

Do Derivative Disclosures Impede Sound Risk Management?*

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Abstract

We model an environment in which firms disclose only one side of a hedging transaction, namely the gain or loss on the forward. However, the firm cannot credibly disclose the other side of the hedging transaction, namely the underlying exposure that is being hedged. We show that because the firm cannot credibly communicate that the exposure from its underlying project is hedgeable, greater transparency in the firm's derivative activities distorts firms' hedging decisions.

The nature of these distortions depend crucially on (i) firms' information quality about their project types and (ii) the market's prior beliefs about whether or not firms have hedgeable projects.

For most reasonable levels of information quality, we find that instead of impeding risk management, derivative disclosures are likely to induce firms to engage in excessive speculation.

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1. Introduction

The spate of highly publicized accounting scandals in recent years has given great impetus to the trend toward greater transparency of a firm's financial activities. The belief is that shining a bright light into the dark corners of firms' accounts—via greater disclosure—will prevent managers from hiding the consequences of their actions from outside investors. Good corporate governance and greater transparency are thus seen as two sides of the same coin. Such an argument is indeed quite compelling in economies with completely frictionless markets and complete contracts. However, when there are imperfections in an economy, the benefits of greater transparency are not so clear.

To understand this, it is useful to draw an analogy from the theory of the second best from welfare economics. When there is more than one imperfection in an economy, removing a subset of these imperfections need not be welfare-improving overall. It is conceivable that the removal of one of the imperfections magnifies the negative effects of the other imperfections to the detriment of overall welfare. Therefore, mandating greater disclosure of a firm's activities without addressing the other imperfections in an environment need not guarantee a welfare improvement.

Our aim in this paper is to shed some light on this second-best perspective by focusing on an environment prone to imperfections in the pricing mechanism and contracting institutions. In particular, we examine a *hedging* environment where the firm cannot credibly disclose the true nature of the project that it undertakes. To see this concretely, consider a firm that can perfectly hedge the risky cash flows from an expected inventory sale by buying a forward contract. Suppose, in the interim, the gain or loss on the forward contract is observed before the terminal cash flows from the inventory sale are realized. If the forward purchase

transaction is viewed *independently* of the inventory sale transaction, then the interim realization of the payoffs from the forward contract makes the firm's cash flows appear more volatile than the firm's actual net cash flows. However, such volatility in the firm's cash flows is artificial, in the sense that the gain or loss on the forward contract will be exactly offset by the loss or gain on the inventory when it is sold. If investors are fully informed of the nature of the firm's project, such volatility is, therefore, a veil, and all investors can see through it. Unfortunately, there are several natural features of a firm's hedging environment that can prevent such volatility from merely being a veil. We list some possible causes.

1. Firms may be endowed with an anticipated exposure that is either hedgeable or unhedgeable. However, firms cannot *credibly* disclose to outsiders about the nature of their exposures.
2. To hedge their anticipated exposures, firms may buy a forward contract and the cash flows of the forward contract may be realized *before* the cash flows from the underlying exposure.
2. Managers and shareholders of firms may have short horizons and therefore dislike *both* short-run and long-run volatility. Thus, a firm with a hedgeable project may not want to hedge when its *perceived* short-run volatility is increased. This occurs because the capital market cannot distinguish a firm that is appropriately hedging by buying a forward contract from another firm that is exacerbating its short-term and long-term volatility—thereby speculating by buying a forward contract.

We take the above features as given and focus on the effect of derivative disclosures on short term volatility. In particular, we assume (i) that shareholders have short horizons, so that managers want to minimize short-term volatility; (ii)

the firm is involved in a cash flow hedge, and the payoffs of the forward contract are realized before the payoffs of the long term project; *(iii)* the firm is endowed with either a hedgeable project or an unhedgeable project, but the firm cannot credibly disclose that it has a hedgeable exposure to the capital market. These three conditions ensure that short term volatility in the firm's cash flows matters.

We model a firm that undertakes a project with a long gestation that takes two periods to yield its terminal cash flows. At date 0, when the manager makes the hedging decision, she does not know for sure whether the cash flows from the project are hedgeable. However, the manager observes a private but noisy signal about the project type. Thus, even though the manager is uncertain about her project type, she is still better informed than the capital market about whether her firm's project is hedgeable or not. Based on her superior information, at date 0, the manager then decides whether or not to hedge the date 2 cash flows from the project by buying a forward contract. The forward contract is a perfect hedge of a firm's hedgeable project because the payoffs from the forward contract are perfectly negatively correlated with the payoffs from the hedgeable project.¹ On the other hand, the payoffs from the project may be unhedgeable in the sense that the payoffs from forward contract are not correlated with the payoffs from the project. To capture the feature that the interim cash flow volatility of a firm that is properly hedging may be higher than the firm's terminal cash flow volatility, we assume that the payoffs from the forward contract are realized at date 1 instead of date 2. Thus, except for the sequential mismatch in the resolution of uncertainty, the payoffs from the forward contract are a perfect hedge of the cash flows from the hedgeable project. The manager makes the hedging decision at date 0 in order to minimize both the date 1 and the date 2 cash flow variances. These variances,

¹We have assumed that the forward is a perfect hedge of the hedgeable project for analytical tractability. This assumption can be easily relaxed to introduce imperfect hedging.

in turn, depend on what information is available to the capital market at date 1. We model two information regimes: a *disclosure regime* in which the firm is required to disclose whether or not it has purchased the forward contract and a *non-disclosure regime* in which the firm does not disclose any information about its derivative activities. Under the disclosure regime, if the firm buys the forward contract, the firm marks the forward contract to market at date 1 and thereby discloses the payoffs on the forward contract. However, when the capital market observes the forward contract at date 1, it is unsure whether or not the firm has a hedgeable project. As discussed earlier, the firm cannot credibly disclose to the market that it has a hedgeable project. This occurs because the information that the firm observes about its project type is imprecise at the time it makes its hedging decision. Therefore, a firm with an unhedgeable project, basing its hedging decision on the best information that it has, could still *incorrectly* hedge and thereby *speculate* unintentionally, by buying the forward contract.

The main result in the paper is that derivative disclosures lead to distortions in a firm's risk management strategy. However, when derivative disclosures are not made, a firm's risk management strategy is socially optimal. The nature of the distortions when derivative disclosures are made depend crucially on (i) the firm's information quality about the project type and (ii) the market's prior beliefs that the firm has a hedgeable project. We show that when firms do not have very precise information about project types or the proportion of firms with hedgeable projects is relatively low, then there is massive underhedging in the economy relative to the social optimum. In fact, we show the existence of a unique equilibrium in which none of the firms buy the forward contract. The intuition behind this result is as follows: any firm that buys the forward must convince the market that it has a hedgeable project. However, when the *ex ante* incidence of hedgeable firms is small or when the firms' information is very noisy, the market

exercises a great deal of scepticism about whether a firm has a hedgeable project. In the face of such scepticism, a firm's best reply is not to buy the forward given the hedging decisions of all the other firms. We show that this is the best reply for *all* the firms resulting in a large social inefficiency. On the other hand, when the information of firms is relatively precise and the proportion of firms with hedgeable projects is relatively high, there is excessive speculation in the economy relative to the social optimum. We show that all firms in the economy buy the forward contract. This occurs because it is now *very easy* to convince the market that it has a hedgeable project by buying the forward contract, so that all firms buy the forward contract. This once again results in a large welfare loss because even the firms with unhedgeable projects buy the forward contract. For intermediate levels of information quality and prior beliefs about the proportion of hedgeable firms, there exists a unique interior equilibrium in which some firms buy the forward contract while the remainder do not. However, the incidence of hedging is sub-optimal.

To get a feel for reasonable levels of information quality in our model, we relate the informativeness of the signals that firms observe with the probabilities of type *I* and type *II* errors. A type *I* error in our context is the error of not buying the forward contract when the firm has a hedgeable project. A type *II* error is buying the forward contract when the firm is unhedgeable. We focus on the decision rule that equates the probabilities of both types of errors. We show that for most reasonable levels of information quality, derivative disclosures will most likely result in excessive speculation rather than underhedging. In fact, underhedging virtually disappears when the type *I* and type *II* errors equal 10%.

Our objective in this paper is to illustrate that enhanced disclosures may not always be beneficial. In that vein, we present a one-sided argument against enhanced disclosures. To be sure, we do not want our model to imply that, in

general, greater transparency is not socially desirable. There is a vast empirical and theoretical literature that has shown the net benefits of enhanced disclosures (see Bushman and Smith (2003) and Leuz and Verrechia (2000) for some very good recent examples). We also believe that there are undeniable benefits to greater transparency in the long run. However, our aim is to redress the balance against the arguments for greater transparency by *deliberately* focusing on a hedging environment where more disclosures can be harmful. As discussed above, we focus on a hedging environment because it seems like a natural setting where the second-best perspective discussed above can be sharply made—firms cannot credibly disclose the nature of their underlying exposures *and* may have short horizons. Given these frictions, greater transparency in the firm’s derivative activities distorts firms’ hedging decisions.

Our disclosure model cannot therefore be used to address the mark-to-market versus historical cost accounting debate for derivative instruments. Statement of Financial Accounting Standards No. 133 (SFAS 133) mandated that all derivatives be marked to market. The standard was very controversial because many industry leaders felt that marking derivatives to market would induce artificial volatility in firms’ reported numbers (see for example Greenspan (1997)). Regulators, however, felt that historical cost accounting, however, also masked a firm’s derivatives activities. Given our stated objective, we show that *disclosing* forward transactions via mark-to-market accounting may distort risk management strategies while *not disclosing* forward transactions as under historical cost accounting may be socially optimal.²

The rest of the paper is organized as follows. We present the model in the next section. Section 3 establishes the socially optimal benchmark regime. Section 4

²See Plantin, Sapra, and Shin (2007) for a model that analyzes the trade-offs of mark-to-market accounting vs. historical cost accounting.

investigates the *non-disclosure regime* in which derivative disclosures are not made at date 1. Section 5 investigates the *disclosure regime* in which firms must disclose whether or not they have purchased the forward contract. Section 6 illustrates the role of information quality and the *ex ante* proportion of hedgeable firms in determining the nature of the risk management distortions in the disclosure regime. Section 7 concludes.

2. The Model

There are three types of projects - a hedgeable project, an unhedgeable project and a forward contract. These projects are stochastic processes that yield the following cash flows at two dates indexed by $t \in \{1, 2\}$.

| | $t = 1$ | $t = 2$ |
|-------------|---------|---------|
| hedgeable | w_1 | w_2 |
| forward | v_1 | v_2 |
| unhedgeable | z_1 | z_2 |

The random variables $\{\tilde{w}_t, \tilde{v}_t, \tilde{z}_t\}$ have distributions given as follows.

| | $t = 1$ | $t = 2$ |
|-------------|-----------------------------------|-----------------------------------|
| hedgeable | $w_1 = 0$ with prob. 1 | $\tilde{w}_2 \sim N(1, \sigma^2)$ |
| forward | $\tilde{v}_1 \sim N(0, \sigma^2)$ | $v_2 = 0$ with prob. 1 |
| unhedgeable | $z_1 = 0$ with prob. 1 | $\tilde{z}_2 \sim N(1, \sigma^2)$ |

where the joint densities are such that the forward contract is a perfect hedge for the hedgeable project, except for a timing mismatch. The date 1 realization is perfectly negatively correlated with the *date 2* realization of the hedgeable contract. The unhedgeable project has cash flows that are uncorrelated with the

other projects. Thus, we have

$$\begin{aligned}\text{corr}(\tilde{v}_1, \tilde{w}_2) &= -1 \\ \text{corr}(\tilde{z}_2, \tilde{v}_1) &= 0 \\ \text{corr}(\tilde{z}_2, \tilde{w}_2) &= 0\end{aligned}$$

There is a continuum of firms in the economy. Each firm is endowed with either a hedgeable project or an unhedgeable project. We call the firm with a hedgeable project, a *hedgeable firm* and the firm with a unhedgeable project, an *unhedgeable firm*.

We wish to capture a realistic feature of a firm's hedging environment: the manager of a firm may not know for sure whether the firm is endowed with a hedgeable project or not. However, the manager of the firm may have better information than the capital market about whether the firm's project is hedgeable or not. She then bases her hedging decision on the best information that she has. Assume that the manager observes a private signal on whether the project is hedgeable or not. If the firm's project is hedgeable, the signal observed by the firm is drawn with density

$$f_H(\cdot)$$

while if the firm's project is not hedgeable, the signal is drawn with density

$$f_N(\cdot)$$

Conditional on the type of project, signals drawn across firms are i.i.d. That is, the signals received by the hedgeable firms are i.i.d. draws with density $f_H(\cdot)$, and the signals received by the unhedgeable firms are i.i.d. draws with density $f_N(\cdot)$. We assume that the signals are informative in the sense that the ratio

$$\frac{f_H(x)}{f_N(x)}$$

is increasing in the signal x . The monotone likelihood ratio property thus holds. Given that higher signals make it more likely that the firm's project is hedgeable, we consider decision rules for the firms in which there is a threshold value x^* of the signal such that a firm chooses to hedge and buy the forward contract if and only if the signal realization x is higher than the threshold x^* . Thus, we construct equilibria in which there is a common threshold x^* and all firms use the following *switching strategy*:

$$\begin{cases} \text{buy forward} & \text{if } x \geq x^* \\ \text{not buy forward} & \text{if } x < x^* \end{cases} \quad (2.1)$$

Although an individual manager does not know whether her firm is hedgeable, it is common knowledge that fraction ρ of the firms is hedgeable. The remainder, $1 - \rho$, of firms is unhedgeable. For a given threshold x^* for the switching strategies of the firms, we denote by h the proportion of those hedgeable firms that decide to buy the forward contract. Denoting by $F_H(\cdot)$ the cumulative distribution function corresponding to f_H , we have

$$h = 1 - F_H(x^*)$$

Similarly, we will denote by s the proportion of *unhedgeable* firms who decide to buy the forward contract.

We introduce a minor technical assumption. Of the group of unhedgeable firms, we will assume that a small proportion $\varepsilon > 0$ of the firms are *speculative firms* in the sense that its managers know that they have the unhedgeable project, but nevertheless always buy the forward contract. These firms' terminal values are given by the sum $v_1 + z_2$. For all our results reported below, we will be taking the limit in which $\varepsilon \rightarrow 0$. The purpose of the perturbation is to enable the market to derive off-equilibrium beliefs when it encounters a deviation by one firm in buying the forward when starting from the status quo in which no firm buys the forward contract.

The proportion s of unhedgeable firms who decide to buy the forward contract consists of the fraction ε of speculative firms that knowingly purchase the forward contract, and those firms with unhedgeable projects that act in good faith, but *incorrectly hedge* due to the realization of their signal. Thus, s is given by

$$s = \varepsilon + (1 - \varepsilon)(1 - F_N(x^*))$$

The joint densities over the types of firms and whether they buy the forward contract or not are then given by

| | hedgeable firms | unhedgeable firms |
|-----------------|-----------------|---------------------|
| buy forward | $h\rho$ | $s(1 - \rho)$ |
| not buy forward | $(1 - h)\rho$ | $(1 - s)(1 - \rho)$ |

The date 2 liquidation values of the two types of firms depend on whether or not they decide to buy the forward contract. The date 2 liquidation values are given by

| | hedgeable firms | unhedgeable firms |
|-----------------|-----------------|-------------------|
| buy forward | $w_2 + v_1 = 1$ | $z_2 + v_1$ |
| not buy forward | w_2 | z_2 |

We will impose the logistic functional form on the signal densities³ so as to simplify the algebra and enable comparative statics analysis. We will assume that the signals of the unhedgeable firms are drawn from the cumulative distribution:

$$F_N(x) = \frac{1}{1 + \exp(-(x - \mu))} \quad (2.2)$$

while the signals of the hedgeable firms are drawn from the cumulative distribution:

$$F_H(x) = \frac{1}{1 + \exp(-(x - \mu - \Delta))} \quad (2.3)$$

where $\Delta > 0$. Thus, μ is the mean of the density f_N for non-hedgeable firms while the mean of the density f_H for hedgeable firms is given by $\mu + \Delta$.

³See Amemiya (1981) for the approximation properties of logistic densities for the normal.

The positive constant Δ is a measure of the informativeness of the signals of the firms. The larger is Δ , the more informative are the signals about whether or not the firm has a hedgeable project, since the signals are drawn from densities that are far apart. We investigate the relationship between the informativeness measure Δ and the probability of type I and type II errors later.

Given our assumptions on the signal densities and assuming that all firms use a switching strategy described by the inequalities in (2.1), we can solve explicitly for the proportion s of speculative firms as a function of the proportion h of hedgeable firms. For any threshold x^* , we have

$$\begin{aligned} s &= \varepsilon + (1 - \varepsilon) \frac{1}{1 + \exp(x^* - \mu)} \\ h &= \frac{1}{1 + \exp((x^* - \mu - \Delta))} \end{aligned}$$

The second equation implies that $x^* = \mu + \Delta + \ln\left(-\frac{h-1}{h}\right)$. Substituting this expression in to the first equation and taking $\varepsilon \rightarrow 0$, we have

$$s = \frac{1}{1 + e^{\Delta} \left(\frac{1}{h} - 1\right)} \tag{2.4}$$

When $\Delta = 0$, the signals are uninformative about the project type so that a hedgeable firm is indistinguishable from an unhedgeable firm. This implies that $s(h) = h$ so that as the proportion h of hedgeable firms increases, the proportion of speculative firms increases at the same rate. However, as Δ increases, the signals become more informative so that firms with hedgeable projects can better separate themselves from firms with unhedgeable projects - that is, h can be made larger without making s large.

The informativeness measure, Δ , plays a crucial role in determining the optimal hedge ratios in each accounting regime. Intuitively, the less informative the signals are, the more costly it will be for a hedgeable firm that buys a forward

contract (and is therefore properly hedging its terminal cash flows) to distinguish itself from an unhedgeable firm that is exacerbating the riskiness of its terminal cash flows by buying a forward contract.

3. Socially Optimal Hedge Ratio: Benchmark Regime

The social welfare optimum for a risk averse population would be for all firms to minimize their terminal volatility. Suppose each firm knew for sure whether or not its project were hedgeable, then all hedgeable firms should buy the forward contract, and all unhedgeable firms should not buy the forward contract. The socially optimal hedge ratio is $h^*(\rho) = 1$ for all ρ so that the resulting ex ante terminal volatility would be $(1 - \rho)^2 \sigma^2$, the ex ante terminal volatility of the unhedgeable firms.

However, this welfare optimum is unattainable in our environment because each firm faces uncertainty about whether or not its project is hedgeable. We will therefore derive the ex ante socially optimal hedge ratio, $h^*(\cdot)$, given that each firm is uncertain of its project type but observes a noisy private signal about it.

When viewed from date 0, the final liquidation value of the firm is described by the random variable $\tilde{\theta}$ defined as

$$\tilde{\theta} = h\rho \cdot 1 + (1 - h)\rho \cdot \tilde{w}_2 + s(h)(1 - \rho) \cdot (\tilde{z}_2 + \tilde{v}_1) + (1 - s(h))(1 - \rho) \cdot \tilde{z}_2 \quad (3.1)$$

where s is written as a function of h to take into account the explicit dependence of the proportion of speculative firms on the proportion of hedgeable firms as described by equation (2.4).

We assume that the date 0 hedging decision is determined by the manager's ex ante utility function $U(\cdot)$ defined as

$$U(E(\tilde{\theta}), Var(\tilde{\theta})) = E(\tilde{\theta}) - kVar(\tilde{\theta}) \quad (3.2)$$

where $E(\tilde{\theta})$ is the date 0 expected liquidation value of the firm and $Var(\tilde{\theta})$ is the date 0 variance of the firm's final liquidation value and k is a positive constant. If the shareholders of the representative firm have CARA preferences with aggregate risk aversion coefficient k , then their expected utility will take the form described in 3.2. Note that U is increasing in the firm's expected liquidation value and decreasing in the variance of the firm's liquidation value. Because $E(\tilde{\theta}) = 1$, this implies that the socially optimal hedge ratio $h^*(\rho, \Delta)$ minimizes the date 0 variance of the firm's final liquidation value. The variance of the firm's final liquidation value is the quadratic form:

$$V(h, \rho, \sigma^2) = a\Sigma a^T \quad (3.3)$$

where

$$a = [(1-h)\rho \quad s(h)(1-\rho) \quad (1-s(h))(1-\rho)]$$

$$\Sigma = \sigma^2 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

and $s(h)$ is given by equation 2.4.

Substituting for a and Σ in equation (3.3), we get the following expression for the ex ante date 0 variance the firm's final liquidation value:

$$V(h, \rho, \sigma^2) = [(1-h)^2\rho^2 - 2s(h)(1-h)\rho(1-\rho) + (s^2(h) + 1)(1-\rho)^2] \sigma^2 \quad (3.4)$$

Proposition 1. *The socially optimal hedge ratio, $h^*(\rho, \Delta)$, is given by:*

$$h^*(\rho, \Delta) = \frac{1}{2\rho(e^\Delta - 1)} \left(1 + 2\rho(e^\Delta - 1) - \sqrt{(1 + 4\rho(1-\rho)(e^\Delta - 1))} \right)$$

Proof. See Appendix. ■

It can be shown that $\frac{\delta h^*}{\delta \rho} > 0$ for all Δ so that the socially optimal hedge ratio increases as the ex ante proportion of hedgeable firms increases as expected. Similarly, $\frac{\delta h^*}{\delta \Delta} > 0$ for all ρ so that the socially optimal hedge ratio increases when the signals are more informative because the hedgeable firms can better separate themselves from the unhedgeable firms. For the case when $\Delta = 0$, we have $h^*(\rho, 0) = \rho$ so that when the signals about project type are uninformative, the socially optimal hedge ratio equals ex ante proportion of hedgeable firms.

The socially optimum hedge ratio will serve as a benchmark against which we will compare the equilibrium hedge ratios in two information regimes: a *disclosure regime* where a firm is required to disclose whether or not it has purchased the forward contract at the interim date 1 and a *non-disclosure regime* where a firm does not disclose any information about its forward contract until the terminal date 2.

As discussed earlier, if the firm only cares about its terminal volatility, then issues about disclosure or non-disclosure of the forward contract at date 1 are moot. However, we will show that when the manager the firm or the firm's shareholders have short horizon payoffs that depend on both its interim and terminal volatility, the incentives to purchase the forward contract will be perverse: the firm may either under-hedge and thus not undertake sound risk management or over-hedge and thus speculate by taking on excessive risk. For each information regime, we will thus examine the incentives of a firm to purchase the forward contract when the firm's payoffs depend on *both* their interim volatility and terminal volatility.

4. Non-Disclosure Regime

In the non-disclosure regime, at date 0 the firm observes a private signal x about whether it has a hedgeable project or not. At the interim date 1, the firm does not disclose any information about whether or not it has purchased the forward

contract until date 2, when the terminal cash flows from the firm's project are realized. The only information publicly observable at date 1 in the non-disclosure regime is v_1 , the payoff from the forward contract. However, the capital market does not observe whether or not a firm has purchased a forward contract.

Suppose firms are run by short horizon managers who dislike both date 1 and date 2 variance. Their hedging decisions at date 0 is determined by the utility function

$$U\left(E\left(\tilde{V}_1\right), \sigma_1^2\right) \equiv E\left(\tilde{V}_1\right) - k\sigma_1^2 \quad (4.1)$$

where σ_1^2 is the variance of first period market value when viewed from date 0 and $E\left(\tilde{V}_1\right)$ is the date 0 expected value of the firm. Suppose the date 1 value, V_1 , of the firm is given by:

$$V_1 = E(\tilde{\theta}|v_1) - k\text{Var}(\tilde{\theta}|v_1) \quad (4.2)$$

where $\tilde{\theta}$ is the terminal value of the firm given by (3.1). If shareholders in the capital market have CARA preferences and aggregate risk aversion, k , then the market clearing price of the firm at date 1 would take the form in equation (4.2). Because the capital market does not observe whether or not the firm has purchased the forward contract, the information set of the capital market at date 1 consists only of v_1 .

Substituting for $\tilde{\theta}$ in (4.2) yields the following expression for V_1 :

$$V_1 = h\rho \cdot 1 + (1-h)\rho \cdot (1-v_1) + s(h)(1-\rho) \cdot (1+v_1) + (1-s(h))(1-\rho) - k(1-\rho)^2\sigma^2$$

Substituting for V_1 in (4.1) yields:

$$U\left(E\left(\tilde{V}_1\right), \sigma_1^2\right) = 1 - k \left[((1-h)^2\rho^2 - 2s(h)(1-h)\rho(1-\rho) + (s^2(h) + 1)(1-\rho)^2) \sigma^2 \right] \quad (4.3)$$

Thus, the firm chooses h to minimize the following volatility:

$$\sigma^2 \left[((1-h)^2\rho^2 - 2s(h)(1-h)\rho(1-\rho) + (s^2(h) + 1)(1-\rho)^2) \right]$$

But this is exactly the ex ante volatility of the firm's terminal cash flows described by equation (3.4). Hence, the equilibrium hedge ratio in the non-disclosure regime is also the ex ante socially optimal hedge ratio.

We should add immediately that this result is the result of our simplified framework, rather than a normative result that we advocate. Our model has been chosen so as to highlight the perverse nature of disclosures. Hence, the benchmark non-disclosure regime has been chosen deliberately to coincide with the social optimum.

5. Disclosure Regime

In the disclosure regime, the firm observes at date 0 a private signal x about whether or not it has a hedgeable project. However unlike the non-disclosure regime, at date 1, the firm is required to disclose whether or not it has bought the forward contract. Thus, at date 1, the capital market observes not only the payoff v_1 from the forward contract but also whether or not the firm has purchased the forward contract.

5.1. Expected Payoffs of a disclosing firm

We will derive the payoffs of firms that purchase the forward contract. In the disclosing regime, these firms must disclose the forward contract. Suppose at date 1, the firm discloses the outcome of the forward contract. The market then puts conditional probability

$$\frac{h\rho}{h\rho + s(1 - \rho)}$$

that the firm has a hedgeable project, and conditional probability

$$\frac{s(1 - \rho)}{h\rho + s(1 - \rho)}$$

that the firm has an unhedgeable project. Thus, from the market's point of view at date 1, the final liquidation value of the disclosing firm at date 2 is the random variable:

$$\tilde{\theta}_d \equiv \frac{h\rho}{h\rho + s(1-\rho)} \cdot 1 + \frac{s(1-\rho)}{h\rho + s(1-\rho)} \cdot (\tilde{z}_2 + v_1)$$

Therefore, the date 1 expected value of $\tilde{\theta}_d$ given the market's information is:

$$\begin{aligned} y_d &\equiv E\left(\tilde{\theta}_d|v_1\right) = \frac{h\rho}{h\rho + s(1-\rho)} \cdot 1 + \frac{s(1-\rho)}{h\rho + s(1-\rho)} \cdot (E(\tilde{z}_2) + v_1) \\ &= 1 + \frac{s(1-\rho)}{h\rho + s(1-\rho)}v_1 \end{aligned}$$

We can also calculate the conditional volatility of final liquidation value of the firm when viewed from date 1. It is given by:

$$\begin{aligned} \sigma_{d,2}^2 &\equiv E\left(\tilde{\theta}_d - y_d|v_1\right)^2 = E\left(\frac{s(1-\rho)}{h\rho + s(1-\rho)}(\tilde{z}_2 - 1)\right)^2 \\ &= \left(\frac{s(1-\rho)}{h\rho + s(1-\rho)}\right)^2 \sigma^2 \end{aligned}$$

The market value, V_1^d , of the disclosing firm at date 1 is therefore given by:

$$\begin{aligned} V_1^d &= y_d - k\sigma_{d,2}^2 \\ &= 1 + \frac{s(1-\rho)}{h\rho + s(1-\rho)}v_1 - k\left(\frac{s(1-\rho)}{h\rho + s(1-\rho)}\right)^2 \sigma^2 \end{aligned}$$

When viewed from date 0, the payoff from the forward contract, \tilde{v}_1 , is a random variable so that the ex ante expected value of the disclosing firm's interim market value is:

$$\begin{aligned} E(\tilde{V}_1^d) &= 1 + \frac{s(1-\rho)}{h\rho + s(1-\rho)}E(\tilde{v}_1) - k\left(\frac{s(1-\rho)}{h\rho + s(1-\rho)}\right)^2 \sigma^2 \\ &= 1 - k\left(\frac{s(1-\rho)}{h\rho + s(1-\rho)}\right)^2 \sigma^2 \end{aligned}$$

and the volatility of the disclosing firm's interim market value at date 1 is given by

$$Var(\tilde{V}_1^d) = \left(\frac{s(1-\rho)}{h\rho + s(1-\rho)} \right)^2 \sigma^2$$

It is noticeable that for the disclosing firm, the date 1 and the date 2 conditional variances are the same.

The payoffs of a short horizon manager of a disclosing firm who maximizes the expected utility of date 1 cash flows, V_1^d , is then given by:

$$\begin{aligned} U \left(E \left(\tilde{V}_1^d \right), Var \left(\tilde{V}_1^d \right) \right) &\equiv E(\tilde{V}_1^d) - kVar(\tilde{V}_1^d) \\ &= 1 - 2k \left(\frac{s(1-\rho)}{h\rho + s(1-\rho)} \right)^2 \sigma^2 \end{aligned} \quad (5.1)$$

5.2. Expected Payoffs of a non-disclosing firm

Consider now the firms that do not purchase the forward contract and hence do not disclose the forward contract. Conditional on no disclosure of the forward, the probability that the firm is a hedgeable firm is:

$$\frac{(1-h)\rho}{(1-h)\rho + (1-s)(1-\rho)}$$

The conditional probability of the firm being unhedgeable is

$$\frac{(1-s)(1-\rho)}{(1-h)\rho + (1-s)(1-\rho)}$$

Thus, from the market's point of view at date 1, the final liquidation value of the non-disclosing firm at date 2 is the random variable:

$$\tilde{\theta}_n \equiv \frac{(1-h)\rho}{(1-h)\rho + (1-s)(1-\rho)} \cdot \tilde{w}_2 + \frac{(1-s)(1-\rho)}{(1-h)\rho + (1-s)(1-\rho)} \cdot \tilde{z}_2$$

The date 1 expected value of $\tilde{\theta}_n$ given the market's information is

$$\begin{aligned} y_n &\equiv E\left(\tilde{\theta}_n|v_1\right) = \frac{(1-h)\rho}{(1-h)\rho + (1-s)(1-\rho)}(1-v_1) + \frac{(1-s)(1-\rho)}{(1-h)\rho + (1-s)(1-\rho)}E(\tilde{z}_2) \\ &= 1 - \frac{(1-h)\rho}{(1-h)\rho + (1-s)(1-\rho)}v_1 \end{aligned}$$

The conditional volatility of final liquidation value of the non-disclosing firm when viewed from date 1 is given by

$$\begin{aligned} \sigma_{n,2}^2 &\equiv E\left(\tilde{\theta}_n - y_n|v_1\right)^2 = E\left(\frac{(1-s)(1-\rho)}{(1-h)\rho + (1-s)(1-\rho)}(\tilde{z}_2 - 1)\right)^2 \\ &= \left(\frac{(1-s)(1-\rho)}{(1-h)\rho + (1-s)(1-\rho)}\right)^2 \sigma^2 \end{aligned}$$

The market value, V_1^n , of the non-disclosing firm at date 1 is therefore given by:

$$\begin{aligned} V_1^n &= y_n - k\sigma_{n,2}^2 \\ &= 1 - \frac{(1-h)\rho}{(1-h)\rho + (1-s)(1-\rho)}v_1 - k\left(\frac{(1-s)(1-\rho)}{(1-h)\rho + (1-s)(1-\rho)}\right)^2 \sigma^2 \end{aligned}$$

When viewed from date 0, the payoff from the forward contract, \tilde{v}_1 , is a random variable so that the ex ante expected value of the non-disclosing firm's interim market value is:

$$E(\tilde{V}_1^n) = 1 - k\left(\frac{(1-s)(1-\rho)}{(1-h)\rho + (1-s)(1-\rho)}\right)^2 \sigma^2$$

and the volatility of the non-disclosing firm's interim value at date 1 is given by

$$Var(\tilde{V}_1^n) = \left(\frac{(1-h)\rho}{(1-h)\rho + (1-s)(1-\rho)}\right)^2 \sigma^2$$

The payoffs of a short horizon manager of a non-disclosing firm who maximizes the expected utility of date 1 cash flows, V_1^n , is then given by:

$$\begin{aligned} U\left(E\left(\tilde{V}_1^n\right), Var\left(\tilde{V}_1^n\right)\right) &\equiv E(\tilde{V}_1^n) - kVar(\tilde{V}_1^n) \tag{5.2} \\ &= 1 - k\left(\frac{(1-h)^2\rho^2 + (1-s)^2(1-\rho)^2}{((1-h)\rho + (1-s)(1-\rho))^2}\right) \sigma^2 \end{aligned}$$

5.3. Equilibria in the Disclosure Regime

We solve for equilibrium in switching strategies of the form

$$\begin{cases} \text{buy forward} & \text{if } x \geq x^* \\ \text{not buy forward} & \text{if } x < x^* \end{cases}$$

Since each firm's ex ante probability of hedging is a monotonic function of its switching point x_i^* , we can write the ex ante payoffs in terms of the switching points $\{x_i^*\}$. However, it is convenient to work directly with h in our analysis. Because the proportion h of hedgeable firms is a monotonic function of x^* and from equation (2.4), the proportion, s , of unhedgeable firms that buy the forward is monotonic in h , the ex ante payoffs from buying the forward contract and from not buying the forward contract can be written solely as a function of h as follows.

From (5.1), the ex ante payoff, $U_D(h)$ from buying the forward contract is:

$$U_D(h) = 1 - 2k \left(\frac{s(h)(1-\rho)}{h\rho + s(h)(1-\rho)} \right)^2 \sigma^2 \quad (5.3)$$

Similarly, from (5.4), the ex ante payoff, $U_{ND}(h)$ from not buying the forward contract is:

$$U_{ND}(h) = 1 - k \left(\frac{(1-h)^2 \rho^2 + (1-s(h))^2 (1-\rho)^2}{((1-h)\rho + (1-s(h))(1-\rho))^2} \right) \sigma^2 \quad (5.4)$$

The ex ante payoffs $U_D(h)$ and $U_{ND}(h)$ define a normal form, binary action game among the continuum of firms, and our equilibrium notion is the plain Nash equilibrium notion for normal form perfect information games. An equilibrium is a profile of decisions i.e., whether to buy or not to buy the forward contract - one for each firm - such that, one firm's decision maximizes its payoff given the decisions of all the other firms.

We may consider three possible types of equilibrium. The first is when no firm buys the forward contract. Such an equilibrium exists when

$$U_D(0) \leq U_{ND}(0)$$

so that when no-one buys the forward (i.e. $h = 0$), it is better not to buy the forward oneself. The second type of equilibrium is when *every* firm buys the forward. Such an equilibrium exists when

$$U_D(1) \geq U_{ND}(1)$$

so that when everyone buys the forward (i.e. $h = 1$), it is better to buy the forward oneself. Finally, we could also have an interior equilibrium in which there is some fraction h (strictly between zero and one) of firms that buy the forward that makes all firms indifferent between buying the forward or not. In other words

$$U_D(h) = U_{ND}(h)$$

Before we characterize the equilibria in the disclosure regime more fully, let us first note that there is always an equilibrium in the disclosure regime when none of the firms buy the forward contract. In other words, there is always an equilibrium with $h = 0$. To see this, note that

$$U_D(0) = 1 - 2k\sigma^2 < 1 - k \left(\left(\frac{(1-\epsilon)(1-\rho)}{\rho+(1-\epsilon)(1-\rho)} \right)^2 + \left(\frac{\rho}{\rho+(1-\epsilon)(1-\rho)} \right)^2 \right) \sigma^2 = U_{ND}(0)$$

so that $U_D(0) < U_{ND}(0)$. This result, however, rests to a large extent on our technical assumption that there is always a small proportion ε of unhedgeable firms who speculate by buying the forward contract regardless of the signal received. We make this assumption simply for the technical reason that off-equilibrium beliefs must be defined for $h = 0$. For this reason, it would not be warranted to claim any important status to this result.

However, if the *only* equilibrium is the one in which $h = 0$, then such a result would be more noteworthy. For some parameter values, it turns out that the only equilibrium is the one in which $h = 0$.

Proposition 2. *Suppose*

$$\Delta < \ln \left\{ \frac{1-\rho}{\rho} \left(\sqrt{\frac{2}{\rho^2 + (1-\rho)^2}} - 1 \right) \right\}. \quad (5.5)$$

Then there is a unique equilibrium. In this equilibrium, no firm buys the forward contract.

Proof. See Appendix ■

Condition (5.5) defines the region in (ρ, Δ) -space in which none of the firms buy the forward contract in equilibrium. This condition is intuitive, since it is likely to be satisfied when Δ is small (so that the firms' signals are very noisy) and when ρ is small, making it less likely ex ante that buying the forward will fulfil a hedging function. The welfare consequences of the lack of hedging activity in this equilibrium could be potentially very large, especially when the socially optimal level of purchase of the forward contract is large. We explore these issues in more detail in the next section.

It is also important to understand why the ex ante optimal h given by proposition 1 cannot be sustained as an equilibrium. In equilibrium, each firm chooses the action that maximizes its own payoff taking others actions as given. The firm does not take account of the social optimum when making its decision. Any firm that buys the forward must convince the market that it has a hedgeable project. However, when the ex ante incidence of hedgeable firms is small (i.e. ρ is small), or when the signals are very noisy (Δ is small), the market exercises a great deal of scepticism. In the face of such scepticism, a firm's best reply is not to buy the forward. This is the best reply for *all* the firms. Thus, none of them hedge, and the unique equilibrium is the one in which $h = 0$.

At the opposite end of the spectrum, we can also have an equilibrium in which there is *excessive* purchase of the forward contract in the sense that the equilibrium

level of h is higher than the socially optimal level. In particular, we can identify the parameter values in which every firm buys the forward contract, so that $h = 1$.

Proposition 3. *Suppose*

$$\Delta > \ln \left(\frac{2(1-\rho)^2\rho + \rho\sqrt{4(1-\rho)^2 - 1}}{(1-\rho)(1-2(1-\rho)^2)} \right) \quad (5.6)$$

Then there exists an equilibrium in which $h = 1$. There is no interior equilibrium.

Proof. See Appendix ■

In reading this proposition, it should be borne in mind that the right hand side of (5.6) is not well-defined for all values of ρ . In the appendix, we show that the expression is well-defined only for a narrow range of ρ lying between $1 - \frac{1}{\sqrt{2}}$ ($\simeq 0.29$) and 0.5. Provided that the right hand side of (5.6) is well-defined, the inequality defines the region in (ρ, Δ) -space in which all firms buy the forward contract. It shows that for a relative large values of ρ and Δ , every firm in the economy hedges. This overhedging occurs for analogous reasons as that described for the $h = 0$ case. There is a preponderance of hedgeable firms in the economy (ρ is large), and firms have precise signals. Thus, a firm needs to do little to convince the market that it has a hedgeable project. However, the problem is that it is now *too easy* to convince the market. However precise the signal, pushing h up to 1 means that s (the incidence of inadvertent speculation by unhedgeable firms) is also pushed up to 1. This social inefficiency is not taken into account by the individual firms.

Proposition 4. *Suppose that*

$$\ln \left\{ \frac{1-\rho}{\rho} \left(\sqrt{\frac{2}{\rho^2+(1-\rho)^2}} - 1 \right) \right\} < \Delta < \ln \left(\frac{2(1-\rho)^2\rho + \rho\sqrt{4(1-\rho)^2 - 1}}{(1-\rho)(1-2(1-\rho)^2)} \right)$$

then there is an interior equilibrium in which h is strictly between zero and one. There is precisely one such interior equilibrium.

Proof. See Appendix ■

For intermediate levels of informativeness Δ and for intermediate values of the incidence of hedgeable firms ρ , there is an interior equilibrium. Again, it is worth emphasizing that each firm considers its own payoff, rather than what is socially optimal. The interior equilibrium is possible because for the equilibrium incidence of h , each firm is indifferent between buying the forward and not. The fact that h lies strictly between zero and one is thus quite removed from the reason why the socially optimal level of h is between zero and one. In equilibrium, the proportion h is determined by the indifference condition of the firms, rather than what is socially optimal.

6. Risk Management Distortions in Disclosure Regime

How do the equilibrium hedge ratios in the disclosure regime compare with the social optimum for different values of ρ and Δ ? The larger the divergence between the two, the greater is the social welfare loss that results from the disclosure regime. We can illustrate the nature of the distortions by plotting the socially optimal hedge ratio against the equilibrium hedge ratio in the disclosure regime as the function of ρ , the ex ante incidence of hedgeable firms. We plot three such cases - for $\Delta = 0$, $\Delta = 1$ and $\Delta = 3$.

Figure 6.1 shows that when $\Delta = 0$, (so that the signals observed by the firms are worthless), the socially optimal hedge ratio is given by the 45 degree line. That is, the optimal hedge ratio is given by ρ itself. However, the equilibrium hedge ratio displays a very different shape. It is a jump function that takes the value zero when $\rho < 0.5$, and takes the value 1 when $\rho > 0.5$. The intuition is clear. When the signal is worthless, the only information that the market can rely on is the ex ante incidence ρ . When ρ is less than 0.5, any firm that buys the forward contract will be viewed as being taking an unjustified risk, and will be marked

down. Hence, every firm will refrain from buying the forward. Conversely, when $\rho > 0.5$, the ex ante incidence justifies buying the forward contract. Each firm is in the same situation, and so all firms end up by buying the forward. When ρ is close to 0.5, the social efficiency loss can be substantial.

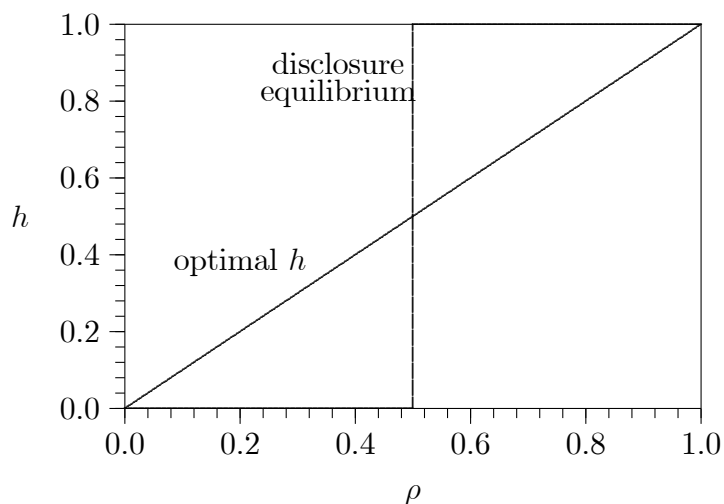


Figure 6.1: disclosure regime vs non-disclosure regime for $\Delta = 0$

Figures 6.2 and 6.3 illustrate that as the level of informativeness Δ of the signals increases to $\Delta = 1$ and then to $\Delta = 3$, the extent of underhedging significantly diminishes. In fact when $\Delta = 3$, underhedging virtually disappears and occurs only for a very small range of values of ρ . On the other hand, the extent of overhedging or speculation becomes more severe and seems to persist as Δ increases from 1 to 3. Figure 6.3 shows that $h^*(\rho, 3) = 1$ for $\rho \geq 0.3$. Given that the level of informativeness is a crucial determinant of the nature of risk management distortions, it is useful to get a feel for reasonable levels of informativeness, Δ , for a representative firm in the economy.

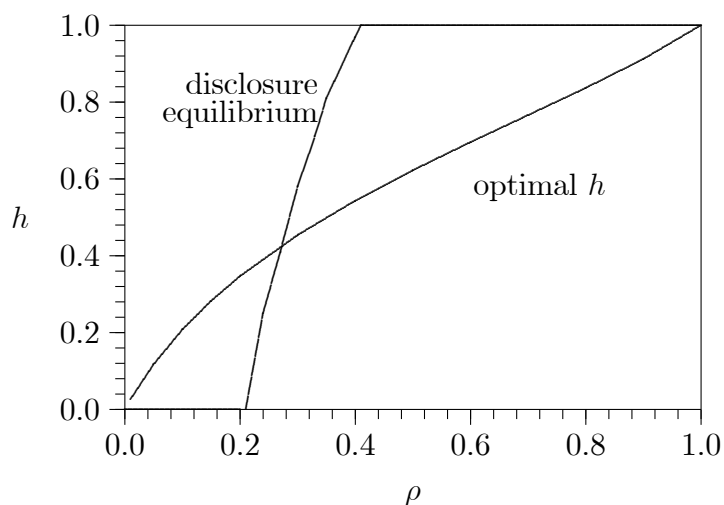


Figure 6.2: disclosure regime vs non-disclosure regime for $\Delta = 1$

6.1. Levels of Informativeness and Error Probabilities

The previous section illustrated the nature of the distortions in a firm's risk management strategy as the informativeness, Δ , of the signals changed. To get a feel for reasonable values of Δ , we need to understand how Δ is related to the error probabilities of a representative firm. One way to do this is to relate Δ with the probabilities of type I and type II errors. A type I error in our context is the error of not buying the forward contract when the firm has a hedgeable project. A type II error is buying the forward when the firm is unhedgeable. For any given decision rule, we can associate the probability of type I and type II errors.

A convenient way to summarize both error probabilities would be to consider the decision rule that equates the probabilities of both types of errors. Then, for any given Δ , we can compute the error probability of committing a type I error (which, by construction is also the probability of a type II error). From the signal densities given by (2.2) and (2.3), the decision rule that would equate the probabilities of type I and type II errors is that which sets the switching point x^*

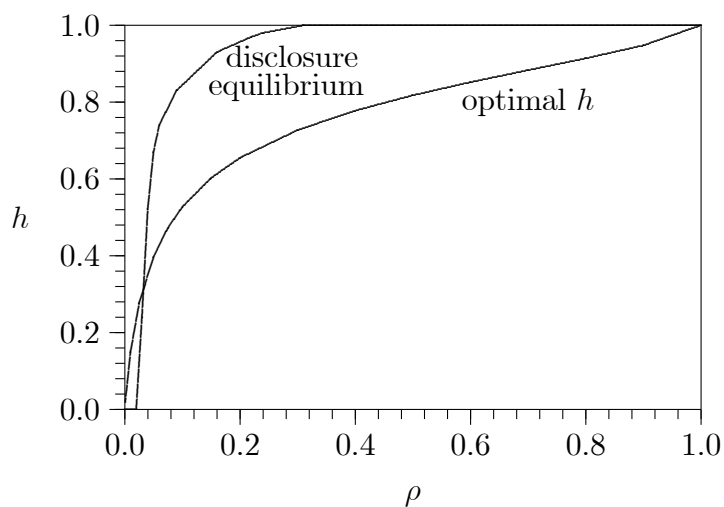


Figure 6.3: disclosure regime vs non-disclosure regime for $\Delta = 3$

to be half way between the means of the two densities. In other words,

$$x^* = \mu + \frac{\Delta}{2}$$

Then, the probability of a type I error is given by the area under F_H to the left of x^* , which is

$$F\left(-\frac{\Delta}{2}\right) \tag{6.1}$$

where $F(\cdot)$ is the c.d.f. of the logistic distribution with mean zero. That is

$$F(x) = \frac{1}{1 + e^{-x}}$$

Figure 6.4 shows how Δ is related to the error probability (6.1).

For an error probability of 20% the corresponding level of Δ is about 3. This corresponds to the level of informativeness shown in Figure 6.3 where we saw

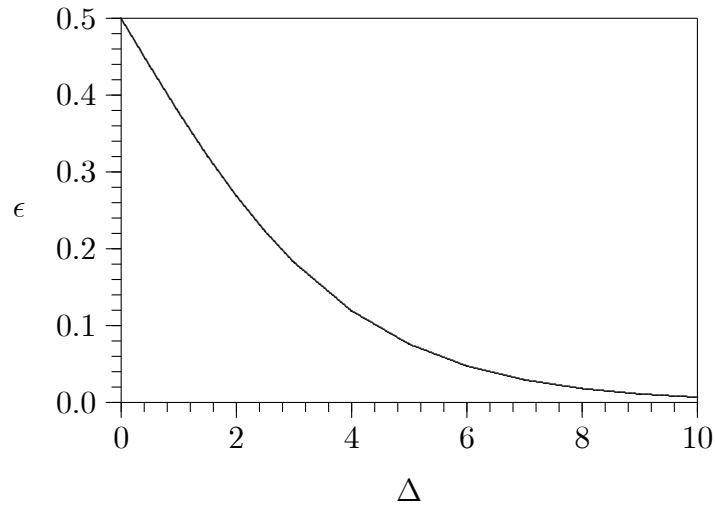


Figure 6.4: Error probabilities

that overhedging or excessive speculation is most likely to be a problem for a large range of values of ρ . Underhedging will only occur for a very small range of values of ρ . Figure 6.4 thus suggests that for most reasonable values of Δ , derivative disclosures will more likely lead to excessive speculation rather than underhedging.

7. Conclusion

Using a simple model, we have examined the claim that derivative disclosures may impede sound risk management. Derivatives disclosures can distort firms' hedging decisions under some cases. The nature of these distortions depend crucially on (i) the firm's information quality about the project type and (ii) the market's prior beliefs that the firm has a hedgeable project. When firms have noisy information about project types or the proportion of firms with hedgeable projects is relatively low, then there is underhedging in the economy relative to the social optimum. On the other hand, when the information of firms is relatively

precise and the proportion of firms with hedgeable projects relatively high, there is excessive speculation in the economy relative to the social optimum. However, for most reasonable levels of information quality, we find that instead of impeding risk management, derivative disclosures are likely to induce firms to engage in excessive speculation.

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Appendix

Proof of Proposition 1

The ex ante date 0 variance of the liquidation value is given by

$$V(h, \rho, \sigma^2) = [(1-h)^2 \rho^2 - 2s(h)(1-h)\rho(1-\rho) + (s^2(h) + 1)(1-\rho)^2] \sigma^2$$

where $s(h)$ is given by equation (2.4). Solving the first order condition $\frac{d}{dh}V = 0$ for h yields the following four roots:

$$h_1 = \frac{1}{2(-\rho + \rho e^\Delta)} \left(1 + 2\rho e^\Delta - 2\rho + \sqrt{(1 + 4\rho e^\Delta - 4\rho - 4\rho^2 e^\Delta + 4\rho^2)} \right),$$

$$h_2 = \frac{1}{2(-\rho + \rho e^\Delta)} \left(1 + 2\rho e^\Delta - 2\rho - \sqrt{(1 + 4\rho e^\Delta - 4\rho - 4\rho^2 e^\Delta + 4\rho^2)} \right),$$

$$h_3 = \frac{1}{2\rho} \frac{2\rho e^\Delta + 2\sqrt{(\rho^2 e^\Delta - \rho e^\Delta)}}{-1 + e^\Delta},$$

$$h_4 = \frac{1}{2\rho} \frac{2\rho e^\Delta - 2\sqrt{(\rho^2 e^\Delta - \rho e^\Delta)}}{-1 + e^\Delta}$$

Because $\rho < 1$, h_3 and h_4 are complex roots and are therefore not relevant. Similarly, h_1 is not relevant because it lies outside the unit interval. Finally, h_2 lies strictly between 0 and 1 and is therefore the relevant root for our purposes.

Proof of Proposition 2

From (5.3), the expected payoff from buying the forward contract is given by:

$$U_D(h) = 1 - 2k \left(\frac{s(h)(1-\rho)}{h\rho + s(h)(1-\rho)} \right)^2 \sigma^2$$

where $s(h)$ is given by equation (2.4). Letting $\epsilon \rightarrow 0$ and $h \rightarrow 0$ we get:

$$\begin{aligned} U_D(0) &= \lim_{h \rightarrow 0, \epsilon \rightarrow 0} 1 - 2k \left(\frac{s(h)(1-\rho)}{h\rho + s(h)(1-\rho)} \right)^2 \sigma^2 \\ &= 1 - 2k \left(\frac{1-\rho}{\rho e^\Delta + 1 - \rho} \right)^2 \sigma^2 \end{aligned}$$

From (5.4), the expected payoff from not buying the forward contract is given by:

$$U_{ND}(h) = 1 - k \left(\left(\frac{(1-s(h))(1-\rho)}{(1-h)\rho + (1-s(h))(1-\rho)} \right)^2 + \left(\frac{(1-h)\rho}{(1-h)\rho + (1-s(h))(1-\rho)} \right)^2 \right) \sigma^2$$

Letting $h \rightarrow 0$ and $\epsilon \rightarrow 0$, we get:

$$U_{ND}(0) = \lim_{h \rightarrow 0, \epsilon \rightarrow 0} U_{ND}(h) = 1 - k [\rho^2 + (1-\rho)^2] \sigma^2$$

The $h = 0$ boundary is defined by the following equation:

$$U_{ND}(0) = U_D(0)$$

which holds if and only if

$$2 \left(\frac{1-\rho}{\rho e^\Delta + 1 - \rho} \right)^2 = \rho^2 + (1-\rho)^2$$

Re-arranging gives the desired condition

$$\Delta = \ln \left\{ \frac{1-\rho}{\rho} \left(\sqrt{\frac{2}{\rho^2 + (1-\rho)^2}} - 1 \right) \right\}$$

Proof of Proposition 3

From (5.3):

$$U_D(1) = \lim_{h \rightarrow 1, \epsilon \rightarrow 0} U_D(h) = 1 - 2k(1-\rho)^2 \sigma^2$$

Similarly, from (5.4):

$$U_{ND}(1) = 1 - k \left(\frac{\rho^2 + (1-\rho)^2 e^{2\Delta}}{(\rho + (1-\rho) e^\Delta)^2} \right) \sigma^2$$

The $h^*(\rho, \Delta) = 1$ boundary region is defined by the following equation:

$$\begin{aligned} U_D(1) &= U_{ND}(1) \\ 2(1-\rho)^2 &= \left(\frac{\rho^2 + (1-\rho)^2 e^{2\Delta}}{(\rho + (1-\rho)e^\Delta)^2} \right) \end{aligned}$$

Note that because $\frac{\rho^2 + (1-\rho)^2 e^{2\Delta}}{(\rho + (1-\rho)e^\Delta)^2} < 1$ we have $2(1-\rho)^2 \leq 1$. This gives us a lower bound on ρ , namely, $\rho \geq 1 - \frac{1}{\sqrt{2}} \simeq 0.29$. Meanwhile,

$$2(1-\rho)^2 = \left(\frac{\rho^2 + (1-\rho)^2 e^{2\Delta}}{(\rho + (1-\rho)e^\Delta)^2} \right)$$

is a quadratic in e^Δ . Solving the equation for e^Δ yields the following two roots where:

$$e^\Delta = \frac{-2(1-\rho)^2 \rho - \rho \sqrt{4(1-\rho)^2 - 1}}{(1-\rho)(2(1-\rho)^2 - 1)}$$

and

$$e^\Delta = \frac{-2(1-\rho)^2 \rho + \rho \sqrt{4(1-\rho)^2 - 1}}{(1-\rho)(2(1-\rho)^2 - 1)}$$

For these roots to be real, this implies that $\sqrt{4(1-\rho)^2 - 1} \geq 0$. Equivalently, $\rho \leq 0.5$. The second root is not feasible because $\frac{-2(1-\rho)^2 \rho + \rho \sqrt{4(1-\rho)^2 - 1}}{(1-\rho)(2(1-\rho)^2 - 1)} < 1$.

Therefore $\Delta = \ln \left(\frac{-2(1-\rho)^2 \rho - \rho \sqrt{4(1-\rho)^2 - 1}}{(1-\rho)(2(1-\rho)^2 - 1)} \right)$ defines the the boundary for the $h = 1$ region. Taking together with the lower bound on ρ , a necessary condition for the solution to Δ to be well-defined is

$$1 - \frac{1}{\sqrt{2}} \leq \rho \leq 0.5$$

Proof of Proposition 4

For a unique interior solution h^* , the payoff functions, $U_D(h)$ and $U_{ND}(h)$ should intersect only once with:

$$U_D(h) > U_{ND}(h) \text{ for all } h < h^*$$

and

$$U_D(h) < U_{ND}(h) \text{ for all } h > h^*$$

It can easily be shown that $U_D(h)$ is increasing in h for all ρ and $\Delta > 0$. The condition, $\rho \leq 0.5$ guarantees $U_{ND}(h)$ is also increasing in h for all Δ . Therefore for a unique interior equilibrium, we must show that:

$$U_D(0) > U_{ND}(0)$$

and

$$U_D(1) < U_{ND}(1)$$

But $U_D(0) > U_{ND}(0)$ implies that :

$$\left(\frac{1-\rho}{\rho e^\Delta + 1 - \rho} \right)^2 < \rho^2 + (1-\rho)^2$$

which as shown from Proposition 2 is defined by the region:

$$\Delta > \ln \left\{ \frac{1-\rho}{\rho} \left(\sqrt{\frac{2}{\rho^2 + (1-\rho)^2}} - 1 \right) \right\}$$

Similarly $U_D(1) < U_{ND}(1)$ implies that:

$$2(1-\rho)^2 > \left(\frac{\rho^2 + (1-\rho)^2 e^{2\Delta}}{(\rho + (1-\rho) e^\Delta)^2} \right)$$

which from Proposition 3 is defined by the region:

$$\Delta < \ln \left(\frac{-2(1-\rho)^2 \rho - \rho \sqrt{4(1-\rho)^2 - 1}}{(1-\rho)(2(1-\rho)^2 - 1)} \right)$$