for $Q \ge Q_{4p}$. The optimal design is pictured in **Figure 7**.

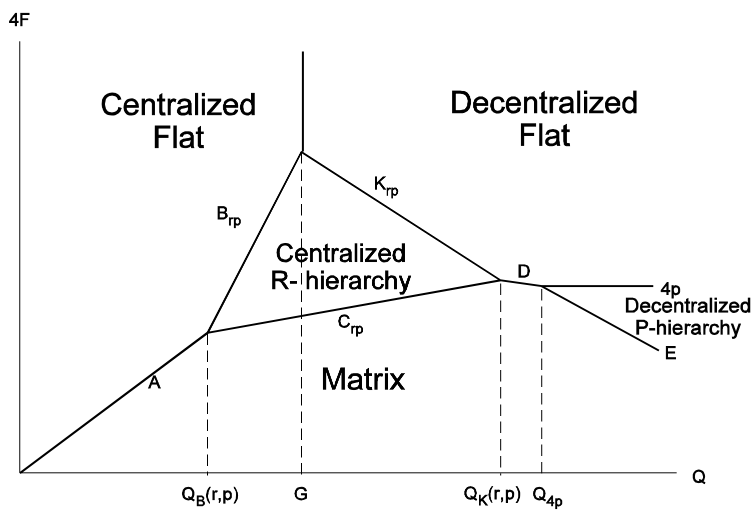

Figure 7: Optimal Organization Design for $G > Q_B(r,p) \ge J$

Case 3: $G > J > Q_B(r,p)$

For $Q \ge G$, H is the same as in Case 2 above. The only difference in this case is in the behavior of H for $Q < G$. Consequently, we have

$$H(Q) = \begin{cases} A(Q), \; for \; Q \le Q_B(r,p), \\ B_{pr}(Q), \; Q_B(r,p) < Q < J, \\ B_{rp}(Q), \; J \le Q < G, \\ K_{rp}(Q), \; for \; G \le Q < Q_K(r,p), \\ D(Q), \; for \; Q_K(r,p) \le Q < Q_{4p}, \\ 4p, \; for \; Q \ge Q_{4p}, \end{cases} \tag{22}$$

and

$$L(Q) = \begin{cases} A(Q), \; for \; Q \le Q_B(r,p), \\ C_{pr}(Q), \; Q_B(r,p) < Q < J, \\ C_{rp}(Q), \; J \le Q < Q_K(r,p), \\ D(Q), \; for \; Q_K(r,p) \le Q < Q_{4p}, \\ E(Q), \; for \; Q \ge Q_{4p}. \end{cases} \tag{23}$$

The optimal organization design for 4F between L(Q) and H(Q) is the centralized P-hierarchy for $Q_B(p,r) \le Q < J$, the decentralized P-hierarchy for $Q \ge Q_{4p}$, and the centralized R-hierarchy for $J \le Q < Q_K(r,p)$. The optimal design is pictured in **Figure 5** (in the text).

Case 4: $J > Q_B(r,p)$ and $G \le Q_B(p,r)$

Since $G \le Q_B(p,r)$, $H(Q) = A(Q)$ for $Q \le G$. Moreover, using (8), $K_{xy}(G) = B_{xy}(G) \le A(G) = D(G)$. Since D(Q) is less steep than $K_{xy}(Q)$, this implies that $K_{xy}(Q) < D(Q)$ for all $Q > G$. The rest of this case follows as in Case 1 above. Therefore H and L are given by (18) and (19), and the optimal design is the same as for Case 1 above and pictured in **Figure 6**.

Case 5: $J > Q_B(r,p)$, $J > G > Q_B(p,r)$, and $\min\{J, 2r+r^2s\} \ge Q_K(p,r)$

Since $J > Q_B(r,p)$, $G < J$, so $H(Q) = \max\{A(Q), B_{pr}(Q)\}$ for $Q \le G$. Also, using (8), $K_{pr}(G) = B_{pr}(G) > A(G) = D(G)$, so $Q_K(p,r) > G$. Since $Q_K(p,r) \le J$ and D(Q) is less steep than $K_{rp}(Q)$, $D(Q) \ge K_{xy}(Q)$ for $Q \ge Q_K(p,r)$. Finally, since $Q_K(p,r) \le 2r+r^2s$, $Q_{4p} > Q_K(p,r)$. Therefore,

$$
H(Q) = \begin{cases} A(Q), \ for \ Q \le Q_B(p,r), \\ B_{pr}(Q), \ Q_B(p,r) < Q < G, \\ K_{pr}(Q), \ for \ G \le Q < Q_K(p,r), \\ D(Q), \ for \ Q_K(p,r) \le Q < Q_{4p}, \\ 4p, \ for \ Q \ge Q_{4p}, \end{cases} \tag{24}
$$

and

$$
L(Q) = \begin{cases} A(Q), \ for \ Q \le Q_B(p,r), \\ C_{pr}(Q), \ Q_B(p,r) < Q < Q_K(p,r), \\ D(Q), \ for \ Q_K(p,r) \le Q < Q_{4p}, \\ E(Q), \ for \ Q \ge Q_{4p}. \end{cases} \tag{25}
$$

The optimal organization design for 4F between L(Q) and H(Q) is the centralized P-hierarchy for $Q_B(p,r) \le Q < Q_K(p,r)$ and the decentralized P-hierarchy for $Q \ge Q_{4p}$. The optimal design is pictured in **Figure 8**.

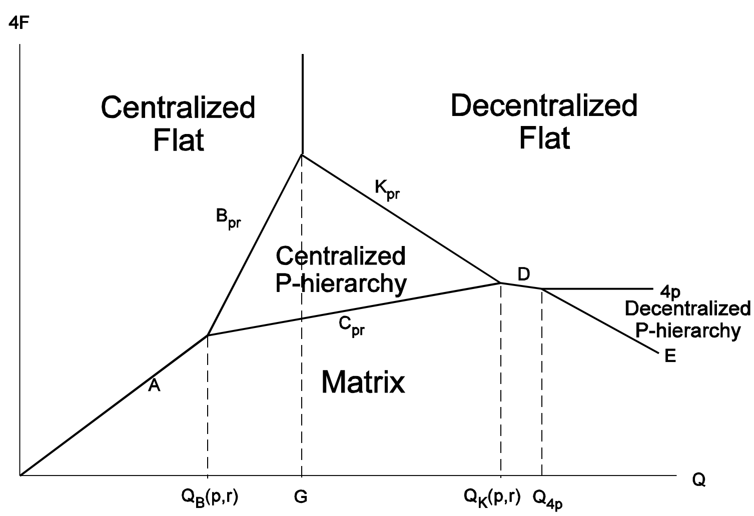

Figure 8: Optimal Organization Design for $J > Q_B(r,p)$, $J > G > Q_B(p,r)$ and $\min\{J,2r+r^2s\} \ge Q_K(r,p)$

Case 6: $J > Q_B(r,p)$, $J > G > Q_B(p,r)$, $2r+r^2s \leq \max\{J,Q_K(p,r)\}$

This case is the same as the previous case, except that, since $K_{pr}(2r+r^2s) = 4p$, for all Q such that $D(Q) \geq K_{pr}(Q)$, $D(Q) < 4p$. Consequently,

$$H(Q) = \begin{cases} A(Q), \ for\ Q \leq Q_B(p,r), \\ B_{pr}(Q),\ Q_B(p,r) < Q < G, \\ K_{pr}(Q),\ for\ G \leq Q < 2r+r^2s, \\ 4p,\ for\ Q \geq 2r+r^2s, \end{cases} \tag{26}$$

and

$$L(Q) = \begin{cases} A(Q),\ for\ Q \leq Q_B(p,r), \\ C_{pr}(Q),\ Q_B(p,r) < Q < 2r+r^2s, \\ E(Q),\ for\ Q \geq 2r+r^2s. \end{cases} \tag{27}$$

The optimal organization design for 4F between L(Q) and H(Q) is the centralized P-hierarchy for $Q_B(p,r) \leq Q < 2r+r^2s$ and the decentralized P-hierarchy for $Q \geq 2r+r^2s$. The optimal design is pictured in **Figure 9**.

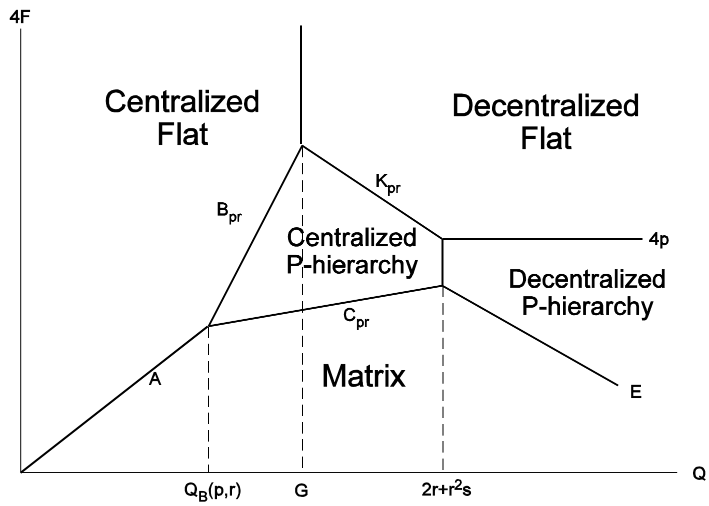

Figure 9: Optimal Organization Design for $J > Q_B(r,p)$, $J > G > Q_B(p,r)$ and $2r+r^2s \le \max\{J, Q_K(p,r)\}$

Case 7: $J > Q_B(r,p)$, $J > G > Q_B(p,r)$, and $J \le 2r+r^2s$

Note that, by Lemma 2, $J \le 2r+r^2s$ implies that $T > Q_K(r,p)$. This case is similar to the previous one, except that, since $Q_{4p} > T > Q_K(r,p)$, $D(Q) \ge \max\{K_{rp}(Q), 4p\}$ for $Q_K(r,p) \le Q < Q_{4p}$. Therefore,

$$H(Q) = \begin{cases} A(Q), \; for \;\; Q \le Q_B(p,r), \\ B_{pr}(Q), \;\; Q_B(p,r) < Q < G, \\ K_{pr}(Q), \; for \;\; G \le Q < J, \\ K_{rp}(Q), \; for \;\; J \le Q < Q_K(r,p), \\ D(Q), \; for \;\; Q_K(r,p) \le Q < Q_{4p}, \\ 4p, \; for \;\; Q \ge Q_{4p}, \end{cases} \tag{28}$$

and

$$L(Q) = \begin{cases} A(Q), \ for \ Q \le Q_B(p,r), \\ C_{pr}(Q), \ Q_B(p,r) < Q < J, \\ C_{rp}(Q), \ J \le Q < Q_K(r,p), \\ D(Q), \ Q_K(r,p) \le Q < Q_{4p}, \\ E(Q), \ for \ Q \ge Q_{4p}. \end{cases} \tag{29}$$

The optimal organization design for 4F between L(Q) and H(Q) is the centralized P-hierarchy for $Q_B(p,r) \le Q < J$, the centralized R-hierarchy for $J \le Q < Q_K(r,p)$, and the decentralized P-hierarchy for $Q \ge Q_{4p}$. The optimal design is pictured in **Figure 10**.

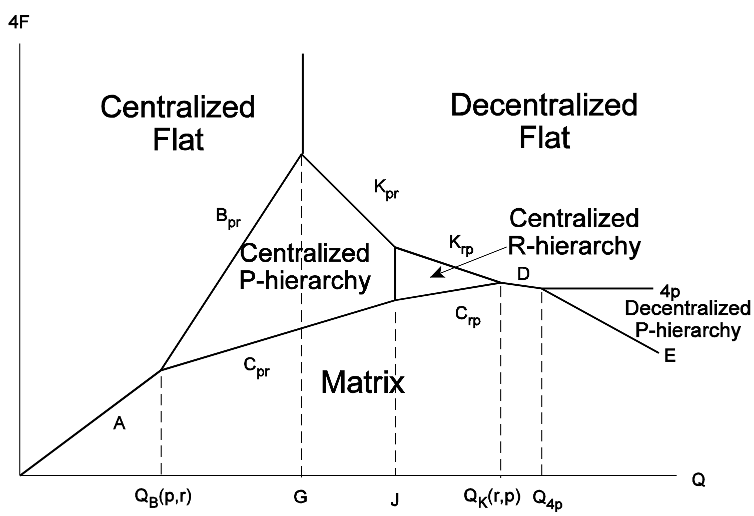

Figure 10: Optimal Organization Design for $J > Q_B(r,p)$, $J > G > Q_B(p,r)$, and $J \le 2r+r^2s$

The above cases exhaust the possible configurations.

Finally, we state Proposition 4 formally and prove it.

Proposition 4. Let [X] = {(Q,F)|the optimal design given synergy s is X}, for X = DF (decentralized flat structure), CF (centralized flat structure), M (matrix), DP (decentralized P-hierarchy), CP (centralized P-hierarchy) and CR (centralized R-hierarchy). Consider an increase in synergy gain from s to s′ and denote by [X]′ the region corresponding to [X] given synergy s′. Then [DF]′ ⊂ [DF], [DP]′ ⊂ [DP], [M] ⊂ [M′], [CP] ⊂ [CP]′, [CR] ⊂ [CR]′, and [CF] ⊂ [CF]′. Moreover at least one of [DF]′ ⊂ [DF] and [DP]′ ⊂ [DP] is strict.

Proof. Denote the functions H, L, G, J, and T evaluated using s′ instead of s by H′, L′, G′, J′ and T′, respectively. It is easy to check that G′ > G, T′ > T, J′ = J, H′ ≥ H with equality for Q ≤ G and strict inequality for Q > G, and L′ ≥ L with equality for Q ≤ G.

First suppose Q ≤ G. Then clearly (Q,F) ∈ [X] implies (Q,F) ∈ [X]′ for all F and X. Thus [X]′∩{(Q,F)|Q ≤ G} = [X]∩{(Q,F)|Q ≤ G}. Therefore, we need only show the result for Q > G, so assume Q > G for the remainder of the proof.

Now suppose (Q,F) ∈ [DF]′. Then H′ > H implies that (Q,F) ∈ [DF]. Thus [DF]′ ⊂ [DF]. If (Q,F) ∈ [M], then L′ ≥ L implies that (Q,F) ∈ [M]′. Thus [M] ⊂ [M]′.

Next suppose (Q,F) ∈ [CP]. Using **Table 2**, since C is independent of s, we have that H′(Q) > H(Q) > 4F > L(Q) = L′(Q) in this case. Therefore, since J′ = J, (Q,F) ∈ [CP]′. This establishes that [CP] ⊂ [CP]′. Inspection of **Figure 5** to **Figure 10** reveals that whenever (Q,F) ∈ [CR], L(Q) = C_{rp}(Q). Since C is independent of s and T′ > T, we have that (Q,F) ∈ [CR] implies that (Q,F) ∈ [CR]′, i.e., [CR] ⊂ [CR]′.

Now suppose (Q,F) ∈ [DP]′. Then E(Q) < E′(Q) < 4F < 4p, and either T′ < Q < $2p+p^2s'$ or Q ≥ $2p+p^2s'$. If the former, then T < Q, so (Q,F) ∈ [DP]. If the latter, then Q > $2p+p^2s$, so (Q,F) ∈ [DP]. Therefore, [DP]′ ⊂ [DP].

Finally, [CF]∩{(Q,F)|Q > G} = ∅ ⊂ [CF]′. Also, since G and T are strictly increasing in s and H is strictly increasing in s for Q > G, [X] ≠[X]′ for at least one X. Suppose [DF]′ = [DF] and [DP]′ = [DP]. This implies that [M]∪[CP]∪[CR]∪[CF] = [M]′∪[CP]′∪[CR]′∪[CF]′. But the fact that [X] ⊂ [X]′

for X = M, CP, CR, and CF implies that [X] = [X]′ for X = M, CP, CR, and CF. This contradicts the fact that [X] $\neq$[X]′ for at least one X.

Q.E.D.