Temporal Risk and Probability Weights: A Descriptive Model of Delayed Resolution of Uncertainty

by
George Wu

George Wu
University of Chicago
Graduate School of Business
1101 E.58th Street
Chicago, IL 60637
(773) 834-0519
george.wu@gsb.uchicago.edu

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Abstract

Many meaningful decision problems are choices among gambles with delayed resolution of uncertainty (DRU). Notably, students of decision making have concentrated their attention largely on gambles with instantaneously resolved uncertainty. A descriptive theory is proposed for evaluating gambles with DRU such that period gambles (*i.e.*, gambles resolved at the same time) are evaluated by cumulative prospect theory (Tversky and Kahneman, 1992). In this context, the probability weighting function can depend on the timing of uncertainty resolution and thus captures both attitudes toward risk and preferences for early or delayed resolution of uncertainty. Stochastic stationarity conditions are used to relate the period weighting functions. It is argued that a second-order stochastic stationarity condition adapted from axiomatizations of hyperbolic discounting will produce an analogous property of hyperbolic probability discounting. The model makes some sharp predictions for decision behavior in delayed resolution settings, most notably preferences for delayed resolution for small probabilities of gains and most losses. In addition, this model may provide some insight into the origins of the nonlinearity of the prospect theory probability weighting function.

Key words: temporal resolution of uncertainty, nonexpected utility theory, cumulative prospect theory, probability weights, stochastic stationarity.

1. Introduction

Many meaningful real world decision problems involve *delayed* resolution of uncertainty. In many medical decisions — *e.g.*, choices between various forms of prenatal diagnosis (Cantor, 1991) or decisions to test for AIDS or Huntington's disease (Brody, 1988; Herrnstein, 1990) — the decision maker must decide whether or not she wishes to have the true state of health revealed early, generally at a cost. In insuring a house against fire or a car against damage, the true state of nature (whether the car is ultimately wrecked) is not revealed immediately after the decision is made but sometime in the future (when the car is wrecked or the insurance expires). As a final example, high school seniors may apply to college Early Action or Early Decision to find out about their future a few months early and avoid several extra months of anxiety or worry.

Although the problem has not been ignored altogether, most of the models of delayed resolution of uncertainty (DRU) are not appealing *descriptive* models. First, most treatments of the problem do not consider primitive preferences for early versus late resolution. Instead, these works deal with induced preferences, or the planning benefits of early resolution (*e.g.*, Markowitz, 1959; Mossin, 1969; Spence & Zeckhauser, 1972; Machina, 1984). There are often important planning benefits to early resolution; for example, certain medical treatments may lower the severity or probability of a disease. Planning benefits notwithstanding, individuals may have purely *psychological* reasons for preferring early or delayed resolution. The models that do consider DRU at a primitive level rely on axioms that have been violated systematically in static and/or dynamic domains. For example, Kreps & Porteus (1978) impose an intertemporal version of the substitution axiom (for catalogs of substitution violations in static contexts, see Camerer, 1995). Similarly, Chew & Epstein's (1989) recursive utility relies on the betweenness axiom, an axiom which is violated systematically in atemporal settings (Camerer & Ho, 1994). Other temporal axioms such as dynamic consistency (Kreps & Porteus, 1978 and Chew & Epstein, 1989) have not been tested explicitly in settings of delayed resolution of uncertainty, but violations of dynamic consistency over deterministic consumption streams suggest that these axioms are probably descriptively inadequate as well (*e.g.*, Ainslie, 1975; Thaler, 1981; Loewenstein, 1987; Benzion, Rapoport, & Yagil, 1989; Loewenstein & Thaler, 1989; Loewenstein & Prelec, 1992).

This paper argues that a descriptively accurate model of DRU must, at a minimum, account for (1) framing effects (distinguishing between gambles coded as gains and losses); (2) risk aversion and risk seeking in both the gain and loss domains; (3) preference for early and delayed resolution in both the gain and loss domains; (4) the dramatic increase in anxiety as the resolution date approaches. The first two properties require that a model of DRU inherit the generally observed properties of preferences in static settings (see Camerer, 1995). The models that seem to best describe risky choice behavior belong to the prospect theory family (Kahneman & Tversky, 1979, and Tversky & Kahneman, 1992).[1] In prospect theory, outcomes are segregated into gains and losses, and the weights assigned to outcomes do not typically correspond to the probability of receiving those outcomes. In this paper, we extend cumulative prospect theory (Tversky & Kahneman, 1992) to lotteries with DRU, thus permitting the probability weighting functions for gains and losses to depend on the timing of resolution.

A model designed to accommodate preferences for timing of resolution of uncertainty is the *anxiety model* (Wu, 1999). In this model, an unresolved gamble induces *anxiety*, a negative utility state, or *hope*, a positive utility state, in the period prior to uncertainty resolution. Thus, risk aversion is interpreted in this temporal context as having two components: (generally diminishing) marginal utility *and* anxiety aversion. Like prospect theory and the models of Quiggin (1982), Yaari (1987), and Luce (1988), the anxiety model is rank-dependent in form, although the outcome space does not distinguish between gains and losses — concentrating instead on final asset positions.

In the anxiety model, the probability weighting function depends on the timing of uncertainty resolution, thus capturing preferences for early versus late resolution of uncertainty. To illustrate, consider two gambles with identical probability distributions over final consequences but different dates of uncertainty resolution. If earlier resolution is preferred for all probability distributions, then the probability weighting function is convex; if delayed resolution is preferred then the weighting function is concave. Thus the shape of the typical prospect theory weighting function, concave then convex, has an additional temporal interpretation, summarized in the following table:

		Atemporal Gambles		Temporal Gambles (DRU)	
		Gains	Losses	Gains	Losses
small p	overweighted ($w(p) > p$)	risk-seeking	risk-aversion	hope (delay-seeking)	anxiety (resolution-seeking)
large p	underweighted ($w(p) < p$)	risk-aversion	risk-seeking	anxiety (resolution-seeking)	hope (delay-seeking)

Several experiments on DRU support the overweighting of small probabilities and temporal reflection effects. First, a study by Lovallo & Kahneman (1992) offers evidence for non-linear probability weights in the gain domain. They asked University of California-Berkeley undergraduates whether they wanted to have a drawing for a vacation for two to Hawaii held immediately or in 1 week. When the probability of winning the vacation was .90, 30 of 33 subjects (92%) preferred immediate resolution; when the probability of winning was .10, 20 of 36 subjects (56%) preferred immediate resolution ($p < .05$). In a second experiment, Chew & Ho (1994) found that 78% of subjects preferred to know about hypothetical test results immediately when they expected to do well, compared to 45% when they expected to do poorly.

We tested for the temporal reflection effect directly on subjects recruited at the Boston Museum of Science and from a continuing education program at the Harvard School of Public Health. Half the subjects were given the following question, inspired by Tversky & Kahneman's (1981) invariance violations:

> Question 1 (*n*=85): Imagine that you are working for a consulting company. Weeks ago, you were told that your next assignment would be an extremely exciting job with high visibility. Unfortunately, another project is understaffed and needs one additional member. The project, you hear, is especially boring, and the manager is considered an ogre. Since neither you nor your 2 fellow co-workers wants to take the boring assignment, your boss has decided to have a drawing to determine who will get the assignment. Of course, there is a *1 in 3 chance* that you will be assigned to the boring job. You are preparing to leave for 2 weeks of vacation. Would you prefer to have the drawing held:
>
> <u>now</u> [55%] <u>when you return</u> [45%]

In Question 1, outcomes are framed negatively with respect to the reference point, "an extremely exciting job with high visibility." When the question was framed as a loss, about half the subjects wanted to delay resolution, presumably because finding out that they would be stuck with the boring job, a loss, might ruin their vacation.

We asked a second set of subjects a second question, framing outcomes as gains:

> Question 2 (*n*=84): Imagine that you are working for a consulting company. Weeks ago, you were told that your next assignment would be an especially boring project managed by someone considered to be an ogre. Fortunately, another project is understaffed by two members. The other project is extremely exciting and has high visibility. Since you and your 2 fellow co-workers all want to take the exciting assignment, your boss has decided to have a drawing to determine who will get the assignment. Of course, there is a *2 in 3*

chance that you will be assigned to the exciting job. You are preparing to leave for 2 weeks of vacation. Would you prefer to have the drawing held:

now [77%] when you return [23%]

When the question was framed as a gain, the majority of people wanted to know the outcome before they left for vacation. Although the two problems are identical in terms of final outcomes, a 1/3 chance of an exciting job and 2/3 chance of a boring job, subjects react differently when they are faced with the two problems, often wishing to avoid information about losses, while at the same time strongly preferring to find out about gains immediately (p=.001, binomial test).[2]

In §2, we review cumulative prospect theory and the anxiety model and describe why neither of these approaches is adequate as a descriptive model of DRU. In §3, we describe some additional axioms within the framework of cumulative prospect theory that restrict the probability weighting function, the most important of which are Stochastic Stationarity and Second-Order Stochastic Stationarity. Within the framework of the anxiety model, Stochastic Stationarity (akin to stationarity assumptions in discounted utility, *e.g.*, Koopmans, 1960) restricts the probability weighting functions to be defined iteratively. Stochastic Stationarity is probably too strong an assumption for a descriptive model; this paper argues that we should expected failures of Stochastic Stationarity in choices between lotteries with delayed resolution that are analogous to failures of stationarity in choices over consumption streams. Imposing a weaker assumption called Second-Order Stochastic Stationarity (based on Prelec, 1989) permits "non-linear" probability discounting that is similar to the hyperbolic discounting that has been observed over deterministic consumption streams (*e.g.*, Ainslie, 1975). The major result is a sign-, rank-, and resolution-dependent utility representation for gambles with delayed resolution. Finally, in §4, we discuss the model more generally, and present some testable hypotheses of the model. Proofs are left for the Appendix.

2. Review of Cumulative Prospect Theory and the Anxiety Model

We begin by establishing some notation. Let X denote the set of possible outcomes, where x_0 is the status quo or reference point. For simplicity, we also assume that all outcomes are received at some time $T > 0$. Although this assumption is admittedly restrictive, the purpose of this paper is to understand primitive preferences for early versus delayed resolution. To the extent that preferences for timing of payments, while quite important, obscure these issues, they are left asides.

Define a *temporal prospect* to be $f = [(p_{-m}, x_{-m};...; p_{-1}, x_{-1}; p_0, x_0; p_1, x_1...; p_n, x_n), r]$, where $x_i \in X$, $x_i < x_{i+1}$ for all $i = -m,..., n-1$, p_i denotes the probability of receiving x_i, and $r < T$ denotes the date of uncertainty resolution. In other words, $x_{-m},..., x_{-1}$ are negative outcomes or *losses* with respect to the reference point, and $x_1,..., x_n$ are positive outcomes or *gains*. Of course, $\sum_{i=-m}^{n} p_i = 1$. The space of all temporal prospects is denoted L, *i.e.*, L $=\{ [(p_{-m}, x_{-m};...; p_{-1}, x_{-1}; p_0, x_0; p_1, x_1; ...; p_n, x_n), r]: r \leq T, x_i < x_{i+1}, x_i \in X$ for $i = -m,..., n-1 \}$. A prospect f can be parsed into non-positive and non-negative component gambles, $f^- =$ $[(p_{-m}, x_{-m};...; p_{-1}, x_{-1}; \sum_{i=0}^{n} p_i, x_0), r]$ and $f^+ =$ $[(\sum_{i=-m}^{0} p_i, x_0, p_1, x_1; ... ; p_n, x_n), r]$, respectively. When neither the cumulative probability of a loss nor the cumulative probability of a gain sums to one, both f^- and f^+ are normalized by adding the total probability of a gain and the total probability of a loss respectively to the probability of the reference point, x_0.

In addition, it is useful to describe just the probability distribution of a temporal prospect. Let $[f]$ denote the probability distribution of a temporal prospect, *i.e.*, $[f]=(p_{-m},x_{-m};...;\ p_n,x_n)$. Finally, let $[f]_r$ denote a temporal prospect with the probability distribution given by $[f]$ resolved at *r*, *i.e.*, $[f]_r$ = $[(p_{-m},x_{-m};...;\ p_n,x_n),r]$.

The valuation of f^- and f^+ is given by the following rank-dependent form, so-called because the weight assigned to x_i depends on the rank of x_i with regards to the prospect's other outcomes:

$$V(f^-)=\sum_{i=-m}^{-1} v(x_i)\left[w^-\left(\sum_{j=-m}^{i} p_j\right)-w^-\left(\sum_{j=-m}^{i-1} p_j\right)\right], \tag{2.1}$$

$$V(f^+)=\sum_{i=1}^{n} v(x_i)\left[w^+\left(\sum_{j=i}^{n} p_j\right)-w^+\left(\sum_{j=i+1}^{n} p_j\right)\right], \tag{2.2}$$

and

$$V(f)=V(f^-)+V(f^+). \tag{2.3}$$

Equations (2.1) through (2.3) are known collectively as *cumulative prospect theory*, CPT (Tversky & Kahneman, 1992; for an axiomatization, see Wakker & Tversky, 1993). The value function, $v(\cdot)$, is assumed to be S-shaped value function (concave for positive *x*, convex for negative *x*), and steeper for losses than gains, . In addition, for simplicity, $v(0)=0$. The weighting functions, $w^+(\cdot)$ and $w^-(\cdot)$ are also inverse S-shaped, typically concave in the range of p < .35) and convex for p > .35 (Tversky & Kahneman, 1992; Camerer & Ho, 1994, Wu &

Gonzalez, 1996). Furthermore, $w^{+}(\cdot)$ and $w^{-}(\cdot)$ seem to be remarkably similar in shape, consistent with the reflection effect (Kahneman & Tversky, 1979; Camerer, 1992).

Note that the specification of outcomes in CPT does not include the timing of uncertainty resolution. Thus two lotteries with the same probability distribution over final outcomes but different resolution dates have the same utility. The natural extension of CPT to temporal gambles follows:

$$V(f^{-}) = \sum_{i=-m}^{-1} v(x_i)\left[\pi^{-}\left(\sum_{j=-m}^{i} p_j, r\right) - \pi^{-}\left(\sum_{j=-m}^{i-1} p_j, r\right)\right], \tag{2.4}$$

$$V(f^{+}) = \sum_{i=1}^{n} v(x_i)\left[\pi^{+}\left(\sum_{j=i}^{n} p_j, r\right) - \pi^{+}\left(\sum_{j=i+1}^{n} p_j, r\right)\right], \tag{2.5}$$

and

$$V(f) = V(f^{-}) + V(f^{+}). \tag{2.6}$$

All gambles resolved at some time r satisfy CPT. Note that if $\pi^{+}(\cdot, r)$ and $\pi^{-}(\cdot, r)$ are independent of r, then the representation reduces to (2.1)-(2.3). However, if the weights depend on the timing of uncertainty resolution, (*i.e.*, $\pi^{+}(p,r) \neq \pi^{+}(p,r')$ ≠ π⁺(p,r′) for some r and r'), then some axiom is required to characterize $\pi^{+}(\cdot,\cdot)$ and $\pi^{-}(\cdot,\cdot)$ more precisely. [Since $\pi^{+}(\cdot,\cdot)$ and $\pi^{-}(\cdot,\cdot)$, are qualitatively similar, unless otherwise noted, remarks about $\pi^{+}(\cdot,\cdot)$ also apply to $\pi^{-}(\cdot,\cdot)$.]

In the anxiety model, the following axiom was used to restrict the form of the weighting functions:

Stochastic Stationarity (SS) Axiom:
If $[(p, x; 1-p, y), 0] \sim [(q, x; 1-q, y), 1]$ then, for all k>0, $[(p, x; 1-p, y), k]$ ~ $[(q, x; 1-q, y), k+1]$.

This axiom is in the spirit of stationarity assumptions employed in discounted utility (*e.g.*, Koopmans, 1960). Within the framework of the anxiety model, the rationale for SS is that the anxiety induced by an unresolved gamble should be "constant" over time. Put differently, if a gamble is resolved at *t*=5, then the anxiety felt at *t*=1 should not be that different from the anxiety felt at *t*=2, etc. Probability discounting should only depend on the differences in resolution dates, not the timing of resolution.

In Wu (1999), expected utility holds for instantaneously resolved gambles, *i.e.*, $\pi^{+}(p,0)=p$. The result of imposing SS within this framework is that $\pi^{+}(\cdot)$ is defined iteratively. The iterative form defines a "period probability discount function," $a^{+}(p)$, to be equal to $\pi^{+}(p,1)$ and then scales all resolution date-dependent weighting functions recursively: $\pi^{+}(p,r)=a^{+}\left(\pi^{+}(p,r-1)\right)$ for all $r>1$. For example, $\pi^{+}(p,1)=a^{+}(p)$, $\pi^{+}(p,2)=a^{+}\left(a^{+}(p)\right)$, …, $\pi^{+}(p,r)=$ $a^{+}\left(\ldots r \text{ times} \ldots \left(a^{+}\left(p\right)\right)\ldots\right)$.

To see why SS is descriptively inadequate, consider the following four gambles, $[f]_0$ = $[(.15,0;.85,1000),0]$, $[g]_1$ = $[(.10,0;.90,1000),1]$, $[f]_{12}$ = $[(.15,0;.85,1000),12]$, and $[g]_{13}$= $[(.10,0;.90,1000),13]$, where all the dates of resolution are in units of months from today. Preferences which violate SS are $[f]_0 \succ [g]_1$ and $[g]_{13} \succ [f]_{12}$. In looking at $[f]_0$, the uncertainty is resolved immediately. However, with $[g]_1$, the uncertainty is resolved in 1 month, and resolution at 1 month looks like an eternity compared to resolution immediately. On the other hand, comparing $[f]_{12}$ and $[g]_{13}$, both lotteries are resolved in the relatively distant future. They are close enough in resolution date that this is not a distinguishing feature of the prospects.

However, since $[g]_{13}$ provides a higher chance at winning 1,000, $[g]_{13}$ is preferred. Put differently, sensitivity to differences in resolution dates decreases as the date of resolution is pushed into the future. In other words, anxiety *depends on the time to resolution and is not constant in time*. It is not hard to think of many real-world instances in which anxiety builds up non-linearly as the date of resolution approaches. March 15 was known to Caesar as the Ides of March, but to medical students in the U.S., it is known as Match Day. On that day, fourth-year students find out the residency program where they will spend the next two or three years of their lives. Not knowing the outcome of their "match" becomes increasingly unbearable to students as March 15 approaches.

In sum, we expect discounting of probabilities to violate SS. However, non-linear impatience is not a new idea to students of temporal choice. There are many examples of systematic violations of Koopman's stationarity axiom. For example, many people prefer $10 today to $11 in a week but would rather have $11 in a year and a week than $10 in a year. Ainslie (1975) has shown that such temporal preferences are well-captured by a hyperbolic (*not* exponential) discounting function. In the next section, we replace SS by a weaker axiom called Second-Order Stochastic Stationarity.

3. A Descriptive Anxiety Model

The objects of choice are the $n+m+1$-outcome gambles described in Section 2. Recall that *X* denotes that outcome set, and L denotes the set of lotteries. Let L_t denote the subset of L resolved at *t*, *i.e.*, $\mathsf{L}_t = \{[(p_{-m}, x_{-m};...; p_{-1}, x_{-1}; p_0, x_0; p_1, x_1;...p_n, x_n), t] : x_i < x_{i+1}, x_i \in X$ for $i = -m,..., n-1)\}$. We denote gambles *f*, *g*, etc. The following assumptions are then needed:

Assumption 1 (A1): Preferences on $\mathcal{L}_\tau$ satisfy cumulative prospect theory.

The next proposition follows immediately.

Proposition 1: If A1 holds, then preferences on $\mathcal{L}_\tau$ can be represented by (2.4)-(2.6).

The next assumption places restrictions on $\pi^+(\cdot,\cdot)$ and $\pi^-(\cdot,\cdot)$. In the last section, we argued that Stochastic Stationarity was overly restrictive. The following relaxation of SS is based on an axiom of Prelec (1989) in the context of choice over consumption streams; a similar axiom, called *timing regularity*, is due to Harvey (1991).

Assumption 2 (A2) Second-Order Stochastic Stationarity [SOSS]):
There exists $\alpha \geq 1$, such that if $[(p,x;1-p,y),0] \sim [(q,x;1-q,y),1]$, then, for all $k \geq 0$, $[(p,x;1-p,y),k] \sim [(q,x;1-q,y),1+\alpha k]$.

SS is the special case of SOSS with α=1. Thus, SOSS generalizes SS, as Prelec's Second-Order Stationarity generalizes Stationarity. To understand SOSS, consider again the following two temporal lotteries, $[f]_0 = [(.15,0\ ;\ .85,1000),0\text{ months}]$ and $[g]_1 = [(.10,0\ ;\ .90,1000),1\text{ months}]$. First, suppose that the payment will be made in one year. In addition, suppose our decision maker judges $[f]_0$ to be indifferent to $[g]_1$; the decision maker is indifferent between finding out the result immediately (hence foregoing an additional .05 chance at winning 1,000) and waiting one month to have the uncertainty resolved. A large reason for the size of the probability premium is that $[f]_0$ is resolved immediately, and our decision maker will avoid one month of worry. Now consider two slightly different temporal lotteries, $[f]_1 = [(.15,0\ ;\ .85,1000),1\text{ month}]$ and $[g]_x = [(.10,0\ ;\ .90,1000),x\text{ months}]$. The decision maker must choose x so that $[f]_1 : [g]_x$. Since $[f]_1$ is now also delayed in time, a good part of the appeal of $[f]_0$ has also disappeared.

To make $[g]_x$ just as good as $[f]_1$, a longer delay is needed to compensate for the .05 probability premium. Suppose that if $x = 3$ months, then $[f]_1 \sim [g]_3$. SOSS requires that for all $y \geq 0$, $[f]_y$ = $[(.15, 0 ; .85, 1000), y \text{ months}] \sim [g]_{1+2y} = [(.10, 0 ; .90, 1000), 1+2y \text{ months}]$.

To see how SS and SOSS differ in a second more informal way, we look at the lotteries depicted in Figures 1 and 2. Figure 1a depicts $[f]_0$ and $[g]_1$, while Figure 1b depicts the same lotteries, with each lottery resolved one period later. The arrows on the decision trees (in the pre-resolution period) denote both the anxiety induced by the unresolved lotteries and the magnitude of the anxiety. In other words, the arrows depict the *stream of anxiety*. One rationale for SS is that if anxiety increases exponentially until the lottery is resolved, then a basic principle of cancellation requires that preferences between $[f]_0$ and $[g]_1$ should be the same as preferences between $[f]_1$ and $[g]_2$. First look at $[f]_0$ and $[g]_1$ in Figure 1a. Since $[f]_0$ and $[g]_1$ are judged indifferent by assumption, then the aversive effects of anxiety in $[g]_1$ just compensate for the .05 lower chance at winning $1,000. SS also requires that $[f]_1$ and $[g]_2$ in Figure 1b be indifferent. If the stream of anxiety from $t = 0$ to $t = 1$ induced by $[f]_1$ is the same as that induced by $[g]_2$ in the same time period (as is the case in Figure 1b), then preferences for $[f]_1$ and $[g]_2$ should not depend on these identical anxiety flows. Since the two gambles are the same in all other respects as $[f]_0$ and $[g]_1$ (just displaced by one period), $[f]_1$ should also be indifferent to $[g]_2$.

Insert Figures 1 and 2 about here

However, it is quite strong to assume that these two anxiety flows are identical. Suppose that anxiety increases more rapidly as the timing of resolution approached. Figure 2 depicts such a situation. Now, most of the anxiety is felt just prior to the resolution of uncertainty and builds up dramatically as the resolution date approaches; as a result, the anxiety induced by $[f]_1$ and $[g]_2$ are very different for $t \in [0,1]$. The cancellation principle cannot be used. So to make $[g]_x$ indifferent to $[f]_1$, the date of resolution of $[g]_x$ must be pushed back further than to $t = 2$, perhaps $t = 3$ or even $t = 4$.

The following proposition results from assuming that SOSS holds.

Proposition 2: If A1 and A2 are satisfied, then preferences on L_t satisfy (2.4)-(2.6) with the additional restrictions that:

(i) $\pi^+\left(p, \sum_{i=0}^{r} \alpha^i\right) = a^+\left(\mathbf{K}\, r{+}1 \text{ times } \mathbf{K}\left(b^+(p)\right)\mathbf{K}\right)$ for $\alpha \geq 1$;

(ii) $\pi^-\left(p, \sum_{i=0}^{r} \alpha^i\right) = a^-\left(\mathbf{K}\, r{+}1 \text{ times } \mathbf{K}\left(b^-(p)\right)\mathbf{K}\right)$ for $\alpha \geq 1$;

where $a^+(\cdot)$, $b^+(\cdot)$, $a^-(\cdot)$, and $b^-(\cdot)$ are defined as follows: $b^+(p) = \pi^+(p,0)$, $b^-(p) = \pi^-(p,0)$, $a^+\left(b^+\left(p\right)\right) = \pi^+(p,1)$, and $a^-\left(b^-\left(p\right)\right) = \pi^-(p,1)$.

If Stochastic Stationarity is satisfied, then there is an additional restriction on α.

Proposition 3: If Stochastic Stationarity is also satisfied, then $\alpha = 1$ in Proposition 2.

As α gets closer to 1, preferences approach linear probability discounting.

All gambles can be evaluated by assessing four probability weighting functions and a constant for both gains and losses. First, $b^+(\cdot)$ and $b^-(\cdot)$ can be thought of as baseline weighting functions for instantaneously resolved gambles.[3] Second, $a^+(\cdot)$ and $a^-(\cdot)$ are interpreted as "period" probability discounting functions. If SS is assumed, a lottery resolved at *t* is evaluated by applying the period weighting functions, one at a time, to find preferentially indifferent gambles resolved at $t-1$, $t-2$, etc.

To illustrate this model, suppose that we wish to evaluate $[(.6,0;.4,100),15]$. Proposition 2 gives results in terms of iterative functions. For simplicity, let $a^{(+1)} = p$ denote $a^{+}(p)$, the first-iterative function, and $a^{(+n)}(p) = a^{+}\left(a^{+(n-1)}(p)\right)$, the n-th iterative function. Let $\alpha = 2$, and note that $\pi^{+}(p,15) = \pi^{+}(p,1+2+4+8)$. Hence, by Proposition 2, $\pi^{+}(p,15) = a^{+}\left(\pi^{+}(p,7)\right) = a^{(+2)}\left(\pi^{+}(p,3)\right) = a^{(+3)}\left(\pi^{+}(p,1)\right) = a^{(+4)}\left(b^{+}(p)\right)$. Thus, $V([(.6,0;.4,100),15]) = v(100)\pi^{+}(.6,15) = v(100)a^{(+4)}\left(b^{+}(p)\right)$.

The model is easily extended to arbitrary and non-integer resolution dates. Note that $\sum_{i=0}^{r}\alpha^{i} = \left(\alpha^{r-1}-1\right)/\left(\alpha-1\right)$. Given α, we can determine r for any resolution date t:

$$r = \ln\left(t(\alpha-1)+1\right)/\ln(\alpha)-1. \tag{3.1}$$

The weighting function proposed by Prelec (1998) is especially convenient[4]:

$$a^{+}(p) = \exp[-(-\ln p)^{\gamma}] \tag{3.2}$$

and

$$\pi^{+}(p,0) = b^{+}(p) = \exp[-(-\ln p)^{\beta}]. \tag{3.3}$$

We can use (3.1)-(3.3) to evaluate any lottery. From Proposition 2, $\pi^{+}(p,1) = a^{+}\left(b^{+}(p)\right)$. Taking natural logs of both sides of (3.3) and substituting $-\ln\left(b^{+}(p)\right)$ into (3.2) for $-\ln(p)$, we get

$$\pi^{+}(p,t) = \exp[-(-\ln p)^{\beta\gamma^{r+1}}], \tag{3.4}$$

where r is given in (3.1).

To illustrate, suppose that $[(.3,0;.7,100),0]$ **:** $[(.25,0;.75,100),1]$ and $[(.3,0;.7,100),1] \sim [(.25,0;.75,100),2.5]$. Furthermore, suppose for simplicity that $b^{+}(p) = p$ for all p. The two indifference relations imply that $\alpha = 1.5$ and $\pi^{+}(.70,0) = \pi^{+}(.75,1)$, while (3.3) and (3.4) imply that $\gamma = \ln(.75)/\ln(.70) = .806$. Thus, $\pi^{+}(p,t) = \exp[-(-\ln p)^{.806^{r'}}]$, where $r' = \ln(.5t+1)/\ln(1.5)$.

We can apply the model to the job question described in the Introduction. In Question 1, the “extremely exciting job” is the reference point and the prospects are framed as a loss: subjects can choose to have the uncertainty resolved immediately, $[(\frac{1}{3},\text{Bad Job};\frac{2}{3},0),0]$, or in two weeks, $[(\frac{1}{3},\text{Bad Job};\frac{2}{3},0),2]$. Using Proposition 2, “2 weeks” is preferred to “now” if $a^{-}\left(b^{-}\left(\frac{1}{3}\right)\right)v(\text{Bad}) > b^{-}\left(\frac{1}{3}\right)v(\text{Bad})$ or, since $v(\text{Bad}) < 0$, $a^{-}\left(b^{-}\left(\frac{1}{3}\right)\right) > b^{-}\left(\frac{1}{3}\right)$. If we assume that $a^{-}(\cdot)$ and $b^{-}(\cdot)$ have the standard prospect theory shape — concave below and convex above p^{*} for p^{*} between .30 and .40 — then immediate resolution is preferred if $p^{*} < 1/3$ and delayed resolution is preferred if $p^{*} > 1/3$. This is consistent with our empirical finding that 45% of subjects prefer to delay the gamble.

The second question is framed as a gain, since the reference point is “an especially boring project.” Subjects must choose between immediate resolution, $[(\frac{1}{3},0;\frac{2}{3},\text{Good}),0]$, and resolution in two weeks, $[(\frac{1}{3},0;\frac{2}{3},\text{Good}),2]$. Again, using Proposition 2, the former is preferred if $a^{+}\left(b^{+}\left(\frac{2}{3}\right)\right) < b^{+}\left(\frac{2}{3}\right)$, which is consistent with empirically observed weighting functions: $a^{+}(p) < p$ and $b^{+}(p) < p$ for $p > p^{*}$.

Finally, it is worth understanding how to reconcile these different representations with the extensive knowledge we have on preferences for instantaneously resolved lotteries. A minimal requirement for a descriptive model of DRU is that it be consistent with empirical findings for static lotteries. The weighting functions $b^{+}(\cdot)$ and $b^{-}(\cdot)$ correspond to the standard static probability weighting functions, $w^{+}(\cdot)$ and $w^{-}(\cdot)$, in cumulative prospect theory.

4. Discussion

4.1. Links with Behavioral Decision Theory

The descriptive model of DRU presented in the previous section has several implications for behavioral decision theory. First, the extra freedom provided by the descriptive anxiety model may explain a few behavioral decision theoretic puzzles. For example, Schoemaker (1980) found that subjects were willing to pay more to avoid a .01 chance at losing $-10,000 when the problem was stated in an insurance frame than when it was stated in a neutral monetary frame. Such behavior is not surprising if we think about the nature of insurance risk. When decision makers insure themselves against the negative effects of some particular state of nature (*e.g.*, car wrecks, fires, theft, etc.), the underlying uncertainty is nearly always delayed in nature. One interpretation for the discrepancy in stated certainty equivalents is that the insurance frame invokes delayed resolution heuristics, even if the timing of uncertainty is not explicitly stated; the rationale for insurance purchases is not just loss aversion, but also a desire to avoid unwanted anxiety. Furthermore, the date of uncertainty resolution is itself uncertain. Recall that hyperbolic anxiety implies that anxiety increases dramatically as the date of resolution approaches. In the case of an ambiguous date of resolution, the anxiety could be chronically high, since such backwards induction is not possible. A high school senior who finds out which colleges have admitted her some time the week of April 9-15 probably suffers more total anxiety than the high school senior who finds out the same information for sure on April 15.

Second, the anxiety model offers insight into lottery (*i.e.*, lotto/megabucks) purchasing behavior. Explanations of simultaneous purchases of lottery tickets and insurance have evolved from Friedman & Savage (1948) (*i.e.*, convex/concave portions of the utility function) to prospect theory (*i.e.*, overweighting of extreme probabilities, Kahneman & Tversky, 1979). Lotto tickets offer extremely small chances (on the order of 1 in 1,000,000 to 1 in 23,000,000) of winning extremely large monetary prizes (sometimes well over $10,000,000). Surveys of lottery purchasers indicate that the majority of lotto purchases are planned (Clotfelter & Cook, 1989). Thus, even someone who purchases a lotto ticket minutes before the winning number is drawn receives the psychic benefit of dreaming about wealth from the moment she has *decided* to buy the ticket.[5]

Last, as mentioned earlier, the descriptive anxiety model provides an additional clue about the origins of prospect theory's probability weighting function. Suggestions as to the weighting function's origin include venture theory (Einhorn & Hogarth, 1990), a process model that explains the shape in terms of anchoring and adjustment. Kahneman & Tversky (1984) have offered a psychophysical explanation for the shape of the weighting function:

> an increase from 0% to 5% appears to have a larger effect than an increase from 30% to 35%, which also appears smaller than an increase from 95% to 100%. These considerations suggest a category-boundary effect: A change from impossibility to possibility or from possibility to certainty has a bigger impact than a comparable change in the middle of the scale.

An alternative explanation for the origin of probability weights is offered in terms of the primitive psychological emotions of hope and anxiety. One way of interpreting the probability weighting function is in terms of cognitive attention. As the decision maker endures the unresolved uncertainty, she thinks about the various outcomes. If the cognitive attention she gives outcome x_i is t_i, then the *relative* cognitive attention given to x_i is $q_i = t_i / \sum_{j=-m}^{n} t_j$. If $q_i < p_i$, then x_i is given less cognitive attention than expected; likewise, if $q_i > p_i$, then x_i is given more cognitive attention than expected. For both gains and losses, the anxiety model would predict higher than expected attention to both extreme gains and extreme losses. However, cognitive weights have a very different effect if the outcomes are low- or high-valued. When the outcome is high-valued, then the excessive cognitive attention manifests itself in terms of hope, a psychological effect of positive utility to the decision maker. Likewise, if the outcome is low-valued, then excessive cognitive attention corresponds to anxiety, a negative utility effect. The anxiety model thus explains the weighting function partially in terms of the psychophysics of hope and anxiety. Only a small chance at winning $10,000,000 is necessary to give lotto ticket purchasers hope of a new life style. Similarly, very small chances of disastrous events (such as airplane crashes) can cause great worry for airplane travellers. Indeed, there is no reason that hope and anxiety should be linear in probabilities. Although the mathematical expectation of a .000002 chance at winning $10,000,000 is twice as high as that of a .000001 chance at winning the same $10,000,000, the former gamble may not provide twice as much hope (psychological expectation) as the latter.

Finally, one interpretation about static lotteries provides some hints about the origins of the probability weighting function. If hope and anxiety are primitive psychological effects induced by lotteries with delayed resolution, decision makers should not experience these psychological reactions when confronted with an instantaneously resolved lottery. One interpretation is that when decision makers deal with the atypical event of a gamble with instantaneous resolution, they use heuristics honed by their more typical experience in dealing with lotteries with delayed resolution.

4.2. Framing and Temporal Risk

Any model of sign-dependent choice depends critically on the reference point. Unlike typical choice experiments in which subjects accept a frame uncritically, in this domain, decision makers have ample time to set their frame actively. Do decision makers in fact frame a prospect actively, and, if they do, what reference point or points do they use?

First, it is natural to think of the reference point as some level of expectation about the outcome of the relevant temporal prospect. Often the reference point may be the status quo. In turn, the reference point drives three distinct psychological components: dread/anticipation; anxiety/hope; and disappointment/elation. Dread/anticipation (Loewenstein, 1987) can be thought of as the consumption of the reference point: the higher the reference point, the greater the anticipatory utility; the lower the reference point, the greater the dread. On the other hand, anxiety/hope is the pre-resolution psychological reaction to variation around the reference point. Disappointment/elation (Bell, 1985) is the post-resolution psychological effect of receiving outcomes better or worse than expected. All three contribute to the total psychological utility of a temporal prospect.

If the reference point is optimistic relative to mathematical expectation, then utility is gained from consuming the reference point. Loewenstein (1987) terms this “savoring of expectation”. In addition, since outcomes are predominantly losses (with regard to the reference point), there will be hope rather than anxiety. However, high expectations open up the likelihood of disappointment; it is likely that the true outcome will be worse than expected (*e.g.*, a worker receives a $500 bonus when she was expecting a $1,000 bonus). Bell terms the resulting psychological reaction to an outcome below-expectation *disappointment*. On the flip side, setting high expectations lowers the chance of receiving higher than expected outcomes (*elation*). Thus, in setting expectations, a hedonic maximizer, *i.e.*, someone who maximizes the sum of all psychic utility, must balance the benefit of reducing excessive anxiety against the possibility of post-resolution disappointment.

Loewenstein & Linville (1986) argue that people do, in fact, adjust their expectations to balance pre-resolution (anxiety and anticipation) and post-resolution (disappointment) utility. If resolution is imminent, then the balance between pre-resolution and post-resolution effects tips in favor of the latter: individuals typically have low expectations. If, on the other hand, the resolution date is distant, then anxiety and anticipation loom larger; since optimism lowers the resulting anxiety, expectations are set high.

The framing of temporal prospects provides one explanation for the overwhelming failure of potential victims of Huntington's Disease to elect for a test for the genetic marker of the disease. A child of a parent with Huntington's Disease has a 50% chance of inheriting the disorder. The disorder is untreatable, incurable, and leads to progressive mental and physical degeneration. Symptoms — involuntary movements of the body, depression, and increasing forgetfulness — appear as early as the age of 30 but can appear as late as one's 50's. Even though the genetic marker has been available for over ten years, fewer than ten percent of the population at risk has chosen to undergo the procedure.[6] Certainly, early resolution has possibly substantial planning benefits — early resolution potentially impacts many important decisions, such as whether to have children, start a new career, buy a house, etc. Thus, failure to test is even more dramatic.

The two outcomes of Huntington's Disease, each equally likely, can be called "No Disorder" (ND) and "Disorder" (D). Those at-risk might use either outcome as the reference point. ND, is the more plausible reference point, largely because it is, in some sense, the status quo. An individual who uses ND as the reference point judges life optimistically, thereby reducing the anxiety of the long years of wait. Optimistic framing implies that the temporal prospect is coded as a loss, $[(.5,D;.5,0),t]$. Such an individual is averse to early resolution and will not test for the genetic marker. Furthermore, if they did elect for the test, they would be penalized for setting expectations so optimistically: if they find out they have the disorder, they would probably be devastated. On the other hand, their elation in the event that they do not have the disorder would almost certainly be of a smaller magnitude than their potential disappointment.

If D, the disease, is the reference point, then the prospect is coded as a gain, $[(.5,0;.5,ND),t]$. Daily life viewed through a pessimistic frame is exceedingly difficult, full of anxiety and worry. Such an individual would prefer to find out immediately to enduring years of waiting for symptoms of the disorder to appear. Low expectations minimize the possibility of dramatic disappointment, but also open the door for the extraordinary elation that comes with finding out that the disorder was not inherited.[7]

4.3. Empirical Validation

We turn to predictions of the model. For a static gamble, cumulative prospect theory predicts when we find risk-seeking or risk-averse behavior. One working assumption is that $a^{+}(\cdot)$

and $b^{+}(\cdot)$ have the same qualitative concave-convex shape. If this assumption holds, then the anxiety model has several implications for gambles with the same probability distribution, but different dates of resolution:

1. When we observe risk aversion (seeking) holds for a static gamble, we should observe preferences for early (delayed) resolution of uncertainty over the respective delayed gamble.
2. As the date of resolution is pushed further and further into the future, the probability weighting function approaches a step function (see Figure 3).

Insert Figure 3 about here

Testing the anxiety model is more difficult than first thought might suggest. First, we must take care not to confound planning effects with anxiety effects. We want a study of primitive preferences for delayed gambles, not induced preferences. In most static lottery questionnaires, the outcome space is monetary. Early resolution of uncertainty for monetary lotteries permits consumption smoothing, a notable benefit. An individual who prefers to have resolution resolved immediately may do so for planning reasons, even though at a primitive level she prefers to delay resolution. Perhaps planning benefits known only to the subject are one reason the experimental results discussed in the introduction are not more dramatic. However, preference for delayed resolution of uncertainty is strong proof for primitive preferences for delays, since there can never be any planning benefits in delaying resolution.

Second, delayed receipt of outcomes must necessarily accompany delayed resolution. However, in many real world problems with delayed resolution, the consequence of choice is received immediately after the uncertainty is resolved. For example, you find out that you have the flu and you suffer from the flu; your company promotes you, and you assume the better job; a jury proclaims you innocent, and you go home a free person. Thus questionnaire problems can easily become artificial or implausible, or perhaps, even more importantly, suspicious. If your payment is determined by the outcome of a coin flipped today, you might wonder why you won't be paid until next month. On the other hand, if the coin flip and the payment both occur in a month, then this seems more natural.

The nature of premiums for early resolution presents a third difficulty. In the model presented here, we assume that individuals who suffer from anxiety will sacrifice some probability of winning to have uncertainty resolved earlier. On the contrary, it is possible that behavior in delayed resolution contexts are driven by rules, like "I won't lower my chances of winning just to know a few days earlier" or, similarly, "I won't assume any additional chance of loss, merely because I'm too weak to wait a week." It's easy to imagine a high school student tormenting over whether she will get into a good college: "It's driving me crazy; I'd give just about anything to know whether I got into college." It's equally easy imagining that the same student would prefer to wait and suffer, rather than find out immediately, but reduce her chances of admission.

When anxiety aversion is pitted directly against a principle (non-compensatory rule) generated perhaps by extreme loss aversion, anxiety loses out. If such rule-based reasoning describes behavior in this domain correctly, then a model that explains preferences in terms of probability premiums will necessarily fail. Such behavior can be explained, however, as a *saliency mismatch* (Prelec and Herrnstein, 1991). A mismatch in saliency occurs when "one element of the cost-benefit pair is vivid and easy to imagine, whereas the other is not." Compare the following two temporal prospects, $[f]_0 = [(.02, \text{sick}; .98, \text{well}),\ 0\ \text{weeks}]$ and $[g]_1 = [(.01, \text{sick}; .99, \text{well}),\ 1\ \text{week}]$. An individual who prefers $[g]_1$ to $[f]_0$ is not willing to assume an extra .01 chance at becoming sick in order to avoid a week of anxiety. The probabilities of illness in this choice are much more salient than the anxiety the individual will suffer from waiting to know. On the other hand, an individual tormented by the idea of being sick might spend a whole day in a doctor's office waiting for a doctor to analyze her. She might even agree that the costs of spending an entire day waiting to see a doctor is in fact higher than assuming an additional .01 chance at becoming sick. In this case, the choice is between $[h]_0 = [(.01, \text{sick+wait}; .99, \text{well+wait}),\ 0\ \text{weeks}]$ and $[g]_1 = [(.01, \text{sick}; .99, \text{well}),\ 1\ \text{week}]$. Now there is no mismatch in saliency. She must simply decide between dedicating a day to finding out her health status immediately or spending a week troubled by whether she is ill. Furthermore, in choosing $[h]_0$, she does not violate any rules she might have against increasing her risk of illness to appease her psyche.

Rule-governed behavior makes testing the anxiety model difficult, but not impossible. If decision makers are governed by rules like the one described above, strictly speaking, the model cannot be correct, since anxiety aversion cannot be measured in terms of probability premiums. However, there still can be truth in the qualitative predictions of the model, *e.g.*, hyperbolically increasing anxiety. To test these predictions, it is necessary to introduce some external yardstick (*e.g.*, time spent, money in the case uncertainty about health status, etc.) to measure the anxiety premium.

A fourth problem, the method of uncertainty resolution, is suggested by a study by Berwick & Weinstein (1985). Many currently or recently pregnant women express a strong desire not to know the sex of their fetus. In fact, 43% of patients surveyed would pay a positive sum of money not to know the information. However, when their ultrasound technician knows the sex of their fetus, many of these patients also want to know. More generally, this study suggests that a lottery that has been resolved (but is not known to you) is not the same psychologically as an unresolved lottery with the same *ex ante* probability distribution. A decision maker might not want to know about a possible loss, since the benefits of hope might outweigh any planning benefits you might get from knowing earlier. However, hope is a fragile state, bordering at times on self-deception; if someone else knows the true outcome (you have already been fired or not fired; the doctor tested for a disease incidentally while performing another procedure), then hope seems less psychologically plausible, and delaying information about the outcome is now no longer as compelling.

Appendix

Proof of Proposition 1: Immediate. ¾

Proof of Proposition 2: First, we prove the result for gains. Using Proposition 1, let $b^+(p) = \pi^+(p,0)$ and $b^-(p) = \pi^-(p,0)$ for all p. Then scale the "period" weighting function, $a^+(\cdot)$ and $a^-(\cdot)$, in the following way: $a^+\left(\pi^+\left(p,0\right)\right) = \pi^+(p,1)$ and $a^-\left(\pi^-\left(p,0\right)\right) = \pi^-(p,1)$ for all p. Let $[f] = (p,0;1-p,x)$, $[g] = (q,0;1-q,x)$, and $[h] = (s,0;1-s,x)$ such that $[f]_0 \mathbf{:} [g]_1$ and $[h]_0 \mathbf{:} [f]_1$. By SOSS, for some $\alpha \geq 1$, $[f]_r \mathbf{:} [g]_{1+\alpha r}$ and $[h]_r \mathbf{:} [f]_{1+\alpha r}$. Simplifying $[f]_0 \mathbf{:} [g]_1$ using Proposition 1, we get $V([f]_0) = \pi^+(p,0)v(x)$ and $V([g]_1) = \pi^+(q,1)v(x)$, which in turn implies

$$\pi^+(p,0) = b^+\left(p\right) = \pi^+\left(q,1\right) = a^+\left(b^+\left(q\right)\right) \tag{1}$$

by the definitions given above. By SOSS,

$$[f]_1 \mathbf{:} [g]_{1+\alpha} \tag{2}$$

and

$$[f]_{1+\alpha} = [g]_{\alpha(\alpha+1)+1} = [g]_{1+\alpha+\alpha^2}\,. \tag{3}$$

We can simplify (2), using Proposition 1:

$$\pi^+(q,1+\alpha) = \pi^+(p,1) = a^+\left(b^+\left(p\right)\right). \tag{4}$$

Substituting (1) in (4) and simplifying, we get $\pi^+(q,1+\alpha) = a^+\left(a^+\left(b^+\left(q\right)\right)\right)$. Similarly, we can simplify (3):

$$\pi^+(q,1+\alpha+\alpha^2) = \pi^+(p,1+\alpha)\,. \tag{5}$$

By the same argument as before, using $[h]_0 \mathbf{:} [f]_1$,

$$\pi^+(p,1+\alpha) = \pi^+(s,1) = a^+\left(b^+\left(s\right)\right) = a^+\left(a^+\left(b^+\left(p\right)\right)\right). \tag{6}$$

Simplifying (5), using (1) and (6), we get $\pi^+(q,1+\alpha+\alpha^2) = a^+\left(a^+\left(a^+\left(b^+\left(q\right)\right)\right)\right)$. By induction, we can see that $\pi^+\left(q,\sum_{i=0}^{r-1}\alpha^i\right) = a^+\left(\mathbf{K}\, r \text{ times}\, \mathbf{K}\left(b^+(q)\right)\mathbf{K}\right)$. Repeating the same argument for $\pi^-(\cdot,\cdot)$, we get the desired result. ¾

Proof of Proposition 3: Under Proposition 2, A2 implies that $b^+(p) = \pi^+(p,0) = b^-(p) = \pi^-(p,0)$

$= p$. ¾

Proof of Proposition 4: Substituting $\alpha = 1$ in (7), we get the desired representation. ¾

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Figures

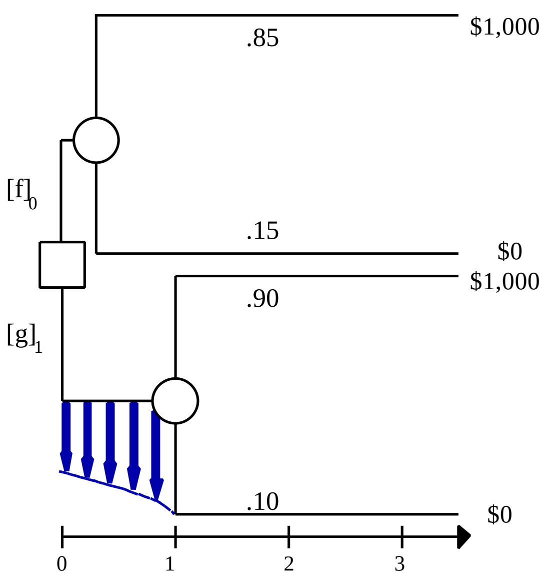

Figure 1a: $[f]_0$ and $[g]_1$ with constantly increasing anxiety

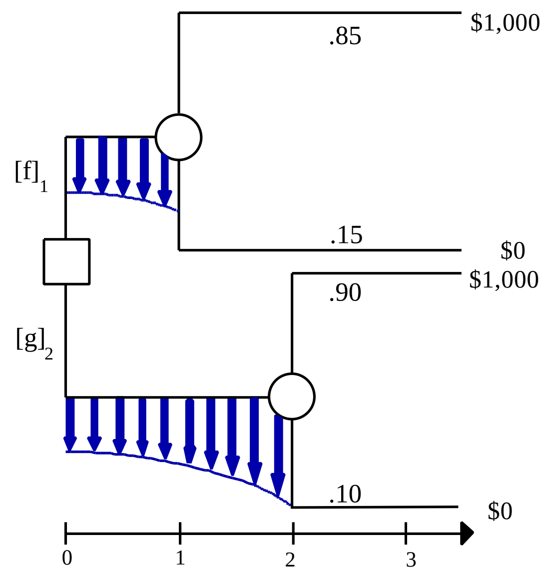

Figure 1b: $[f]_1$ and $[g]_2$ with constantly increasing anxiety

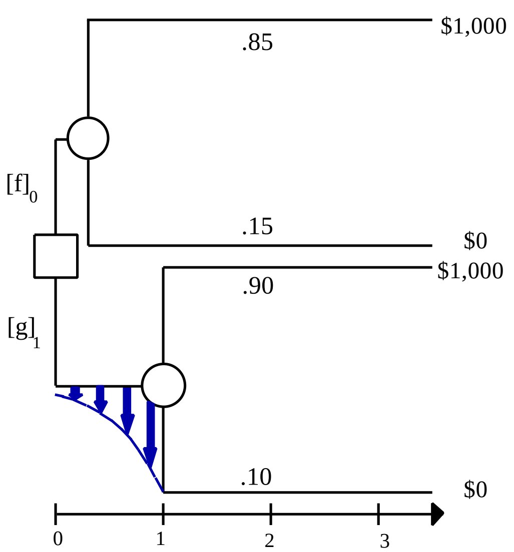

Figure 1c: $[f]_0$ and $[g]_1$ with hyperbolically increasing anxiety

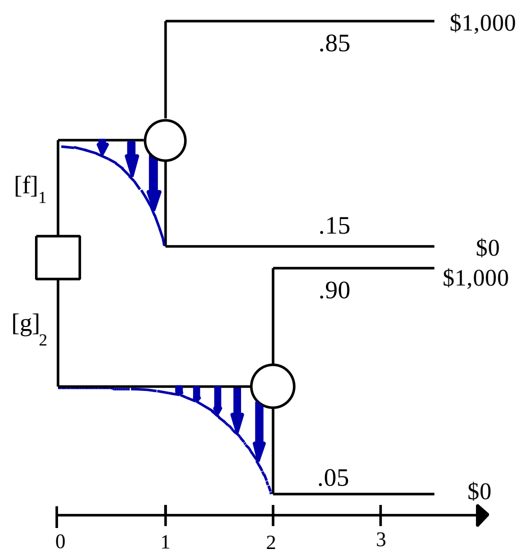

Figure 1d: $[f]_1$ and $[g]_2$ with hyperbolically increasing anxiety

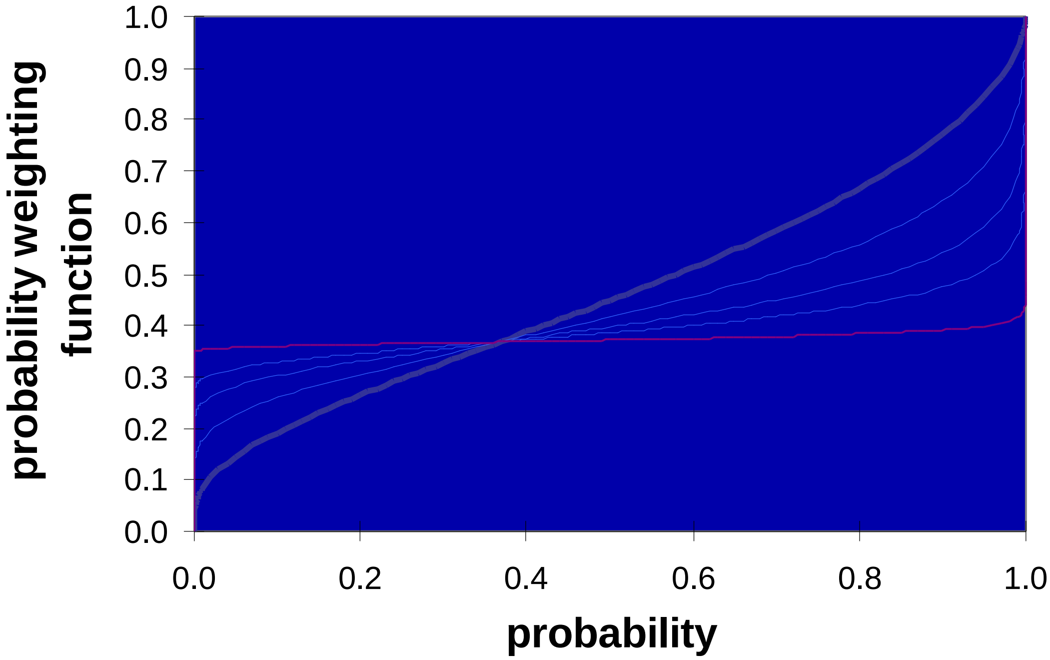

Figure 3: Iterative probability weighting functions

Endnotes

[1] We do not suggest that prospect theory is a perfect model of choice under uncertainty. Tversky & Kahneman (1992) suggest that separability of probabilities and outcomes probably does not hold strictly. See also Camerer (1989) and Wu (1994).

[2] Two separate surveys were conducted and then pooled for simplicity. A binomial test of shows significance at the .05 level for both subject pools taken individually.

[3] We could obtain a more general result by assuming that SOSS holds only for gambles that are strictly gains or strictly losses. In this case, the constants in Propositions 2 and 3 would be sign-dependent, α^- and α^+, rather than α.

[4] Prelec's function is concave below $1/e \approx .37$, and convex above that value.

[5] In direct contrast, purchases of instant tickets (*i.e.*, scratch tickets) are largely impulsive. At first glance, such behavior seems to be difficult to reconcile with the anxiety model, since risk-seeking behavior should accompany preferences for delayed resolution. However, it is possible that excessive impatience for cash (a major factor, since instant tickets are typically paid in full immediately upon winning) overwhelms the suspense/hope gained from delaying scratching.

[6] The costs of testing for the genetic marker are not inconsequential, in terms of time or money. Brody (1988) reports that, because of variants in the same genetic marker, the test requires blood from one related Huntington victim plus six to eight other family members. In addition, testing can be performed at only 20 sites nationwide and costs about $4,000.

[7] Brody (1988) reports anecdotal evidence consistent with the interpretation given. She quotes Dr. Jason Brandt, a psychologist at John Hopkins: "No one who got a positive result has become severely

depressed, given up hope, required psychotropic medication or hospitalization and there has been no talk of suicide or family disruption."