Do Mandatory Hedge Disclosures Discourage or Encourage Excessive Speculation?

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Abstract

In order to shed some light on the desirability of hedge disclosures, I investigate the consequences of hedge disclosures on a firm's risk management strategy. Several major results emerge from this analysis. First, greater transparency about a firm's derivative activities is not necessarily a panacea for imprudent risk management strategies. I show that such transparency actually induces the firm to take excessive speculative positions in the derivative market. Second, I show that the firm may choose a prudent risk management strategy in the absence of hedge disclosures. However, the selection of a prudent risk management comes at a cost. The firm's production policy is distorted in the absence of hedge disclosures.

These findings suggest that the FASB must carefully investigate the trade-offs between production distortions and risk management distortions in evaluating the desirability of mandatory hedge disclosures for all firms.

1 Introduction

In this paper, I investigate the informational effects of mandatory hedge disclosures on a firm's risk management strategy. I model an environment in which the firm is simultaneously choosing the underlying transactions and managing the risks of those transactions. It is important to study this *simultaneous* choice because the choice of inherent risk and its management are inextricably linked. If risk management is altered by mandatory hedge disclosures, a firm could be induced to choose its *real* transactions differently.

Specifically, I study an environment in which a firm initially commits resources to producing a commodity whose price is uncertain. The firm's choice of production quantity determines the magnitude of its inherent risk. The firm can manage this risk by taking positions in the futures market when the firm chooses its production plan. The firm has private information about the commodity spot price and could use this information to choose its position in the futures market. Thus, the firm's futures trade could consist of both hedging and speculative components.

To study the interaction between risk management and production, I examine risk management in the context of a futures market with basis risk. Basis risk occurs because hedging works imperfectly in practice such that a firm always faces some residual risks even after it has hedged its operational risks. Basis risk is introduced in my model when futures contracts mature before the revenues from production are realized. In the absence of basis risk, Danthine's [1978] "separation" result would hold, causing the firm's production choice to become independent of the firm's beliefs about commodity spot prices. I show that when basis risk is present the firm's production is affected by both its risk management strategies and private beliefs.

The firm's choices are motivated by the desire to influence its value in the capital market. To make valuation relevant to the firm's shareholders, I assume that, for consumption purposes, current shareholders need to liquidate their holdings prior to the realization of the firm's terminal cash flows. In equilibrium, the capital market price is affected by both the gains and losses from the firm's futures position as well as the assessed distribution of commodity spot prices. The latter assessment is conditioned by inferences drawn from public disclosures regarding the firm's observable decisions and cash flows. From a valuation perspective, disclosure of the firm's futures choices has both direct and indirect effects on its capital market price. The direct effect arises from the observed gains or losses on the firm's futures trades. The indirect effect arises because the firm's futures and production decisions are based on its private information about conditions in the commodity spot market.

I examine three regimes in this paper. In the first regime, all information, including the firm's signal, is public. This full information setting provides a benchmark that characterizes prudent risk management. In the second regime, the regime with mandatory hedge disclosures, the firm's signal is private, but its production and futures choices are disclosed. In the third regime, the regime with no hedge disclosures, only the firm's production choice is observable, and there is no disclosure of the firm's futures trades.

The main result derived in this paper is that when the firm's futures trades are disclosed, the firm's risk management strategy is adversely affected. Rather than being more prudent in its risk management, the firm is induced to take extreme positions in the futures market which are interpreted as excessive speculation. This result is significant given the debate over the effect of mandatory hedge disclosures on a firm's risk management strategy. Although regulators generally believe that hedge disclosures would discourage imprudent risk management strategies because of increased transparency (Jenkins [1997]), my model shows that this is not necessarily the case. To the contrary, mandatory hedge disclosures may actually encourage excessive speculation.

The intuition behind this result is as follows. Because the firm is concerned about its capital market value, it has a natural incentive to make decisions that imply that the firm has received a high private signal by the capital market. A high private signal indicates that the commodity spot price will be high, which leads to high capital market prices. To convey that it has received a high signal credibly, the firm is induced to take a position in the futures market that is consistent with this message. However, extreme positions in the futures market that are not consistent with the firm's true beliefs are costly because they result in expected losses in the firm's futures trades. These losses are directly priced by the capital market. Thus, there is a disciplinary effect on the disclosure of futures gains or losses. However, in equilibrium this discipline needs to be magnified for credibility to be sustained. The firm's futures trades need to be sufficiently extreme. Because the firm's observed production is also affected by its private signal and is, therefore, informative one would expect that the firm would be induced to overproduce. Surprisingly, it turns out that even though the firm's production choice has informational content and the firm deviates from prudent risk management, the firm's production policy is not distorted from first best.

In the regime in which the firm's futures position is not disclosed, the actual gains or losses from futures trades cannot be directly priced by the capital market. One would expect that the excess derivative speculation that is induced by disclosures would disappear and that the firm would revert to a policy of prudent risk management. However, even though the firm's actual futures position cannot be observed, the capital market must form beliefs about the firm's risk management policy and take into account possible gains or losses in the futures market. Intuitively, because inferences about the firm's private signal can only be drawn from the firm's observed production in this regime and because the firm's private signal, in turn, guides its risk management choice, one would expect that the firm's production policy would be distorted relative to the first best regime. It turns out that the capital market rationally uses the firm's observed production to infer the firm's derivative position and price the firm accordingly. Thus, the firm's production policy communicates information about two elements: the commodity spot price and the firm's risk management decisions. The major result derived in the regime with no hedge disclosures stokes the debate regarding the desirability of hedge disclosures. I show that when hedge disclosures are not mandated, the firm may indeed adopt a policy of prudent risk management. However, prudent risk management comes at a cost: the firm's production policy is distorted.

These findings shed some light on the desirability of hedge disclosures mandated by the Financial Accounting Standard Board (FASB) through its Statement of Financial Accounting Standards, *Accounting for Derivative Instruments and Hedging Activities* (SFAS 133). First, greater transparency about a firm's derivative activities is not always a panacea for imprudent risk management strategies. My model shows that hedge disclosures may very well have the opposite and undesirable effect of inducing excessive speculation. Second, in the regime without mandatory hedge disclosures, regulators need to be aware that a policy of prudent risk management can be optimal but that such a policy is obtained at a cost: the firm's production decision is now distorted. This analysis suggests that regulators may need to investigate the trade-offs between production distortions and risk management distortions more carefully before mandating hedge disclosures for all firms.

This study adds to the small but growing literature on the *real effects* of hedge disclosures. Kanodia *et al* [2000] examine the desirability of hedge disclosures in terms of their effect on the informational efficiency of the futures price and thereby on industry output. They show that in the absence of appropriate hedge disclosures, the futures price confounds hedge-motivated trades with speculative trades. This inefficiency leads to a significant downward bias in aggregate industry output and in the equilibrium futures price. My study investigates hedge disclosures from the capital market's valuation perspective, rather than the efficiency of futures prices. Melumad, Weyns, and Ziv [1999] study the effects of alternative hedge accounting standards on the hedging choices of a firm in the presence of capital markets. My analysis differs significantly from Melumad *et al* both in terms of methodology and results. Melumad *et al* show that information about the firm's asset endowments, revealed at an interim date, affects the firm's incentives to hedge at the initial date. As in my model, current shareholders liquidate the firm at an interim date, and, therefore, the firm cares about its market value at the interim date. However, in Melumad *et al* the firm does not choose its inherent risk but is exogenously endowed with a random quantity of a risky asset whose return can be hedged in a forward market.

2 The Model

I consider the decisions of a single firm in a competitive industry that produces a single good. There are three dates in the economy: dates 0, 1, and 2, respectively. The firm is owned at date 0 by N risk-averse shareholders, each having constant absolute risk aversion, ρ. The firm irreversibly commits resources to production at date 0, but the actual production, q, is only available for sale later at date 2. At date 2 a spot market opens, and the good is sold at the prevailing spot price. From the perspective of date 0, the period 2 spot price $\widetilde{p}$ is a random variable. Therefore, the firm is exposed to price risk. To manage this risk exposure, the firm has the opportunity to trade futures contracts in a futures market that opens at date 0.

I want to capture an important institutional feature of futures markets. A firm faces basis risk whenever it trades in the futures market. Basis risk arises for two primary reasons. First, the asset whose price is being hedged may not be exactly the same as the asset underlying the futures contract. Second, the timing of cash flow realizations from the underlying asset may not coincide with the expiration date of the futures contract. The presence of basis risk has important implications for disclosure rules.

I introduce basis risk by requiring that all futures contracts be closed out at date 1 instead of date 2 when the good is available for sale. The firm's date 0 futures trade is denoted by z,[1] and the date 0 and date 1 futures prices are denoted by f_0 and f_1, respectively. Because futures contracts are closed out at date 1, the firm incurs a gain or loss of $z(f_0 - f_1)$ at date 1. However, the revenues from production are generated at date 2 at a price of $\widetilde{p}$, which is imperfectly correlated with $\widetilde{f}_1$.[2] Thus, the firm is exposed to basis risk. In order to focus on the interaction between the capital market pricing rule and the firm's risk management strategy, I assume that the spot price $\widetilde{p}$ and the date 0 and date 1 futures prices, f_0 and $\widetilde{f}_1$, respectively, are all exogenously given. In all three regimes, I assume that the firm's production choice, q, is publicly revealed at date 1 in addition to the realization of the new futures price, f_1. Thus, the information available to the capital market at date 1 consists of f_0, f_1, q, and potentially z, depending on the disclosure rules in effect.

The terminal profit of the firm is a random variable described by:

$$\widetilde{\pi} = \underbrace{\widetilde{p}q - c(q)}_{production\ profits} + \underbrace{z(f_0 - \widetilde{f}_1)}_{futures\ profits} \tag{1}$$

Equation (1) shows that the date 2 profit is made up of two components: production profits and futures profits. Production profits are the result of the difference between the revenues from the sale of the product on the spot market at the price $\widetilde{p}$ and the production cost denoted by $c(q)$. I assume the cost function $c(q)$ is strictly increasing and convex. Futures profits result from the gain or loss on the futures position, given by the product of the futures position z and the difference in the period 0 and period 1 futures prices (that is, $f_0 - f_1$).

Before choosing q and z the firm observes a signal, y, at date 0, about the date 2 spot price, p. A high value of y indicates that the commodity spot price at date 2 is also likely to be high. Because the futures market and spot market relate to the *same* underlying product, a high value of y implies that the futures price, f_1, at date 1 is also likely to be high. However, because the signal y provides *direct* information about conditions in the spot market but only *indirect* information about the conditions in the futures market, y is likely to be more informative about the spot price, $\widetilde{p}$, than about the futures price, $\widetilde{f}_1$. The indirect effect arises from the correlation between the futures price and the spot price. The preceding

[1] The firm's futures trade, z, can be thought of as the number of futures contracts supplied by the firm. However, z can be positive or negative because nothing prevents the firm from being long ($z < 0$) in the futures market.

[2] In principle, the firm may want to undertake an additional futures trade at date 1 after the futures market reopens, as in Anderson and Danthine (1983). In my model, I do not allow such "dynamic hedging" for analytical tractability.

arguments are captured in the following assumption:

Assumption 1 $Cov(\widetilde{p}, \widetilde{f}_1) > 0$, and $Cov(\widetilde{p}, \widetilde{y}) \geq Cov(\widetilde{f}_1, \widetilde{y}) > 0$

For the mathematical development of the model, I make the following distributional assumption:

Assumption 2 *$(\widetilde{p}, \widetilde{f}_1, \widetilde{y})$ is a multivariate normal random variable with mean vector $\mu = (\mu_p, \mu_f, \mu_y)$ and variance-covariance matrix $\sum$. The correlations among $\widetilde{p}, \widetilde{f}_1$, and $\widetilde{y}$ are positive but less than one.*

Although the covariance between $\widetilde{p}$ and $\widetilde{f}_1$ ensures that there are genuine hedging opportunities available to the firm, the imperfect correlation between $\widetilde{p}$ and $\widetilde{f}_1$ implies that these hedges are not perfect because there is basis risk. In the analysis that follows, I construct a benchmark first best regime that characterizes prudent risk management. Then I compare the regimes with and without hedge disclosures against this benchmark regime.

3 Full Information Regime: Benchmark for Prudent Risk Management

I derive the equilibrium production and futures market positions of the firm when all of its current shareholders hold the firm until the cash flows from production are realized at date 2. Throughout this paper, I assume that the manager makes all decisions in the interests of the current shareholders. This assumption allows me to abstract away from incentive issues and focus exclusively on disclosure issues from a valuation perspective.[3] Because $\widetilde{p}$, $\widetilde{f}_1$, and $\widetilde{y}$ are jointly normal, the conditional distribution of $\widetilde{\pi}$ given y is also normal. Given that all shareholders have constant absolute risk aversion, the expected utility of terminal cash flows, $\widetilde{\pi}$, of current shareholders can be expressed in terms of the mean and variance of $\widetilde{\pi}$, so that the firm's objective function is:[4]

$$\underset{q,z}{Max}\; E(\widetilde{\pi}|y, f_0) - \lambda Var(\widetilde{\pi}|y, f_0) \tag{2}$$

[3] In principle, hedge disclosures could also be investigated from the contracting perspective of efficiently motivating managers to provide effort. Because such efficient contracting concerns did not seem to dominate the FASB deliberations on SFAS 133, I focus on the interaction among capital market prices, hedge disclosure rules, and the firm's risk management decisions because such interaction was repeatedly mentioned during those FASB deliberations.

[4] The objective function in equation (2) can also be obtained by viewing the firm as maximizing a surrogate "group utility function" as described in Wilson's "Theory of Syndicates" [1968].

where $\lambda \equiv \frac{\rho}{N}$ is the aggregate risk aversion of the current shareholders. It can also be shown that maximizing the objective function in equation (2) is equivalent to maximizing the date 0 market value of the firm, provided that the capital market has all of the information possessed by the manager.

Because the firm's manager acts in the best interests of the current shareholders and all shareholders hold shares of the firm until the terminal cash flows are realized, it is irrelevant whether or not the shareholders observe the manager's information $\widetilde{y}$. Substituting for $\widetilde{\pi}$ from equation (1), the maximization in equation (2) is equivalent to:

$$\underset{q,z}{Max}\ \underbrace{qE(\widetilde{p}|y) - c(q) + z[f_0 - E(\widetilde{f}_1|y)]}_{\text{expected profits}} - \lambda\underbrace{[q^2Var(\widetilde{p}|y) + z^2Var(\widetilde{f}_1|y) - 2qzCov(\widetilde{p},\widetilde{f}_1|y)]}_{\text{variance of profits}} \quad (3)$$

Differentiating equation (3) with respect to z and q, yields the following first order necessary conditions which are also sufficient because the objective function is concave.

$$f_0 - E(\widetilde{f}_1|y) - 2\lambda zVar(\widetilde{f}_1|y) + 2\lambda qCov(\widetilde{p},\widetilde{f}_1|y) = 0 \quad (4)$$

$$E(\widetilde{p}|y) - c'(q) - 2\lambda qVar(\widetilde{p}|y) + 2\lambda zCov(\widetilde{p},\widetilde{f}_1|y) = 0 \quad (5)$$

Solving equation (4) for the optimal futures position z^* yields:

$$z^* = \underbrace{\beta_f\, q^*}_{\text{pure hedge component}} - \underbrace{s^*}_{\text{speculative component}} \quad (6)$$

where

$$\beta_f \equiv \frac{Cov(\widetilde{p},\widetilde{f}_1|y)}{Var(\widetilde{f}_1|y)} \quad \text{and} \quad s^* \equiv \frac{E(\widetilde{f}_1|y) - f_0}{2\lambda Var(\widetilde{f}_1|y)} \quad (7)$$

Equation (6) shows that the optimal futures position of the firm, z^*, consists of two separate components: a pure hedge component and a speculative component. Because covariances and variances do not depend on the realization of the conditioning information for normally distributed random variables, the coefficient $\beta_f \equiv \frac{Cov(\widetilde{p},\widetilde{f}_1|y)}{Var(\widetilde{f}_1|y)}$ on the optimal production, q^*, in the first component in the expression for z^* is a constant. The imperfect correlation between $\widetilde{p}$ and $\widetilde{f}_1$ implies that β_f is a constant strictly between zero and one. Thus, the hedge component of the optimal futures trade in equation (6) implies that the firm is selling futures equivalent to a constant fraction of its optimal production and is, therefore, hedging only a fraction of its production. The speculative component of the optimal futures trade in equation (6) has the same form as the demand of a risky asset (Grossman [1977]). In addition, the speculative component indicates that the firm will use its beliefs about the difference between the current futures price f_0 and the date 1 futures price $\widetilde{f}_1$ to speculate in the futures market at date 0. Thus, equation (6) indicates that the firm hedges by selling futures equal to some fixed proportion of its output in the futures market and then readjusts its position in the futures market by means of an informationally motivated trade.

I define prudent risk management by the optimal futures position, z^*, of the firm in the full information regime:

$$\underbrace{z^* = \beta_f\, q^* - \ s^*}_{prudent\ risk\ management}$$

Although the controversy surrounding SFAS 133 has been described in terms of the effects of mandated hedge disclosures on prudent risk management strategy, what constitutes a prudent risk management strategy is not well defined. Using z^* as a benchmark for prudent risk management strategy is appropriate because it shows that, absent any incentive effects and in the presence of basis risk, the firm manages its risks by hedging a fraction β_f of its production in the futures market. Then the firm adjusts away from this hedge position through an informationally motivated trade (or speculative trade), s^*, that is consistent with the firm's beliefs and that is in the best interests of its shareholders. This benchmark for prudent risk management will be used to compare the equilibrium allocations in the other regimes.

Equation (6) characterizes the optimal futures position, given the firm's optimal production, q^*. Inserting the optimal z^* from equation (6) into equation (5) and substituting for β_f from equation (7), it can be shown that the optimal condition for q^* satisfies:

$$\underbrace{c'(q^*) + 2\lambda q^* Var(\widetilde{p}|f_1, y)}_{\text{marginal cost of production}} = \underbrace{E(\widetilde{p}|y) + \beta_f[\ f_0 - E(\ \widetilde{f}_1|y)]}_{\text{marginal revenue of production}} \tag{8}$$

The left-hand side of equation (8) is the marginal cost associated with production. The term $c'(q)$ is the direct marginal cost of producing the good, and the term $2\lambda q Var(\widetilde{p}|f_1, y)$ is the marginal risk premium. This risk premium captures the effect of production on the residual risk faced by the firm after the firm has reduced its price risk through its optimal hedge. In the absence of basis risk, the marginal risk premium would be zero and Danthine's separation result would hold. To see this, replace $\widetilde{f}_1$ with $\widetilde{p}$ in equation (8) and note that the variance term $Var(\widetilde{p}|f_1, y)$ on the left-hand side of equation (8) then becomes zero. From equation (7), $\widetilde{f}_1 = \widetilde{p}$ implies that β_f equals one so that equation (8) becomes:

$$c'(q) = f_0 \tag{9}$$

Equation (9) clearly illustrates the separation result. Production is determined entirely by the futures price and not by the firm's (i.e., the shareholders') beliefs about the commodity spot market or by shareholders' risk preferences.

The right-hand side of equation (8) is the marginal revenue of production. To see why this is the case, the right-hand side of equation (8) can be re-expressed as follows:

$$\beta_f[E(\widetilde{p}|y) + f_0 - E(\widetilde{f}_1|y)] + (1 - \beta_f)E(\widetilde{p}|y) \tag{10}$$

Equation (10) indicates that in the presence of basis risk the firm behaves as if it sells a proportion $(1-\beta_f)$ of its production in the spot market at the expected spot price $E(\widetilde{p}|y)$ and hedges the remaining proportion β_f of its production by selling futures at date 0. The effective spot price from hedging the remaining fraction β_f of its production is $E(\widetilde{p}|y) + f_0 - E(\widetilde{f}_1|y)$. Each unit that is sold in the futures market at date 0 earns f_0. At date 1 the firm closes out its futures position by purchasing futures at a date 0 expected futures price of $E(\widetilde{f}_1|y)$. The hedged production is sold in the spot market at date 2 at the date 0 expected spot price of $E(\widetilde{p}|y)$. Therefore, the effective spot price is $E(\widetilde{p}|y) + f_0 - E(\widetilde{f}_1|y)$.

Equation (8) demonstrates that, unlike Danthine's separation result (1978), the firm's optimal production depends on both the aggregate risk aversion of its shareholders and on the information, $\widetilde{y}$, about the commodity spot price. Given the dependence of the optimal production q^* on $\widetilde{y}$, equation (6) implies that the firm's information, $\widetilde{y}$, will affect the optimal futures trade through both the pure hedge and speculative components. The relationships between the firm's information $\widetilde{y}$ and its production and futures trade are important in a setting in which y is the firm's private information. Therefore, it is useful to examine how the reduced form expressions for the full information production schedule $q^*(y)$ and the full information futures schedule $z^*(y)$ vary with y. In order to do so, I make the following assumption:

Assumption 3 *The cost function is quadratic; that is,* $c(q) = kq^2$, *where* $k > 0$.

Substituting $c'(q) = 2kq$ in equation (8) and solving for q^* yields:

$$q^*(y) = \frac{E(\widetilde{p}|y) + \beta_f\ (f_0 - E(\widetilde{f}_1|y)}{2k + 2\lambda Var(\widetilde{p}|f_1, y)} \tag{11}$$

Because $Var(\widetilde{p}|f_1, y)$ is not a function of y and $\widetilde{p}$, $\widetilde{f}_1$, and $\widetilde{y}$ are jointly normal, differentiate the preceding expression to obtain:

$$\frac{dq^*}{dy} = \frac{cov(\widetilde{p}, \widetilde{y}) - \beta_f cov(\widetilde{f}_1, \widetilde{y})}{(2k + 2\lambda Var(\widetilde{p}|f_1, y))var(\widetilde{y})} \tag{12}$$

Now $cov(\widetilde{p}, \widetilde{y}) \geq cov(\widetilde{f}_1, \widetilde{y})$ and $\beta_f < 1$ imply the following lemma:

Lemma 1 *In the first-best regime, the firm's production is strictly increasing and linear in the signal* y*.*

From equation (6):

$$\frac{dz^*}{dy} = \beta_f \frac{dq^*}{dy} - \frac{ds^*}{dy} \tag{13}$$

Equation (13) demonstrates that the firm's signal, y, also has two opposing effects on the firm's optimal choice of its futures position z. Lemma 1 implies that the firm's production q^* increases in y, causing a hedge-motivated increase in z^*. However, higher values of y signal a higher value for $\widetilde{f}_1$ inducing an increase in the firm's speculative position s^* because the full information speculative schedule, $s^*(y)$, is increasing in y. An increase in s^* in turn leads to a speculative-motivated decrease in z^*. Because both $\frac{dq^*}{dy}$ and $\frac{ds^*}{dy}$ do not depend on y, the optimal futures schedule $z^*(y)$ is unambiguously increasing or decreasing in y depending on which one of these two forces dominates. If the hedge-motivated component dominates the speculative component, $z^*(y)$ increases in y. Conversely, if the speculative component exceeds the hedge-motivated component, $z^*(y)$ decreases in y. Using the preceding expressions, I obtain a necessary and sufficient condition under which $z^*(y)$ is unambiguously increasing or decreasing in y.

Lemma 2 $\frac{dz^*}{dy} \lesseqgtr 0$ *if and only if the following condition holds:*

$$\frac{\beta_f}{k + \lambda Var(\widetilde{p}|f_1, y)} Cov(\widetilde{p}, \widetilde{y}) \lesseqgtr \left(\frac{\beta_f^2}{k + \lambda Var(\widetilde{p}|f_1, y)} + \frac{1}{\lambda Var(\widetilde{p}|f_1, y)} \right) Cov(\widetilde{f}_1, \widetilde{y})$$

Because $Cov(\widetilde{p}, \widetilde{y}) \geq Cov(\widetilde{f}_1, \widetilde{y})$, the condition in lemma 2 indicates how much larger $Cov(\widetilde{p}, \widetilde{y})$ must be compared to $Cov(\widetilde{f}_1, \widetilde{y})$ in order for the hedge-motivated component of z^* to dominate its speculative component. Conversely, the condition also shows that in order for the speculative component of z^* to dominate its hedge-motivated component, $Cov(\widetilde{p}, \widetilde{y})$ must not be too much larger than $Cov(\widetilde{f}_1, \widetilde{y})$.

3.1 Early Liquidation by the Firm's Shareholders

My main objective in this paper is to investigate the effect of mandatory hedge disclosures on the firm's production and risk management decisions. If, as assumed in the previous section, current shareholders hold their shares in the firm until all the cash flows in the firm are realized at date 2, disclosure issues become moot. The firm's manager, who represents the current shareholders, will make all decisions in their best interests, irrespective of whether or not the firm's superior information, y, is publicly known. On the other hand, if the current shareholders inelastically[5] liquidate their holdings in the firm to a group of prospective shareholders, this early liquidation by the firm's current shareholders now sets up a potential conflict between current and future shareholders. This conflict arises because current shareholders are concerned *only* with the cash flows from liquidating their holdings. On the contrary, prospective shareholders, who buy into the firm and hold until the terminal date, are concerned with the liquidation value of the firm *and* the terminal cash flows of the firm.

I first derive the firm's equilibrium production and futures decisions in a regime in which the firm's current shareholders inelastically liquidate all their holdings at date 1 to a succeeding generation of shareholders with the same degree of aggregate risk aversion λ. I show that, provided the firm's information $\widetilde{y}$ is public, the conflict between current and prospective shareholders is fully resolved, and the benchmark equilibrium decisions derived previously continue to be valid. However, when current shareholders (or the firm's manager who represents the current shareholders) are endowed with information that is superior to that of potential shareholders, disclosure issues become important because they affect the cash flows from early liquidation. Intuitively, prospective shareholders will use the firm's *observed* decisions to extract the firm's superior information and determine the price that they are willing to pay for the firm. Hedge disclosure issues then become important because the observed decisions of the firm depend on which disclosure rules are in effect. The firm, acting in the current shareholders interests, responds to the pricing rule of the capital market and chooses its decisions at date 0. Therefore, the price determined in the capital market could affect the firm's decisions made in the interests of current shareholders.

I first derive the equilibrium price at which the firm will be traded at date 1. Given that $\widetilde{y}$ is public information, prospective shareholders assess the distribution of $\widetilde{\pi}$ conditional on y, q, f_1, and z. Because the conditional distribution of $\widetilde{\pi}$ is normal, the equilibrium price $\varphi(q, z, y, f_1)$ of the firm at date 1 is given by:

$$\varphi(q, z, y, f_1) = E(\widetilde{\pi}|q, z, y, f_1) - \lambda Var(\widetilde{\pi}|q, z, y, f_1)$$

[5] Inelastic liquidation ensures that the early liquidation by current shareholders conveys no information to prospective shareholders as it does in Leland and Pyle [1977].

where $\widetilde{\pi} = \widetilde{p}q - c(q) + z(f_0 - f_1)$ at date 1. Substituting for $\widetilde{\pi}$ in the equilibrium price function yields:

$$\varphi(q, z, y, f_1) = qE(\widetilde{p}|f_1, y) - c(q) + z(f_0 - f_1) - \lambda q^2 Var(\widetilde{p}|f_1, y) \tag{14}$$

Equation (14) reflects the fact that at date 1 the only uncertainty in terminal profits comes from the spot price $\widetilde{p}$. The firm's gains or losses from its futures position, $z(f_0 - f_1)$, have already been realized. Additionally, prospective shareholders condition their beliefs on both the public information, y, and on the futures price, f_1, that has been observed at date 1.

From the perspective of date 0, the equilibrium date 1 price, $\widetilde{\varphi}$, is a normally distributed random variable. The firm's current shareholders collectively consume this price $\widetilde{\varphi}$, which is realized at date 1, rather than the firm's terminal cash flows. At date 0 the firm chooses its production and futures trade to maximize the following objective function:

$$\underset{q,z}{Max}\ E(\widetilde{\varphi}|y) - \lambda Var(\widetilde{\varphi}|y) \tag{15}$$

After substituting for $\varphi(q, z, y, f_1)$ from equation (14) in equation (15), the firm's objective function is:

$$\begin{aligned} & qE[E(\widetilde{p}|f_1, y)|y] - c(q) + z[f_0 - E(\widetilde{f}_1|y)] - \lambda q^2 Var(\widetilde{p}|f_1, y) - \lambda q^2 Var[E(\widetilde{p}|f_1, y)|y] \\ & - \lambda z^2 Var(\widetilde{f}_1|y) + 2\lambda qz Cov[E(\widetilde{p}|f_1, y), \widetilde{f}_1|y] \end{aligned} \tag{16}$$

If equation (16) is compared with the firm's objective function described in equation (3), in which the current shareholders *do not* liquidate their holdings at date 1, an important difference emerges. If current shareholders liquidate their holdings at date 1, they are no longer *directly* concerned with the uncertainty in $\widetilde{p}$ but rather with the uncertainty in the firm's interim market value $\widetilde{\varphi}$. The firm's trade in the futures market is a hedge of this latter uncertainty rather than a hedge of the *primitive* uncertainty underlying the spot price. Because the date 1 market value $\widetilde{\varphi}$ of the firm reflects all the relevant information in the economy and the aggregate risk aversion of the current and prospective shareholders is the same, the firm's equilibrium decisions will remain unchanged. Early liquidation should not affect the firm's equilibrium decisions.

Theorem 1 *In a full information regime, the pricing rule in the capital market is such that the firm's equilibrium production level and futures trades are unaffected by early liquidation by current shareholders.*

Theorem 1 shows that the conflict between current and prospective shareholders is fully resolved when there are no informational differences between these two groups of shareholders. In the next two sections, I show that when the firm has information superior to that of the capital market, the conflict between current and prospective shareholders will not be fully resolved, and therefore, the baseline result of Theorem 1 will no longer hold.[6] Intuitively, this result is true because prospective shareholders must infer the firm's superior information from the firm's *observed* decisions in order to value the firm. In the full information regime, given that all the value relevant information is public, the firm's decisions have no informational value, and early liquidation does not impact the firm's decisions at date 0. In the next two regimes, given that the firm has superior information to the capital market, the firm's decisions will naturally acquire a new *informational* value apart from their *real* value on affecting the underlying cash flows of the firm. The capital market extracts the firm's superior information from the observed decisions of the firm and values the firm. The firm which makes all decisions in the best interests of its current shareholders, chooses its production level and futures trade in response to the capital market valuation rule. The firm's observed decisions and its capital market valuation are, therefore, naturally linked and will be determined simultaneously. This link will imply that the firm's observed decisions could, in turn, be affected by the informational content of the pricing rule at date 1.

4 Regime with Mandatory Hedge Disclosures

In this section, I investigate the *informational* effects of these new hedge disclosures on a firm's production and risk management decisions.[7] It is important to understand what value relevant information these gains and losses on derivatives disclose about the firm. First, there is a direct effect arising from the actual gains or losses on the firm's derivative trades. Second, because the prices of publicly traded derivatives

[6] Even though the firm is maximizing its cash flows from liquidation at date 1 and may seem to be behaving "myopically," rather than maximizing its terminal cash flows at date 2, the implication of Theorem 1 is that as long as the pricing rule at date 1 in the full information economy is *appropriately informed* about the future states of the economy, no distortion should occur in the firm's decisions. In fact, this baseline result of the full information regime can be generalized. It can be shown that in a dynamic model with an infinite horizon, maximizing the terminal cash flows of the firm in a full information economy is equivalent to maximizing the price of the firm for every period. (For example, see, Kanodia, [1980]). In the regimes with private information, it will become clear that if the shareholders do not liquidate until the terminal date, then there would be no distortions in the firm's decisions. Thus, the distortions in my environment are due to the informational content of the price at date 1 *and* because the manager is concerned about the firm's date 1 market value.

[7] Statement of Financial Accounting Standards No 119, *Disclosure about Derivative Financial Instruments and Fair Value of Financial Instruments,* (SFAS 119) did require footnote disclosures of a firm's gains and losses on most derivative instruments, even though the disclosure requirements were not as comprehensive as those of SFAS 133. Recognition of these derivative gains or losses on the firm's balance sheet mandated by SFAS 133 or disclosure of these derivative gains or losses in the footnotes, mandated by SFAS 119, are informationally equivalent in my model. Therefore, to the extent that the capital market could also observe $z(f_0 - f_1)$ from the footnote disclosures, my model applies to SFAS 119 as well.

are readily available, these gains and losses reveal the firm's derivative position. Given that a firm makes production and derivative choices based on its superior information about conditions in the commodity market, observing a firm's derivative position may potentially reveal this superior information to the capital market. The latter effect is indirect because it is based on capital market inferences drawn from observing the firm's derivative position.

In my model, the firm's cash flows are realized at date 2. At date 1, the gain or loss on the futures position is $z(f_0 - f_1)$. As indicated earlier, mandatory hedge disclosures reveal $z(f_0 - f_1)$ and also z because f_0 and f_1 are publicly available futures prices. To capture the idea that z could reveal the firm's superior information about conditions in commodity markets, I assume that the signal $\widetilde{y}$ is privately known by the firm. To make disclosures issues relevant, I continue to assume that current shareholders liquidate their holdings inelastically at date 1, before the firm's terminal cash flows are realized. In this case, knowledge of $\widetilde{y}$ is important to prospective shareholders because any information about commodity markets is critical in determining the price that they are willing to pay for the firm. Given the conflict of interest between current and prospective shareholders, it is apparent that direct disclosure of $\widetilde{y}$ is not credible.[8] However, observation of the firm's production and futures trades could possibly reveal the firm's information to prospective shareholders. The price at which the firm is traded in the capital market depends on the firm's choice of q and z for two reasons. First, these choices affect the underlying cash flows of the firm. Second, these choices communicate information. The pricing rule in the capital market determines the decision rules $q(y)$ and $z(y)$ that are in the best interests of the firm's current shareholders. In equilibrium, the inferences of prospective shareholders must be rational in the following sense. These inferences and the capital market price that results from these inferences should be consistent with the decision rules actually chosen by the firm.

I now construct a fully revealing equilibrium using the methodology developed in Kanodia and Lee [1998]. Given that the firm's choice of z is revealed through mandatory hedge disclosures at date 1 and given that the firm's choice of production, q, is publicly known at date 1, the information available to the capital market at date 1 is the triple $\{q, z, f_1\}$. Given mean-variance pricing, the equilibrium price

[8] The lack of credibility creates a demand for mechanisms that would enhance the credibility of communication between the firm and the capital market. In this study, the firm employs one such communication device, namely disclosure of its decision rules. This device indirectly reveals its private information but at a cost. Whether or not there are other less costly communication devices is an open question that merits further investigation.

conditional on this information is:

$$\varphi(q,z,f_1) = qE(\widetilde{p}|f_1,q,z) - c(q) + z(f_0 - f_1) - \lambda q^2 Var(\widetilde{p}|f_1,q,z)$$

The gains or losses from the firm's futures position, $z(f_0 - f_1)$, have already been realized at date 1. Thus, the only random variable whose distribution is assessed by the capital market is the commodity spot price $\widetilde{p}$. The distribution of $\widetilde{p}$ is conditioned by the observed date 1 futures price f_1 because $Cov(\widetilde{p},\widetilde{f}_1) > 0$. The conditioning on $\{q,z\}$ arises from the fact that q and z are chosen by the firm after privately observing y. Suppose that the capital market makes a *perfect* inference that $y = \widehat{y}$ given some observed production and futures trade pair, $\{q,z\}$. I will show later that such perfect inference is indeed feasible. The equilibrium price at date 1 must then be:

$$\varphi(q,z,f_1,\widehat{y}) = qE(\widetilde{p}|f_1,\widehat{y}) - c(q) + z(f_0 - f_1) - \lambda q^2 Var(\widetilde{p}|f_1,\widehat{y}) \tag{17}$$

Because $\left(\widetilde{p},\widetilde{f}_1,\widetilde{y}\right)$ is jointly normal:

$$E(\widetilde{p}|f_1,\widehat{y}) = \beta_f\ f_1 + \beta_y \widehat{y} + A$$

where,

$$\beta_y \equiv \frac{Cov(\widetilde{p},\ \widetilde{y}\ |f_1)}{Var(\widetilde{y}|f_1)} \text{ and } A \equiv \mu_p - \beta_f\ \mu_f - \beta_y \mu_y$$

and β_f is defined in equation (7). Therefore, the firm views $\varphi(q,z,f_1,\widehat{y})$ as:

$$\varphi(q,z,f_1,\widehat{y}) = q\left[\beta_f\ f_1 + \beta_y\widehat{y} + A\right] - c(q) + z(f_0 - f_1) - \lambda q^2 Var(\widetilde{p}|f_1,\widehat{y}) \tag{18}$$

From the firm's perspective at date 0, the only uncertainty in $\varphi(\cdot)$ arises from the uncertain date 1 futures price, $\widetilde{f}_1$. The uncertainty in $\widetilde{p}$ is priced out by prospective shareholders in the capital market, and the firm takes the capital market's pricing rule as a given. The firm assesses the distribution of $\widetilde{f}_1$ conditional on the *true* value of $\widetilde{y}$ regardless of the market's belief $\widehat{y}$. The firm knows that if the true value of y is low, the value of $\widetilde{f}_1$ is also likely to be low.

Given some $\{q,z\}$ and given the market's inference $\widehat{y}$ at date 0, the firm, which has observed $\widetilde{y} = y$, must expect a date 1 capital market value of:

$$E[\varphi(q,z,\widetilde{f}_1,\widehat{y})|y] = q[\beta_f\ E(\widetilde{f}_1|y) + \beta_y\widehat{y} + A] - c(q) + z[f_0 - E(\widetilde{f}_1|y)] - \lambda q^2 Var(\widetilde{p}|f_1,\widehat{y})$$

and a price variance of:

$$Var[\varphi(q,z,\widetilde{f}_1,\widehat{y})|y] = (\beta_f\ q - z)^2\, var(\widetilde{f}_1|y)$$

Therefore, the net expected payoffs of the firm's current shareholders are described by:

$$\begin{aligned}\Psi(q,z,\widehat{y}\;;y) &\equiv E[\varphi(q,z,\widetilde{f}_1,\widehat{y})|y] - \lambda Var[\varphi(q,z,\widetilde{f}_1,\widehat{y})|y] \\ &= q[\beta_f\; E(\widetilde{f}_1|y) + \beta_y\widehat{y} + A] - c(q) + z[\;f_0 - E(\widetilde{f}_1|y)] - \lambda q^2 Var(\widetilde{p}|f_1,\widehat{y}) \\ &- \lambda(\beta_f\, q - z)^2\, var(\widetilde{f}_1|y)\end{aligned} \tag{19}$$

In the preceding expression, the capital market's inference $\widehat{y}$ should not be regarded as a constant, but rather as an inference that varies with the observed values of q and z. By responding to the effect of its production and futures trade choice $\{q,z\}$ on its capital market value, the firm is *implicitly* taking into account the effect of its choices on the capital market's inference $\widehat{y}$. To consider the possibility of a fully revealing equilibrium, I examine how the firm's marginal rates of substitutions between the capital market inference, $\widehat{y}$, and the firm's futures position, z, and between the capital market inference, $\widehat{y}$, and the firm's production choice, q, change with the firm's true type y (that is, with the signal y the firm has observed).

Given a fixed production level, such as $q = \overline{q}$:

$$\frac{\delta\widehat{y}}{\delta z} = -\frac{\Psi_z}{\Psi_{\widehat{y}}} = -\frac{\Big(f_0 - E(\widetilde{f}_1|y) + 2\lambda[\beta_f\,\overline{q} - z]\,Var(\widetilde{f}_1|y)\Big)}{\beta_y\,\overline{q}} \tag{20}$$

The numerator of equation (20) is strictly decreasing in y. Because $\beta_y > 0$, and if $\overline{q} > 0$, the marginal rate of substitution between z and $\widehat{y}$ is strictly increasing in the firm's true type, y, for all z and $\widehat{y}$. Therefore, the single crossing property required for the separation of types is satisfied. In fact, if the numerator in equation (20) is positive, so that the net expected payoff is increasing in z (which turns out to be the case), then $\frac{\delta\widehat{y}}{\delta z} < 0$. Because $\frac{\delta\widehat{y}}{\delta z}$ is negative and increases in y, a high type is willing to take a smaller position in the futures market in order to induce a given increase in $\widehat{y}$ than a low type. Intuitively, this result is true because the low type wants to be inferred as a high type because a high inferred value of the signal y indicates that the spot price, $\widetilde{p}$, will be high. For such an inference to be credible, $\frac{\delta\widehat{y}}{\delta z} < 0$ implies that the low type has to choose a small futures position. However, taking a small position in the futures market may be costly because the expected futures price at date 1, which depends on the true type, may not be high. A small date 1 futures price may imply expected losses in the firm's futures trade.[9] However, the higher the type, the larger the expected date 1 futures price is. Therefore, the

[9] In order to choose a small value of z at date 0, equation 6 indicates that the firm needs to take a large, long speculative

smaller expected losses in futures trade are at date 1 allow a high type to separate itself from a low type.

Similarly, the marginal rate of substitution between $\widehat{y}$ and q for a fixed futures trade, $z = \overline{z}$, is given by:

$$\frac{\delta \widehat{y}}{\delta q} = -\frac{\Psi_q}{\Psi_{\widehat{y}}} = -\frac{\beta_f \; E(\widetilde{f}_1|y) + \beta_y \widehat{y} + A - c'(q) - 2\lambda q Var(\widetilde{p}|f_1, \widehat{y}) - 2\lambda \beta_f [\beta_f \, q - \overline{z}] \, Var(\widetilde{f}_1|y)}{\beta_y \, q}$$

If $q > 0$, the marginal rate of substitution between q and $\widehat{y}$ is strictly decreasing in y for all q and $\widehat{y}$. It turns out that when $q > 0$, $\Psi_q < 0$, so that $\frac{\delta \widehat{y}}{\delta q} > 0$. Because the marginal rate of substitution is strictly both decreasing in y and positive, this implies that a high type is willing to overproduce much more compared to a low type in order to separate itself from the low type. However, overproducing may be costly because the firm may incur losses in the spot market. These losses arise because the expected spot price, which depends on the firm's true type, may be small, resulting in small expected revenues. However, these expected losses will be smaller for the high type than for the low type. Thus, separation of the two types is also possible through their production choice q.

The preceding discussion implies that a fully revealing equilibrium is indeed sustainable and suggests that such revelation could occur from the firm's equilibrium production schedule $q(y)$, its equilibrium futures schedule $z(y)$, or some function of both equilibrium production and futures schedules: $q(y)$ and $z(y)$, respectively. Any such equilibrium schedule of decisions $\{q(y), z(y)\}$ must be incentive compatible. The firm of type y^* does not have the incentive to deviate from $\{q(y^*), z(y^*)\}$, given the equilibrium pricing rule in the capital market and the inferences underlying this pricing rule. The incentive compatibility requirements have a special form when the equilibrium is fully revealing. A fully revealing decision schedule $\{q(y), z(y)\}$ must satisfy:

$$\Psi[q(y), z(y), y, y] \geq \Psi[q(\widehat{y}), z(\widehat{y}), \widehat{y}; y], \forall y, \widehat{y} \tag{21}$$

where $\Psi(\cdot)$ are the expected net payoffs of the firm's current shareholders specified in equation (19). The third argument of $\Psi(\cdot)$ is the inferred value of $\widetilde{y}$, and the fourth argument is the true value of $\widetilde{y}$. In specifying equation (21), any type that chooses the pair $\{q(\widehat{y}),\ z(\widehat{y})\}$ is assessed by the market to be of type $\widehat{y}$, regardless of its true type because the firm's decision schedule is viewed as fully revealing.

To better understand the nature of the incentive compatibility requirements described in equation

position in the futures market (buy futures at f_0). If the firm is a low type, this large speculative position has to be closed out at date 1 at a smaller expected futures price $E(\widetilde{f}_1|y)$, resulting in expected losses on the futures trade.

(21), I will work with the indirect payoff functions defined as follows:

$$V(y) \equiv \Psi[q(y), z(y), y; y] \text{ and } V(\widehat{y}, y) \equiv \Psi[q(\widehat{y}), z(\widehat{y}), \widehat{y}; y],$$

where $\widehat{y}$ in the latter expression is the inferred type, and y is the true type of the firm. From equation (19):

$$V(y) = q(y)[\beta_f \, E(\widetilde{f}_1|y) + \beta_y y + A] - c(q(y)) + z(y)[f_0 - E(\widetilde{f}_1|y)] - \lambda q^2(y) Var(\widetilde{p}|f_1, y) \qquad (22)$$
$$- \lambda[\beta_f \, q(y) - z(y)]^2 \, Var(\widetilde{f}_1|y)$$

and

$$V(\widehat{y}, y) = q(\widehat{y})[\beta_f \, E(\widetilde{f}_1|y) + \beta_y \widehat{y} + A] - c(q(\widehat{y})) + z(\widehat{y})[f_0 - E(\widetilde{f}_1|y)] - \lambda q^2(\widehat{y}) Var(\widetilde{p}|f_1, \widehat{y})$$
$$- \lambda[\beta_f \, q(\widehat{y}) - z(\widehat{y})]^2 \, Var(\widetilde{f}_1|y) \qquad (23)$$

Equation (23) can be expressed as:

$$V(\widehat{y}, y) = V(\widehat{y}) - [\beta_f q(\widehat{y}) - z(\widehat{y})] \left(E(\widetilde{f}_1|\widehat{y}) - E(\widetilde{f}_1|y) \right)$$

Because $\widetilde{f}_1$ and y are jointly normal, $E(\widetilde{f}_1|\widehat{y}) - E(\widetilde{f}_1|y) = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}(\widehat{y} - y)$ so that:

$$V(\widehat{y}, y) = V(\widehat{y}) - [\beta_f q(\widehat{y}) - z(\widehat{y})] \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} (\widehat{y} - y) \qquad (24)$$

Thus, the incentive compatibility constraints described in equation (21) are equivalent to:

$$V(y) \geq V(\widehat{y}) - \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} [\beta_f q(\widehat{y}) - z(\widehat{y})](\widehat{y} - y), \forall y, \widehat{y} \qquad (25)$$

Equation (25) indicates that a certain discipline is built into the equilibrium pricing rule in the capital market. The intuition behind this discipline is found in the firm's natural incentive to be viewed as a high type–that is, as a firm that has observed a high value of the signal y. This incentive arises because a high value of y reveals favorable conditions in the commodity spot market, which result in a higher market value at date 1. However, to be inferred as a high type, the firm has to choose a $\{q, z\}$ pair that is consistent with this inference. To be inferred as a high type, suppose the firm chooses an extremely *long* position, z, in the futures market. However, at date 1, the futures price $\widetilde{f}_1$ is realized, and the firm knows that having observed a low y, on average, the expected value of the futures price $E(\widetilde{f}_1|y)$ will be low. The low value of $E(\widetilde{f}_1|y)$ implies that the firm will incur expected losses in its futures trade. These expected losses are priced by the capital market at date 1 and discipline the firm from taking extreme positions in the futures market. To see how the discipline arises from the pricing rule at date 1, note that:

$$\begin{aligned} E[\varphi(\widehat{q},\widehat{z},\widetilde{f}_1)|y] &= \widehat{q}[\beta_f\ E(\widetilde{f}_1|y)+\beta_y\widehat{y}+A]-c(\widehat{q})+\widehat{z}[f_0-E(\widetilde{f}_1|y)]-\lambda\widehat{q}^2Var(\widetilde{p}|f_1,\widehat{y}) \\ &= E[\varphi(\widehat{q},\widehat{z},\widetilde{f}_1)|\widehat{y}]-\frac{Cov(\widetilde{f}_1,\widetilde{y})}{Var(\widetilde{y})}[\beta_f\widehat{q}-\widehat{z}](\widehat{y}-y) \end{aligned}$$

Therefore, if $\beta_f\widehat{q}-\widehat{z}>0$, which turns out to be the case, in equilibrium for positive realizations of $\widehat{q}$, then at date 0 the price that the firm expects in the capital market is strictly lower than the expected price for type $\widehat{y}$. The penalty expression in the expected capital market price, $\frac{Cov(\widetilde{f}_1,\widetilde{y})}{Var(\widetilde{y})}[\beta_f\widehat{q}-\widehat{z}](\widehat{y}-y)$, is the same as the amount by which $V(\widehat{y},y)$ is less than $V(\widehat{y})$.[10]

The incentive compatibility constraints will be used in a programming problem in order to derive the equilibrium decision schedule of the firm. However, the formulation in equation (25) shows that a continuum of incentive compatibility constraints must be considered. Using the following lemma, I will convert the continuum of incentive compatibility constraints into a more tractable form.

Lemma 3 *The decision schedule $\{q(y),z(y)\}$ is incentive compatible if and only if:*

1. *The schedule $\beta_f q(y)-z(y)$ is increasing in y, and*
2. *$V'(y)=\frac{Cov(\widetilde{f}_1,\widetilde{y})}{Var(\widetilde{y})}[\beta_f q(y)-z(y)],\forall\ y$.*

Lemma 3, which is a standard result in adverse selection settings with a continuum of types states that the continuum of incentive compatibility constraints can be collapsed into a differential equation and a monotonicity requirement.

Can a fully revealing equilibrium be constructed using the full information decision schedule? Recall that because the full information decision schedule $\{q^*(y),z^*(y)\}$ is monotonic in y, a fully revealing equilibrium is indeed plausible. However, I show that as long as the firm's production level is not zero, the first best decision schedule $\{q^*(y),z^*(y)\}$ is not incentive compatible. This implies that in an

[10] From this analysis, note that the disciplining force in the pricing rule arises because at date 0 the distribution of the futures price that will be realized at date 1, f_1, depends upon the firm's true type y. Because y is privately observed by the firm, the *nature* of the firm's private information is important is sustaining a fully revealing equilibrium. In the hedging environment modeled in this paper, this disciplining force arises quite naturally. The firm has private information y about its future spot price. Because the spot market and futures market relate to the same underlying commodity, the private information, y, is also informative, albeit indirectly, about the date 1 futures price f_1. Suppose instead that the firm had private information about its production quantity, rather than about its future spot price. Then, unless the distribution of the futures price at date 1 is derived through market clearing, instead of being exogenous as in the model, its distribution would not depend on the firm's quantity. In this case, the disciplinary force of the capital market price would disappear.

asymmetric information regime in which the futures position of the firm is revealed through mandatory hedge disclosures, the firm's decision schedule $\{q(y), z(y)\}$ will be distorted.

Theorem 2 *The first best decision schedule* $\{q^*(y), z^*(y)\}$ *is not incentive compatible for almost all* y *(that is, for all* y *such that* $q(y) \neq 0$*).*

Theorem 2 implies that when the production level of the firm is zero, the firm chooses the full information futures position. Given that the firm's decision schedule is distorted when its production level is not zero, the firm's decisions in a fully revealing equilibrium must be a solution to the following optimal control problem:

$$\underset{q(y), z(y)}{Max} \int_{-\infty}^{\infty} V(y)h(y)dy$$

subject to,

$$V'(y) = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q(y) - z(y)], \tag{26}$$

where $V(y)$ is defined in equation (22) and $h(\cdot)$ is the normal density function over y. In specifying this program, I have ignored the incentive compatibility requirement that $\beta_f q(y) - z(y)$ is increasing in y, so that the programming problem is relaxed. Later, I will verify that the equilibrium decision schedule meets the monotonicity requirement. Let $L(y)$ denote the Lagrange multiplier associated with constraint (26) and let $\{q^{**}(y), z^{**}(y)\}$ denote the equilibrium decision schedule in the current regime.

Differentiating the previous Hamiltonian expression with respect to $q(y)$ and $z(y)$, respectively, I obtain the following first-order necessary conditions:

$$\Psi_q = -\beta_f \, L(y)\frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} \tag{27}$$

where Ψ_q denotes the partial of $\Psi(\cdot)$ with respect to q and is given by:

$$\Psi_q = \beta_f \, E(\widetilde{f}_1|y) + \beta_y y + A - c'(q(y)) - 2\lambda q(y) Var(\widetilde{p}|f_1, y) - 2\lambda\beta_f [\beta_f \, q(y) - z(y)] \, Var(\widetilde{f}_1|y)$$

and

$$\Psi_z = L(y)\frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} \tag{28}$$

where Ψ_z denotes the partial of $\Psi(\cdot)$ with respect to z and is given by:

$$\Psi_z = f_0 - E(\widetilde{f}_1|y) + 2\lambda[\beta_f \, q(y) - z(y)] \, Var(\widetilde{f}_1|y)$$

Equations (27) and (28) imply the following:

$$\Psi_q = -\beta_f \ \Psi_z \tag{29}$$

Substituting the expressions for Ψ_q and Ψ_z in equation (29) and simplifying yields the following expression:

$$E(\widetilde{p}|y) + \beta_f[f_0 - E(\widetilde{f}_1|y)] = c'(q^{**}(y)) + 2\lambda q^{**}(y)Var(\widetilde{p}|f_1, y)$$

This preceding equation is identical to the equilibrium condition satisfied by production in the full information economy described in equation (5). Thus, even though intuition might suggest that the firm will distort its production schedule $q(y)$ in order to influence the capital market's inference about the firm's private information, the preceding result shows that, in equilibrium, the firm's production schedule is not distorted. This result implies that in order to credibly sustain a fully revealing equilibrium, the firm is induced to deviate from a policy of prudent risk management while its production policy is not distorted at all. Because the full information decision schedule $\{q^*(y), z^*(y)\}$ is not incentive compatible, this implies that the risk management strategy $z^{**}(y)$ of the firm must be distorted. Next, I characterize the precise distortions in the risk management strategy $z^{**}(y)$ of the firm.

Given that the production schedule of the firm is not distorted from the production schedule in the full information economy, distortions in the equilibrium futures position of the firm are equivalent to distortions in the speculative position of the firm. To characterize these speculative distortions, I define $s(y) \equiv \beta_f q(y) - z(y)$ where $s(.)$ is the speculative schedule of the firm. Because $q^{**}(y) = q^*(y)$, in the regime with hedge disclosures distortions in the equilibrium futures position $z^{**}(y)$ are equivalent to distortions in the speculative schedule, $s^{**}(y)$. Therefore, instead of characterizing $z^{**}(y)$, I will characterize the equilibrium speculative schedule, $s^{**}(y)$. The equilibrium speculative schedule $s^{**}(y)$ must satisfy the following differential equation (see the Appendix):

$$\Psi_z \frac{ds}{dy} = \beta_y q^*(y), \tag{30}$$

where

$$\Psi_z = f_0 - E(\widetilde{f}_1|y) + 2\lambda s^{**}(y)\, Var(\widetilde{f}_1|y), \text{ and } s^{**}(y) = \beta_f\, q^*(y) - z^{**}(y).$$

Equation (30) is a first order linear fractional differential equation in s. Solving the differential equation allows me to characterize the unique linear equilibrium speculative schedule $s^{**}(y)$.

Theorem 3

1. *The linear equilibrium speculative schedule $s^{**}(y)$ satisfies*

$$s^{**}(y) = \alpha_0 + \alpha_1 y$$

$$\text{where } \alpha_1 \equiv \frac{\left(\frac{Cov(f_1,y)}{Var(y)} + \sqrt{\frac{Cov^2(f_1,y)}{Var^2(y)} + \frac{4\lambda\beta_y^2 Var(f_1|y)}{k+\lambda Var(p|f_1,y)}} \right)}{4\lambda Var(f_1|y)} \text{ and}$$

$$\alpha_0 \equiv \alpha_1 \left(\frac{\beta_f \ f_0 + A}{\beta_y} \right) + \frac{\mu_f - f_0 - \frac{Cov(f_1,y)}{Var(y)} \left(\frac{\beta_f \ f_0+A}{\beta_y} + \mu_y \right)}{2\lambda Var(f_1|y)}$$

2. $\frac{ds^{**}(y)}{dy} > \frac{ds^{*}(y)}{dy} > 0$, *where $s^{*}(y)$ is the speculative schedule in the full information regime.*

3. $s^{**}(y) \neq s^{*}(y)$ *except at $y = y_0$ where y_0 solves $q^{*}(y) = 0$.*

Using Theorem 3, I will now investigate the properties of the speculative schedule $s^{**}(y)$. Result (2) of Theorem 3 implies that $s^{**}(y)$ is increasing in y. Given that $s^{**}(y) = \beta_f q^{*}(y) - z^{**}(y)$, the monotonicity requirement of the equilibrium schedule $s^{**}(y)$ is verified. Result (2) also indicates that the speculative schedule in the regime with hedge disclosures, $s^{**}(y)$, is steeper than the first best speculative schedule $s^{*}(y)$. Result (3) also confirms that the firm chooses the first best speculative position when the firm's production level is zero (that is, at $y = y_0$). Therefore, the two speculative schedules have different slopes, and they intersect only once at the point where the firm's optimal production level is zero.

Insert Figure 1 here

Figure 1 shows that the firm's speculative position in the regime with mandatory hedges disclosures $s^{**}(y)$ is distorted as long as production is nonzero (that is, for all $y \neq y_0$). More precisely, for all values of y such that $y > y_0$, the firm takes an even longer speculative position in the futures market so that $s^{**}(y) > s^{*}(y)$. Similarly, for those values of y when output is negative, the firm takes an even shorter position in the futures market so that for $y < y_0$, $s^{**}(y) < s^{*}(y)$. Thus, mandatory hedge disclosures induce the firm to take excessive speculative positions in the futures market relative to the full information regime.

Given that the first best futures position $z^{*}(y)$ is defined as prudent risk management behavior, the preceding result demonstrates that mandatory hedge disclosures actually deter prudent risk management behavior by inducing firms to take excessive speculative positions in the futures market. The capital market pricing rule may have induced the firm to either overproduce or to engage in excessive speculation or both. The preceding analysis has shown that excessive speculation–not overproduction–is the firm's most efficient and credible response to the capital market's pricing rule, which causes the firm to depart from a policy of prudent risk management.[11]

5 Regime with No Hedge Disclosures

Unlike the regime with hedge disclosures in which both the production level q and the futures position z of the firm are observed at date 1, investors in the regime with no hedge disclosures can only observe the firm's production level q at date 1. Because the futures position, z, of the firm is not observed, the capital market can neither price the actual gain or loss $z(f_0 - f_1)$ on the futures position at date 1, nor use z to infer the firm's private information. However, the capital market must somehow assess the firm's futures position to rationally value the firm. Because the firm's production choice q is revealed at date 1 and its production choice is guided in part by the firm's private signal, it seems reasonable to model the capital market's assessment by a conjectured schedule $\widehat{z}(q)$. The capital market's beliefs about the firm's futures position do not vary with the firm's actual choice of z. However, the beliefs do vary with the firm's choice of q. The firm takes this feature of the capital market's pricing rule into account in determining its optimal production choice.

If the firm's equilibrium production policy $q(y)$ is invertible in y, the capital market infers the value of y by inverting the schedule $q(y)$ to determine the equilibrium capital market price. Thus, the conjectured futures trading strategy $\widehat{z}(q)$ can be expressed as a function of y:

$$\widehat{z}(y) = \widehat{z}(t(q))$$

where $t(\cdot)$ denotes the inverse function of $q(\cdot)$. The equilibrium capital market price can then be expressed as:

$$\varphi(q, f_1, y) = qE(\widetilde{p}|f_1, y) - c(q) + \widehat{z}(t(q))(f_0 - f_1) - \lambda q^2 Var(\widetilde{p}|f_1, y) \tag{31}$$

[11] It is plausible that a contractual agreement between the firm and the current shareholders might mitigate the risk management distortions caused by hedge disclosures. My model is silent on this issue because the incentives of the manager have not been modeled. It is indeed conceivable that an explicit compensation contract between the firm and the current shareholders could mitigate the distortions in the firm's risk management strategy and induce efficient decisions. Such a compensation contract could also benefit future shareholders. However, it is not clear whether such a renegotiation proof contract exists. (See Persons [1994]) The nature of the optimal contract that would survive renegotiation in my environment *and* induce efficient decisions is an open and interesting question that deserves further investigation.

where the dependence of q on y has been suppressed. Equation (31) illustrates that the firm's actual choice of z does not affect the firm's price at date 1 because the firm's futures position is not observed. However, the conjectured gains or losses from futures trades are reflected in the equilibrium capital market price. The firm must, therefore, be indifferent among all choices of z. At first glance, it might appear that because the actual gains or losses from the futures trade are not priced, the firm is unaffected by realizations of f_1 and would, therefore, lose all incentive to speculate. This intuition is incorrect. The firm is affected by the price difference $(f_0 - f_1)$ through the firm's *production* policy because the market's conjecture of its futures position depends on the observed production quantity. Thus, the expression $\widehat{z}(t(q))(f_0 - f_1)$ appears in equation (31).

The firm's production policy, $q(y)$, has a dual inferential role in this regime. First, q is used to infer the firm's private information y. Second, q is used to infer the firm's futures position. Given this dual inferential role, it might seem that the firm would be tempted to overproduce to convey good news to the market. However, with an appropriate capital market conjecture, $\widehat{z}(q)$, this temptation is disciplined by the firm's expectation of the conjectured gain or loss on the futures trade, $\widehat{z}(q)(f_0 - E(\widetilde{f}_1|y))$, where y is the true signal observed by the firm.

Because any equilibrium in the current regime must depend upon the capital market's endogenous expectations about the firm's futures position, multiple equilibria will prevail. Given the existence of multiple equilibria in the regime with no hedge disclosures and given that the regulators' goal is to deter imprudent risk management strategies, is there an equilibrium in this regime when the firm actually chooses a prudent risk management strategy? If such an equilibrium exists in which the firm may actually choose a prudent risk management strategy without any mandatory hedge disclosures, the desirability of mandatory hedge disclosures to curb imprudent risk management loses its appeal. In fact, in the environment modeled in the paper mandatory hedge disclosures have the opposite effect of inducing excessive speculation.

Theorem 4

1. *In the regime with no hedge disclosures when the capital market conjectures the following futures schedule* $\widehat{z}(t(q)) = \beta_f q - s^*(t(q))$, *where* $s^*(.)$ *is the full information speculative schedule and* $t(.)$ *is the inverse of* $q(.)$, *the equilibrium decision schedule is given by* $\{q^{***}(y), s^*(y)\}$. *In this decision*

schedule $s^*(y)$ *is the full information speculative schedule and the optimal production schedule,* $q^{***}(y)$*, satisfies the following first order differential equation:*

$$[(-A+\beta_f\ f_0)-\beta_y y+(2k+2\lambda Var(\widetilde{p}|f_1,y))\,q^{***}(y)]\,\frac{dq}{dy}=\beta_y q^{***}(y)$$

2. *The firm's production policy is given by the following:*

$$q^{***}(y)=\frac{A+\beta_f\ f_0+\beta_y y}{k+\lambda Var(\widetilde{p}|f_1,y)}$$

Theorem 4 demonstrates that it is optimal for the firm to choose a prudent risk management strategy. Not surprisingly in the regime with no hedge disclosures, prudent risk management cannot be attained without a distortion in production. Specifically, the firm's production policy $q^{***}(y)$ will be distorted. Figure 2 illustrates the nature of these distortions in $q^{***}(y)$ relative to the first best production policy, $q^*(y)$.

Insert Figure 2 here

The findings in the regimes with and without mandatory hedge disclosures should help guide regulators in evaluating the desirability of hedge disclosures. First, I have shown that a policy of prudent risk management cannot be attained in the regime with hedge disclosures. This finding undermines the primary objective of SFAS 133–the deterrence of imprudent risk management activities. In fact, hedge disclosures induce firms to engage in excessive speculation. Second, in the regime without mandatory hedge disclosures, regulators need to be aware that a policy of prudent risk management can be optimal, but prudent risk management is obtained at a cost. The firm's production decision is distorted. This analysis suggests that for those industries whose inherent risks are perfectly observable, only a much more careful analysis of the magnitude of the relative trade-offs between production distortions versus speculative distortions will help determine the desirability of mandatory hedge disclosures. Perhaps using industry specific parameters, a careful industry-wide calibration exercise of the magnitude of risk management distortions versus production distortions might provide guidance about which distortion is more costly and hence whether hedge disclosures should be mandated for that specific industry.

6 Extensions and Limitations

This paper sheds some light on the *real effects* of hedge disclosures and can be used as a springboard for further investigation. Clearly more work is warranted in order to investigate the desirability of hedge disclosures from a policy perspective. First, my analysis has shown that mandatory hedge disclosures may discourage prudent risk management strategies while the absence of hedge disclosures fosters prudent risk management, but at a cost: sub-optimal production. However, I do not investigate the relative trade-offs between production and risk management distortions. It is possible that risk management distortions are not as costly as production distortions for a given industry. Thus, mandating hedge disclosures for that particular industry may be warranted. As I pointed out earlier, a careful calibration exercise using primitive parameters from specific industries may help determine the magnitude of the relative trade-offs between production distortions and risk management distortions. Second, my analysis assumes that the firm's exposure to inherent risk captured by the production level, q, is known to the capital market. In a setting in which the firm's level of inherent risk is not known, a firm's incentives to engage in speculation may be different. Investigating a setting in which inherent risk is private would complement the current study and may provide further insights into the effects of the mandatory hedge disclosures on a firm's risk management strategy.

Finally, I have assumed throughout the analysis that the firm is exogenously endowed with private information about future spot market conditions even though such information is probably obtained at a cost. Given that information acquisition is costly, the potential costs and benefits of obtaining such information should be modeled and compared across each disclosure regime. Intuitively, the benefit of obtaining private information about the future spot price results in an increase in expected utility from making informed production and risk management decisions. The cost of obtaining private information consists of the direct exogenous cost of information acquisition and the indirect endogenous costs that arise from distortions in production and risk management decisions. For each disclosure regime, the relative costs and benefits of obtaining such information must be compared to evaluate the desirability of hedge disclosures.

7 Appendix: Proofs of Results

Proof of Theorem 1

The firm's objective function described in equation (16) can be simplified using the following two standard statistical results:[12]

$$E(E(\widetilde{p}|f_1, y)|y) = E(\widetilde{p}|y), \text{ and} \tag{32}$$

$$Var(\widetilde{p}|y) = E(Var(\widetilde{p}|f_1, y)|y) + Var(E(\widetilde{p}|f_1, y)|y)$$

Given normally distributed random variables, the conditional variance of $\widetilde{p}$ is a constant that does not depend upon the realization of the conditioning information so that:

$$Var(\widetilde{p}|y) = Var(\widetilde{p}|f_1, y) + Var(E(\widetilde{p}|f_1, y)|y) \tag{33}$$

Using the properties of conditional expectations, it can be shown that

$$Cov[E(\widetilde{p}|f_1, y), \widetilde{f}_1|y] = Cov(\widetilde{p}, \widetilde{f}_1|y) \tag{34}$$

Using equations (32), (33), and (34) the objective function (16) becomes:

$$qE(\widetilde{p}|y) - c(q) + z[f_0 - E(\widetilde{f}_1|y)] - \lambda[q^2 Var(\widetilde{p}|y) + z^2 Var(\widetilde{f}_1|y) - 2qzCov(\widetilde{p}, \widetilde{f}_1|y)]$$

which is identical to equation (3). ■

Proof of Lemma 3

First I prove the necessity part of the lemma. Consider any two types, y and $\widehat{y}$. The incentive compatibility constraint for type y is:

$$V(y) \geq V(\widehat{y}, y), \forall\, y, \widehat{y}$$

But $V(\widehat{y}, y) = V(\widehat{y}) - \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q(\widehat{y}) - z(\widehat{y})](\widehat{y} - y)$ so that the incentive compatibility constraint (IC) for type y becomes:

$$V(y) \geq V(\widehat{y}) - \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q(\widehat{y}) - z(\widehat{y})](\widehat{y} - y)$$

Similarly, the IC constraint for type $\widehat{y}$ is:

[12] See DeGroot [1970].

$$V(\widehat{y}) \ \geq V(y) - \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q(y) - z(y)](y - \widehat{y})$$

This implies that the IC constraints are equivalent to:

$$\frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q(y) - z(y)](\widehat{y} - y) \leq V(\widehat{y}) - V(y) \leq \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q(\widehat{y}) - z(\widehat{y})](\widehat{y} - y) \tag{35}$$

It follows immediately from (35) that $\widehat{y} \geq y$ implies $\beta_f q(y) - z(y) \leq \beta_f q(\widehat{y}) - z(\widehat{y})$ which proves condition (1). In turn, this monotonicity implies that $\beta_f q(y) - z(y)$ is continuous almost everywhere. Dividing (35) by $(\widehat{y} - y)$ and taking the limit as $\widehat{y} \to y$ yields $V'(y) = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q(y) - z(y)]$, which is condition (2).

Now I prove the sufficiency part of the theorem. Integrating condition (2) using the Fundamental Theorem of Integral Calculus yields:

$$V(y) = C + \frac{Cov(\ \widetilde{f}_1, \widetilde{y})}{Var(\ \widetilde{y})} \int_{-\infty}^{y} [\beta_f q(t) - z(t)] dt$$

where C is the arbitrary integration constant.

Note that since $V(y) - V(\widehat{y}, y) = [V(y) - V(\widehat{y})] - [V(\widehat{y}, y) - V(\widehat{y})]$,

$$\begin{aligned} V(y) - V(\widehat{y}, y) &= \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} \int_{\widehat{y}}^{y} [\beta_f q(t) - z(t)] dt - \frac{Cov(\ \widetilde{f}_1, \widetilde{y})}{Var(\ \widetilde{y})}[\beta_f q(\widehat{y}) - z(\widehat{y})](y - \widehat{y}) \\ &= \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} \int_{\widehat{y}}^{y} \left([\beta_f q(t) - z(t)] - [\beta_f q(\widehat{y}) - z(\widehat{y})]\right) dt \end{aligned}$$

If $\widehat{y} < y$, then condition (1) implies that the above integral is non-negative given that:

$$\beta_f q(t) - z(t) \geq \beta_f q(\widehat{y}) - z(\widehat{y}) \text{ for each } t > \widehat{y}.$$

On the other hand, if $\widehat{y}$ is greater than y, the above integral becomes:

$$\int_{y}^{\widehat{y}} \left([\beta_f q(\widehat{y}) - z(\widehat{y})] - [\beta_f q(t) - z(t)]\right) dt,$$

which is again non-negative. ■

Proof of Theorem 2

I need to show that the Lagrange multiplier $L(y)$ associated with the incentive compatibility constraint (26) is binding for almost all y, i.e., $L(y) \neq 0, \forall\ y$ such that $q(y) \neq 0$. Suppose, in order to obtain a contradiction, that for some $\widetilde{y} = \overline{y}$, $L(\overline{y}) = 0$ and $q^*(\overline{y}) \neq 0$. This implies that the first best production level, $q^*(\overline{y})$, and the first best futures position, $z^*(\overline{y})$, must be incentive compatible. Given $\{q^*(\overline{y}), z^*(\overline{y})\}$:

$$V(\overline{y}) = q^*(\overline{y})[\beta_f\, E(\widetilde{f}_1|\overline{y}) + \beta_y \overline{y} + A] - c(q^*(\overline{y})) + z^*(\overline{y})[f_0 - E(\widetilde{f}_1|\overline{y})] \tag{36}$$
$$- \lambda q^{*2}(\overline{y}) Var(\widetilde{p}|f_1, \overline{y}) - \lambda[\beta_f\, q^*(\overline{y}) - z^*(\overline{y})]^2\, var(\widetilde{f}_1|\overline{y})$$

For $\{q^*(\overline{y}), z^*(\overline{y})\}$ to be incentive compatible, $V(\overline{y})$ must satisfy the following condition:

$$V\,'(\overline{y}) = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q^*(\overline{y}) - z^*(\overline{y})] \tag{37}$$

But since $V(\overline{y})$ is maximized at $q(y) = q^*(\overline{y})$ and $z(y) = z^*(\overline{y})$, using the *Envelope theorem* to differentiate $V(\overline{y})$ with respect to $\overline{y}$ yields:

$$V\,'(\overline{y}) = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f\, q^*(\overline{y}) - z^*(\overline{y})] + \beta_y q^*(\overline{y}) \tag{38}$$

Equations (37) and (38) imply that for $q^*(\overline{y})$ and $z^*(\overline{y})$ to be incentive compatible, the following condition must hold:

$$\frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f\, q^*(\overline{y}) - z^*(\overline{y})] + \beta_y q^*(\overline{y}) = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}[\beta_f q^*(\overline{y}) - z^*(\overline{y})]$$

which, after cancelling common terms, yields $\beta_y q^*(\overline{y}) = 0$, which is impossible since $\beta_y > 0$ and $q^*(\overline{y}) \neq 0$.

■

Derivation of the differential equation satisfied by $s^{**}(y)$:

Because $V(y) = \Psi(q(y), z(y), y)$, differentiate $V(y)$ with respect to y, to obtain:

$$V'(y) = \Psi_q q'(y) + \Psi_z z'(y) + \Psi_y, \tag{39}$$

where Ψ_q and Ψ_z are given above with $\beta_f q(y) - z(y)$ replaced by $s(y)$. Ψ_y denotes the partial of $\Psi(\cdot)$ with respect to y and is given as follows:

$$\Psi_y = \frac{Cov(\,\widetilde{f}_1, \widetilde{y})}{Var(\,\widetilde{y})} s(y) + \beta_y q(y)$$

Recall that the incentive compatibility constraint requires that:

$$V'(y) = \frac{Cov(\,\widetilde{f}_1, \widetilde{y})}{Var(\,\widetilde{y})} s(y) \tag{40}$$

Equating the right-hand sides of equations (39) and (40) results in:

$$\Psi_q q'(y) + \Psi_z z'(y) + \Psi_y = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} s(y)$$

Using $\Psi_q = -\beta_f \Psi_z$ from equation (29) and the above equation for Ψ_y, the equilibrium speculative schedule $s^{**}(y)$ must satisfy the following differential equation:

$$\Psi_z \frac{ds}{dy} = \beta_y q^*(y), \text{ where}$$

$$\Psi_z = f_0 - E(\widetilde{f}_1|y) + 2\lambda s^{**}(y)\, Var(\widetilde{f}_1|y), \text{ and } s^{**}(y) = \beta_f\, q^*(y) - z^{**}(y).$$

Proof of Theorem 3

I first rewrite equation (30) in the form:

$$\frac{ds}{dy} = \frac{b_0 + b_1 y}{c_0 + c_1 y + c_2 s}, \text{ where} \tag{41}$$

$$b_0 \equiv \frac{\beta_y(\beta_f\ f_0 + A)}{2k + 2\lambda Var(\widetilde{p}|f_1, y)},\ b_1 \equiv \frac{\beta_y^2}{2k + 2\lambda Var(\widetilde{p}|f_1, y)},\ c_0 \equiv -\mu_f + f_0 + \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}\mu_y,$$
$$c_1 \equiv -\frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})}, \text{ and } c_2 \equiv 2\lambda Var(\widetilde{f}_1|y)$$

The linear fractional differential equation (41) can be transformed into a homogeneous form by changing the variables y to Y and s to S related by the equations: $y = Y + h$, $s = S + k$ where h and k are constants and $h = -\frac{b_0}{b_1}$, and $k = \frac{c_1 b_0 - c_0 b_1}{b_1 c_2}$

The homogenous equation:

$$\frac{dS}{dY} = \frac{b_1 Y}{c_1 Y + c_2 S} \tag{42}$$

can now be solved using the substitution $S = uY$.

Using the above substitution, (42) becomes:

$$u + Y\frac{du}{dY} = \frac{b_1}{c_1 + c_2 u}$$

The above equation is a separable equation which after rearranging, can be integrated to yield the following one parameter family of solutions:

$$\int \frac{c_1 + c_2 u}{b_1 - c_1 u - c_2 u^2} du = \ln Y + C \tag{43}$$

where C is an arbitrary constant.

Using the substitution $\sqrt{b_1 + \frac{c_1^2}{4c_2}} \tanh\theta = \sqrt{c_2}(u + \frac{c_1}{2c_2})$, the left hand side of integral (43) becomes:

$$\int \left(\frac{c_1 + c_2 u}{b_1 - c_1 u - c_2 u^2} \right) du = \frac{-1}{2} \ln(b_1 - c_1 u - c_2 u^2) + \frac{c_1}{\sqrt{4b_1 c_2 + c_1^2}} \tanh^{-1} \left(\frac{2c_2 u + c_1}{\sqrt{4b_1 c_2 + c_1^2}} \right)$$

Using the above result, (43) becomes:

$$\frac{-1}{2} \ln(b_1 - c_1 u - c_2 u^2) + \frac{c_1}{\sqrt{4b_1 c_2 + c_1^2}} \tanh^{-1} \left(\frac{2c_2 u + c_1}{\sqrt{4b_1 c_2 + c_1^2}} \right) = \ln Y + C$$

Substituting for $u = \frac{S}{Y}$ in the preceding expression and using $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$, the above expression can be re-arranged to yield the following equation:

$$\left(\frac{\sqrt{4b_1 c_2 + c_1^2} Y - 2c_2 S(Y) - c_1 Y}{\sqrt{4b_1 c_2 + c_1^2} Y + 2c_2 S(Y) + c_1 Y} \right)^{\frac{-c_1}{\sqrt{4b_1 c_2 + c_1^2}}} = C \left[b_1 Y^2 - c_1 S(Y) Y - c_2 S^2(Y) \right]$$

Setting the numerator of the left-hand side to zero and solving for $S(Y)$ yields the linear solution $S(Y) = \frac{\sqrt{4b_1 c_2 + c_1^2} - c_1}{2c_2} Y$ to the differential equation. Recalling that $S = s - k$ and $Y = y - h$, the linear solution to the differential equation therefore becomes:

$$s^{**}(y) = \frac{\sqrt{4b_1 c_2 + c_1^2} - c_1}{2c_2} y + \frac{\sqrt{4b_1 c_2 + c_1^2} - c_1}{2c_2} (\frac{b_0}{b_1}) + \frac{c_1 b_0 - c_0 b_1}{b_1 c_2}$$

Substituting for the values of $b_0, b_1, c_0, c_1,$ and c_2 yields part (1) of Theorem 3.

Using $s^{**}(y)$,it is very straightforward to verify parts (2) and (3) of Theorem (3). ■

Proof of Theorem 4

For any given conjectured futures schedule, $\widehat{z}((q(y))$, the IC condition that the production schedule $q(y)$ must satisfy can be characterized by the following condition:

$$W\ '(y) = \frac{Cov(\widetilde{f}_1, \widetilde{y})}{Var(\widetilde{y})} \left[\beta_f q(y) - \widehat{z}(q(y)) \right], \text{ where} \tag{44}$$

$$W(y) = q(y)[\beta_f\ E(\widetilde{f}_1|y) + \beta_y y + A] - c(q(y)) + \widehat{z}(q(y))[f_0 - E(\widetilde{f}_1|y)] - \lambda q^2(y) Var(\widehat{p}|f_1, y)$$
$$- \lambda \left[\beta_f q(y) - \widehat{z}(q(y)) \right]^2 Var(\widetilde{f}_1|y)$$

The capital market conjectures the following futures schedule for a given q

$$\widehat{z}(t(q)) = \beta_f q - s^*(t(q)),$$

where $s^*(.)$ is the full information speculative schedule.

Substituting the above conjecture in (44) and simplifying, the IC condition becomes:

$$\Big[\beta_f\ E(\widetilde{f}_1|y)+\beta_y y+A-c'(q(y))-2\lambda q(y)Var(\widetilde{p}|f_1,y)+\beta_f[f_0-E(\widetilde{f}_1|y]\Big]\,q'(y)$$
$$+\ \Big[E(\widetilde{f}_1|y)-f_0-2\lambda Var(\widetilde{f}_1|y)s^*(y)\Big]\,s'^*(y)+\beta_y q(y)=0$$

Because $s^*(y)$ is the full information speculative schedule, this implies that:

$$E(\widetilde{f}_1|y)-f_0-2\lambda Var(\widetilde{f}_1|y)s^*(y)=0$$

so that the optimal production schedule, $q^{***}(y)$, must satisfy the following differential equation:

$$[-(A+\beta_f\ f_0)-\beta_y y+(2k+2\lambda Var(\widetilde{p}|f_1,y)q^{***}(y)]\,\frac{dq}{dy}=\beta_y q^{***}(y)$$

which proves part (1) of Theorem 4. Using techniques similar to those used for solving the linear differential equation in Theorem (3), the linear solution to the above differential equation can be established.

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Figure 1: Speculative Distortions in the Presence of Mandatory Hedge Disclosures

y: Value of signal about the commodity spot price privately observed by firm.
$s^*(y)$: Speculative schedule in the full information regime.
$s^{**}(y)$: Speculative schedule in regime with mandatory hedge disclosures.
$q^*(y)$: Production schedule in the full information regime.
y_o: Value of y at which $q^*(y) = 0$.

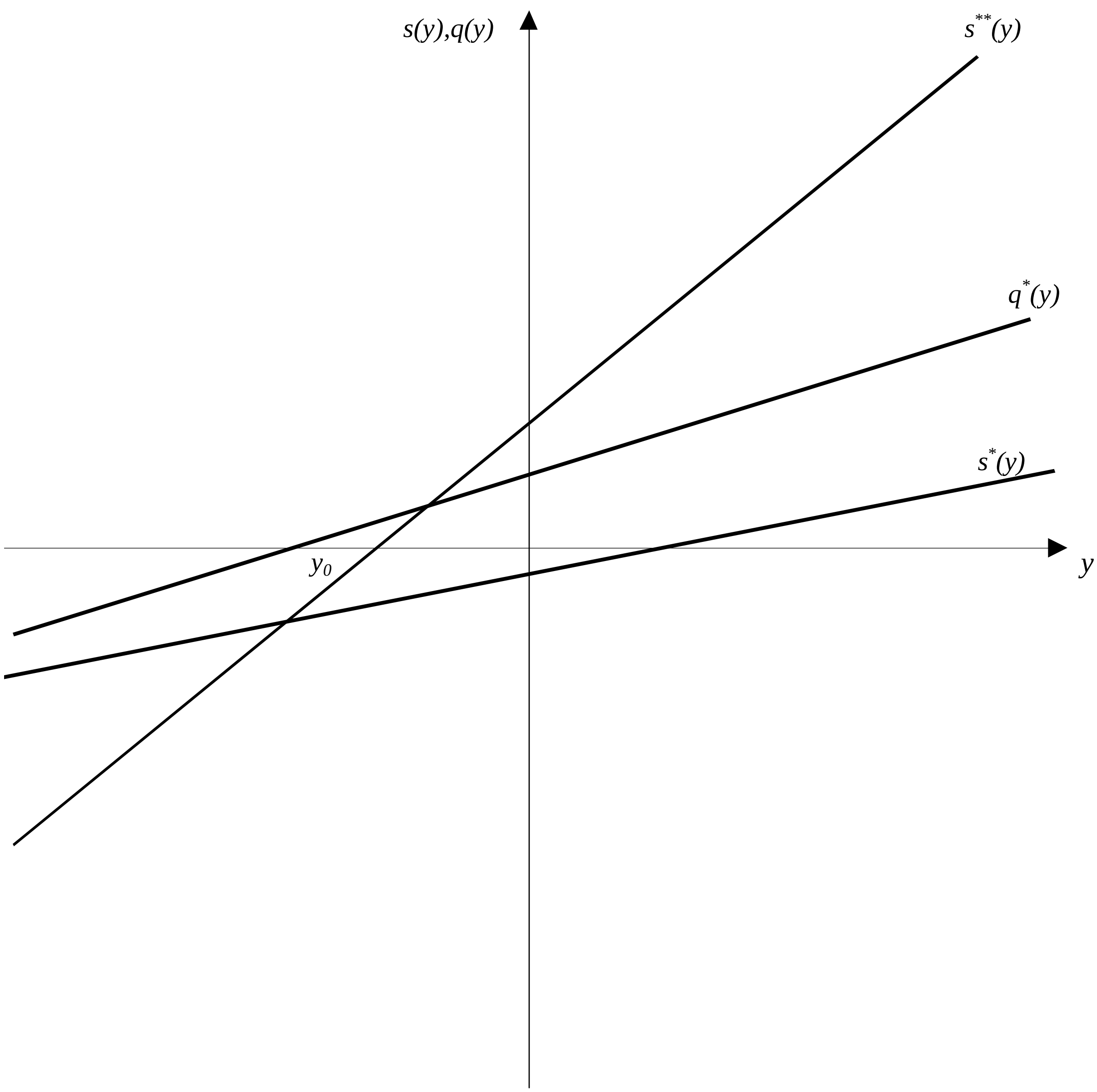

Figure 2: Production Distortions in the Absence of Hedge Disclosures

y: Value of signal about commodity spot price privately observed by firm.
$q^*(y)$: Production schedule in the full information regime.
$q^{***}(y)$: Production schedule in the regime with no hedge disclosures.
y_o: Value of y at which $q(y) = 0$.
$s^*(y)$: Speculative schedule in the full information regime.