An Empirical Test of Gain-Loss Separability in Prospect Theory*

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Abstract We investigate a basic premise of prospect theory, that the valuation of gains and losses is separable. In prospect theory, gain-loss separability implies that a mixed gamble is valued by summing the valuations of the gain and loss portions of that gamble. Two experimental studies demonstrate a systematic violation of the double matching axiom, an axiom that is necessary for gain-loss separability. We document a reversal between preferences for mixed gambles and the associated gain and loss gambles—mixed gamble A is preferred to mixed gamble B, but the gain and loss portions of B are preferred to the gain and loss portions of A. The observed choice patterns are consistent with a process in which individuals are less sensitive to probability differences when choosing among mixed gambles than when choosing among either gain or loss gambles.

Key Words Risky choice, prospect theory, mixed gambles, double matching, probability weighting function.

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1 Introduction

Prospect theory distinguishes itself from the classical theory of decision under risk, expected utility theory, in taking change in wealth rather than absolute wealth to be the relevant carrier of value (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). This distinction has been applied with enormous success to applications in business, finance, law, medicine, and political science (e.g., Barberis, Huang, and Santos, 2001; Camerer, 2000; Jolls, Sunstein, and Thaler, 1998; McNeil et al., 1982; Quattrone and Tversky, 1988). Indeed, most important real world decisions are mixed gambles, involving some possibility of gain and some possibility of loss (MacCrimmon and Wehrung, 1990; March and Shapira, 1987).

This article investigates how individuals choose among mixed gambles by examining a fundamental assumption of prospect theory, gain-loss separability. Simply stated, gain-loss separability requires that preferences for gains be independent of preferences for losses and, more strongly, that the valuation of a mixed gamble be the sum of the valuations of the gain and loss portions of that gamble. A failure of gain-loss separability has practical as well as theoretical implications. First, empirical findings gleaned from the numerous studies of single-domain gambles will not necessarily generalize to the domain of mixed gambles. Second, procedures to elicit loss aversion coefficients usually assume separability between gains and losses and thus may require modification (e.g., Abdellaoui, Bleichrodt and Paraschiv, 2007; Schmidt and Traub, 2002). Finally, models that relax gain-loss separability will necessarily be more complex than models that assume separability.

We examine gain-loss separability by testing an axiom known as *double matching*, a necessary condition for gain-loss separability within prospect theory (Tversky and Kahneman, 1992). We define double matching formally in the next section, but, before doing so, we motivate and illustrate our primary test of double matching with an example. We gave participants a choice between the following two gambles:

Problem 1 ($n = 81$)

$$H = \begin{pmatrix} .50 \text{ chance at } \$4200 \\ .50 \text{ chance at } \$\text{-}3000 \end{pmatrix} \text{ vs. } L = \begin{pmatrix} .75 \text{ chance at } \$3000 \\ .25 \text{ chance at } \$\text{-}4500 \end{pmatrix}$$

[52%] [48%]

Levy and Levy (2002a) ran a variant of this choice problem in which the gain in H was \$4500 (Experiment 1, Test I). In our study, a slight majority of participants preferred H to L (the percentage of participants choosing each option is given in the square brackets). Gain-loss separability requires that mixed gamble H be preferred to mixed gamble L if the gain portion of H is preferred to the gain portion of L and the loss portion of H is preferred to the loss portion of L. In the next section, we show that this idea is precisely what is captured by the double matching axiom.

To test gain-loss separability, we decomposed gambles H and L into gain gambles, H^+ and L^+, and loss gambles, H^- and L^-. The same participants were also asked to choose between H^+ and L^+ and between H^- and L^-. The gambles and choice percentages are shown below:

Problem 2 ($n = 81$)

$$H^+ = \begin{pmatrix} .50 \text{ chance at } \$4200 \\ .50 \text{ chance at } \$0 \end{pmatrix} \text{ vs. } L^+ = \begin{pmatrix} .75 \text{ chance at } \$3000 \\ .25 \text{ chance at } \$0 \end{pmatrix}$$

[15%] [85%]

Problem 3 ($n = 81$)

$$H^- = \begin{pmatrix} .50 \text{ chance at } \$0 \\ .50 \text{ chance at } \$\text{-}3000 \end{pmatrix} \text{ vs. } L^- = \begin{pmatrix} .75 \text{ chance at } \$0 \\ .25 \text{ chance at } \$\text{-}4500 \end{pmatrix}$$

[37%] [63%]

A majority of participants preferred the gain portion of L to the gain portion of H, as well as the loss portion of L to the loss portion of H. Nonetheless, a slight majority of participants chose mixed gamble H to mixed gamble L, thus violating double matching.

More generally, we hypothesize that individuals are less sensitive to differences in probabilities when choosing between mixed gambles (Problem 1) than when faced with a choice between gain or loss gambles (Problems 2 or 3). Indeed, if individuals are completely insensitive to probabilities when faced with Problem 1, H will *appear* to dominate L, since the best outcome in H (+4200) is higher than the best outcome in L (+3000), and the worst outcome in H (-3000) is higher than the worst outcome in L (-4500).

The remainder of the paper presents a more detailed empirical investigation of gain-loss separability and demonstrates that the violation of gain-loss separability illustrated above is not unique.

In Section 2, we review prospect theory, present a formal definition of double matching, and review relevant empirical findings. Section 3 presents our primary empirical test of double matching. This investigation consists of over 30 variants of the problem presented in the Introduction, the vast majority showing the same pattern of violations of gain-loss separability. We also formalize an error model analysis and show that our choice patterns are systematic and not due to random error. In addition, estimates of the probability weighting function to the choice data support our psychological interpretation that individuals are less sensitive to probability differences when making choices among mixed gambles than when faced with gambles involving all gains or all losses. In Section 4, we establish the robustness of our double matching violations by employing a gamble configuration that differs from the problem presented above. Section 5 concludes with a discussion of the implications of this research, potential psychological drivers of our results, and some directions for future research.

2 Prospect Theory and Gain-Loss Separability

2.1 Prospect Theory Basics

Both versions of prospect theory, original prospect theory (Kahneman and Tversky, 1979) and cumulative prospect theory (Tversky and Kahneman, 1992), assume gain-loss separability. We present cumulative prospect theory here because that theory applies to gambles with an arbitrary number of outcomes.

We denote a gamble $P = (x_1, p_1; \ldots; x_k, p_k; 0, p_0; x_{k+1}, p_{k+1}; \ldots; x_n, p_n)$, where p_i indicates the probability of outcome x_i. Outcomes are rank-ordered, $x_1 > \ldots > x_k > 0 > x_{k+1} > \ldots > x_n$, where $x_1, \ldots, x_k$ are *gains* and $x_{k+1}, \ldots, x_n$ are *losses* relative to a reference point or neutral outcome 0. A gamble is a *mixed gamble* if there is a strictly positive probability of a gain and a strictly positive probability of a loss, i.e., $\sum_{i=1}^{k} p_i > 0$ and $\sum_{i=k+1}^{n} p_i > 0$.

A mixed gamble can be parsed into a *gain gamble*,

$$P^{+} = \left(x_1, p_1; \ldots; x_k, p_k; 0, p_0 + \sum_{i=k+1}^{n} p_i \right),$$

and a *loss gamble*,

$$P^- = \left(0, \sum_{i=1}^{k} p_i + p_0; x_{k+1}, p_{k+1}; \ldots; x_n, p_n\right),$$

such that the probability of a loss (gain) is moved to the neutral outcome 0 so that probabilities for any gain (loss) gamble sum to one. A *single-domain gamble* is either a gain gamble or a loss gamble. Preferences for gambles are denoted in the standard fashion: $P \succeq (\succ)\, Q$ denotes that P is weakly (strictly) preferred to Q. P is indifferent to Q (denoted $P \sim Q$) if $P \succeq Q$ and $Q \succeq P$.

In cumulative prospect theory, the utility of a mixed gamble is simply the sum of the utilities of the gain and loss gambles:

$$U(P) = U(P^+) + U(P^-). \tag{2.1}$$

(2.1) represents preferences in the sense that $P \succeq Q$ if and only if

$$U(P) = U(P^+) + U(P^-) \geq U(Q) = U(Q^+) + U(Q^-). \tag{2.2}$$

A necessary condition for the separability of gains and losses in (2.2) is the *double matching axiom* (Tversky and Kahneman, 1992, p. 318).

Double Matching For all mixed gambles, P and Q, if $P^+ \sim Q^+$ and $P^- \sim Q^-$, then $P \sim Q$.

The necessity of double matching for the cumulative prospect theory representation follows immediately from (2.2). Wakker and Tversky (1993, p. 157) invoke a slightly weaker axiom called *gain-loss consistency* and show that this axiom is usually implied by other axioms and thus can often be dropped.

P^+ and P^- are valued as follows:

$$U(P^+) = \pi^+(p_1)v(x_1) + \sum_{i=2}^{k}\left[\pi^+\left(\sum_{j=1}^{i} p_j\right) - \pi^+\left(\sum_{j=1}^{i-1} p_j\right)\right] v(x_i), \tag{2.3}$$

and

$$U(P^-) = \pi^-(p_n)v(x_n) + \sum_{i=k+1}^{n-1}\left[\pi^-\left(\sum_{j=i}^{n} p_j\right) - \pi^-\left(\sum_{j=i+1}^{n} p_j\right)\right]v(x_i), \tag{2.4}$$

where $v(\cdot)$ is a *value function*, and $\pi^+(\cdot)$ is the *probability weighting function* for gains and $\pi^-(\cdot)$ is the probability weighting function for losses. In addition, the value function is scaled such that $v(0) = 0$ and exhibits *loss aversion* if $-v(-x) > v(x)$ for all x.[1] Both $\pi^+(\cdot)$ and $\pi^-(\cdot)$ are non-decreasing with $\pi^+(0) = \pi^-(0) = 0$ and $\pi^+(1) = \pi^-(1) = 1$. In (2.3) and (2.4), the value of each outcome is multiplied by a *decision weight* that typically differs from the probability of that outcome. The representation is called *rank-dependent* because the decision weight attached to an outcome depends on the rank of that outcome (Quiggin, 1982). The representation for losses in (2.4) "mirrors" that for gains in the sense that the weighting function begins with the most extreme outcomes first (the worst loss in (2.4) and the best gain in (2.3)) and then works toward the reference point.

2.2 Empirical Tests of Mixed Gambles

A large number of studies have highlighted the descriptive shortcomings of expected utility theory (for reviews, see Camerer 1995; Fox and See, 2003; Luce, 2000; Machina, 1987; Starmer 2000; Wu, Zhang, and Gonzalez, 2004). Prospect theory can explain most of these empirical findings, although Birnbaum and his colleagues have documented a series of patterns that cannot be accommodated by cumulative prospect theory (e.g., Birnbaum, 2004; Birnbaum and McIntosh, 1996; Birnbaum and Navarette, 1998).

Tversky and Kahneman (1992) conducted the first comprehensive investigation of the shape of the value and probability weighting functions. They found that the value function was concave for gains, convex for losses, and exhibited loss aversion, and that the probability weighting function was inverse S-shaped (first concave, then convex) for both gains and losses, with the weighting function for losses slightly less curved and more elevated than the weighting function for gains. Many subsequent studies have replicated these basic results using both choice and matching tasks

[1] See Abdellaoui, Bleichrodt, and Paraschiv (2007) for other definitions of loss aversion.

(e.g., Abdellaoui, 2000; Bleichrodt and Pinto, 2000; Camerer and Ho, 1994; Etchart-Vincent, 2004; Fehr-Duda, de Gennaro, and Schubert, 2006; Gonzalez and Wu, 1999; Lattimore, Baker, and Witte, 1992; Tversky and Fox, 1995; Wu and Gonzalez, 1996).

A variety of empirical studies have involved mixed gambles. However, most of these studies were not explicitly designed to compare choices between mixed gambles and single-domain gambles, but instead to test the expected utility model (e.g., Mosteller and Nogee, 1951; Davidson, Suppes, and Siegel, 1957), or to measure loss aversion (e.g., Abdellaoui, Bleichrodt and Paraschiv, 2007; Brooks and Zank, 2005; Schmidt and Traub, 2002; see also Köbberling and Wakker, 2005). Another set of studies used duplex gambles, which involve the joint receipt of a simple gain gamble and a simple loss gamble, to test whether decision makers used certain information processing strategies (e.g., Payne and Braunstein, 1971; Slovic and Lichtenstein, 1968). Slovic (1967) postulated an axiom called duplex decomposition (see also Luce, 2000) that requires that a mixed gamble be indifferent to the joint receipt of the gain and loss portions of the mixed gamble. Preliminary empirical studies have been mixed about the validity of duplex decomposition (e.g., Cho, Troung, and Haneda, 2005).

A smaller number of studies have investigated direct properties of mixed gambles. Payne, Laughhunn, and Crum (1980) showed a reversal of preference depending on whether a mixed gamble was translated by adding or subtracting a constant amount from each outcome, a finding consistent with loss aversion (see, also, Payne, Laughhunn, and Crum, 1981). Chechile and Butler (2000, 2003) tested whether mixed gambles could be described by a bilinear form, of which prospect theory is a special case. Participants were given one mixed gamble, and were asked to provide a probability of winning for a second mixed gamble so that they were indifferent between the two gambles. While these data appear to be inconsistent with a bilinear form, the analytical procedure used in these investigations is quite complex and relies on parametric assumptions about the form of the value function and the probability weighting function. Payne (2005) documented choice patterns for mixed gambles that are inconsistent with cumulative prospect theory assuming typical parameter values. Payne argued that these patterns demonstrate that individuals sometimes use a heuristic of selecting the mixed gamble with the highest probability of a gain.

Levy and Levy (2002a,b) presented several empirical patterns that they argued contradicted prospect theory's S-shaped value function and instead supported an inverse S-shaped value function, convex for gains and concave for losses, as originally proposed by Markowitz (1952). A variant of one of the patterns was presented in the introduction. Wakker (2003) and Baucells and Heukamp (2004) critiqued their analysis and showed that these patterns are consistent with prospect theory if the probability weighting function is sufficiently inverse S-shaped (see, also, Baucells and Heukamp, 2006). Baltussen, Post, and van Vliet (2006) studied a variant of Levy and Levy's (2002a) Experiment 2. Their data are consistent with the functional form of cumulative prospect theory, but requires a probability weighting function that is much more linear for gains than observed in other studies and more curved for losses than for gains, contrary to the previously reviewed studies.

3 Study 1

In this section, we present the results of the first of two empirical investigations of gain-loss separability. We employed a within-participants test of the double matching axiom using variants of the example presented in the introduction. The empirical test used a choice between "simple" mixed gambles with one gain and one loss outcome, $H = (g, p; l, 1-p)$ and $L = (g', p'; l', 1-p')$, where $g > g' > 0$ and $0 > l > l'$. H and L are mnemonic for "High" and "Low" in the sense that H has higher outcomes than L, a higher gain ($g > g'$) and a less severe loss ($l > l'$).

To test double matching, we parsed H and L into gain gambles and loss gambles and asked the same participants to choose between H^+ and L^+, as well as H^- and L^-. Choice patterns of $H \succ L$, $L^+ \succ H^+$, and $L^- \succ H^-$ (HL^+L^-, for short) or $L \succ H$, $H^+ \succ L^+$, and $H^- \succ L^-$ (LH^+H^-, for short) violate double matching.

3.1 Method

We recruited 453 students from The University of Chicago to participate in this study. The study consisted of 6 separate surveys. Each survey contained 4 to 7 tests of double matching (12 to 21 questions) as well as several filler questions to bring the total number of questions to approximately 20 to 25 binary choices. The first 3 surveys were administered in the form of survey booklets. For each survey, 6 booklets were created in which we randomized the order of the choices and counterbalanced the location of the gambles in each pair (left or right side of sheet). The final

	H Gamble				*L* Gamble				Choice Percentages			Double Matching Violations		Error Rates		Likelihood Ratio Test			
Test	g	p	l	$1-p$	g'	p'	l'	$1-p'$	$\%H$	$\%H^+$	$\%H^-$	$\%HL^+L^-$	$\%LH^+H^-$	$P(H\|L^+L^-)$	$P(L\|H^+H^-)$	Asym.	Mixed	n	Survey
1	150	0.30	-25	0.70	75	0.80	-60	0.20	22.2%	9.9%	17.3%	12.3%	1.2%	16.4%	50.0%	0.048	0.040	81	1
2	1800	0.05	-200	0.95	600	0.30	-250	0.70	21.0%	17.3%	14.8%	6.2%	2.5%	8.6%	66.7%	0.359	0.383	81	1
3	1000	0.25	-500	0.75	600	0.50	-700	0.50	28.3%	11.9%	20.3%	11.9%	0.0%	16.7%	0.0%	0.012	0.018	60	2
4	200	0.30	-25	0.70	75	0.80	-100	0.20	33.3%	18.1%	22.2%	16.7%	2.8%	25.5%	50.0%	0.024	0.024	72	3
5	1200	0.25	-500	0.75	600	0.50	-800	0.50	43.1%	20.8%	25.0%	25.0%	6.9%	37.5%	55.6%	0.001	0.000	72	1
6	750	0.40	-1000	0.60	500	0.60	-1500	0.40	51.4%	26.4%	25.0%	23.6%	4.2%	41.5%	50.0%	0.000	0.000	72	3
7	4200	0.50	-3000	0.50	3000	0.75	-6000	0.25	51.9%	14.8%	37.0%	28.4%	2.5%	50.0%	28.6%	0.000	0.000	81	1
8	4500	0.50	-1500	0.50	3000	0.75	-3000	0.25	48.3%	16.7%	46.7%	21.7%	6.7%	43.3%	50.0%	0.010	0.001	60	2
9	4500	0.50	-3000	0.50	3000	0.75	-6000	0.25	58.3%	16.7%	55.0%	18.3%	3.3%	45.8%	28.6%	0.001	0.010	60	2
10	1000	0.30	-200	0.70	400	0.70	-500	0.30	51.3%	47.5%	27.5%	17.5%	6.3%	41.2%	35.7%	0.017	0.026	80	4
12	3000	0.01	-490	0.99	2000	0.02	-500	0.98	59.3%	42.0%	35.8%	16.0%	2.5%	43.3%	16.7%	0.001	0.036	81	1
11	4800	0.50	-1500	0.50	3000	0.75	-3000	0.25	54.2%	33.3%	44.4%	15.3%	6.9%	45.8%	62.5%	0.022	0.049	72	3
13	2200	0.40	-600	0.60	850	0.75	-1700	0.25	51.7%	38.3%	41.7%	11.7%	5.0%	30.4%	27.3%	0.092	0.136	60	2
14	2000	0.20	-1000	0.80	1700	0.25	-1100	0.75	57.6%	33.9%	47.5%	15.3%	3.4%	56.3%	40.0%	0.021	0.029	59	1
15	1500	0.25	-500	0.75	600	0.50	-900	0.50	51.3%	51.3%	32.5%	10.0%	5.0%	29.6%	28.6%	0.003	0.005	80	4
16	5000	0.50	-3000	0.50	3000	0.75	-6000	0.25	65.0%	42.5%	42.5%	15.0%	1.3%	46.2%	7.1%	0.000	0.060	80	4
17	1500	0.40	-1000	0.60	600	0.80	-3500	0.20	58.8%	47.5%	41.3%	16.3%	8.8%	44.8%	35.0%	0.019	0.094	80	4
18	2025	0.50	-875	0.50	1800	0.60	-1000	0.40	71.7%	51.7%	41.7%	16.7%	1.7%	62.5%	8.3%	0.000	1.000	60	2
19	600	0.25	-100	0.75	125	0.75	-500	0.25	57.5%	55.0%	43.8%	11.3%	5.0%	42.9%	20.0%	0.147	1.000	80	4
20	5000	0.10	-900	0.90	1400	0.30	-1700	0.70	40.0%	46.7%	53.3%	5.0%	6.7%	27.3%	36.4%	1.000	1.000	60	2
21	700	0.25	-100	0.75	125	0.75	-600	0.25	71.3%	58.8%	47.5%	11.3%	1.3%	45.0%	4.0%	0.000	1.000	80	5
22	700	0.50	-150	0.50	350	0.75	-400	0.25	63.3%	58.3%	48.3%	10.0%	8.3%	54.5%	33.3%	0.195	1.000	60	2
23	1200	0.30	-200	0.70	400	0.70	-800	0.30	70.0%	58.8%	50.0%	17.5%	5.0%	63.6%	13.8%	0.002	1.000	80	5
24	5000	0.50	-2500	0.50	2500	0.75	-6000	0.25	78.8%	53.8%	55.0%	18.8%	3.8%	75.0%	11.1%	0.000	0.000	80	6
25	800	0.40	-1000	0.60	500	0.60	-1600	0.40	57.5%	63.8%	51.3%	10.0%	11.3%	38.1%	27.3%	0.396	0.144	80	4
26	5000	0.50	-3000	0.50	2500	0.75	-6500	0.25	71.3%	61.3%	58.8%	8.8%	3.8%	50.0%	10.0%	0.027	1.000	80	5
27	700	0.25	-100	0.75	100	0.75	-800	0.25	72.5%	57.5%	63.8%	7.5%	7.5%	37.5%	18.2%	0.086	1.000	80	6
28	1500	0.30	-200	0.70	400	0.70	-1000	0.30	75.0%	58.8%	62.5%	10.0%	5.0%	42.1%	11.1%	0.010	1.000	80	6
29	1600	0.25	-500	0.75	600	0.50	-1100	0.50	72.5%	60.0%	68.8%	6.3%	6.3%	55.6%	15.6%	0.138	1.000	80	5
30	2000	0.40	-800	0.60	600	0.80	-3500	0.20	65.0%	66.3%	62.5%	3.8%	7.5%	33.3%	18.8%	0.888	1.000	80	5
31	2000	0.25	-400	0.75	600	0.50	-1100	0.50	80.0%	62.5%	68.8%	8.8%	3.8%	53.8%	7.9%	0.004	1.000	80	6
32	1500	0.40	-700	0.60	300	0.80	-3500	0.20	77.5%	63.8%	67.5%	8.8%	5.0%	46.7%	10.0%	0.017	1.000	80	6
33	900	0.40	-1000	0.60	500	0.60	-1800	0.40	70.0%	73.8%	61.3%	2.5%	10.0%	25.0%	22.2%	0.888	1.000	80	5
34	1000	0.40	-1000	0.60	500	0.60	-2000	0.40	77.5%	71.3%	70.0%	10.0%	5.0%	61.5%	8.7%	0.009	1.000	80	6
All									58.7%	45.3%	45.9%	13.0%	4.9%	38.2%	19.3%	0.000	0.000		

Table 1: Gamble parameters and choice percentages for 34 tests of double matching (Study 1). Tests are ordered by $(\%H^+ + \%H^-)/2$. The table also reports two additional measures of double matching violations, as well as the results of likelihood ratio test described in Section 3.3. The last row shows choice percentages and measures of double matching violations, aggregated over the 34 tests.

3 surveys were run on computer, with the order of choices randomized and location of gambles counterbalanced for each participant.[2] Participants were paid \$3 to \$4 for completing the survey booklet.

All choices were hypothetical. There were two major reasons for not using monetary incentives. First and foremost, mixed gambles involve the possibility of loss. Using gambles with real losses imposes obvious practical and methodological limitations, including the necessity of limiting the stakes of the gambles and the probability of loss (e.g., Thaler and Johnson, 1990, p. 653). Second, the conclusions drawn from hypothetical choice studies do not appear to be qualitatively different from those in which participants were paid based on their choices (Camerer, 1989; Camerer and Hogarth, 1999), although participants were generally more risk-averse for real choices than for hypothetical choices.

The entire empirical study of double matching involved 34 tests (listed in Table 1). We varied the gambles in several respects to test the robustness of gain-loss separability. The expected value was sometimes identical for both options, but generally was not. The probability of a gain varied substantially across our mixed gambles, ranging from .01 to .80. Hence, the probability of a loss varied from .20 to .99. We also varied the relative attractiveness of H^+ and L^+, and H^- and L^-, to create tests in which the majority of participants would choose L^+ over H^+ and L^- over H^-, and others in which the majority of participants would prefer H^+ and H^-. To do so, some of the tests were variants of others (e.g., Tests 7, 9, 16, 24, and 26), in which the outcome values were varied to change the relative attractiveness of H^+ and H^-.[3]

3.2 Summary Results

Choice models are generally formulated in terms of deterministic axioms, but statistical tests should reflect the possibility that decision makers make errors when deciding (Luce and Suppes, 1965). Thus, the statistical testing of binary choice data like ours poses notable challenges (Iverson and Falmagne, 1985; Myung, Karabatsos, and Iverson, 2005). We deal with these difficulties by first

[2] In a pre-test of our computer program, we replicated 12 of the questions used in the original 3 survey booklets. The correlation between the choice percentages for the booklet and computer was .96.

[3] Test 7 was presented in the introduction. The mixed gambles (but not the gain or loss gambles) in Tests 8, 9, and 18 were studied by Levy and Levy (2002a,b).

presenting four measures of double matching violations, each of which provides a slightly different view of the data. We take these measures to be suggestive, as it is possible that some of these results could be produced by random error. In Section 3.3, we develop several error models to determine whether the pattern of data are systematic or caused by random error. This analysis also provides a statistical test for our data and shows that random error cannot account for the full pattern of results.

Choice percentages for all gambles are found in Table 1.[4] We let $\%H$ denote the percentage of participants who chose mixed gamble H over mixed gamble L. Similarly, $\%H^+$ and $\%H^-$ denote the percentage of participants who chose H^+ over L^+, and H^- over L^-, respectively. Earlier, we suggested that the violation of double matching in our introductory example was consistent with a process in which participants were less sensitive to probability differences for mixed gambles than for gain gambles or loss gambles. Such a process favors mixed gamble H over mixed gamble L, since H has a higher upside as well as a lower downside. Indeed, this process is consistent with the empirical pattern, $\%H > \max(\%H^+, \%H^-)$, observed in 29 of the 34 tests (sign test: $p < .0001$).

Three alternative measures examine how mixed gamble H is "advantaged" in choice relative to mixed gamble L. First, we look at the relative proportion of the two types of double matching violations, HL^+L^- and LH^+H^- (with $\%HL^+L^-$ and $\%LH^+H^-$ denoting the respective percentages of each violation). In our introductory example (Test 7), 30.9% of participants violated double matching, with 28.4% choosing H, L^+, and L^- and only 2.5% choosing L, H^+, and H^-. Table 1 contains double matching violation rates for each of the 34 tests. Overall, 28 of the 34 tests showed the same pattern: $\%HL^+L^- > \%LH^+H^-$ (sign test: $p < .0001$). Across all 34 tests, HL^+L^- violations occurred 2.7 times more often than LH^+H^- violations ($\%HL^+L^- = 13.0\%$ and $\%LH^+H^- = 4.9\%$).

Next, we consider the relationship between two conditional probabilities, $P(H|L^+L^-)$ and $P(L|H^+H^-)$. The pattern $\%HL^+L^- > \%LH^+H^-$ could reflect a disproportionate percentage of participants preferring L^+ and L^- relative to H^+ and H^-. However, we find, consistent with our hypothesized process, that $P(H|L^+L^-) > P(L|H^+H^-)$ for 26 of the 34 tests (sign test: $p < .01$).

[4]The e-companion (http://mansci.journal.informs.org/) contains choice percentages for all 8 possible choice patterns, HH^+H^-, HH^+L^-, etc.

Moreover, across all tests, $P(H|L^+L^-) = 38.2\%$ is almost twice as large as $P(L|H^+H^-) = 19.3\%$. These conditional probabilities are also reported in Table 1.

Finally, we examine the likelihood of choosing H over L for the two "indeterminate" choice patterns where double matching does not apply: H^+L^- and L^+H^-. Our hypothesized story suggests that H is advantaged relative to L, or $P(H|H^+L^-) > .5$ and $P(H|L^+H^-) > .5$, while the null hypothesis requires that the two conditional probabilities equal .5. These conditional probabilities show the same pattern as the other measures: $P(H|H^+L^-) > .5$ for 26 of the 34 tests (sign test: $p < .001$), while $P(H|L^+H^-) > .5$ for 25 of the 34 tests (sign test: $p < .01$). Aggregated over the 34 tests, $P(H|H^+L^-) = 63.4\%$ and $P(H|L^+H^-) = 61.2\%$. We report $P(H|H^+L^-)$ and $P(H|L^+H^-)$ for each of the 34 tests in the e-companion.

3.3 Statistical Tests

Could our violations of double matching be a statistical artifact generated by random error? Random error models have been proposed to explain expected utility violations (Ballinger and Wilcox 1997; Hey, 1995; Hey and Orme 1994), overconfidence (Brenner 2000; Erev, Wallsten, and Budescu, 1994), and better-than-average effects (Burson, Larrick, and Klayman, 2006; Krueger and Mueller, 2002). For example, the introductory example might be explained by a "regression effect": if $\%H^+$ and $\%H^-$ are close to 0, then $\%H$ will generally be closer to .5, and hence larger than either $\%H^+$ or $\%H^-$.

To rule out random error explanations for our violations of double matching, we compare a *random* error model in which the error rate for choosing among gambles is identical for gain, loss, and mixed gambles (the *null error model*) with a *systematic* error model motivated by our hypothesized process that individuals are less sensitive to probability differences when choosing among mixed gambles than when choosing among single-domain gambles (the *asymmetric error model*). We also consider a second asymmetric error model in which the error rate for choosing among mixed gambles is higher than the error rate for choosing among gain or loss gambles (the *mixed error model*). The error analysis provides a formal statistical test for the data in Study 1 and shows that the asymmetric error model provides a significantly better account of the double matching violations than either the null or mixed error models. In addition, the analysis shows that whereas the mixed error model is consistent with some of our double matching violations, it cannot account for the full empirical pattern of data.[5]

The three proposed error models consist of assumptions about preference types, satisfaction of the double matching axiom, and error rates. These assumptions are described below.

We first make an assumption about preference types. We assume that there are four types, $\theta_{H^+H^-}$, $\theta_{H^+L^-}$, $\theta_{L^+H^-}$, and $\theta_{L^+L^-}$. Each type has "underlying preferences," though their "revealed preferences" reflect some error (see below). Type $\theta_{H^+H^-}$ prefers H^+ over L^+ and H^- over L^-, type $\theta_{H^+L^-}$ prefers H^+ over L^+ and L^- over H^-, etc.

We next assume that "underlying preferences" satisfy double matching. Type $\theta_{H^+H^-}$ prefers H^+ over L^+ and H^- over L^-, and thus H over L, whereas type $\theta_{L^+L^-}$ prefer L^+ over H^+ and L^- over H^-, and hence L over H. We consider the other two types, $\theta_{H^+L^-}$ and $\theta_{L^+H^-}$, "indeterminate" and assume that they choose according to their gain or loss preferences with equal probability.

Finally, we assume that decision makers choose with error. Let ϵ_S be the error rate for single-domain gambles, such that $P(L^+ \succ H^+|\theta_{H^+H^-}) = P(L^- \succ H^-|\theta_{H^+H^-}) = P(H^+ \succ L^+|\theta_{L^+L^-}) = P(H^- \succ L^-|\theta_{L^+L^-}) = \epsilon_S$. The remaining conditional probabilities are defined analogously.

The three proposed error models differ in their treatment of the choice between mixed gambles H and L. Let $\epsilon_L = P(L \succ H|\theta_{H^+H^-})$ and $\epsilon_H = P(H \succ L|\theta_{L^+L^-})$. The three error models below make different restrictions on the relationship among the three error rates, ϵ_S, ϵ_L, ϵ_H:

- "Null" model: $\epsilon_S = \epsilon_L = \epsilon_H$;
- "Mixed" error model: $\epsilon_L = \epsilon_H > \epsilon_S$;
- "Asymmetric" error model: $\epsilon_H > \epsilon_S = \epsilon_L$.

The null model assumes that the same error rate applies to all gambles, whereas the mixed model assumes that a higher error rate applies to mixed gambles than single-domain gambles, perhaps because mixed gambles are more complicated than single-domain gambles. The asymmetric model

[5] In the e-companion, we also derive implications of each of the three error models for the four measures of double matching violations discussed in Section 3.2. This analysis shows that the mixed error model is consistent with the observed double matching violations when $(\%H^+ + \%H^-)/2$ is less than 50%, but not when this average exceeds 50%.

allows a different error rate for type $\theta_{L^+L^-}$ than type $\theta_{H^+H^-}$. The empirical pattern, $\%H > \max(\%H^+, \%H^-)$, as well as the hypothesized process suggests that $\epsilon_H > \epsilon_L = \epsilon_S$.[6]

To test the significance of each double matching violation, we employ a likelihood ratio test, which performed a similar role in Tversky (1969, 1972) and Wu, Zhang, and Abdellaoui (2005). We first describe the details of the procedure and then provide the results of the statistical test.

Let $\mathbf{p}$ denote the vector of probabilities of the underlying types. Furthermore, let $L_0(D; \mathbf{p}, \epsilon_S)$ be the likelihood of the data D under the null model for a given set of type parameters $\mathbf{p}$ and error rate, ϵ_S, with $L_0^*(D) = \max_{\mathbf{p},\epsilon_S} L_0(D|\mathbf{p}, \epsilon_S)$ denoting the maximum likelihood for the null model. We also conduct maximum likelihood estimations on the mixed and asymmetric error models and use a likelihood ratio test to determine whether the extra parameters used in those models provides a significantly better fit. For example, let $L_A(D; \mathbf{p}, \epsilon_S, \epsilon_H)$ be the likelihood of the data under the asymmetric model, with $L_A^*(D) = \max_{\mathbf{p},\epsilon_S,\epsilon_H} L_A(D|\mathbf{p}, \epsilon_S, \epsilon_H)$ the maximum likelihood for the asymmetric model. Then the statistic $2 \ln [L_A^*(D)/L_0^*(D)]$ is distributed approximately $\chi^2(1)$ (e.g., Mood and Graybill, 1963). In all cases, $0 \leq \epsilon_S, \epsilon_H, \epsilon_L \leq \frac{1}{2}$.

We fit the choice patterns for each of the 34 double matching tests using this procedure. Table 1 contains a comparison of the mixed and asymmetric error models relative to the null error model. This analysis shows that the asymmetric model provides a significantly better account of our choice data than either the null or mixed error models. The asymmetric model shows a significantly better fit than the null model in 71% of the tests (using the conventional $p < .05$ standard). In contrast, the mixed model fits better than the null model in only 41% of the tests (again using $p < .05$). Note that the asymmetric and mixed model perform similarly when $(\%H^+ + \%H^-)/2 < .5$. However, when $(\%H^+ + \%H^-)/2 > .5$, only 1 of 14 tests are significant with the mixed model, while 8 of 14 tests are significant with the asymmetric model. The e-companion contains a more complete reporting of the likelihood ratio test, include a table of the estimated error rates, as well as a detailed discussion of the direction of the estimated error rates.

[6]Since the two indeterminate types, $\theta_{H^+L^-}$ and $\theta_{L^+H^-}$, choose like type $\theta_{H^+H^-}$ or type $\theta_{L^+L^-}$ with equal probability, $P(H \succ L|\theta_{H^+L^-}) = P(H \succ L|\theta_{L^+H^-}) = (1/2)P(H \succ L|\theta_{H^+H^-}) + (1/2)P(H \succ L|\theta_{L^+L^-}) = \frac{1}{2}(1 - \epsilon_L + \epsilon_H)$. For the asymmetric model, $\epsilon_H > \epsilon_L$, and thus $P(H \succ L|\theta_{H^+L^-}) > \frac{1}{2}$.

3.4 Probability Weighting Function Estimation

We formally test the hypothesis that violations of double matching result from a diminished sensitivity to probability differences for mixed gambles relative to gain or loss gambles by fitting a probability weighting function to our double matching test data. We use a probability weighting function specification proposed by Tversky and Kahneman (1992), and used by used by Abdellaoui (2000), Camerer and Ho (1994), and Wu and Gonzalez (1996):

$$\pi(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}. \tag{3.1}$$

The parameter γ captures the degree of curvature and elevation. There is no probability distortion if $\gamma = 1$, and more pronounced curvature and lower elevation with smaller values of γ.

To test the hypothesis that the probability weighting function is more curved for mixed gambles than gain or loss gambles, we let γ differ for single-domain gambles (gains or losses) (γ_S) and mixed gambles (γ_M). Our hypothesis requires that $\gamma_M < \gamma_S$.

Since our data are binary, we employ a stochastic choice analysis akin to Baucells and Heukamp (2004), Camerer and Ho (1994), and Wu and Gonzalez (1996). This "representative-agent" approach assumes that all respondents have the same underlying preferences but make errors in their choices. For a discussion of the assumptions behind this method, see McFadden (1981).

Let $P(A \succ B)$ be the probability of choosing option A over option B. We assume that this probability is a function of the utility of option A, $U(A)$, and the utility of option B, $U(B)$: $P(A \succ B) = f(U(A), U(B))$ (see Luce and Suppes, 1965). We use a logistic function to describe this probabilistic relation,

$$P(A \succ B) = \frac{1}{1 + \exp(-\mu(U(A) - U(B))}. \tag{3.2}$$

The scaling parameter μ captures the sensitivity of choices to utility differences. If $\mu = 0$, all choices are decided by a coin toss (i.e., $P(A \succ B) = .5$ for all A and B). As $\mu \to \infty$, $P(A \succ B) \to 1$ when $U(A) > U(B)$, and $P(A \succ B) \to 0$ when $U(A) > U(B)$.

We assume that the utility of a gamble is determined by cumulative prospect theory (Eqns.

(2.1), (2.3)-(2.4)). We follow Tversky and Kahneman (1992) in using a power function specification for the value function,

$$v(x) = \begin{cases} x^{\alpha_G}, & x > 0 \\ -\lambda(-x)^{\alpha_L}, & \text{otherwise} \end{cases}.$$

The power function is invariant to multiplicative scaling and becomes more concave for gains as α_G decreases and more convex for losses as α_L decreases. The loss aversion coefficient, λ, captures the extent to which losses loom larger than gains.

Our stochastic choice specification chooses the best fitting γ_S and γ_M, where the objective function is to maximize the likelihood of the data. Let $j = 1, \ldots, 34$ indicate a test of double matching, with n_j denoting the number of participants for test j, and $\%H_j$, $\%H_j^+$, and $\%H_j^-$, denoting the percentage of participants who chose H_j over L_j, H_j^+ over L_j^+, and H_j^- over L_j^-, respectively. Then the objective function is:

$$\begin{aligned} \text{Max}_{\mu,\gamma_S,\gamma_M} \prod_{j=1}^{34} & P(H_j \succ L_j)^{(n_j)(\%H_j)}(1 - P(H_j \succ L_j))^{(n_j)(1-\%H_j)} \times \\ & P(H_j^+ \succ L_j^+)^{(n_j)(\%H_j^+)}(1 - P(H_j^+ \succ L_j^+))^{(n_j)(1-\%H_j^+)} \times \\ & P(H_j^- \succ L_j^-)^{(n_j)(\%H_j^-)}(1 - P(H_j^- \succ L_j^-))^{(n_j)(1-\%H_j^-)}. \end{aligned}$$

Thus, the likelihood is maximized over the 7593 choices in Table 1. The optimization problem was implemented using a nonlinear optimization procedure written in MATLAB, with μ, γ_S, and γ_M left as free parameters.

We conducted a variety of analyses to test our hypothesis. We held α_G, α_L, and λ constant in our analyses, first setting $\lambda = 2$ and $\alpha_G = \alpha_L = .5$ and then performing sensitivity analysis on these parameters to test the robustness of the results. We fixed these parameters at these levels in our base analysis for several reasons. First, α and γ cannot be identified uniquely with the simple gambles used in our study (see, e.g., Prelec, 1998). Second, Abdellaoui (2000) and Tversky and Kahneman (1992) estimated power function coefficients that were nearly identical for gains and losses. Third, we used a power function coefficient of .5 because this estimate was produced by Wu and Gonzalez (1996) using a similar procedure. Fourth, the loss aversion coefficient reflects the parameter estimates of Tversky and Kahneman (1992), as well as the endowment effect studies of Kahneman, Knetsch, and Thaler (1990).

The estimation results support our hypothesis that the weighting function is more curved for mixed gambles than single-domain gambles, thereby lending support to our hypothesis that participants are less sensitive to intermediate probabilities with mixed gambles than with single-domain gambles. Our weighting function estimate for single-domain gambles, $\hat{\gamma}_S$, was .67, close to the parameter estimate of .71 from Wu and Gonzalez (1996). In contrast, the parameter value for mixed gambles was considerably lower, $\hat{\gamma}_M = .55$, a difference that was statistically significant ($t = 620.9, p < .0001$). (The best-fitting scaling parameter was found to be $\hat{\mu} = .18$.) These estimates are plotted in Figure 1. We conducted a variety of sensitivity analyses to test whether this finding was robust and found that $\hat{\gamma}_M$ was significantly lower than $\hat{\gamma}_S$ for all combinations of values of α from .3 to 1.0 and λ from 1 to 2.5 (see e-companion for details).

To further investigate the robustness of our analysis, we replaced the form in Eq. (3.2) with the weighting function proposed by Prelec (1998), $\pi(p) = \exp(-(-\ln p)^{\beta})$. The Prelec weighting function is the identity function when $\beta = 1$, approaches a step function as $\beta \to 0$, and has a fixed point at $1/e \approx .368$ (i.e., $\pi(1/e) = 1/e$). We found qualitatively identical results ($\hat{\beta}_S = .61$, $\hat{\beta}_M = .47$, $t = 589.6$, $p < .0001$), with the Prelec function performing slightly worse in terms of log likelihood.

We also estimated a separate set of models in which we allowed γ in the Tversky and Kahneman weighting function to differ for gain gambles, loss gambles, and mixed gambles. Specifically, we let γ_{GS}, γ_{LS}, $\gamma_{GM} = \gamma_{GS} - \delta$, and $\gamma_{LM} = \gamma_{LS} - \delta$ be the parameters for gain gambles, loss gambles, the gain portions of mixed gambles, and the loss portions of mixed gambles, respectively. Thus, δ captures the difference in curvature between mixed gambles and single-domain gambles. This specification yielded similar results to the analyses presented earlier. Contrary to most previous studies, the parameter for losses, $\hat{\gamma}_{LS} = .66$, was substantially lower than for gains, $\hat{\gamma}_{GS} = .76$ (however, see Baltussen et al., 2006), but most critically, the parameter capturing the difference between single-domain gambles and mixed gambles, $\hat{\delta} = .14$ was significantly positive ($t = 587.9$, $p < .0001$), indicating a more curved weighting function for mixed gambles than for single-domain gambles. We also produced similar results assuming the weighting function proposed by Prelec (1998).

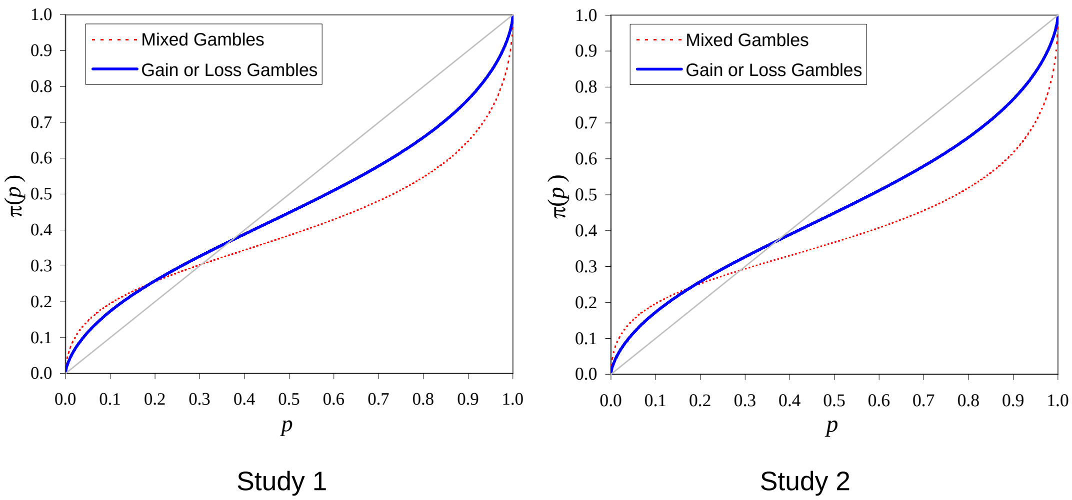

Figure 1: Probability weighting functions, $\pi(p)$, for mixed gambles and single-domain gambles, estimated from Study 1 and Study 2 choice data using a stochastic choice functional and the Tversky and Kahneman (1992) function.

4 Study 2

We suggested at the end of Section 3.4 that the violations of gain-loss separability in Study 1 were due to a diminished sensitivity to probabilities for mixed gambles than single-domain gambles. However, all the gambles in Study 1 have a particular configuration—the gain and loss outcomes in the H gambles are larger than the respective gain and loss outcomes in the L gamble. Thus, an alternative explanation for our observed violations of gain-loss separability is that participants may be using an "extreme outcome heuristic." This heuristic, while interesting, clearly has less generality than our explanation in terms of diminished sensitivity to probabilities. Study 2 addresses this confound by introducing an *indirect* choice test of gain-loss separability.

Consider the following sets of gambles,

$$H^{+} = \begin{pmatrix} p \text{ chance at } \$x \\ 1-p \text{ chance at } \$0 \end{pmatrix} \text{ vs. } L^{+} = \begin{pmatrix} q \text{ chance at } \$y \\ 1-q \text{ chance at } \$0 \end{pmatrix},$$

and

$$[H^+, C^-] = \begin{pmatrix} p \text{ chance at } \$x \\ 1-p-r \text{ chance at } \$0 \\ r \text{ chance at } \$\text{-}z \end{pmatrix} \text{ vs. } [L^+, C^-] = \begin{pmatrix} q \text{ chance at } \$y \\ 1-q-r \text{ chance at } \$0 \\ r \text{ chance at } \$\text{-}z \end{pmatrix},$$

where $x > y$ and $p < q$. H^+ and L^+ are gain gambles, with the mixed gambles, $[H^+, C^-]$ and $[L^+, C^-]$, created by concatenating the loss gamble, $C^- = (1-r, 0; r, -z)$, to H^+ and L^+, respectively. A violation of gain-loss separability occurs if $H^+ \succ$ (resp. $\prec$) L^+ but $[L^+, C^-] \succ$ (resp. $\prec$) $[H^+, C^-]$. If decision makers are more sensitive to probability differences when choosing among single-domain gambles than mixed gambles as in Study 1, then the relative preference for q chance at $\$y$ over p chance at $\$x$ is greatest when the choice involves two gain gambles than when it involves two mixed gambles.

Of course, direct choice between these gambles confounds our diminished sensitivity to probabilities story with the extreme outcome heuristic. In addition, the choice between $[H^+, C^-]$ and $[L^+, C^-]$ may promote cancellation or editing of common outcomes (e.g., Kahneman and Tversky, 1979; Wu, 1994). To circumvent these concerns, we compare H^+ and L^+ each with one alternative, A^+, and $[H^+, C^-]$ and $[L^+, C^-]$ separately with a second alternative B. The second alternative B could be a mixed gamble, a sure thing gain or loss, or a gain or loss gamble. This indirect test thus involves a comparison of the choice percentages for four pairs of gambles, $P(H^+, A^+)$, $P(L^+, A^+)$, $P([H^+, C^-], B)$, and $P([L^+, C^-], B)$. Note that the design of this study has an advantage and disadvantage relative to the design used in Study 1. The indirect test uses a different configuration of gambles than Study 1 and hence investigates the generality of our violations of gain-loss separability. On the other hand, the test is indirect and more complicated than the test used in Study 1. Thus any violations of gain-loss separability will be less transparent than those observed in our previous study.[7]

4.1 Methods

We recruited 102 students from the University of Chicago to participate in this study. As in the previous study, we employed a within-participants design. Participants made binary choices

[7]Since the loss portion of the gambles coincide, this design also tests sign-comonotonic independence (see Wakker and Tversky, 1993).

between 24 unique gamble pairs (see Table 4). The choices of H^+ versus A^+ and L^+ versus A^+ were used in two separate tests (Tests 1 and 2, 3 and 4, etc.) and thus our design produced 8 total tests of gain-loss separability. The gambles were presented on a computer, with the order of the gambles randomized and the location of each gamble in the pair (left or right) counterbalanced for each participant. Participants were paid \$3 for completing the task. As with the previous study, all choices are hypothetical.

4.2 Summary Results

Choice percentages for Study 2 are shown in Table 2. In examining the results, we first present a broad overview of the results by examining choice percentages. Choice percentages are suggestive although imprecise. Thus, we subsequently develop a formal test of whether gain-loss separability is violated.

Gain-loss separability requires that adding the common loss C^- not change preferences between H^+ and L^+, i.e., $H^+ \succ$ (resp. $\prec$) L^+ if and only if $[H^+, C^-] \succ$ (resp. $\prec$) $[L^+, C^-]$. Thus, $P(H^+, A^+) < P(L^+, A^+)$ and $P([H^+, C^-], B) > P([L^+, C^-], B)$ constitutes a violation of gain-loss separability in the predicted direction. We observe exactly these patterns in tests 1, 3, and 4. Moreover, the second differences, $(P(H^+, A^+) - P(L^+, A^+)) - (P([H^+, C^-], B) - P([L^+, C^-], B))$, are negative as predicted for 7 of the 8 tests and for the average of all tests.

4.3 Statistical Tests

These measures constitute weak and imprecise tests of double-matching: Violations can occur in the absence of choice percentage reversals, and it is unclear how to interpret second differences in choice percentages. To test gain-loss separability formally, we develop two stochastic choice models, one which constrains preferences to follow gain-loss separability and one which does not. We then test whether the unconstrained model provides a better fit to the choice data. We begin by making the simplifying but standard assumption that the probability that prospect S is chosen over prospect T is captured by the logit model, $P(S \succ T) = \frac{1}{1+\exp(-\mu(U(S)-U(T)))}$.

Under this assumption, fitting the choice percentages in Table 2 involves estimating 6 free parameters for each of the 8 tests: $U(H^+)$, $U(L^+)$, $U([H^+, C^-])$, $U([L^+, C^-])$, $U(A^+)$, and $U(B)$

Test	H^+		L^+		C^-		A^+		B				$\%(H^+ \succ A^+)$	$\%(L^+ \succ A^+)$	$\%([H^+, C^-] \succ B)$	$\%([L^+, C^-] \succ B)$	Likelihood Ratio Test
	x	p	y	q	$-z$	r	w	s	x'	p'	$-y'$	q'					p-value
1	4200	0.50	3000	0.75	-3500	0.25	1500	0.95			-2000	0.05	35.3%	50.0%	72.5%	70.6%	0.095
2	4200	0.50	3000	0.75	-3500	0.25	1500	0.95			-25	1.00	35.3%	50.0%	73.5%	73.5%	0.155
3	2025	0.50	1800	0.60	-1500	0.25	1350	0.75	200	0.50	-500	0.30	35.3%	46.1%	86.3%	77.5%	0.024
4	2025	0.50	1800	0.60	-1500	0.25	1350	0.75			-600	0.05	35.3%	46.1%	83.3%	76.5%	0.052
5	1200	0.25	600	0.50	-750	0.50	400	0.80			-50	1.00	28.4%	22.5%	65.7%	56.9%	0.888
6	1200	0.25	600	0.50	-750	0.50	400	0.80			-1000	0.20	28.4%	22.5%	75.5%	81.4%	0.159
7	3000	0.01	1000	0.10	-500	0.98	500	0.20			-300	0.99	23.5%	61.8%	57.8%	78.4%	0.120
8	3000	0.01	1000	0.10	-500	0.98	500	0.20			-600	0.75	23.5%	61.8%	57.8%	88.2%	1.000
Average													30.6%	45.1%	70.8%	76.2%	0.028

Table 2: Gamble parameters and choice percentages for 8 tests of gain-loss separability (Study 2). Results of statistical test of gain-loss separability are reported in the last column, with p-values from likelihood-ratio test provided for each of the 8 tests separately and for the aggregate data. Note: For gamble B, we omit a $1 - p' + q'$ chance at \$0.

(since $U(\cdot)$ can be rescaled multiplicatively, we are free to set $\mu = 1$). However, gain-loss separability imposes the restriction that $U([H^+, C^-]) = U(H^+) + U(C^-)$ and $U([L^+, C^-]) = U(L^+) + U(C^-)$, leaving 5 free parameters: $U(H^+)$, $U(L^+)$, $U(C^-)$, $U(A^+)$, and $U(B)$. To test whether gain-loss separability is violated, we thus test whether the unconstrained 6 parameter model provides a significantly better fit to the data than the 5 parameter model that imposes gain-loss separability.

We fit both models to the choice data using maximum likelihood estimation. (Additional details are found in the e-companion.) We first estimate each model separately for the 8 tests of gain-loss separability. We then use the likelihood-ratio test to determine whether the unconstrained model fits the data significantly better than the constrained model. The last column of Table 2 shows p-values for each of the 8 tests. The estimates for 6 of the 8 tests are in the hypothesized direction, with 1 of the 8 tests significant at the 0.05 level, and 2 others are marginally significant ($p < 0.10$). Of course, the 8 tests are not independent since some of the choices appear in two tests.

Finally, we estimate parameters for the choices of all 8 tests simultaneously. The unconstrained model yields a significantly better fit ($p < .03$) than the constrained model, adjusting for the 4 additional parameters (one for each $U(C^-)$).

4.4 Probability Weighting Function Estimation

We followed the same procedure outlined in Section 3.4 to estimate probability weighting functions for mixed gambles (γ_M) and single-domain gambles for the data from Study 2. Each of the 102 participants in Study 2 made 24 choices, thus the likelihood is maximized over the 2448 choices in Table 2. All other aspects of the procedure were identical.

As with Study 1, our base analysis assumed that $\alpha = \alpha_G = \alpha_L = 0.5$ and $\lambda = 2$. The resulting parameter estimates were very close to the Study 1 estimates: $\hat{\gamma}_S = .68$ and $\hat{\gamma}_M = .52$. More critically, the difference between $\hat{\gamma}_S$ and $\hat{\gamma}_M$ was statistically significant ($t = 8.28, p < .0001$). The best-fitting scaling parameter was $\hat{\mu} = .28$, somewhat higher than the estimate of Study 1. The estimates for the Prelec weighting function were also remarkably close to those found for the Study 1 data: $\hat{\beta}_S = .64$ and $\hat{\beta}_M = .44$ ($p < .0001$). The estimates for the Tversky and Kahneman function are plotted in Figure 1.

We also conducted sensitivity analyses on α and λ to test whether these differences were robust.

The e-companion shows that $\hat{\gamma}_S > \hat{\gamma}_M$ for most values of α and λ. However, unlike Study 1, $\hat{\gamma}_S < \hat{\gamma}_M$ when $\alpha \geq .9$. It is important to note, however, that the fits for $\alpha \geq .9$ were notably worse than the fits when α was low.[8]

5 General Discussion

Studies 1 and 2 demonstrated systematic violations of double matching, an axiom that is necessary for gain-loss separability. These violations are consistent with a process in which individuals are less sensitive to differences in probabilities when choosing among mixed gambles than when choosing among gambles with either all gains or all losses. Direct support for this interpretation is provided by our estimates of the probability weighting functions for Studies 1 and 2. In both studies, we found that the weighting function was indeed more curved for mixed gambles than for single-domain gambles.

Below, we offer two possible psychological explanations, one affective and one cognitive, for our violations of gain-loss separability. We conclude with some final remarks about the contribution of our paper.

5.1 Affective Explanation

Affect-based accounts have been evoked to explain a number of decision making phenomena (cf., Rottenstreich and Shu, 2004; Slovic et al., 2002). Rottenstreich and Hsee (2001) found that the probability weighting function was significantly more curved for lotteries over affect-rich objects (such as a hypothetical kiss from a favorite movie star or a hypothetical electric shock) than for lotteries over affect-poor objects (such as monetary gains). To the extent that losses are inherently affect-rich, there may be a "spill-over" to gains in which mixed gambles evoke an affective rather than a calculative mindset (Hsee and Rottenstreich, 2004). Such a mindset would lead to less sensitivity to intermediate probability differences for mixed gambles relative to single-domain gambles.

[8] To illustrate the difference in fit, we computed the mean absolute difference (MAD) between the actual choice percentages and the fitted choice percentages. The MAD for $\alpha = 0.5$ and $\lambda = 2$ was .096, compared to a MAD of .192 for $\alpha = 0.9$ and $\lambda = 2$.

5.2 Cognitive Explanation

We suggest that mixed gambles are more complex psychologically than single-domain gambles. Losses are not merely the opposite of gains, but gains and losses appear to be processed in different parts of the brain (Bechara et al., 1997; Breiter et al., 2001), and seem to be distinct psychologically and not just two ends of a continuum (Larsen et al., 2004). As a result, rules that may be used to simplify single-domain gambles may not translate well to simplifying mixed gambles. For example, while decision makers may use an editing operation such as combination to simplify single-domain gambles (e.g., Tversky and Kahneman, 1979), such a heuristic is not readily applicable to mixed gambles, where the outcomes are "apples and oranges."

Decision makers may deal with the complexity introduced by mixed gambles by using constructive decision making processes to simplify the choice (e.g., Payne, Bettman, and Johnson, 1992). We discussed one heuristic that may be used for choosing among mixed gambles—the reduction of a mixed gamble to extreme outcomes, the best gain and the worst loss. Of course, whereas this heuristic can explain the results of Study 1, it does not account for the violations of double matching in Study 2. This heuristic might be even more appealing for mixed gambles with a large number of outcomes. Indeed, Birnbaum and Bahra (2007) produced violations of double matching with gambles that are more complex than the ones used in our investigation. These violations are consistent with an "extreme outcome heuristic." This account also recalls March and Shapira's (1987) finding that managers evaluate uncertain prospects in terms of the "worst or best (plausible) case" (p. 1411).[9]

To the extent that heuristics are used to simplify mixed gambles, other mixed gamble configurations may encourage the use of other heuristics. Indeed, Payne (2005) found that participants faced with a multiple-outcome mixed gamble preferred to improve the $0 outcome rather than the best gain, contrary to some parametric specifications of cumulative prospect theory. These results suggest that, individuals may use a heuristic of maximizing the probability of gain, or minimizing the probability of loss, when faced with a complicated mixed gamble. It does not appear that our participants are using a heuristic of this sort to evaluate our gambles, since this heuristic would

[9]It is interesting to note that an extreme outcome heuristic will produce behavior that resembles the use of Hurwicz's (1951) "pessimism-optimism index criterion" of maximizing the weighted average of the maximum and the minimum for each option.

generate a violation of double matching in the opposite direction of our empirical findings. However, this heuristic is surely applied in some choice situations. Process tracing methods such as Mouselab may help researchers understand when and why decision makers employ a probability heuristic or an extreme outcome heuristic (e.g., Payne, Bettman, and Johnson, 1992).

5.3 Concluding Remarks

In the last 50 years, a large body of empirical research has investigated how decision makers choose among risky gambles. Most of these findings can be accommodated by prospect theory. An S-shaped value function and inverse S-shaped probability weighting function can model the reflection effect, the fourfold pattern of risk preferences, the common-ratio and common-consequence effects, as well as the generalization of these findings from risk to uncertainty. However, the majority of the existing empirical evidence has involved single-domain gambles. The emphasis on these gambles is sensible—they are easy for research participants to understand and can be studied in hypothetical situations as well as played out for real payoffs.

The study of single-domain gambles is justified if the understanding gleaned from these investigations extends to the domain of mixed gambles. Our study indicates that mixed gamble behavior is described well by an S-shaped value function and an inverse S-shaped probability weighting function. However, gain-loss separability fails, and hence different parameter values are needed for mixed gambles than single-domain gambles. As a result, findings inferred from studies of single-domain gambles may not extend automatically to mixed gambles.

Our violations of gain-loss separability appear to be systematic. Nevertheless, we do not regard our results as providing a general picture of how decision makers choose among mixed gambles. The mixed gambles in Study 1 have a very special configuration that may contribute to a "perceived" dominance: the highest outcome in H is better than the highest outcome in L, and the lowest outcome in H is also better than the lowest outcome in L. Even though Study 2's indirect test of double matching demonstrates that this configuration is not a necessary condition for producing violations of gain-loss separability, Payne's (2005) findings suggest that other heuristics might operate as well. Although a comprehensive study of gain-loss separability is beyond the scope of this paper, we encourage extensions of our tests to mixed gambles with different structures.

Thus, even though future research will surely qualify the account of mixed gambles developed here, we nevertheless see our paper as moving us a step closer toward a fuller understanding of this important and understudied choice domain. In addition, we have proposed cognitive and affective explanations for the violations of gain-loss separability but have not provided direct evidence for either explanation. The role that these psychological accounts and others play in the general evaluation of mixed gambles awaits further investigation.

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