A Note on the Suboptimality of Dollar-Cost Averaging as an Investment Policy

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I. Introduction

The widespread notion that dollar-cost averaging can help an investor minimize the risk of investing all of one's capital in the market at an inappropriate time is aptly stated by Malkiel [4, p. 242]:

> Periodic investments of equal dollar amounts in common stocks can substantially reduce (but not avoid) the risks of equity investment by insuring that the entire portfolio of stocks will not be purchased at temporarily inflated prices. The investor who makes equal dollar investments will buy fewer shares when prices are high and more shares when prices are low.

The following example clarifies the meaning of dollar-cost averaging (hereafter DCA). We assume for simplicity that there are only two investment opportunities which we call A and B. We consider an investor who has no wealth and who inherits wealth in shares of asset A. The hypothetical investor believes that his optimal long-run portfolio consists of equal dollar investments in A and B. According to DCA the investor should refrain from immediately selling half of his wealth in A in order to invest the proceeds in B; instead, he should sell less than half of his wealth now and sell additional shares of A at future times, while continuing to invest the proceeds in asset B, until he attains the optimal portfolio composition. If the same investor were to inherit the wealth in shares of asset B rather than asset A, he should likewise refrain from immediately selling half of his wealth in B in order to invest the proceeds in A; instead, he should accomplish the shift in his portfolio composition over a number of time periods.

The questionable rationale behind the DCA scheme is the following: the investor recognizes the possibility that the price of asset A may be temporarily inflated or depressed relative to the price of asset B, but does not know which is the case. Then the act of shifting wealth from asset A to asset B involves a fair gamble on a temporary shift of the prices of A and B relative to their "fair values." By shifting wealth from asset A to asset B in more than one stage, the investor replaces one major gamble by a number of smaller gambles and, presumably, reduces his risk. Two properties are commonly associated with DCA.

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Property 1: DCA is a nonsequential investment policy.

The nonsequential nature of DCA is considered by its proponents to be the key to its success. Malkiel [4, p. 243] states:

> And a critical feature of the plan is that you have both the cash and the courage to continue to invest during bear markets as regularly as you do in better periods. No matter how pessimistic you are (and everybody else is), and no matter how bad the financial and world news is, you must not interrupt the plan or you will lose the important benefit of insuring that you buy at least some of your shares after a sharp market decline.

Also Cohen, Zinbarg and Zeikel [1, p. 48] stress:

> The important thing is to stick to your schedule--to buy, even though the price keeps falling, which, psychologically, is usually hard to do. This brings your average cost down, and any subsequent rise will yield a significant capital gain. To engage in dollar-cost averaging successfully, you must have both the funds and the courage to continue buying in a declining market when prospects may seem bleak.

Property 2: The DCA policy depends not only on the total wealth of the investor but also on the composition of his wealth.

Referring to our earlier example, according to DCA, if the investor inherits his wealth in shares of A, he will immediately liquidate less than half of his wealth in A and use the proceeds to buy shares of B. Thus his investment decision at time zero is to hold a larger market value of asset A than asset B. If, however, the investor inherits the same amount of wealth in shares of B, he will immediately liquidate less than half of his wealth in B to buy shares in A. Now his investment decision at time zero is to hold a larger market value of asset B than asset A, and this decision is clearly dependent upon the initial composition of his wealth..

We now define a DCA policy as an investment policy which has properties P1 and P2.[1]

II. Critique of Dollar-Cost Averaging

Given a DCA policy which specifies the investment program over the relevant planning horizon, a sequential investment policy, which specifies the same investment at time zero as the DCA policy, but defers future decisions for some later time, weakly dominates the DCA scheme because it allows future decisions to be made with the benefit of information that will become available in the future, e. g., price realizations. Moreover, unless one makes very limiting assumptions on the utility function and probability distributions, the DCA policy will not, in general, coincide with the optimal sequential policy at all future times. These statements hold true even in the presence of transactions costs and taxes. Therefore we state

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[1]Later on we discuss a class of policies which we call "gradual policies" but which are referred to as DCA policies by some authors.

<u>Proposition 1</u>: <u>The dollar-cost averaging policy is dominated by a sequential</u>
                    <u>optimal investment policy</u>.

In view of the above result it seems ironic that proponents of DCA go to great lengths to emphasize that the investor must have the courage to follow a nonsequential policy, irrespective of new information which becomes available.

We now examine DCA within the subset of nonsequential policies. If there are computational costs associated with solving the investment problem sequentially, it might conceivably be preferable to confine the selection of an investment policy to the subset of nonsequential policies.[2] We proceed to show that DCA is also dominated by a nonsequential optimal investment policy. But first we specify our model. We make the standard perfect market assumptions:

A.1.   The investor is a price taker, i.e., his transactions do not affect market prices.

Large institutional investors or mutual funds may conceivably affect market prices if they transact in sufficiently large blocks. This consideration may induce an investor to use DCA in executing a contemplated large shift in his portfolio in order to avoid driving up (down) prices when he is buying (selling). Empirical evidence (Scholes [7]) suggests that large transactions have little discernible effect on market prices. In any case, proponents of DCA do not base their argument on the reaction of market prices to the transaction. Our assumption A.1. eliminates peripheral effects and focuses our attention on the purported primary advantages of DCA.

Whereas it is generally acknowledged that the investor's optimal strategy must take into account the effect of personal taxes, it is not clear whether taxes strengthen or weaken the case for DCA. To avoid unnecessary complications we assume:

A.2.   The investor pays no personal tax on regular income and capital gains.

Transactions costs (brokerage commission and price spread) in general influence the investment decision, but one would expect that these costs would weaken the case for DCA: by spreading his transactions over a number of periods, the investor reduces his bargaining power to negotiate a favorable transaction cost rate. In any case, since transactions costs are not central to the issue of DCA, we assume:

A.3.   There are no transactions costs in buying or selling assets.

Furthermore we make the following standard economic assumptions:

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[2]Neave and Wiginton [5] provide upper bounds on the difference in expected utility of sequential and nonsequential optimal policies.

[3]We exclude optimization criteria which contradict expected utility maximization, for example, minimax regret. Pye [6] discussed the implications of a minimax regret objective on investment policies.

A.4. The investor maximizes his expected utility of consumption. The marginal utility of consumption is positive.

We define

β_t: state of the world at time t.

$R(\beta_{t+1}) \equiv \{R^1(\beta_{t+1}), R^2(\beta_{t+1}), \ldots, R^n(\beta_{t+1})\}^{tr}$: one plus rate of return on assets from time t to t + 1, assuming that state β_{t+1} is realized.

I: n x 1 unit vector.

$w \equiv (w_1, w_2, \ldots, w_n)$: endowment at time zero.

$W \equiv wI$: total endowed wealth at time zero.

$H(\beta_t)$: exogenous income at time t.

$X_t \equiv (X_t^1, X_t^2, \ldots, X_t^n)$: investor's wealth in the assets at time t and after consumption at time t. X_t is a decision variable.

T: investor's lifetime; in general it may be stochastic or infinite.

$C_T \equiv (c_0, c_1, \ldots, c_T)$: consumption vector over lifetime. C_T is a decision variable.

$U(C_T \mid \beta_T)$: utility of lifetime consumption.

We define β_t to be the state of the world at time t, as perceived by the investor under consideration. The set β_t summarizes all the relevant state variables which are known to the investor at time t. Thus β_t may be considered as the information set of the investor and may include past price history of assets, economic indices, inside information, and the subjective beliefs of the investor. We need not assume that the information set β_t is common to all investors. At time t, β_{t+1} is stochastic; the investor's subjective probability distribution of β_{t+1} is conditional upon β_t. In general the distribution of $R(\beta_{t+1})$ at time t is conditional upon β_t since the distribution of β_{t+1} is conditional upon β_t. Thus price need not be a martingale.

We may wish to impose various nonnegativity constraints on the investment variables X_t. We denote the set of investment policies $\{X_0, X_1, \ldots, X_T\}$ which conform to these constraints by Ω. For example, we may wish to specify that borrowing is prohibited or that selling some assets short is prohibited. This will be expressed by non-negativity constraints on the respective variables. Alternatively we may wish to specify that borrowing is permitted but that the borrowing and lending rates differ. To do so we use two different variables to denote borrowing and lending and we restrict the first

variable to negative values and the second to positive values. We also specify different rates of return for these two assets.

The problem is formulated as in Fama [2]

$$\max_{\{C_T, X_0, X_1, \ldots, X_T\} \varepsilon \Omega} E_{\beta_0, w} U(C_T | \beta_T)$$

$$\text{s.t.} \quad X_0 I = wI - c_0$$

$$X_{t+1} I = X_t R(\beta_{t+1}) \mp H(\beta_{t+1}) - c_{t+1}, \quad t=0, 1, \ldots, T-1$$

$$X_T I = 0.$$

We note that the choice on an optimal nonsequential policy depends on the endowment, w, only through the term wI, where wI = W is initial wealth. Therefore an optimal nonsequential policy is invariant to the composition, w, of the endowed wealth. By our definition of DCA, a DCA policy depends nontrivially on the composition of wealth. We may exclude the fortuitous case where the DCA investment decisions for all wealth compositions, happen to be optimal nonsequential policies. Sufficient (but not necessary) conditions to exclude this case are that the utility is strictly concave in consumption and that no two assets are identical. It then follows that the utility is strictly concave in the investment vectors and, therefore, optimal investment is unique (see also Fama [2,3]). Given two different initial compositions of endowment with equal total wealth, the respective investments according to a DCA policy will be different and, therefore, at least one of them will be suboptimal. We now state

Proposition 2: The dollar-cost averaging policy is dominated by an optimal non-sequential investment policy.

Proponents of DCA may argue that our model formulation presupposes detailed knowledge of asset price behavior, whereas DCA requires very little such information. Such arguments are invalid: If the investor has limited information, this fact will be reflected by a limited set of information β_t. Our propositions 1,2 are independent of the content of the set β_t. Second, if the investor has little knowledge of the probabilities of future events (e. g. price changes), this fact will be reflected by a diffuse distribution of β_{t+1}, conditional upon β_t. Our propositions 1,2 are independent of the form of the distribution.

Where, then, does the intuitive rationale of DCA fail? Its rationale is that the investor replaces one major gamble on a temporary shift of prices by a number of smaller gambles and thus diversifies risk. The fault of this argument is misrepresentation of the state of the world, before a decision is made. DCA implies that an investor with all his endowment in asset A is in some way different from an investor with all his endowment in asset B, but otherwise identical. DCA ignores the simple fact that the latter

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investor may costlessly convert his endowment from asset A to asset B before he considers the optimal investment decision. Both investors face the same prospects irrespective of the composition of their endowment, and any claims of gambles on temporarily overpriced or underpriced prices are simply fallacious. We do not claim that the investor should not incorporate in his optimal decision his beliefs on whether assets are overpriced or underpriced. What we do claim is that these beliefs lead to the same optimal portfolio irrespective of the composition of initial wealth.

Finally we contrast DCA with a set of policies which we call "gradual policies." We define an investment policy as "gradual," if the transition from one portfolio to another is accomplished in more than one stage.[4] The term DCA has been used interchangeably in the past to denote the policies which we define as DCA policies, and the policies which we define as gradual policies. The source of confusion is the fact that DCA policies are usually (but not necessarily) gradual policies. Referring to our earlier example, the investor, who inherits his wealth in shares of asset A and who follows a DCA policy, will accomplish the transition to his optimal portfolio in more than one period. In this sense, his policy is also a gradual policy. However, DCA policies are not necessarily gradual policies, and gradual policies are not necessarily DCA policies. We criticized DCA policies as suboptimal policies but we had nothing to say regarding gradual policies. We may easily show by example that optimal sequential or nonsequential policies may well be gradual.[5] However this fact does not lend support to DCA policies.

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[4]Pye [6] examined the optimality of gradual policies. He assumed that the investor maximizes expected utility of end-of-the-horizon wealth; the utility function is strictly concave; and prices follow a stationary arithmetic random walk. He proved that the optimal nonsequential policy is not gradual, i.e., that the investor adjusts his portfolio composition only at the beginning and end of the time horizon. Pye also proved that gradual policies are optimal in a model where the investor minimizes maximum regret, an objective which is inconsistent with expected utility maximization Also Neave and Wiginton [5] proved that gradual policies are optimal in a model of expected utility maximization, where the utility is locally convex. We demonstrate in footnote 5 that one need not postulate an objective other then maximization of a strictly concave utility function in order to show that an optimal investment policy may be gradual. Whereas many different assumptions will give the required result, we show that nonstationarity of the return generating process leads to an optimal policy which is gradual.

[5]The claim is substantiated by the following example. The investor is endowed with \$20, has time horizon T = 2 and utility function $U(c_0, c_1, c_2 | \beta_2) \equiv \ln c_2$, where $c_0, c_1, c_2 \geq 0$. There are only two assets: a riskless asset with rate of return zero and a risky asset. The price per share of the risky asset at time zero is \$10. Over the time period (0,1) the price may increase by \$1.10 or decrease by \$1.00, each with probability half. Over the time period (1,2) and irrespective of the realization in the previous period, the price may increase or decrease by \$1.00, each with probability half.

Clearly it is suboptimal to hold the risky asset over the time period (1,2); this follows from the fact that the expected return is zero over (1,2) and from the risk aversion of the investor. The optimal nonsequential policy (which, in this example, coincides with the optimal sequential policy) is of the form "sell value x of the stock now, and invest the proceeds in the riskless asset, and sell the remainder at time one." The wealth is consumed at t = 2. The only nongradual investment policies are specified by x = 0 or x = 20.

[5] (Continued)

The optimal nonsequential policy is gradual and is specified by $x = 10.91$ if the shares are divisible, or $x = 10$ (i.e., one share) if the shares are indivisible:

Policy: sell stock x now, sell remainder at time one.

x	Expected utility
20	2.995732
10	2.996856
10.91	2.996867
0	2.995232

Thus one need not assume a minimax objective function or locally convex utility function in order to show that an optimal sequential or nonsequential policy may be gradual.

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