A prospect theory model of goal behavior

George Wu∗
Chip Heath†
Richard Larrick‡

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Goals have a powerful effect on performance: higher goals typically produce better performance. Previous research has proposed three key mechanisms to explain these results: effort, persistence, and attention. We present a formal model that relates these mechanisms to a single underlying process. Our model assumes that goals divide performance into two regions, gains and losses, and that the resulting gains and losses are coded according to a prospect theory value function (Kahneman & Tversky, 1979). This simple model explains the stylized findings in the goal setting literature, while also offering several new testable predictions.

∗ Graduate School of Business, University of Chicago, george.wu@gsb.uchicago.edu
† Graduate School of Business, Stanford University, heath_chip@gsb.stanford.edu
‡ Fuqua School of Business, Duke University, larrick@duke.edu
Much research on goal setting has shown that higher goals tend to lead to higher performance (see, Locke & Latham, 1990, for a review). These consistent findings have been explained as arising from three underlying mechanisms: goals encourage individuals to expend more effort, persist longer, and direct attention to the goal at the expense of other activities. We show how these three mechanisms—effort, persistence, and attention—can be understood as outcomes of a single underlying process. We model the effect of goals on performance by positing that goals divide the world into losses and gains as described by the prospect theory value function (Kahneman & Tversky, 1979). In doing so, our model integrates the three underlying mechanisms proposed by previous researchers and offers some new testable implications.

This paper provides a formal model for Heath, Larrick, & Wu’s (1999) “goals as reference points” approach. Heath et al. showed how the major results in the goal setting literature can be explained by an approach in which goals inherit the properties of the prospect theory value function. That paper also developed and tested some new predictions of the value function on goal behavior, including implications of reference-dependence, loss aversion, and diminishing sensitivity.

Our paper is organized as follows. In Section 1, we present a brief review of the goal setting literature, in particular the three mechanisms proposed by Locke & Latham (1990)—effort, persistence, and attention. In Section 2, we present our prospect theory model of goal behavior and show how effort and persistence can be interpreted within this framework. In Section 3, we show how our model explains the final mechanism, attention. We then develop some new implications of our model. In Section 4, we discuss the relationship between loss aversion and “piling up”, the tendency of performance to exceed moderately challenging goals.
by a small amount. In Section 5, we discuss how the beneficial effects of subgoaling can be interpreted in terms of the value function. We conclude in Section 6.

1. Literature

In many areas in psychology, key theoretical results are demonstrated by a handful of studies. The literature on goals is exceptional in that its key findings have been replicated so many times in so many domains. “Goal setting studies have been conducted with 88 different tasks including bargaining, driving, faculty research, health promoting behaviors, logging, maintenance and technical work, managerial work, management training, and safety.” (Locke & Latham, 1991, p. 212). A decade ago when Locke and Latham (1990) conducted their comprehensive book-length review, researchers had conducted 239 laboratory and 156 field studies involving over 40,000 people; the number has increased since then.

The most frequently researched topic in the goal literature is how the content of goals affects performance. Research has typically found that people perform better when they are given goals that are specific and difficult. Mento, Steel, & Karren (1987) suggested that “If there is ever to be a viable candidate from the organizational sciences for elevation to the lofty status of a scientific law of nature, then the relationships between goal difficulty, difficulty / specificity, and task performance are most worthy of serious consideration” (p. 74).

For a number of years, the goal literature focused only on documenting the fact that goal difficulty and specificity affected performance in predictable ways (Naylor & Ilgen, 1984). More recently, researchers have spent time trying to understand why goals work as they do. The most widely cited explanation of the performance results is that of Locke & Latham (1990, 1991) who attribute these findings to a small number of “mediators or causal mechanisms” that are “relatively direct and automatic consequences of goal-directed activity” (Locke & Latham, 1991,
Below, we review the three mechanisms proposed by Locke and Latham (1990)—effort, persistence, and attention. While we applaud their focus on mechanisms in a literature that has historically not focused much on explaining its own robust empirical results (e.g., see the critique of Naylor & Ilgen, 1984), we propose that the mechanisms explained by Locke and Latham can themselves be explained at a deeper level as arising from a single underlying process. In the remainder of the paper, we propose a prospect theory model of goal behavior that explains and integrates Locke and Latham’s three “separate” mechanisms.\(^\S\)

**Goals Increase Effort**

First, Locke & Latham (1990) have argued that goals increase “effort or energy expenditure (i.e., intensity)...” They propose that “(t)his is the core explanation of the goal difficulty effect” (p. 227). A wide variety of experiments have demonstrated that goals increase effort. On physical tasks, goal subjects exercise at a higher rate (Bandura & Cervone, 1983), squeeze a grip harder (Botterill, 1977), lift more weight (Ness & Patton, 1979), and pedal more rapidly on a bicycle (Roberts & Hall, 1987). On cognitive tasks, goal subjects react faster (Locke *et al.*, 1970), perform better on well-rehearsed tasks like addition (Bryan & Locke, 1967) and subtraction (Bandura & Schunk, 1981). Subjects even perform better on less-routinized tasks like brainstorming (Garland, 1982) and anagrams (Sales, 1970). Even when the time period is exceedingly constrained, subjects with higher goals recorded substantially more creative uses for a common object even when they were given only one minute to perform (Locke, 1982).

\(^\S\) As Locke and Latham have noted (1990), the individual results summarized below are often hard to classify as the result of only one mechanism. For example, it is not possible to completely separate the effects of effort and persistence since the same amount of work can be performed by varying either rate of work or time on task. In some cases, people work at a higher rate and for a longer period of time (Latham & Locke, 1975), and in other cases
**Goals increase Persistence**

The second mediating mechanism according to Locke and Latham is persistence: “goals motivate individuals to persist in their activities through time. Hard goals ensure that individuals will keep working longer than they would with vague or easy goals. Hard or challenging goals inspire the individual to be tenacious in not settling for less than could be achieved” (Locke & Latham, 1990, p. 95).

Like effort, persistence is found across a wide variety of tasks. On physical tasks, high goal people compressed a hand dynamometer longer (Hall, Weinberg, & Jackson, 1987), and persisted longer in a classic pain tolerance task (immersing one’s arm in very cold water; Stevenson, Kanfer, & Higgins, 1984). People also persist longer on mental tasks. On anagrams, Sales (1970) found that high goal subjects worked longer and rested less. On a complex mirror maze, high goal subjects completed more than twice as many trials as subjects without goals (Singer, Korieneck, Jarvis, McCloskey, & Candeletti, 1981). On a prose comprehension task, high goal people spent more time studying than subjects with do best goals (Laporte & Nath, 1976). High goal people were also less likely to quit when they encounter difficulties. On bargaining tasks, high goal subjects bargained longer (Neale & Bazerman, 1985; Siegel & Fouraker, 1960), and held their positions longer rather than compromising (Huber & Neale, 1987).

**Goals direct Attention**

Third, Locke & Latham (1990) suggested that goals “direct activity toward actions which are relevant to it at the expense of actions which are not goal-relevant” (Locke & Latham, 1991, p. 227). For example, Locke & Bryan (1969) gave subjects objective feedback about several people adjust effort to the amount of time allowed (Bryan & Locke, 1967). One of the advantages of our model is
aspects of their driving performance on a driving course (e.g., steering, braking, acceleration), but gave them goals on a single performance dimension. Scores changed only on the dimension for which a goal was set. Other studies show similar results, e.g., Nemeroff & Cosentino (1979), Rothkopf & Billington (1979), and Wyer, Srull, Gordon, & Hartwick (1982).

Because goals refocus attention, when people are assigned a goal on one dimension, they may decrease performance on another. Organ (1977) found that subjects who were given a proofreading goal learned less about the content of the passage than subjects with no goal. When people are assigned a difficult quantity goal, they sometimes decrease work quality. For example, when subjects in one experiment were asked to list objects that were hard, white and edible; under the pressure of a high goal, many subjects listed objects that were hard and white, but not very edible (Bavelas & Lee, 1978). Other similar results involved tasks such as sentence composition, typing, and generating creative ideas.

2. A prospect theory model for goal setting

The basic model for goal-setting is classical in most respects: individuals balance benefits and costs by equating marginal benefits and marginal costs. The main departure is with our assumption of the prospect theory value function for the benefit function.

Preliminaries

We assume that an individual has a utility function that is additively separable in costs and benefits. In particular, the utility function $U(x)$ has the following form,

\begin{equation}
U(x) = b(x) - c(x),
\end{equation}

where $b(x)$ is the benefit of achieving $x \geq 0$ units, and $c(x)$ is the cost of achieving $x$ units.
We consider a special case of the optimization of Eq. (1):

**Assumption 1 (Myopic optimization):** The individual stops performing when the marginal cost of obtaining an additional unit first exceeds the marginal benefit of obtaining that unit, *i.e.*, the optimal performance is given by \( x^* = \min(x_1, ..., x_n) \), where \( x_1, ..., x_n \) are solutions to \( b'(x_i) = c'(x_i) \). If \( c'(0) < b'(0) \), then the optimal performance is given by \( x^* = 0 \).

We require the assumption of myopic optimization because solutions to \( b'(x_i) = c'(x_i) \) with a prospect theory value function are not necessarily unique. The last part is required to eliminate the “starting problem”, *i.e.*, situations in which the marginal cost of obtaining the first unit is higher the marginal benefit obtained from that unit (see, Heath, Larrick, & Wu, 1999). However, we return to this issue in Section 5 when we discuss subgoals and the starting problem.

A myopic individual will sometimes behave differently than a forward looking one.

**Assumption 1’ (Forward Looking):** The individual chooses the performance level \( x \) that maximizes Eq. (1), *i.e.*, the optimal performance is given by \( x^* = \arg \max_x b(x) - c(x) \).

Next, we make a standard restriction that the cost function, \( c(\cdot) \), be strictly positive and increasing.

**Assumption 2 (Increasing marginal costs):** For all \( x \), the cost function, \( c(x) \) has the following properties: \( c(x) > 0 \) and \( c'(x) > 0 \).

Assumptions 1 and 2 are classical. Our main departure is the following assumption that benefits are defined with respect to the goal \( g \).

**Assumption 3 (Prospect theory value function):** The benefit function, \( b_g(x) \), given a particular goal \( g \), is given by \( v(x - g) \), where \( v(\cdot) \) is defined as follows:

1. \( v(0) = 0 \);
2. \( v'(x) > 0 \);
3. \( v(x) < -v(-x) \) for \( x > 0 \);
4. \( v'(x) > 0 \) for \( x < 0 \), and;
5. \( v'(x) < 0 \) for \( x > 0 \).
Assumption 3 requires that performance is coded relative to a goal, with outcomes short of the goals deemed losses, and outcomes exceeding the goal deemed gains; that losses loom larger than gains; and that gains become relatively less pleasurable as they increase and losses become relatively less painful as they become larger. These properties can be thought of as reference dependence, loss aversion, and diminishing sensitivity, respectively (Kahneman & Tversky, 1979; Tversky & Kahneman, 1991, 1992). Together, Assumption 3 requires that the prospect theory value function have the characteristic S-shape as shown in Figure 1.

![Figure 1](image)

**Figure 1**: A typical prospect theory value function

Let $x'(g; c(), v())$ be the optimal performance given a goal $g$, assuming a cost function $c()$, and a value function $v()$. For simplicity, when $c()$ and $v()$ remain the same, we refer to $x'(g)$ for simplicity.
Results

Our model predicts that performance will be an inverted U-shaped function of the goal level: performance increases as goal levels increase for "low" goals and decreases as goal levels increase for "high" goals. We formalize this statement in the following proposition:

**PROPOSITION 1:** Let Assumption 1, 2, and 3 hold. Then,

(i) \( \frac{dx^*(g)}{dg} > 0 \) if and only if \( v'(x^*(g) - g) < 0 \) if and only if \( x^*(g) > g \);

(ii) \( \frac{dx^*(g)}{dg} < 0 \) if and only if \( v'(x^*(g) - g) > 0 \) if and only if \( x^*(g) < g \).

Proposition 1 indicates that increasing a goal improves absolute performance if individuals exceed the initial goal (the goal is "easy"), while the opposite is true if individuals do not exceed the initial goal (the goal is "hard"). As first blush, the second result seems at odds with previous empirical findings which show that more difficult goals typically increase average performance. Locke & Latham (1991) report that 91% of 192 studies of goal difficulty have found that higher goals produce higher performance. Note, however, that Proposition 1 captures how individual performance changes as a function of different goals, while the studies of goal difficulty measure how collective performance changes as a result of different goals. Thus, Proposition 1 is consistent with the goal difficulty result if increased performance by individuals who are challenged by a higher goal overcomes the decreased performance by individuals who fall further behind their goal. Heath, Larrick, & Wu (1999, pp. 100-102) show that an asymmetry of this type occurs under reasonable parametric assumptions—higher goals produce slightly worse performance for low ability individuals, but dramatically improved performance for high ability individuals.
Figure 2: Goals of 30 and 40 with a shallow marginal cost function. The higher goal produces higher performance.

We illustrate Proposition 1 graphically and then provide a formal proof. Figure 2 depicts the marginal benefit functions for an individual with a goal of 30, $b'_{30}(x)$, and for an individual with a goal of 40, $b'_{40}(x)$. Also depicted is a marginal cost function, $c'_1(x)$. Note that both individual exceed their goals. In other words, the marginal cost function intersects the marginal benefit function for each above their respective goals. However, the individual with a goal of 30 attains 35 units, while the individual with a goal of 40 attains 42 units, consistent with the (i) part of Proposition 1.

Next consider the same marginal benefit functions in Figure 3. Figure 3 differs from Figure 2 only in the steepness of the associated cost function, $c'_2(x)$. Since marginal costs are now higher, both individuals attain fewer units. Moreover, the individual with a goal of 30 falls short of her goal, attaining 19 units. Consistent with the second part of Proposition 1, the individual with a goal of 40 units does even worse, 17 units.
Figure 3: Goals of 30 and 40 with a steep marginal cost function. The lower goal produces higher performance.

We now prove Proposition 1 formally. To do so, we establish the following Lemma.

**LEMMA:** Let Assumptions 1, 2, and 3 hold. Then \( v''(x^*(g) - g) < c''(x^*(g)) \).

**PROOF OF LEMMA:** Since \( x^*(g) \) is the optimal performance given goal \( g \), \( v'(x^*(g) - g) = c'(x^*(g)) \). By Assumption 1 and 2, \( v'(x^*(g) - g - \varepsilon) > c'(x^*(g) - \varepsilon) > 0 \) for \( \varepsilon > 0 \). Therefore, \( v''(x^*(g) - g) < c''(x^*(g)) \).

We now use this Lemma to prove Proposition 1.

**PROOF OF Proposition 1:** Assumption 3 requires that \( v''(x^*(g) - g) < 0 \) if and only if \( x^*(g) > g \), and \( v''(x^*(g) - g) > 0 \) if and only if \( x^*(g) < g \). Thus it remains to be proved that \( dx^*(g)/dg < (>) 0 \) if and only if \( v'(x^*(g) - g) > (<) 0 \).

Rewriting Eq. (2) under Assumption 3, we have

\[
(3) \quad c'(x^*(g)) = v'(x^*(g) - g).
\]

Using the implicit function theorem and differentiating Eq. (3) with respect to \( g \), we get:
(4) \[ c^*(x^*(g)) \frac{dx^*(g)}{dg} = v^*(x^*(g) - g) \left( \frac{dx^*(g)}{dg} - 1 \right). \]

Rewriting Eq. (4) yields

(5) \[ \frac{dx^*(g)}{dg} = \frac{v^*(x^*(g) - g)}{v^*(x^*(g) - g) - c^*(x^*(g))}. \]

We use the Lemma to prove the result. By the Lemma, we know that \( v^*(x^*(g) - g) - c^*(x^*(g)) < 0 \), which indicates that \( \frac{dx^*(g)}{dg} \) and \( v^*(x^*(g) - g) \) must have opposite signs. Thus, \( \frac{dx^*(g)}{dg} < 0 \) if and only if \( v^*(x^*(g) - g) > 0 \), and \( \frac{dx^*(g)}{dg} > 0 \) if and only if \( v^*(x^*(g) - g) < 0 \).

Proposition 1 implies that if an individual surpasses her goal, she could have performed even better had she increased her goal by a small amount. From Eq. (5) and \( v^*(x^*(g) - g) - c^*(x^*(g)) < v^*(x^*(g) - g) \), we note that for \( x^*(g) > g \), \( \frac{dx^*(g)}{dg} < 1 \). In other words, increasing a goal by 10 units will increase performance by less than 10 units.

It is also important to note that it is critical that \( v(\cdot) \) be strictly concave above the reference point and strictly convex below it. It is easy to see from Eq. (5) that \( \frac{dx^*(g)}{dg} = 0 \) if and only if \( v(\cdot) \) is linear. Thus, a piecewise linear value function with loss aversion will not predict that higher goals lead to higher performance.

Finally, although it appears that the first and second parts of the Proposition are inconsistent, they are not. They would be inconsistent for \( v'(\cdot) \) continuous. However, Assumption 3 requires that the first derivative of the benefit function be discontinuous. Proposition 1 requires that the result hold for \( x^*(g) > 0 \) and for \( x^*(g) < g \), because there is a discontinuity at \( x^*(g) = g \).
Interpreting effort and persistence

To interpret the empirical results on effort and persistence, we posit two strictly increasing functions, \( x = h(e) \) and \( x = k(t) \). The function \( h(\cdot) \) indicates how many units of performance is produced by a particular level of effort \( e \), while \( k(\cdot) \) indicates how many units of performance is produced by working for a particular time \( t \). We can re-write Eq. (2) as \( b'(h(e)) = c'(h(e)) \) and \( b'(k(t)) = c'(k(t)) \) respectively and solve for the optimal \( e \) and \( t \). The results will be identical since \( h(\cdot) \) and \( k(\cdot) \) are strictly increasing. To see the required result, let \( e^*(g) \) be the optimal effort for a goal \( g \). The First Order Condition is now obtained by differentiating the following equation with respect to \( e^*(g) \):

\[
c'(h(e^*(g)))h'(e^*(g)) = v'(h(e^*(g))-g)h'(e^*(g)).
\]

After implicitly differentiating with respect to \( g \) and re-arranging, this becomes

\[
\frac{de^*(g)}{dg} = \frac{v'(h(e^*(g))-g)h'(e^*(g))}{v'(h(e^*(g))-g)h'(e^*(g))-c'(h(e^*(g)))h'(e^*(g))}.
\]

Since \( h'(e^*(g)) \) can be factored out of the numerator and denominator and, by assumption, \( de^*(g)/dg \) is always the same sign as \( dx^*(g)/dg \), the results of Proposition 1 follow immediately. The analysis for persistence is identical and hence is omitted.

3. Goals and Attention

We noted above that goals direct attention from one activity to another. Studies have shown that individuals given a goal on, say, quantity, improve quantity but not other performance dimensions, such as quality. We show that this result also follows from a prospect theory model of goal behavior.
Let there be $n$ dimensions of performance. For simplicity, we let the first dimension denote the dimension with the goal intervention. We denote the performance on dimension $i$, $x_i$.

As before, we assume that an individual has a utility function that is additively separable in costs and benefits. Moreover, we assume that the benefit function is additively separable in the unidimensional benefit functions. In particular, the utility function $U(x)$ has the following form,

$$U(x) = \sum_{i=1}^{n} w_i v_i(x_i) - c_i(x_i)$$

where $v_i(x_i)$ and $c_i(x_i)$ are the value and cost of attaining $x_i \geq 0$ units and $w_i$ is the weight for dimension $i$.

We assume that performance, $(x_1, \ldots, x_n)$, is a function of time allocated to each dimension, i.e., $x_i = f_i(t_i)$. We let $t_i$ denote the time devoted to dimension $i$ and impose the constraint that $\sum_{i=1}^{n} t_i \leq T$, the total amount of time available. We assume that there are positive returns to an additional unit of time devoted to dimension $i$.

**Assumption 4 (Positive returns on time):** For all $i$, $f'(t_i) > 0$.

We also make an assumption about the cost function that is analogous to Assumption 2, cost is an increasing function of the time expended on a particular dimension:

**Assumption 5 (Cost function):** The cost function has the following properties: for all $i$, $c_i(\cdot) > 0$ and $c'_i(\cdot) > 0$.

We next consider the form of the value function. Recall that the first dimension is the one with the goal intervention.

**Assumption 6 (Value function):** The value function has the following properties:

(i) $v_i(\cdot)$ has the properties of $v(\cdot)$ in Assumption 3;
(ii) for all \( i > 1 \), \( \nu'_i(x) > 0 \), \( \nu''_i(x) < 0 \).

Assumption 6 requires that the value function for the first dimension is defined with respect to a goal \( g \) and inherits the properties of the prospect theory value function. The value functions for all other dimensions are concave and increasing.

Finally, we assume that individuals maximize (6) by choosing the best allocation of time, \((t^*_1, \ldots, t^*_n)\), i.e., the maximization problem is equivalent to:

\[
\max_{t_1, \ldots, t_n} U(x_1, \ldots, x_n) = w_1 \nu_1 (f_1 (t_1) - g) + \sum_{i=2}^{n} w_i \nu_i (f_i (t_i)) - \sum_{i=1}^{n} c_i (f_i (t_i)).
\]

We prove the following Proposition that relates goals to attention:

**Proposition 2:** Let Assumptions 1 and 4 through 6 hold. Then

(i) if \( x^*_1 = f_1 (t^*_1) > g \) then \( dt^*_1 (g) / dg > 0 \);

(ii) if \( x^*_1 = f_1 (t^*_1) < g \) then \( dt^*_1 (g) / dg < 0 \).

**Proof of Proposition 2:** The first order condition is

\[
w_j \nu'_j \left( f_j (t^*_j) - g \right) f'_j (t^*_j) - c'_j (t^*_j) = w_j \nu'_j \left( f_j (t^*_j) \right) f'_j (t^*_j) - c'_j (t^*_j) = \lambda,
\]

for all \( j = 1, \ldots, n \), where \( \lambda \) is the Lagrange multiplier for the constraint on time. If \( \sum_{i=1}^{n} t^*_i = T \), then \( \lambda \geq 0 \), otherwise \( \lambda = 0 \). Using the implicit function theorem and differentiating Eq. (8) with respect to \( g \), we get:

\[
c'_1 (f_1 (t^*_1 (g))) \frac{dt^*_1 (g)}{dg} = w_1 \nu'_1 (f_1 (t^*_1 (g)) - g) \left( \frac{dt^*_1 (g)}{dg} - 1 \right).
\]

Rewriting Eq. (9), we get

\[
\frac{dt^*_1 (g)}{dg} = \frac{w_1 \nu'_1 (f_1 (t^*_1 (g)) - g)}{w_1 \nu'_1 (f_1 (t^*_1 (g)) - g) - c'_1 (f_1 (t^*_1 (g)))}. \]

The rest of the proof follows by applying the logic used in the proof of Proposition 1.

We might expect a change in the goal on the targeted dimension to affect performance on the other dimensions, when the time constraint is binding, \( \sum_{i=1}^{n} t^*_i = T \). Indeed, this is the case.
The following Proposition establishes that in the presence of a time constraint, increasing the goal on one dimension results in lower performance on another dimension.

**PROPOSITION 3**: Let Assumptions 1 and 4 through 6 hold. Then for \( j \neq 1 \), if \( x_i^* = f_i(t_i^*) > g \), then \( dt_j^*(g)/dg \leq 0 \).

**PROOF OF PROPOSITION 3**: Implicitly differentiating (8) with respect to \( g \) for \( j = 1 \) and \( j \neq 1 \), we get:

\[
(10) \quad w_j v_j'(f_j(t_j^*(g))) - g \left( \frac{dt_j^*(g)}{dg} - 1 \right) - c_j'(f_j(t_j^*(g))) \frac{dt_j^*(g)}{dg} =
\]

\[
= w_j v_j'(f_j(t_j^*(g))) \frac{dt_j^*(g)}{dg} - c_j'(f_j(t_j^*(g))) \frac{dt_j^*(g)}{dg}.
\]

If \( x_i^* = f_i(t_i^*) > g \), then from Proposition 2, we know that \( dt_i^*(g)/dg > 0 \), and, from above, that \( \frac{dt_j^*(g)}{dg} < 1 \). Therefore, it is clear that the left-hand side of Eq. (10) is negative. Since \( w_j v_j'(f_j(t_j^*(g))) - c_j'(f_j(t_j^*(g))) = \lambda \geq 0 \), \( dt_j^*(g)/dg \leq 0 \).

\[\blacksquare\]

4. Piling Up

In this section, we show that a new implication called “piling up” follows from certain reasonable assumptions concerning the value function, in particular, assumptions about loss aversion. Informally, piling up implies that performance for “moderately” challenging goals will tend to exceed the goal, but only by a very little. In other words, performance will tend to “pile up” around the goal.

To illustrate, consider low, medium, and high ability producers, who must exert high, medium, and low cost respectively to produce the same number of units of performance (see Figure 4). Next, consider a goal of 10 units. This goal is “easy” in the sense that the low, medium, and high ability producers all exceed the goal by a generous margin, completing 21, 26, and 33 units respectively.
We now consider the “medium” goal of 35 units depicted in Figure 5. The goal of 35 units is more challenging – all three of our individuals exceed the goal, but never by much. Our high ability producer completes 41 units, exceeding the goal by 6 units, whereas the medium and low ability producers complete 36 and 35 units respectively.

Finally, we turn to a “high” goal of 60 units (Figure 6). This goal is sufficiently challenging that only the high ability producer completes the goal, achieving 61 units. The medium and low ability producers achieve 32 and 24 units respectively.

In this demonstration, low and high goals yield the highest variation in performance, whereas the medium goal has the lowest spread. In addition, note that the medium goal induces performance that exceeds the goal of 35 units by a small amount. In other words, performance “piles up” around the goal. Note also that this numerical example is consistent with the goal difficulty effect: the highest goal of 60 units leads to the highest average performance, even though two of the three producers fall short of the goal. The goal setting literature generally reports means but not distributions of performance. One exception is Garland (1985), who found that the variance for an extremely challenging goal was 75% higher than the variance for a somewhat less challenging goal (see Heath, Larrick, & Wu, 1999, pp. 101-102).

We tie the intuitive notion of piling up to the value function and show how piling up is related to loss aversion. First, we formalize the notion of piling up. The definition defines a set of goals in which performance exceeds a particular goal by \( \delta \) or less.

**Definition:** For a particular cost function, \( c(\cdot) \), and a particular value function, \( v(\cdot) \), a set of goals, \( G(\delta;c(\cdot),v(\cdot)) \), exhibits \( \delta \)-piling up if for all \( g \in G(\delta;c(\cdot),v(\cdot)) \), \( 0 \leq x^*(g;c(\cdot),v(\cdot)) - g \leq \delta \).

To show that “piling up” is an implication of loss aversion, we first make a simplifying assumption about loss aversion that follows Tversky & Kahneman (1991).
Assumption 7 (Constant Loss Aversion): The value function, $v()$, exhibits constant loss aversion, i.e., for all $x > 0$, $-v(-x) = kv(x)$.

Figure 4: An easy goal with low, medium, and high ability producers

Figure 5: A medium goal with low, medium, and high ability producers
We prove the following Proposition on piling up.

**Proposition 4**: Let Assumption 1, 2, and 3 hold. Also, let Assumption 7 hold for \( v_1(\cdot) \) and \( v_2(\cdot) \). Finally, let \( v_1(x) = v_2(x) \) for \( x > 0 \), \( -v_1(-x) = k_1v_1(x) \) and \( -v_2(-x) = k_2v_2(x) \) for \( x < 0 \), where \( k_1 < k_2 \). Then, for all \( \delta \), \( G(\delta; c(\cdot), v_1(\cdot)) \subseteq G(\delta; c(\cdot), v_2(\cdot)) \).

We interpret Proposition 4 as requiring that the set of goals that satisfy \( \delta \)-piling up weakly increases as the loss aversion coefficient increases, all else equal. Proposition 4 assumes that individuals have the same marginal cost function, \( c(\cdot) \), the same value function, but may differ on individual goals. As the loss aversion coefficient increases, the number of individuals who exceed their goal by a small amount increases as well.

![Figure 6: A high goal with low, medium, and high ability producers](image)

**Proof of Proposition 4**: We prove the result by contradiction. Assume that Proposition 4 is false. Then there exists some \( g' \), such that \( g' \in G(\delta; c(\cdot), v_1(\cdot)) \) but \( g' \notin G(\delta; c(\cdot), v_2(\cdot)) \). If \( g' \notin G(\delta; c(\cdot), v_2(\cdot)) \), then either \( x^*(g'; c(\cdot), v_2(\cdot)) - g' > \delta \) or \( x^*(g'; c(\cdot), v_2(\cdot)) - g' < 0 \). Consider the first case. By Assumption 1, \( v_2'(x^*(g'; c(\cdot), v_2(\cdot)) - g') = c_2'(x^*(g'; c(\cdot), v_2(\cdot)) - g') \), but
\[ v'_2(x) > c'_2(x) \quad \text{for all } x < x^*(g';c'(\cdot),v_2(\cdot)). \] However, \[ v'_2(x^*(g';c(\cdot),v_1(\cdot)) - g') = c'_2(x^*(g';c(\cdot),v_1(\cdot)) - g') \] since and \( v_2(\cdot) \) coincide in the gains. Thus, \[ x^*(g';c(\cdot),v_1(\cdot)) - g' < 0 < \delta. \] In this case, Assumption 1 implies that there exists \( x < g' \) such that \( v'_2(x) = c'_2(x) \). However, since \( k_1 < k_2 \), \( v'_1(x) < c'_2(x) \), which implies that \( x^*(g';c(\cdot),v_1(\cdot)) < g' \) which is a contradiction.

An alternative formulation of piling up is found below. This proposition begins with a different premise. We assume that Individuals 1 and 2 have the same value function but different cost functions, \( c_1(\cdot) \) and \( c_2(\cdot) \). Proposition 5 requires that there exists a sufficiently large loss aversion coefficient that if Individual 1 surpasses the goal by a small amount, then Individual 2 with a steeper cost function will also surpass the goal.

**Proposition 5:** Let Assumption 1 hold, Assumption 2 hold for \( c_1(\cdot) \) and \( c_2(\cdot) \), and Assumptions 3 and 7 hold for \( v_1(\cdot) \) and \( v_2(\cdot) \), where \(-v_1(-x) = k_1v_1(x) \) for \( x < 0 \), \( v_1(x) = v_2(x) \) for \( x > 0 \), and \(-v_2(-x) = k_2v_2(x) \) for \( x < 0 \), where \( k_1 < k_2 \). Also, let \( c_2(x) = f(c_1(x)) \) for \( f(\cdot) \) strictly increasing and \( c_2(\cdot) \) be bounded above for all \( g \in G(\delta;c_1(\cdot),v_1(\cdot)) \). Then there exists a finite \( k_2 \) such that if \( g \in G(\delta;c_1(\cdot),v_1(\cdot)) \), then \( g \in G(\delta;c_2(\cdot),v_2(\cdot)) \).

**Proof of Proposition 5:** If \( g \in G(\delta;c_2(\cdot),v_1(\cdot)) \), then the Proposition clearly holds. Thus, we must consider the case in which \( g \in G(\delta;c_1(\cdot),v_1(\cdot)) \) and \( g \not\in G(\delta;c_2(\cdot),v_1(\cdot)) \). The latter implies that for some \( x_1 < 0 \), \( v'_1(x_1) = c'_2(x_1) \). Thus, if \( k_1 < k_2 \), \( v'_2(x_1) > c'_2(x_1) \). The \( k_2 \) that establishes Proposition 5 can be constructed similarly: \[ \frac{k_2}{k_1} = \max_{x < 0} \frac{c'_2(x)}{v'_1(x)} + \varepsilon \] for some arbitrarily small \( \varepsilon \). Such a \( k_2 \) can always be found and is finite since \( c_2(\cdot) \) is bounded above.

**5. Subgoals and the Starting Problem**

The “goals as reference points” approach suggests that individuals far away from the goal may give up because they do not feel that they are making sufficient progress toward the goal. Heath, Larrick & Wu (1999) called this the “starting problem” and presented some empirical evidence supporting this implication.
The starting problem is relevant when forward looking and myopic optimization produce different levels of performance. Figure 7 illustrates such a situation. Since \( c'(0) > v'(0) \), under myopic optimization, this individual stops immediately. Note, however, that \( c'(x) \) is quite flat, and \( c'(x) < v'(x-g) \) for \( x > 35 \). Indeed, the global optimum is \( x^* = 51 \).

More formally, the starting problem follows from diminishing sensitivity of the value function in losses. Since \( v'(x-g) \) might be close to zero for large \( g \), it is possible that \( c'(x) > v'(x-g) \) for small \( x \). The starting problem depicted in Figure 7 can be overcome if the goal of 50 is broken down into subgoals. For example, suppose that an individual sets five subgoals of 10. Now, the individual is never more than 10 units away from the goal. This transformation is illustrated in Figure 8. The saw-toothed marginal benefit function is written \( b'_{0,10,10,10,10}(x) \) to indicate the division of the goal of 50 into 5 subgoals of 10. The optimal performance under subgoaling is \( x^* = 51 \), even in the case of myopic optimization.

More generally, suppose that a goal \( g \) is decomposed into \( n \) subgoals of \( g_s \) such that \( g = ng_s \). The subgoals serve to redefine the marginal benefit function:

\[
b'_{g,\ldots,g}(x) = \begin{cases} 
  v'(x-ig_s), & (i-1)g_s < x < ig_s, \\
  v'(x-g), & x > g 
\end{cases},
\]

where \( i=1,\ldots,n \) indexes the subgoals. It is easy to see that subgoaling cannot decrease performance and may help. To see this, it is sufficient to note that \( b'_{g,\ldots,g}(x) \geq b'_g(x) \) for all \( x \). Of course, the two functions coincide for \( x > g \). For \( x < g \), \( v'(x-ig_s) \geq v'(x-g) \) for all \( i \geq 1 \) since \( x-ig_s \leq x-g \) for \( i=1,\ldots,n \) and \( v''(x) > 0 \) for \( x < 0 \).
Figure 7: An illustration of the starting problem. Myopic optimization produces 0 units, whereas the global optimum is 51 units.

Figure 8: The benefit of subgoaling. A goal of 50 is broken down into 5 subgoals of 10. The optimal performance is 51 units.
6. General Discussion

We proposed a very simple model to explain the stylized findings in the goal setting literature. The model adopts the “goals as reference points” approach developed in Heath, Larrick, & Wu (1999) to explain how higher goals lead to higher performance. We also explain how effort, persistence, and attention can be interpreted as mechanisms within this framework. The framework also generates some new predictions, such as the tendency of performance to “pile up” around moderately challenging goals and the performance benefits of subgoals.

By systematically explaining three major classes of empirical results, our model adds to previous approaches in the literature which have implicitly or explicitly used particular assumptions about the shape of the value function to explain an occasional result. For example, a linear value function was used by Locke & Latham (1991, p. 222) to explain why goals affect satisfaction, but Proposition 1 shows that diminishing sensitivity is necessary to explain how higher goals increase performance. Moreover several researchers have implicitly or explicitly proposed step-function value functions to explain various aspects of goal-setting phenomena (e.g., Simon, 1955; Campion & Lord, 1982). Such functions do not exhibit diminishing sensitivity and hence have the same problem as linear value functions. What we find exciting about this approach is that the value function, which has received robust support in many other areas of research, contains properties that seem essential for explaining the empirical results of the goal-setting literature—not only diminishing sensitivity in Proposition 1, but also loss aversion in Propositions 4 and 5. By proposing a way to link the value function with the goal setting literature, we provide a way of unifying two large empirical literatures with a single underlying theoretical approach.
We see two main advantages of this modeling approach: parsimony and precision. Other attempts to explain the stylized findings in the goal setting literature have evoked at least three mechanisms: effort, persistence, and attention. Our model offers a more parsimonious story, subsuming all three of these mechanisms. The model also has the benefit of precision. Although it is simple in structure, it is nonetheless powerful enough to make new behavioral predictions as illustrated by the results on piling up and subgoaling.

We close by suggesting some directions for extending our model. Other researchers have tried to explain goal setting results in terms of expectancies: higher goals lead to higher expectancies, and hence increased performance. Although we believe that expectancies play a role in goal setting, in keeping with our emphasis on parsimony and precision, we have not added an expectancy component to our current model. An expectancy story would be more complex and is not needed to explain the goal setting literature’s major findings. Furthermore, we currently lack a precise model to link expectations and behavior. However, we suspect that there are other goal-setting phenomena that are not well-explained by a value function account, and future research might investigate the implications of an expectancy-based model akin to our value function-based model.

Another direction to extend the current work would be to consider endogenous goals. For the most part, the goal setting literature has considered externally-determined goals even though we know that individuals use goals to regulate their own behavior, whether they are studying, exercising, or performing thankless chores. Endogenous goal setting is a topic that deserves more attention and might be modeled by extending our framework to include a principal-agent setup (e.g., Grossman & Hart, 1983), akin to the planner-do models used by Thaler & Shefrin (1981). The planner would declare a goal, and the doer would respond
optimally to that goal using the framework developed here. This setup might be complicated by the planner’s imperfect understanding of the doer’s “type” (the doer might be a low ability or high ability type), and thus the planner and doer can be seen as playing a game of incomplete information (Harsanyi, 1967-68).

Applications of prospect theory to economics, finance, law, medicine, and public policy have yielded important insights (e.g., Barberis, Huang, & Santos, 2001; Bowman, Minehart, & Rabin, 1997), but most of these have used the status quo as a reference point. Recently, however, there is a growing body of literature that has investigated non-status quo reference points (e.g., Camerer et al., 1997; Goette, Huffman, & Fehr, 2004; Hardie, Fader, & Johnson, 1993; Heath, Huddart, & Lang, 1999). For example, Camerer et al. showed evidence for income targeting for New York City cab drivers. Many cab drivers appear to set daily earnings targets (i.e., an income goal), going home when they exceed their daily goal. This seemingly plausible behavior creates a paradox: drivers exceed their goal more rapidly on favorable days (e.g., a rainy day with lots of conventions in town), so they end up working fewer hours on “high wage” days and more hours on “low wage” days — a violation of standard labor theory. By understanding more precisely the influence of goals on behavior, we will be in a better position to understand a number of important organizational and economic phenomena.
References


