An Intermediation-Based Model of Exchange Rates *

Semyon Malamud† and Andreas Schrimpf‡

This version: November 8, 2017

Abstract

We develop a general equilibrium model of decentralized international financial markets. In our model, financial intermediaries bargain with their customers and extract endogenous rents for providing access to foreign securities. The behavior of intermediaries, by tilting state prices, generates an endogenous, non-linear risk structure in exchange rates. We show how this risk structure leads to (i) deviations from covered interest rate parity (CIP) and a link between these deviations and differences in monetary policy conduct across countries; (ii) currency crash risk and skewness; (iii) an explicit link between monetary and stabilization policies and safe haven properties of exchange rates. Crash risk and the shock absorption role of exchange rates are amplified whenever intermediation capacity drops. A global US dollar factor emerges endogenously in the equilibrium pricing kernel and is a key driver of international risk premia.

Keywords: exchange rates, dollar, covered interest parity deviations, currency skew, currency crashes

JEL Classification Numbers: E44, E52, F31, F33, G13, G15, G23

---

*We thank Ana Babus, Saki Bigio, Darrell Duffie, James Kemp, Arvind Krishnamurthy, Steve Mobs, Pierre-Olivier Weill, and especially Colin Ward, as well as seminar participants at UCSD, UCLA, and conference participants at the 2nd LAEF conference on OTC markets for helpful comments. Semyon Malamud acknowledges the financial support of the Swiss National Science Foundation and the Swiss Finance Institute. Parts of this paper were written when Malamud visited BIS as a research fellow. The views in this article are those of the authors and do not necessarily represent those of the Bank for International Settlements (BIS).

†Swiss Finance Institute, EPF Lausanne, and CEPR; E-mail: semyon.malamud@epfl.ch

‡Bank of International Settlements (BIS) and CEPR; Email: andreas.schrimpf@bis.org
1 Introduction

The goal of this paper is to develop a macroeconomic general equilibrium model with international financial markets subject to intermediation frictions. In our model, intermediaries use their market power to extract rents from their customers by providing them with access to trading foreign financial instruments. This rent extraction distorts international risk sharing and alters the dynamics of international risk premia and exchange rates. We show how this simple intermediation friction helps account for some of the major anomalies in foreign exchange and international capital markets including the violation of uncovered interest parity (UIP), the profitability of carry trades, the safe haven properties of exchange rates, currency crash risk, and the breakdown of covered interest parity (CIP).

International financial markets are highly decentralized. The trading of key financial instruments – such as sovereign and corporate bonds, spot foreign exchange (FX) rates, FX forwards and swaps, and most other derivatives used for hedging purposes typically occurs over-the-counter (OTC) through financial intermediaries.\(^1\) Trading in such markets is subject to frictions, whereby a handful of global intermediaries exert significant market power. To study the effect of these market imperfections on exchange rates and the macroeconomy, we introduce an imperfectly competitive intermediation sector into a classical cash-in-advance model a-la Lucas (1982). Our model features an economy with multiple countries and partially integrated financial markets. Each country is populated by two classes of agents, customers (households) and specialists (intermediaries). We introduce some realistic features of segmentation in our model. While customers have free access to local markets for simple local securities (such as local nominal risk free bonds and the local stock market), they have

\(^1\)Indeed, a large part of the trading in global securities and derivatives markets occurs over-the-counter, with bank dealers as major suppliers of intermediation services. For example, daily turnover in interest rate swaps reached almost USD 2 trillion per day in April 2016, while daily trading volume in the global FX market exceeds USD 5 trillion, according to the most recent BIS statistics on global OTC derivatives markets. See, BIS (2016). Trading in global OTC markets dwarfs the volume that is traded, e.g. on equities or futures exchanges. In OTC markets, an identical asset is typically traded at different prices at a given point in time, depending on the identity of the trading counter-parties.
to go via intermediaries in order to gain access to foreign assets and financial instruments in the dealer-to-customer (D2C) market segment. Upon contact, intermediaries take into account customers’ optimal demand for foreign financial assets and use their bargaining power to extract rents and charge markups for providing insurance against (or, speculative bets on) different states of the world economy. At the same time, intermediaries use the dealer-to-dealer (D2D) market to rebalance their inventories and share risks.

Since intermediaries are the marginal investors in international financial markets, their wealth dynamics emerge as key determinants of international risk premia and the behavior of exchange rates. Since intermediaries’ wealth dynamics are determined by the markups they charge to customers, in equilibrium these markups enter directly into the global pricing kernel and, hence, emerge as an important determinant of exchange rates. When a high markup state is realized, intermediaries’ wealth goes up, while the marginal utility of consumption drops. Due to the cash-in-advance constraint, the value of the local currency moves one-to-one with this marginal utility, implying that the currency depreciates and (potentially) crashes in a high markup state. Thus, currency crashes become a self-fulfilling prophecy that arise due to the inability of competitive intermediaries to internalize the pecuniary externality generated by their markups. When such crashes materialize at times of low ex-ante intermediation capacity, their severity is amplified even further.

In our model, heterogeneity in risk properties of exchange rates is to a large extent determined by two factors: differences in the conduct of monetary and stabilization policies across countries, and differences in their intermediation capacities. In a way, intermediaries and the monetary authority play complementary roles by determining the allocation of nominal risk across states. The monetary authority does so by pursuing stabilization policies that adjust the monetary policy stance in response to local and global shocks. By changing

---

2Specifically, local pricing kernels are given by local intermediaries’ marginal utilities, while exchange rates are given by the ratio of these kernels.

3In particular, consumption risk in our model is also endogenous and depends on the level of financial development (captured by the intermediation capacity), as in Acemoglu and Zilibotti (1997).
the risk structure of the economy, such policies affect customers’ demand for insurance
from intermediaries. In turn, intermediaries determine the price of this insurance which
feeds back into consumption allocation and determines the passthrough of the stabilization
policy into state prices. This changes the nature of the risk allocation between customers
and intermediaries, and impacts intermediaries’ demand for currencies, determining the
transmission of stabilization policies into the exchange rates. One of our major goals
is to understand how these channels influence the response of exchange rates to global
macroeconomic conditions; in particular, the so-called safe haven properties of exchange
rates.

The term “safe haven” is commonly used for currencies that tend to appreciate at
times of global economic downturns, usually accompanied with stock market crashes. In
standard, frictionless monetary models, policies that aggressively ease monetary policy in
global economic downturns naturally lead to currency depreciation: An increase in the
money supply leads to an immediate drop in the value money. These effects are particularly
strong for countries with large intermediation sectors because these countries naturally serve
as insurance providers to the rest of the world and hence suffer the most during crisis
periods. This is what Maggiori (2013) calls the “reserve currency paradox”. We show
that intermediation markups have the potential to (at least partially) resolve this paradox:
While countries with larger intermediation sector indeed suffer more from global downturns,
their markups are also more sensitive to global conditions; when this markup channel is
strong enough, it overturns the standard risk sharing channel and implies that the currency
appreciates.

Our model also allows us to study the impact of non-fundamental shocks in the form of
monetary policy uncertainty on exchange rates. Customers in a country where monetary
policy is highly uncertain contact intermediaries to buy insurance against this source of
uncertainty. Intermediaries charge markups for providing this insurance, limiting customers’
ability to allocate risk across states and forcing the currency to depreciate at time of a global crash. Thus, the currency of a country where there is little monetary policy uncertainty naturally emerges as a safe haven. The underlying mechanism is characteristic to our model: customers’ expectations about future policy create demand pressure in the D2C market, determining equilibrium markups and the risk properties of exchange rates.

In our model, the international transmission of shocks is determined exclusively by trade in real goods. As a result, the domestic pricing kernel depends crucially the (trade weighted) exchange rate index against the local currency. For example, the US dollar pricing kernel depends on the global dollar index. When this dollar index depreciates (i.e., dollar weakens), global demand for US goods goes up, while the US domestic consumption decreases. As a result, insurance against dollar depreciation states becomes valuable, and the US dollar pricing kernel loads negatively on the dollar index. In particular, currencies with a high loading on the dollar index command a positive dollar risk premium and have higher expected returns, consistent with the empirical evidence.

Interestingly enough, we show that, together with the contemporaneous dollar factor, the interaction of international trade with intermediation markups leads to the emergence of a second “dollar-like” factor given by the present discounted value of future dollar factor values.

In our model, customers willing to borrow or lend in a foreign currency cannot do so directly and have to do so through intermediaries. For example, they can do it by borrowing in the local currency and then entering an FX swap contract with the intermediary in order to borrow dollars synthetically; the corresponding indirect rate of borrowing dollars may be different from the rate at which intermediaries can borrow dollars directly. Such deviations from covered interest rate parity (CIP) have been a pervasive phenomenon in the post-crisis period. The sign of these deviations is determined by customers’ desire to borrow or lend

\footnote{See also Gourinchas et al. (2017) who show that the US dollar index is negatively related to global trade when US Dollar is an invoicing currency.}

\footnote{See, Verdelhan (2017) and Pozdeev (2017).}

\footnote{CIP states that the interest rates implicit in foreign exchange swap markets coincide with the corre-}
in the foreign currency, which (due to market fragmentation) creates a price pressure in the swap market. We show explicitly how CIP deviations are intimately linked to differences in monetary policy conduct across countries: When monetary policies in two countries start diverging, a non-zero currency basis against the US dollar (defined as the difference between the FX swap-implied and the spot dollar rates) may emerge, and this basis is typically positive if the US monetary/stabilization policies react more aggressively to global economic conditions. Furthermore, we show that a non-zero basis always arises when countries differ in at least one of the following: (i) intermediation capacity; (ii) stock market volatility; (iii) monetary policy uncertainty.

We show how the dollar present value factor penetrates from the pricing kernel into the cross-currency basis. In our model, this link between the dollar and the basis arises because customers consider dollar assets as a hedge against the loss of purchasing power because they suffer less from this global demand effect. As an illustration, consider a country for which the global demand for domestic goods is more sensitive than that for US goods. Then, customers in that country perceive dollar assets as an insurance device because they suffer less from this global demand pressure. As a result, they contact intermediaries to use the US dollar as an insurance against future drop in purchasing power. This creates a price pressure in the D2C market, and intermediaries exploit it by charging higher markups, giving rise to a positive basis. While the forces underlying the emergence of the basis are purely macroeconomic and trade-driven, those forces affect exchange rates through intermediaries’ balance sheet: A stronger dollar redistributes wealth across intermediaries in different countries, depending on their dollar exposures.

**Roadmap.** The remainder of the paper is structured as follows. Section 2 provides an overview of the relevant literature. Section 3 describes the model. Section 4 provides corresponding spot interest rates. The breakdown of CIP even for some of the world’s most liquid currency pairs is one of the most surprising recent developments in global financial markets. See, for example, Du et al. (2016), Avdjiev et al. (2016), Borio et al. (2016), and Rime et al. (2017).
equilibrium characterization. Section A.1 solves for the equilibrium in the frictionless case. Section 5 investigates the link between intermediation frictions and various exchange rate anomalies. Section 6 studies the global pricing kernel and the impact of global trade on CIP deviations. Section 7 concludes.

2 Literature Review

The literature on general equilibrium models of exchange rates is vast. Most papers either assume complete international financial markets (see, for example, Lucas (1982); Cole and Obstfeld (1991), Dumas (1992); Backus et al. (1992); Backus and Smith (1993); Obstfeld and Rogoff (1995); Pavlova and Rigobon (2007); Verdelhan (2010); Colacito and Croce (2011); Hassan (2013)) or an exogenously specified market incompleteness in the form of portfolio constraints (see, for example, Chari et al. (2002); Corsetti et al. (2008); Pavlova and Rigobon (2008)), unspanned risk factors (Pavlova and Rigobon (2010, 2012), Farhi and Gabaix (2016), Brunnermeier and Sannikov (2017)) or limits to market participation (Alvarez et al. (2002, 2009) and Bacchetta and Van Wincoop (2010)). By contrast, in our model market incompleteness and limits to international risk sharing are endogenous, and are determined by equilibrium intermediation markups.

The most closely related to ours are the papers by Maggiori (2013), Gabaix and Maggiori (2015), and Itskhoki and Mukhin (2017).\footnote{Several papers (see, for example, Jeanne and Rose (2002), Evans and Lyons (2002), Hau and Rey (2006), Bruno and Shin (2014)) study the impact of frictions on exchange rates without modelling macroeconomic fundamentals such as exports and imports of multiple goods. Instead, they focus on how the behaviour and incentive structure of intermediaries shapes market outcomes in foreign exchange.} Maggiori (2013) considers a two country model characterized by an asymmetry in financial intermediation capacity: In his model, one country (US) has a better developed (i.e., less credit constrained) intermediation sector. During global crises, US suffers heavier losses (through wealth transfers to the rest of the world) because of its role as a global insurer, leading to asymmetric international risk
Maggiori (2013) highlights how these effects lead to a “reserve currency paradox”, forcing the US dollar to depreciate in bad times and hence playing against the role of the US dollar as a global safe asset. Our model allows us to look at the reserve currency paradox from a different angle. First, we show that the markup channel does have the potential to resolve the reserve currency paradox. Specifically, intermediation markups in countries with larger capacity are more sensitive to global conditions. As a result, at times of a global crash, intermediaries in such countries suffer more, their marginal utility spikes, and the currency appreciates. Second, we link the safe haven properties of exchange rates to monetary policy uncertainty. In particular, we show that US dollar may arise as a global safe haven currency if US monetary policy features a lower amount of uncertainty and/or reacts more aggressively to deteriorating global macroeconomic conditions. In this case, US intermediaries endogenously sell more insurance against global crisis states, which in turn makes them suffer more when such a crisis arrives. Since the ratio of intermediaries’ marginal utilities emerges as a key determinant of exchange rates in our model, the US dollar appreciates in bad states.

Gabaix and Maggiori (2015) develop a general equilibrium model of exchange rates based on the limited risk bearing capacity of financial intermediaries. In their model, intermediaries demand a risk premium for holding currency risk originating in global imbalances. Gabaix and Maggiori (2015) show that this simple intermediation friction has a major impact on equilibrium exchange rates dynamics; in particular, their model is able to rationalize

---

8 Kindleberger (1965), Despres et al. (1966), Caballero et al. (2008), Mendoza et al. (2009), and Chien and Naknoi (2015) also emphasize differences in financial development across countries as an important source of global imbalances.

9 That is, a currency that appreciates at a time of a global crisis.

10 The importance of intermediation frictions for the transmission and the amplification of shocks in domestic markets has been acknowledged in many papers. See, for example, Holmstrom and Tirole (1997), Bernanke et al. (1999), Gertler and Kiyotaki (2010), He and Krishnamurthy (2011, 2013, 2014), Adrian and Boyarchenko (2012), Brunnermeier and Sannikov (2014, 2016), Adrian et al. (2014), Rampini and Viswanathan (2015), He et al. (2016b), Korinek and Simsek (2016), Piazzesi and Schneider (2016), Bianchi and Bigio (2016), Bigio and Sannikov (2016), Malamud and Schrimpf (2016), and Coimbra and Rey (2017).
many of the important stylized facts about exchange rates, and link these stylized facts to intermediaries’ balance sheets.

Both in Gabaix and Maggiori (2015) and in our paper, imperfections arise from price pressure effects in the D2C market segment. However, the nature of this price pressure in our model is different from that in Gabaix and Maggiori (2015) and stems from imperfect competition and an endogenous market fragmentation. In particular, in contrast to models with exogenously specified limits to market participation, our model shows how barriers to international trade (intermediation markups) arise endogenously and are determined by forces of supply and demand such as customers’ “reaching for yield” (e.g., through a carry trade) and “flight to safety” whereby customers are attracted by “safe haven” currencies. Finally, in Gabaix and Maggiori (2015) the dynamics of intermediaries’ risk bearing capacity is specified exogenously, while in our model it is endogenous, and is proportional to intermediaries’ net worth. Negative shocks to this net worth occur whenever states against which intermediaries sell a lot of insurance are realized, leading to a redistribution of wealth and affecting state-contingent risk premia and exchange rates dynamics.

Itskhoki and Mukhin (2017) develop a dynamic model similar to that of Gabaix and Maggiori (2015), in which noise traders in the bond markets give rise to exogenous small but persistent shocks to international bond markets. They show that a model with such financial shocks alone is quantitatively consistent with the empirically observed joint dynamics of exchange rates and macro variables. Importantly, Itskhoki and Mukhin (2017) use log-linear approximations for their analysis, making it difficult to address non-linear effects such as tail risks and state contingent risk premia, which are key to our analysis.

Our paper is also linked to the literature on the international monetary policy spillovers. For example, Rey (2013) discusses the classical Mundellian trilemma and finds that US monetary policy shocks have a significant impact even on economies with large financial

---

11 Such as those of Alvarez et al. (2002, 2009) and Bacchetta and Van Wincoop (2010).
12 The Mundellian trilemma states that flexible exchange rates are able to perfectly absorb foreign monetary policy shocks in the presence of free capital mobility.
markets, questioning the conventional wisdom that flexible exchange rates are enough to guarantee monetary autonomy in a world of large capital flows, contrary to the conventional wisdom (see, e.g., Obstfeld and Taylor (2004)). In our model, we show explicitly how monetary policy shocks are transmitted internationally, and how they are linked to local and global intermediation capacities. In particular, we are able to explicitly compute the global matrix of monetary transmission (see, Shin (2017)) and show that the impact of monetary shocks in a given country on exchange rates between other countries is proportional to these countries’ relative intermediation capacities.  

In our model, intermediation markups charged for insurance against some states of the world can become prohibitively high, making pricing kernel and exchange rates exhibit behaviour reminiscent of “rare disasters” (see, e.g., Barro (2006)). As Farhi and Gabaix (2016) demonstrate, rare disaster risk has a first order impact on equilibrium exchange rates, and can be used to explain a wide array of international asset pricing puzzles. In particular, Farhi and Gabaix (2016) show that a currency with a lot of disaster risk has a very high put price and a high implied volatility. Furthermore, Farhi and Gabaix (2016) also show that the most risky currencies (defined as currencies that have the largest exposure to global disaster risk) have a positive correlation with world stock market returns, whereas the least risky currencies have a negative correlation, consistent with the findings of Lustig et al. (2011). Our model is also able to generate similar phenomena, but the disasters arise endogenously.

Our paper is also related to the recent work by Farhi and Maggiori (2017). In this paper, they develop a model of the international monetary system and study the role of global safe asset providers in determining the structure of this system. In our model, the nature of local monetary policy, in particular its responsiveness to local and global shocks is a major

---

13 In our model, this result is driven by the fact that balance sheets of (imperfectly competitive) intermediaries play a key role in the international transmission of shocks. See Miranda-Agrippino and Rey (2015) for some supporting evidence.

14 See also He et al. (2016a), who investigate the emergence of endogenous safe assets in a global games framework.
determinant of the safety properties of local currencies. Our model implies that the status for a given currency of being a globally safe asset is intimately linked to expectations about the future state-contingent conduct of monetary policy. To the best of our knowledge, this implication is unique to our model and is different from other models of currency stabilization, such as that of Hassan et al. (2016).\textsuperscript{15}

Finally, our paper is also related to the recent literature on deviations from the CIP. See, for example, Du et al. (2016), Avdjiev et al. (2016), Borio et al. (2016), and Rime et al. (2017). To the best of our knowledge, our model is the first macroeconomic model that derives CIP deviations endogenously, through price pressure effects in imperfect international financial markets. In particular, we are able to shed some light on the origins of CIP deviations, as well as on their signs and their cross-section.

\section{The Model}

We consider a standard, international multiple goods monetary economy with intra-temporal\textsuperscript{16} cash-in-advance constraints, as in Lucas (1982). Time is discrete, $t = 0, 1, \cdots, T$, and the information structure is characterized by a probability space $(\Omega, P)$ equipped with a filtration $(\mathcal{F}_t)_{t \geq 0}$. There are $N$ countries, indexed by $i = 1, \cdots, N$. Country $i$ produces a tradable good, also indexed by $i$. Country $i$ tradable good is produced by an endowment process $X_{i,t}$, $i = 1, \cdots, N$, $t \geq 0$. The government of country $i$ controls the total money supply, $M_{i,t}$.

Each country is populated by two classes of agents, $I$-agents (intermediaries, or, dealers) and $H$-agents (households, or, customers) that have different, time-varying, stochastic time discount factors $\Psi^H_{i,t}$, $\Psi^I_{i,t}$, $i = 1, \cdots, N$.\textsuperscript{17} We assume that all agents derive utility from

\textsuperscript{15}Hassan et al. (2016) show how state-contingent monetary/stabilization policies impact the risk properties of exchange rates.
\textsuperscript{16}That is, agents only need to hold cash within the period for consumption needs, and do not store cash inter-temporally.
\textsuperscript{17}Pavlova and Rigobon (2007) interpret $\Psi^H_{i,t}$ as a common shock to households demand for consumption
consumption $C_{i,t}^{bundle}$ of a country-specific bundle of tradable goods. All agents endowed with standard, inter-temporal, logarithmic preferences

$$E \left[ \sum_{t=0}^{T} \Psi_{i,t}^{J} \log C_{i,t}^{bundle} \right], \ J = I, H, \ i = 1, \cdots, N, \quad (1)$$

where

$$C_{i,t}^{bundle,I,H} = \prod_{k=1}^{N} (C_{i,k,t}^{I,H})^{\theta_{i,k,t}}, \ i = 1, \cdots, N$$

is the country-specific tradable goods consumption bundle. Here, $C_{i,k,t}^{I,H}, \ k = 1, \cdots, N$ is the time-$t$ consumption of country-$k$ tradable good in country $i$ by the corresponding agents’ class $I, H$. As is common in international finance literature, we allow the preference parameters $\theta_{i,k,t}$ to be time-varying, capturing potential taste and demand shocks. Without loss of generality, we normalize these preference parameters so that

$$\sum_{k} \theta_{i,k,t} = 1$$

for all $i = 1, \cdots, N$.

Denote by $P_{i,k,t}$ the nominal price of good $k$ in country $i$, in the units of the local currency. The cash-in-advance constraint implies that the total nominal expenditures for country $i$ tradable good’s endowment $X_{i,t}$ always equals money supply:

$$P_{i,i,t} X_{i,t} = M_{i,t}, \ i = 1, \cdots, N, \ t \geq 0. \quad (2)$$

18See, for example, Pavlova and Rigobon (2007, 2008, 2010); Gabaix and Maggiori (2015) for similar preference specifications.
The following lemma characterizes the optimal choice of money-consumption bundles.

**Lemma 1**  
*Given the total nominal expenditure in the units of currency* $i$, 

$$C_{i,t}^{I,H} \equiv \sum_{k=1}^{N} p_{i,k,t} C_{i,k,t}^{I,H},$$

*the optimal consumption bundle of the respective agent class is given by*

$$C_{i,k,t}^{I,H} = C_{i,t}^{I,H} P_{i,k,t}^{-1} \theta_{i,k,t}, \quad i, k = 1, \ldots, N.$$  

(3)

We assume that all class $I$ agents (intermediaries) from each country $i = 1, \ldots, N$ have a direct access to a frictionless, complete, centralized, international dealer-to-dealer (D2D) market. We interpret these agents as specialists who possess a technology that allows them to issue and trade general state-contingent claims (a full set of Arrow securities) with other agents. Below, we will always use currency of country 1 (US dollars) as the reference currency and use $\$ to denote the corresponding economic variables. Since markets are complete, the prices of all financial securities traded in the inter-dealer market can be encoded in a single, international US dollar nominal pricing kernel $M_{\$,t,t+1}^{I}$ quoted in the units of currency 1, so that the time-$t$ US dollar price $q_t$ of a state-contingent claim with a dollar payoff $Y_{t+1}$ is given by

$$q_t = E_t[M_{\$,t,t+1}^{I} Y_{t+1}].$$

In the sequel, we will refer to $M_{\$,t,t+1}^{I}$ as the (US dollar) dealer-to-dealer (D2D) pricing kernel. In stark contrast to class-$I$ agents, class $H$ agents (henceforth, customers) of a given country $i$ do not have a direct access to the inter-dealer market, except for the possibility to trade the claim on their endowment $X_{i,t}$ (the stock index of country $i$) as well as one-period nominal risk-free bonds of their respective country. Customers willing to trade any other security
need to contact an intermediary (an intermediation firm) and bargain over the counter in a
dealer-to-customer (D2C) market. For example, customers can borrow or lend in their local
currency at market rates, but those willing to borrow or lend in a foreign currency need to
do so through intermediaries.\footnote{In our model, trading foreign stocks can also be done only through intermediaries. This assumption allows us to capture the fact that trading and owning foreign stocks often involves significant amounts of intermediation. For example, a US investor can invest in foreign stocks through American Depository Receipts (ADRs). But, in reality this transaction goes through an intermediary (a custodian bank) who is in charge of actually holding the ADR. The custodian charges intermediation fees for maintaining the ADR records, collecting the dividends paid out by the foreign issuer, converting it into US dollars and depositing into the stockholder’s account. Thus, effectively, ADR is an OTC contract between the investor and the custodian bank. Similarly, short selling a stock (both local and foreign) always involves intermediation, whereby the short seller has to go to an intermediary who then needs to locate a stock owner to borrow the stock. See, e.g., \citet{Duffie2005}.}

Following \citet{He2013}, we assume class-\(I\) agents are specialists who
run intermediation firms. The objective of such a firm is to maximize the firm value (that is, the present discounted value of intermediation markups) under the D2D pricing kernel. Since markets are complete, the risk neutral firms’ objective coincides with that of the risk averse specialists who run it: Indeed, both the firm and the specialist’s objective is to maximize the present value of revenues under the unique pricing kernel. Therefore, in the future we identify class-\(I\) agents with the intermediation firm they run and we call them intermediaries.\footnote{Importantly, specialists are the only shareholders of intermediaries and hence markups are not rebated back to customers: By assumption, customers (class-\(H\) agents) can only freely trade claims on their wealth and short term bonds. This assumption is made for simplicity and can be relaxed at the cost of unnecessary complications of the analysis.} We formalize the details of the bargaining protocol in the following assumption (see Figure 1 below for a graphical description).

**Assumption 1** *In the beginning of each period \(t\), each customer of country \(i\) is matched with an intermediary of the same country and requests quotes for prices of all one-period-ahead state-contingent claims.\footnote{The assumption of trading only one period claims with intermediaries is standard in the literature. As \citet{Brunnermeier2016} argue, this is without loss of generality if old contracts are indexed on contemporaneous economic conditions.} Intermediary quotes a one period ahead country-specific D2C pricing kernel \(M^H_{t,t+1}\) in the local currency and has full bargaining power in choosing*
Figure 1: Graphical description of market structure in our model for the two country case (country $i$ and country $j$). RFQ denotes the request for quote protocol commonly used in D2C segments of OTC markets.

$M_{i,t,t+1}^{H}$ due to search frictions: If the customer rejects the offer, he can trade country $i$ endowment claims and country $i$ one-period risk free bonds in the country $i$ centralized market with other country $i$ investors, and then has to wait one more period until he is matched with another intermediary. The quotes are binding: After receiving the quote, the customer chooses an optimal bundle of state-contingent claims, and the intermediary sells this bundle to the customer at the quoted prices.

The key mechanisms in our model depend crucially on the ability of intermediaries to extract rents. The assumption of monopolistic competition is made for tractability reasons and can be relaxed; for example, our results can be easily adjusted to allow for a different
bargaining protocol with a bargaining power below one, such as the Nash protocol that is commonly used in the literature on OTC markets. See, Duffie et al. (2005), Duffie et al. (2007)) and Lagos and Rocheteau (2009). However, some papers (see, for example, Petersen and Rajan (1995)) argue that monopolistic competition in the intermediation sector is a closer approximation to the reality due to switching and relationship costs. See, also, Sharpe (1997), Kim et al. (2003), Bolton et al. (2016), Brunnermeier and Koby (2016), Duffie and Krishnamurthy (2016), and Acharya and Plantin (2016).

Assumption 1 implies that we can reformulate the bargaining problem in terms of the local currency nominal D2C state prices \( M_{i,t,t+1}^H \) quoted by the country \( i \) intermediary to a country \( i \) customer. Even though customers can only trade one-period claims, market completeness implies that agents can effectively replicate any stream of nominal expenditures in the local currency, \( (C_t^H)_{t \geq 0} \), with the prices of \( t \)-period ahead Arrow-Debreu claims given through the nominal local currency-denominated stochastic discount factor \( M_{i,0,t}^H = M_{i,0,1}^H M_{i,1,2}^H \cdots M_{i,t-1,t}^H \). The multi-period D2D dollar pricing kernel is defined similarly: \( M_{\$0,t}^I = M_{\$0,1}^I M_{\$1,2}^I \cdots M_{\$t-1,t}^I \).

We denote by \( \mathcal{E}_{i,t} \) the US dollar price of the currency of country \( i \), that is, whenever \( \mathcal{E}_{i,t} \) goes up, the local currency of country \( i \) appreciates against the US dollar. Then, the country \( i \) D2D pricing kernel denominated in local currency are linked to dollar D2D pricing kernel through the identity

\[
M_{i,0,t}^I = M_{\$0,t}^I \mathcal{E}_{i,t}/\mathcal{E}_{i,0}.
\] 

Since our focus is on financial market frictions, we abstract from frictions in international

\textsuperscript{22}The new regulatory environment (based on the Dodd-Frank act) is designed to move OTC trading to electronic trading platforms. For example, trading of standardised interest rate swaps in the US has to a large extent moved to so-called swap execution facilities (SEFs). However, most D2C transaction are executed via a request for quote (RFQ) protocol, which is equivalent to an electronic form of OTC trading. The original two-tier market structure persists, with a D2D segment at the core of the market, as in our model. The same is true for fixed income and foreign exchange markets. See Collin-Dufresne et al. (2016), Bech et al. (2016), and Moore et al. (2016).
goods markets and assume that purchasing power parity (the law of one price) always holds.\footnote{Similarly to Itskhoki and Mukhin (2017), we could also introduce shocks to the law of one price and study its impact on exchange rates.}

In this case, nominal goods prices in local currencies satisfy

\[ P_{i,k,t} = P_{\delta,k,t}/\psi_{i,t}, \quad i,k = 1, \cdots, N. \]

We will use \( r_{i,t} \) to denote the short term nominal interest rate, and we will let \( S_{i,t} \) denote the nominal present value of the total endowment, \( X_{i,t} \), of the local, country-\( i \) good. By (2), \( S_{i,t} \) is the present value of total money supply, \( M_{i,\tau} \), \( \tau \geq t \). Hereafter, we interpret this claim as the country-\( i \) stock index and call it the local stock price. By assumption, local customers can freely trade the endowment claim as well as one-period nominal risk-free bonds. This means that the intermediary has to quote fair prices for both instruments: Otherwise, customers would immediately arbitrage away the differences in the quoted and the inter-dealer rate, leading to unbounded losses for the intermediary. Formally, this means that the D2C pricing kernel \( M_{i,t,t+1} \) quoted by the intermediary has to satisfy two constraints relating the short term rate \( r_{i,t} \) and the stock price, \( S_{i,t} \), in the two market segments:

\[ e^{-r_{i,t}} \equiv E_t[M_{i,t,t+1}^H] = E_t[M_{i,t,t+1}^I] \quad (5) \]
\[ S_{i,t} \equiv M_{i,t} + E_t[M_{i,t,t+1}^H S_{i,t+1}] = M_{i,t} + E_t[M_{i,t,t+1}^I S_{i,t+1}]. \quad (6) \]

We will also make the following assumption.

**Assumption 2** We assume that class \( I \) and class \( H \) agents in country \( i \) are endowed with the respective shares \( \alpha_i \) and \( 1 - \alpha_i \) of the total endowment of the country \( i \) tradable good. At time zero, intermediaries pay a cost \( \bar{K}_{i,0} \) to customers to set up intermediation firms. The monetary authority controls money supply through direct rebates to intermediaries.\footnote{For example, as Brunnermeier and Sannikov (2016) argue, controlling the rate on the central bank reserves is effectively equivalent to controlling the supply of central bank money, whereby interest payments on}
We also use \( N_{i,t+1} = \mathcal{M}_{i,t+1} / \mathcal{M}_{i,t} \) and \( N_{i,t,\tau} = \mathcal{M}_{i,\tau} / \mathcal{M}_{i,t} \) to denote the growth in money supply, and we will use the normalization \( E_t[N_{i,t+1}^{-1}] = 1 = \mathcal{M}_{i,0} \) for all \( i = 1, \cdots, N \), \( t \geq 0 \).

By Assumption 2, customers’ time zero nominal net worth is given by \( W^H_{i,0} = (1 - \alpha_i) S_{i,0} + \bar{K}_{i,0} \). Since markets are complete, customers can use trading in the D2C market to attain any state contingent consumption expenditures profile \( (C^H_{i,t})_{t \geq 0} \) in the local currency satisfying the inter-temporal budget constraint:

\[
E \left[ \sum_{t=0}^{T} C^H_{i,t} M^H_{i,0,t} \right] = W^H_{i,0}.
\]

By (3), inter-temporal utility of an agent in a country \( i = 1, \cdots, N \) is given by

\[
E \left[ \sum_{t=0}^{T} \Psi^I_{i,t} \log C^I_{i,t} \right] = E \left[ \sum_{t=0}^{T} \Psi^I_{i,t} \log C^I_{i,t} \right] + \text{constant}.
\]

Therefore, customer’s inter-temporal optimization problem can be rewritten as

\[
\max \left\{ E \left[ \sum_{t=0}^{T} \Psi^H_{i,t} \log C^H_{i,t} \right] : E \left[ \sum_{t=0}^{T} C^H_{i,t} M^H_{i,0,t} \right] = W^H_{i,0} \right\}.
\]

Let

\[
D^H_{i,t} = E_t \left[ \sum_{\tau=0}^{T-t} \Psi^H_{i,t,t+\tau} \right],
\]

where we have defined

\[
\Psi^H_{i,t,t+\tau} = \frac{\Psi^H_{i,t+\tau}}{\Psi^H_{i,t}}
\]

to be the multi-period discount factors. That is, \( D_{i,t} \) is the expected discount factor for the reserves are equivalent to direct money rebates to intermediaries. Note, however, that market segmentation implies that the distribution of money holdings has real effects in our model and hence cannot be neglected.
whole future consumption stream. We also define
\[ D_{i,t}^{H,I} \equiv \frac{D_{i,\tau}^{H,I}}{D_{i,t}^{H,I}}, \ i = 1, \ldots, N. \]

We will also use \( W_{i,t}^{H,I}, \ i = 1, \ldots, N, \) to denote the nominal wealth of the corresponding agents’ class. The following is true.

**Lemma 2**  
Country \( i \) customers’ nominal consumption and wealth dynamics are given by
\[
C_{i,t}^{H} = W_{i,0}^{H} \frac{\Psi_{i,t}^{H} (M_{i,0,t}^{H})^{-1}}{D_{i,0}^{H}}
\]

(7)

and\(^{25}\)
\[
\frac{W_{i,t}^{H}}{W_{i,t-1}^{H}} = (M_{i,t-1,t}^{H})^{-1} \Psi_{i,t-1,t}^{H} D_{i,t-1,t}^{H}, \ i = 1, \ldots, N.
\]

By Assumption 2, class-I agents are initially endowed with \( \alpha_i \) shares of the claim on the aggregate endowment plus the money rebates from the government. In addition, they own the intermediation firms that generate a nominal income flow \( \mathcal{I}_{i,t} \) in the local currency from intermediation mark-ups. Hence, their nominal net worth is given by
\[
W_{i,0}^{I} = -\bar{K}_{i,0} + \alpha_i S_{i,0} + E \left[ \sum_{t=0}^{T} M_{i,0,t}^{I} (\mathcal{I}_{i,t} + (M_{i,t} - M_{i,t-1})) \right],
\]
where \( \bar{K}_{i,0} \) is the time zero (entry) cost of setting up an intermediation firm.\(^{26}\)

As we mention above, we assume that setting up an intermediation firm is costly. We assume that the entry cost entails both a fixed setup cost \( k_{i,0} > 0 \) and a proportional setup cost \( D_{i,t}^{H,I} \).

\(^{25}\)Note that, importantly, our model features a non-constant consumption/wealth ratio, proportional to \( D_{i,t}^{H,I} \).

\(^{26}\)As we explain above, we assume that this nominal cost is immediately transferred to customers at time zero. The assumption that the cost is only incurred at time zero is made for convenience and can be relaxed.
cost \( \kappa_{i,0} \in (0, 1) \), so that the total cost \( \bar{K}_{i,0} \) is linear in the present value of future revenues:

\[
\bar{K}_{i,0} = k_{i,0} + \kappa_{i,0} E \left[ \sum_{t=0}^{T} M_{i,0,t}^I (\mathcal{I}_{i,t} + (M_{i,t} - M_{i,t-1})) \right].
\]

These costs will play no role in the subsequent analysis.\(^{27}\) Importantly, making these costs sufficiently large, we can make \( W \) arbitrarily small. They also allow us to make an important distinction between the size of markups and the actual profitability of the intermediation sector: While the markups (i.e., the spread between the D2C and the D2D pricing kernels) might be high, the actual profit margins might be quite low.

The same argument as that for (7) implies that an intermediary’s optimal consumption expenditures in the local currency are given by

\[
C_{i,t}^I = W_{i,0}^I \frac{\Psi_{i,t} (M_{i,0,t}^I)^{-1}}{D_{i,0}^I}. 
\]

Let us now consider the bargaining problem between a customer and an intermediary. At time \( t \), a country \( i \) customer with the nominal wealth \( W_{i,t}^H \) gets matched with an intermediary who quotes him a one period ahead pricing kernel \( M_{i,t,t+1}^H \) in the local currency. Given this quote, the customer decides how to optimally finance his future excess consumption, \( C_{i,t+1}^H \) through a portfolio of the risk free bond and the stock to be traded in the centralized market, as well as an OTC contract with a state-contingent payoff that he buys in the D2D market. Due to the no-arbitrage constraints for the local stock and bond markets (see Assumption 1), customers are in fact indifferent between trading the stock and bond in the D2D and the D2C market. Hence, without loss of generality we can assume that they directly trade bonds and stocks with intermediaries. Thus, the agent is simply buying the claim on his future wealth, \( W_{i,t+1}^H \), from the intermediary, so that current consumption is the difference

\(^{27}\)One could potentially use them to endogenize the size of the intermediation sector as well as to study the impact of regulations on the endogenous size of intermediation sector and markups.
between the current wealth and the D2C price of the claim on future wealth:

\[ C_{i,t}^H = W_{i,t}^H - E_t[M_{i,t,t+1}^H W_{i,t+1}^H]. \]

The customers’ problem is thus to solve for the optimal interplay between today’s consumption \( C_{i,t}^H \) and tomorrow’s wealth. The following is a direct consequence of Lemma 2.

**Lemma 3** Optimal demand of a country-\( i \) customer in the D2C market is given by

\[ W_{i,t+1}^H (M_{i,t,t+1}^H) = \frac{W_{i,t}^H \Psi_{i,t+1}^H D_{i,t+1}^H \Psi_{i,0,t}^H (M_{i,0,t}^H)^{-1} (M_{i,t,t+1}^H)^{-1}}{D_{i,t}^H}. \]

The intuition behind Lemma 3 is straightforward: A log utility maximizing agent always consumes inversely proportionally to state prices (formula (7)). Furthermore, his decision to allocate wealth across states is driven by the expected discount factor \( \Psi_{i,t+1}^H D_{i,t+1}^H \), which determines the value of the total future stream of consumption in a given state.

The time \( t \) value of the claim on \( W_{i,t+1}^H \) for the intermediary is given by \( E_t[M_{i,t,t+1}^I W_{i,t+1}^H] \), and intermediary’s objective is to maximize the total markup

\[ I_{i,t} = E_t[M_{i,t,t+1}^H W_{i,t+1}^H] - E_t[M_{i,t,t+1}^I W_{i,t+1}^H] \]

given by the difference between the value of the claim \( W_{i,t+1}^H \) under the D2C and the D2D pricing kernels.\(^{28}\) By Lemma 3, the markup maximization problem of the intermediary takes the form

\[ \max_{M_{i,t,t+1}^H > 0} E_t[(M_{i,t,t+1}^H - M_{i,t,t+1}^I) \Psi_{i,t+1}^H D_{i,t+1}^H (M_{i,t,t+1}^H)^{-1}] \] \hspace{1cm} (8)

under the constraints (5)-(6). Denoting by \( \mu_{i,t} \) and \( \lambda_{i,t} \) the Lagrange multipliers for the

\(^{28}\)Indeed, since intermediaries have access to complete D2D markets, their objective is to maximize the present value of cash flows in the D2C market under the D2D pricing kernel. Those cash flows are given by \( E_t[M_{i,t,t+1}^I W_{i,t+1}^H] \) at time \( t \) and by \(-W_{i,t+1}^H\) at time \( t+1 \), and the present value is given by \( I_{i,t} \).
constrains (5) and (6) respectively, and writing down the first order conditions for (8), we get

\[ M_{i,t+1}^I \Psi_{i,t+1}^H D_{i,t+1}^H (M_{i,t+1}^H)^{-2} = \lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t}. \]  

(9)

The intuition behind (9) is as follows: The marginal gain of selling insurance against a state \( x \) is given by the product of the D2D price \( M_I(x) \) and sensitivity of consumer’s consumption to the price \( M_H(x) \). Since consumers have log utility, this sensitivity is given by \(-\Psi_{i,t+1}^H D_{i,t+1}^H (M_{i,t+1}^H)^{-2}\). At the optimum, this marginal gain is equal to the state-contingent shadow cost of constraints (5)-(6), given by \( \lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t} \). The solution to (9) is reported in the following proposition.

**Proposition 4** The optimal pricing kernel quoted by the intermediary is given by

\[ M_{i,t+1}^H = \left( \frac{\Psi_{i,t+1}^H D_{i,t+1}^H}{(\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})^{1/2}} \right)^{1/2}, \]  

(10)

where the Lagrange multipliers \( \lambda_{i,t}, \mu_{i,t} \in \mathbb{R} \) are determined by the conditions

\[ E_t \left[ \frac{(\Psi_{i,t+1}^H D_{i,t+1}^H)^{1/2}(M_{i,t+1}^I)^{1/2}}{(\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})^{1/2}} \right] = E_t[M_{i,t+1}^I]; \]

\[ E_t \left[ \frac{(\Psi_{i,t+1}^H D_{i,t+1}^H)^{1/2}(M_{i,t+1}^I)^{1/2}S_{i,t+1}}{(\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})^{1/2}} \right] = E_t[M_{i,t+1}^I S_{i,t+1}]. \]

Proposition 4 is key to the subsequent analysis. It shows how the bargaining friction and the ability of intermediaries to charge state-contingent markups distorts asset prices and, as a result, distorts equilibrium allocations.\(^{29}\)

The signs of the Lagrange multipliers \( \lambda_{i,t}, \mu_{i,t} \) will play a very important role in the

---

\(^{29}\) Another important consequence of Proposition 4 is the break-down of money neutrality. The mechanism underlying this non-neutrality is related to the Fisher debt deflation theory, whereby unexpected monetary shocks serve as a channel for redistributing wealth between customers and intermediaries. See Malamud and Schrimpf (2016) for details.

---
subsequent analysis. Since these quantities are Lagrange multipliers of constraints (5)-(6),
their signs are determined by the “direction” in which these constraints are binding. Consider
first the Lagrange multiplier \( \mu_{i,t} \) of the constraint (5). One can equivalently interpret (5) as
a pair of inequality constraints

\[
E_t[M^H_{i,t,t+1}] \geq (1 - \varepsilon)E_t[M^I_{i,t,t+1}]
\]
\[
E_t[M^H_{i,t,t+1}] \leq (1 + \varepsilon)E_t[M^I_{i,t,t+1}],
\]

where the parameter \( \varepsilon \) (determining the corridor inside which the intermediary can quote
rates) is arbitrarily small.

The economic intuition behind these constraints is as follows. If customers would like
to invest into risk free assets\(^{30}\), the intermediary will try to push the nominal rate \( e^{r_{t,t}} = \frac{1}{E_t[M^H_{i,t,t+1}]} \) all the way down to its lower bound, determined by the D2D market rate \( \frac{1}{E_t[M^I_{i,t,t+1}]} \), and hence the constraint \( E_t[M^H_{i,t,t+1}] \leq E_t[M^I_{i,t,t+1}] \) will be binding; in this
case, standard Kuhn-Tucker conditions imply that \( \mu_{i,t} > 0 \). By contrast, if customers find it
optimal to borrow from the intermediaries, the latter will try to push the offered rate all the
way up to its upper bound, determined by the D2D market rate \( \frac{1}{E_t[M^I_{i,t,t+1}]} \), and hence
the constraint \( E_t[M^H_{i,t,t+1}] \geq E_t[M^I_{i,t,t+1}] \) will be binding; in this case, standard Kuhn-Tucker
conditions imply that \( \mu_{i,t} < 0 \). Similar intuition applies to (6): In this case, the sign of
\( \lambda_{i,t} \) is determined by whether the customers are trying to buy more of the stock from the
intermediary (in which case \( \lambda_{i,t} > 0 \)) or want to lay off some of their stock holdings (in
which case \( \lambda_{i,t} < 0 \)). Thus, we can naturally interpret these different regimes akin to “risk
on-risk off” scenarios whereby customers move in or our of stocks and bonds: for example,
\( \lambda_{i,t} < 0 < \mu_{i,t} \) corresponds to a “risk off” episode whereby customers move out of stocks (by
selling them to intermediaries) and invest into risk-free bonds.

\(^{30}\)Intermediaries could provide access to (nearly) risk free assets through private money creation: for
example, through bank deposits and money market funds. We abstract from such private money creation in
our model. See, Brunnermeier and Sannikov (2016) for a model featuring an impact of such private money
creation on monetary policy passthrough.
The key role of intermediaries in our model is to offer customers (risky) alternatives to local securities such as international bonds or other, more complex instruments. Customers’ demand for such securities determines the size and the sign of security’s markups (i.e., the spread between the price of the security in the D2C and the D2D market). For a foreign risky security with nominal payoff $X_{t+1}$ in the local currency, the markup is given by

$$U_t(X_{t+1}) \equiv E_t[M_{i,t,t+1}^H X_{t+1}] - E_t[M_{i,t,t+1}^I X_{t+1}].$$

Define

$$\Gamma_{i,t,t+1} \equiv \frac{M_{i,t,t+1}^H}{M_{i,t,t+1}^I} = (\frac{\psi_{i,t,t+1}^H D_{i,t,t+1}^H}{\lambda_i(t(S_{i,t+1}/S_{i,t}) + \mu_{i,t}}) \quad (11)$$

to be the state-contingent markup. Denote by $\tilde{\text{Cov}}_{i,t}^{I,i}$ the covariance under the D2D risk neutral measure in country $i$. Using (5), we arrive at the following result.

**Proposition 5** *[The fundamental markup equation]* Intermediation markups are given by

$$U_t(X_{t+1}) = e^{-r_{i,t}} \tilde{\text{Cov}}_{i,t}^{I,i}(\Gamma_{i,t,t+1}, X_{t+1}).$$

Proposition 5 is a *fundamental markup equation*, akin to the fundamental equation of asset pricing that characterizes risk premia through covariance with the stochastic discount factor. It is very intuitive: Assets that payoff in states with high costs $\Gamma_{i,t,t+1}$ trade at high markups. It is based on the *risk-based* approach to markups, whereby the risk properties of a security’s payoff together with the price pressure created by customers’ demand in the D2C market are key determinants of markups size.
4 Equilibrium

Recall that $\lambda_{i,\tau}(S_{i,\tau+1}/S_{i,\tau}) + \mu_{i,\tau}$ is the shadow cost of intermediation. We will define

$$\Lambda_{i,t} \equiv (\Psi_{i,t}^H/D_{i,t}^H)^{1/2} \prod_{\tau=0}^{t-1} (\lambda_{i,\tau}(S_{i,\tau+1}/S_{i,\tau}) + \mu_{i,\tau})^{1/2}. \quad (12)$$

and

$$\Lambda_{i,t,\tau} \equiv \frac{\Lambda_{i,\tau}}{\Lambda_{i,t}}$$

to the cumulative shadow costs. These costs limit the ability of intermediaries to extract rents: when $\Lambda_{i,t}$ is large, (10) implies that intermediaries suffer. Customers, by contrast benefits from states with high $\Lambda_{i,t}$, because they get low state prices and hence are able to boost their consumption. In fact, combining (7) and (10), we get

$$C^H_{i,t} = C^H_{i,0} (M^I_{i,0,t})^{-1/2} \Lambda_{i,t},$$

showing that customers’ consumption is proportional to the cumulative shadow cost of intermediation.

Recall that $E_{i,t}$ denotes the nominal exchange rate between country $i$ and the US dollar (i.e., the dollar price of country $i$ currency), while $M_{i,t}$ is the total money supply in country $i$. Imposing market clearing in all tradable goods and in the markets for local currencies, we arrive at the following equilibrium characterization.\textsuperscript{31}

\textsuperscript{31}It is sufficient to impose goods markets clearing, as the asset markets clearing then follows automatically: In an Arrow-Debreu economy, consumption in a given state equals the holdings of Arrow securities for the corresponding state.
Theorem 6  Equilibrium prices are pinned down by the equation system

\[ \sum_{i=1}^{N} \left( C_{i,0}^H (M_{i,0,t}^I)^{-1/2} \Lambda_{i,t} + C_{i,0}^I \Psi_{i,t} (M_{i,0,t}^I)^{-1} \right) \theta_{i,k,t} E_{i,t} = M_{k,t} E_{k,t}, \quad k = 1, \ldots, N, \quad (13) \]

with \( M_{i,0,t}^I \) given by (4).

By Lemmas 1, \( (C_{i,0}^H (M_{i,0,t}^I)^{-1/2} \Lambda_{i,t} + C_{i,0}^I \Psi_{i,t} (M_{i,0,t}^I)^{-1})\theta_{i,k,t} E_{i,t} \) is the total consumption expenditure of country \( i \) agents on the tradable good of country \( k \), measured in US dollars. At the same time, by the cash in advance constraint, the global nominal expenditure on these goods in US dollars is given by \( P_{k,k,t} X_{k,t} E_{k,t} = M_{k,t} E_{k,t} \). Thus, equation system (13) simply states that the total demand for the goods of country \( k \), measured in US dollars, equals the US dollar value of the supply of these goods. While it is not possible to analytically solve this system in general, we present analytical results for several important special cases below in order to highlight the key economic mechanisms.

5  Intermediation and Exchange Rate Anomalies

One of the principal goals in this paper is to understand the role of intermediation frictions for various known anomalies in exchange rates and international capital markets. In this context, the impact of heterogeneity in country characteristics on risk properties of exchange rates are crucial. International trade effectively serves as a propagation mechanism for exporting these characteristics to other countries. This effectively gives rise to a full transmission matrix.

In this section, we focus on the diagonal of this transmission matrix. To this end, we shut down international trade\(^{32}\) and focus on the \textit{local risk sharing between domestic customers and domestic intermediaries}. Under international autarky, it is the differences in local risk sharing that drives divergence between local pricing kernels and, as a result, the dynamics of

\(^{32}\)See, for example, Alvarez et al. (2002) who also study exchange rates in an autarky economy: As in the standard, representative agent consumption based asset pricing models, even though there is no trade, prices of all securities are still uniquely defined in equilibrium, precisely at the levels that guarantee no trade.
exchange rates. In this section, we focus on how the international differences in risk sharing between customers and intermediaries are affected by (i) monetary and stabilization policies; (ii) the capacity and leverage of the intermediation sector. In particular, we focus on safe haven properties of exchange rates and CIP deviations. Later on, in Section 6, we introduce international trade and study the impact of trade on exchange rates. All the results from the current section remain valid as long as the international trade channel is not too strong relative to the domestic risk sharing channel.

Consider an autarky economy in which agents only consume domestic goods. In this case, equilibrium market clearing in the domestic money markets can be rewritten as

$$M_t = C_H^{i,0} \Lambda^{i,0,t} (M_{i,0,t}^I)^{-1/2} + C_I^{i,0} \Psi^{i,0,t} (M_{i,0,t}^I)^{-1}.$$  \hspace{1cm} (14)

Absent international trade, our agents populating the pure monetary economy trade to share two types of risk: i) “real” risks in the form of shocks to their discount factors, and ii) nominal risks in the form of monetary policy shocks. Equation (14) shows that, in equilibrium, agents share the aggregate monetary risk $M_t$ as if markets were frictionless and customers had an (endogenous) discount factor $\Lambda^{i,0,t}$ and a risk aversion of two: Indeed, standard first order conditions in an Arrow-Debreu economy imply that the optimal consumption of an agent with constant relative risk aversion of $\gamma$ is proportional to $(M_{i,0,t}^I)^{-1/\gamma}$. Thus, intermediation frictions alter the nature of equilibrium risk sharing in two ways: by affecting the discount factor $\Lambda$ and by making customers effectively more risk averse.

Both effects create an externality: Because intermediaries do not internalize the impact of their markups on equilibrium prices, they end up taking up too much of aggregate risk (because they are effectively two times less risk averse than customers), and they end up with too little of aggregate consumption in those states in which the shadow cost of intermediation is too high.
Solving the quadratic equation (14) for the D2D pricing kernel \( M_{i,0,t}^I \) and using (4), we arrive at the following result.\(^{33}\)

**Proposition 7** The country i equilibrium D2D pricing kernel and the exchange rate are given by

\[
M_{i,0,t}^I = \left( \frac{\Lambda_{i,0,t} + \left( \Lambda_{i,0,t}^2 + 4C_{i,0}^I \Psi_{i,0,t}^I M_{i,t} \right)^{1/2}}{2M_{i,t}} \right)^2
\]

and

\[
\frac{E_{i,t}}{E_{i,0}} = \frac{M_{i,t}^{-2}}{M_{S,t}^{-2}} \left( \frac{\Lambda_{i,0,t}^2 + \left( \Lambda_{i,0,t}^2 + 4C_{i,0}^I \Psi_{i,0,t}^I M_{i,t} \right)^{1/2}}{\Lambda_{S,0,t}^2 + \left( \Lambda_{S,0,t}^2 + 4C_{S,0}^I \Psi_{S,0,t}^I M_{i,t} \right)^{1/2}} \right)^2
\] (15)

Formula (15) shows how intermediation frictions distort the nature of equilibrium allocations and exchange rates. In stark contrast to the frictionless case (formula (32)), exchange rates become non-linear in the money supply and discount factors, and the basic properties described in Corollary 22 break down. This non-linear behaviour of exchange rates depends crucially on two key objects: the exogenous discount factor of intermediaries, \( \Psi^I \), and the endogenous discount factor of customers, \( \Lambda \).

In this section, we use formula (15) to address several well known anomalies in the behaviour of exchange rates.

\(^{33}\)As in Gabaix and Maggiori (2015), in our model intermediaries are marginal investors in the international financial markets, and hence exchange rates are determined by their marginal utilities which can be quite different from those of households.
### 5.1 Intermediation Markups and Exchange Rates for Small Intermediation Capacity

To gain a deeper understanding of equilibrium behaviour and, in particular, the precise nature of currency crashes, we will need an additional technical assumption. To motivate this assumption, we note that, in our model, $C_{i,0}^I$ can be naturally interpreted as a measure of intermediation capacity of class $I$ agents in country $i$. Intuitively, $C_{i,0}^I$ measures how much of the aggregate risk the intermediation sector is able to absorb. In the limit when $C_{i,0}^I \to 0$, intermediation capacity drops and we end up in a situation where intermediaries do not warehouse any risk on their balance sheet and immediately offload accumulated inventory positions in the inter-dealer market.\(^{34}\) We will use $C_{i,0}^I$ as a parameter controlling the intermediation capacity. In the sequel, we will only consider the case when $C_{i,0}^I$ is small.

Absent intermediaries, the D2C pricing kernel is given by

$$M_{i,t,t+1}^{H,*} = N_{i,t+1}^{-1} \Psi_{i,t,t+1}^H,$$

(16)

That is, not surprisingly, the D2C pricing kernel coincides with the pricing kernel in the frictionless economy populated only by customers. In this case, customers are forced to absorb the two types of aggregate risk in our model: i) the risk of innovations to the global discount factor, $\Psi_{i,t,t+1}^H$, and ii) innovations in monetary policy shocks, $N_{i,t,t+1}$. The assumption that endowment claims are equally priced under the two kernels (see (6)) implies that the stock price $S_{i,t}$ in the zero intermediation capacity limit coincides with the stock price in the frictionless market in which only customers are present and they trade directly.

---

\(^{34}\)This is also sometimes labelled as agency model. In the benchmark version of the model, all intermediaries are identical and hence there is no trade in the D2D market: In this case, equilibrium prices adjust in such a way that no trade is optimal and customers simply hold their endowment. Note also that small intermediation capacity can only be achieved by making the entry costs for intermediation firms sufficiently high.
with each other in a centralized market: That is,

\[ S_{i,t}^* = \mathcal{M}_{i,t} D_{i,t}^H. \]  

(17)

Setting \( C_0^I = 0 \) in (27), we then immediately get

\[ M^*_I, t+1 = \mathcal{N}^{-2}_{t+1} (\Psi^H_{i,t+1} / D^H_{k,t,t+1}) (\lambda_{i,t} \mathcal{N}_{t+1} D^H_{k,t,t+1} + \mu_t), \]

Equations (15)-(16) immediately imply that there exists an equilibrium with \( \mu_{i,t} = 0, \lambda_{i,t} = 1 \): Indeed, in this case \( M^*_I, t+1 = M^*_{t+1} \), and hence the no arbitrage conditions (6)-(5) are trivially satisfied, and there are no markups. The intuition behind this result is straightforward: Absent intermediation capacity, there is nobody to share the risks with; hence, there is no trade in the D2C market, and customers just end up holding the stock market.

We will use \( * \) to denote equilibrium variables in this limit. In particular, we will often use intermediaries’ consumption and wealth, relative to that of customers,

\[ C^*_I / C^*_H = C^*_I / C^*_0 \Psi^I_{i,0} / \Psi^H_{i,0}, \]  

\[ W^*_I / W^*_H = W^*_I / W^*_0 D^I_{i,0} / (\Psi^H_{i,0} D^H_{i,0}), \]

(18)

as well as the equilibrium stock price

\[ S^*_i, t = \mathcal{M}_{i,t} D_{i,t}^H. \]  

(19)

Let us define the D2C risk neutral measure

\[ d\tilde{P}^*_{i,t} = \frac{M^H_{i,t,t+1}}{E_t[M^H_{i,t,t+1}]}, \]
in the zero capacity limit, and let us denote by \( \tilde{E} \) and \( \tilde{\text{Cov}} \) the expectation and the covariance under this measure. The following is true.

**Proposition 8** Suppose that \( C^I_{i,0} \) is sufficiently small. Then, there exists a unique equilibrium, in which the shadow costs \( \mu_{i,t}, \lambda_{i,t} \) are given by

\[
\lambda_{i,t} = 1 + \left( W^{*,I/H}_{i,t} - \frac{\tilde{\text{Cov}}(S^*_{i,t+1}, W^{*,I/H}_{i,t+1}, 1/S^*_{i,t+1})}{\text{Cov}(S^*_{i,t+1}, 1/S^*_{i,t+1})} \right) + O((C^I_{i,0})^2) \\
\mu_{i,t} = -\frac{\tilde{\text{Cov}}(W^{*,I/H}_{i,t+1}, S^*_{i,t+1})}{\text{Cov}(S^*_{i,t+1}, 1/S^*_{i,t+1})} + O((C^I_{i,0})^2)
\]

where \( S^*_t \) is defined in (17).

The intuition behind (20) is as follows. The signs (and the size) of the shadow costs \( \lambda_{i,t}, \mu_{i,t} \) depend on the ability of the stock market to serve as an efficient hedge against states with very high state prices. Naturally, \( \lambda_{i,t} \) is positive (at least for small intermediation capacity): intermediation markups force customers to retain significant positive exposure to the domestic stock market leading to an endogenous home bias, simply because trading foreign securities entails (endogenous) transaction costs. The case \( \mu_{i,t} < 0 \) occurs when intermediaries effectively sell equity market exposure to customers. Indeed, by (20), the sign of \( \mu_{i,t} \) coincides with that of \( \tilde{\text{Cov}}(W^{*,I/H}_{i,t+1}, S^*_{i,t+1}) \), where \( W^{*,I/H}_{i,t+1} = (\Psi^I_{i,0,t+1} D^I_{i,0,t+1})/(\Psi^H_{i,0,t+1} D^H_{i,0,t+1}) \). Hence, \( \mu_{i,t} < 0 \) when either the stock market \( S_{i,t+1}^* \) co-moves negatively with intermediaries’ discount factor, \( \Psi^I_{i,0,t+1} D^I_{i,0,t+1} \), or it co-moves positively with customers’ discount factor, \( \Psi^H_{i,0,t+1} D^H_{i,0,t+1} \), or both.

Recall that, by Lemma 3, customers (respectively, intermediaries) value wealth most in those states in which their expected discount factor, \( \Psi^H_{i,0,t+1} D^H_{i,0,t+1} \) (respectively, \( \Psi^I_{i,0,t+1} D^I_{i,0,t+1} \)) is high: In such states, the agents anticipate a high value of the future consumption stream, and therefore they like securities that pay off precisely in those states. In the former case, intermediaries dislike holding stocks because stocks represent a poor hedge against such
“liquidity shocks”; in the latter, customers enjoy holding stocks because they pay off high precisely in the states in which they need cash the most (stocks are a good hedge). A similar logic applies to bonds, whereby the attractiveness of the bond payoff depends on its real returns and the ability of this bond to satisfy future consumption needs.

We will now derive explicit expressions for exchange rates in the case when the intermediation capacity is small. Recall formula (18) and define

\[ \Delta C^{*,I/H}_{i,t,\tau} \equiv C^{*,I/H}_{i,\tau} - C^{*,I/H}_{i,t,\tau}, \quad \Delta W^{*,I/H}_{i,t,\tau} \equiv W^{*,I/H}_{i,\tau} - W^{*,I/H}_{i,t,\tau}. \]  

The following is true.

**Proposition 9** In the limit of small intermediation capacity, exchange rates are given by

\[
\frac{E_{i,\tau}}{E_{i,t}} = \frac{N^{-1}_{i,t,\tau} \Psi^H_{i,t,\tau}}{N^{-1}_{s,t,\tau} \Psi^H_{s,t,\tau}} \left( 1 + \left( (\Delta C^{*,I/H}_{i,t,\tau} - \Delta C^{*,I/H}_{s,t,\tau}) + (\Delta W^{*,I/H}_{i,t,\tau} - \Delta W^{*,I/H}_{s,t,\tau}) \right) + \sum_{s=t}^{\tau-1} \left( (\lambda_{i,s} - \lambda_{s,s}) + (\mu_{i,s} N^{-1}_{i,s+1} (D^H_{i,s,s+1})^{-1} - \mu_{s,s} N^{-1}_{s,s+1} (D^H_{s,s,s+1})^{-1}) \right) \right) + O(C^2_{i,0} + C^2_{s,0}).
\]

In our model, exchange rates are determined by the ratio of intermediaries marginal utilities, consistent with the empirical evidence in He et al. (2016b). Those marginal utilities are endogenous and depend on the intermediation markups. Proposition 9 provides an explicit, closed form expression for equilibrium exchange rates and shows how they deviate from the frictionless benchmark (32). The first term, \(\Delta C^{*,I/H}_{i,t,\tau} - \Delta C^{*,I/H}_{s,t,\tau}\), is a simple discount factor effect that is already present in the frictionless model: The value of currency is determined by the value of consumption; the latter is in turn determined by the size of the discount factor. Intermediaries consume proportionally to their discount factor, and hence the currency appreciates at times when this consumption is high. The second term, \(\Delta W^{*,I/H}_{i,t,\tau} - \Delta W^{*,I/H}_{s,t,\tau}\), while looking similar to the first one, is new and is a subtle outcome of
intermediation frictions: when the relative wealth \( W_{r^*, I/H}^{i,t,\tau} \) is high, intermediaries bid up the value of the aggregate wealth, making also customers richer; as a result, customers’ need for risk smoothing consumption also drops, and intermediaries can extract less rents, making their consumption more valuable. Finally, the last term also reflects intermediaries’ wealth accumulated from trading with customers in the D2C market, and given by cumulative shadow costs on intermediation. This cumulative shadow cost depends on monetary shocks, \( cN_{\xi,s+1}^{-1} \) because monetary policy is redistributive in our model. Depending on the sign of \( \mu_{\xi,s} \), monetary shocks cause a redistribution of wealth between customers and intermediaries, affecting risk premia, pricing kernels, and exchange rates. In turn, the sign of \( \mu_{\xi,s} \) depends crucially on the expectations about future monetary policy conduct. Understanding the impact of these expectations on exchange rates is the main goal of the next subsection.

5.2 Stabilisation policies and exchange rates overshooting

In the frictionless model of Section A.1, expectations about future monetary policy (or central bank forward guidance) do not have any effect on the exchange rates. This is because agents can perfectly share the aggregate risk. By contrast, with intermediation frictions, expectations about the future monetary policy, as captured by the Lagrange multipliers \( \lambda_{i,t}, \mu_{i,t} \), play a key role in determining the nature of risk sharing because these expectations drive customers’ and intermediaries’ incentives to buy insurance and make levered bets in the D2C market. By Proposition 8, the size and the signs of these Lagrange multipliers are determined by the interaction between monetary policy conduct and the local stock market. This is intuitive: stock market valuations reflect the value of customers’ wealth, and monetary policy influences the risk properties of this wealth.

Everywhere in the sequel, we will make the following technical assumption.

**Assumption 3** There exists a Markov process \( \omega_{i,t} \in \mathbb{R}, t \geq 0 \) with a transition density \( p(\omega_{i,t}, \omega_{i,t+1}) \), and two strictly monotone increasing functions \( f_i, i = H, I \), such that \( \Psi_{i,t}^J = \)
\[ \prod_{\tau=1}^t f_i^J(\omega_{i,\tau}), \quad J = I, H. \] Furthermore, the transition density has the monotone likelihood property: \( \frac{\partial}{\partial \omega} \log p(\omega_{i,t}, \omega_{i,t+1}) \) is monotone increasing in \( \omega_{i,t+1} \).

The following lemma is a direct consequence of Assumption 3.

**Lemma 10** There exist monotone increasing functions \( d_i^J(\omega,t), \quad J = I, H \) such that \( \log D_{i,t}^J = d_i^J(\omega_{i,t}) \), \( J = H, I \). Furthermore, \( W_{i,t+1}^{*,I/H} \) is monotone increasing (decreasing) in \( \omega_{i,t+1} \) if so does \( f_i^I(\omega_{i,t+1})/f_i^H(\omega_{i,t+1}) \).

Recall that \( W_{i,t+1}^{*,I/H} \) is the ratio of intermediaries’ and customers’ wealth. Thus, the fact that \( W_{i,t+1}^{*,I/H} \) is monotone increasing means that intermediaries benefit more from the upside (the times and global wealth is high), but also suffer more in the downside, when the global wealth is low. That is, an increasing \( W_{i,t+1}^{*,I/H} \) corresponds to the case when intermediaries are leveraged. In the real world, intermediaries are always leveraged as their business model typically involves borrowing money from customers and then investing into risky assets.

In our model, the dynamics of asset prices and exchange rates are influenced by two primitive risk factors: shocks to discount rates, as captured by the stochastic nature of \( \Psi_{i,t}^H, \Psi_{i,t}^I \), as well as their present values, \( D_{i,t}^H, D_{i,t}^I \); and shocks to monetary policy, as captured by the stochastic nature growth in the money supply, \( N_{i,t+1} \). The former play the role of sentiment/demand shocks: sentiment shocks or demand shocks; the latter are supposed to respond to these demand shocks, stabilizing the economy and protecting it from “overheating” and “depressions”.

By (19), we have \( S_{i,t+1}^{*,I}/S_{i,t}^{*,I} = N_{i,t+1} D_{i,t,t+1}^H \). That is, the stock market return is the product of discount factor shocks and monetary policy shocks. Thus, a policy that sets the money growth, \( N_{i,t+1} \), to be monotone decreasing in \( D_{i,t,t+1}^H \) naturally dampens volatility of the stock prices.\(^{35}\) Following Hassan et al. (2016), we will use the term ”stabilization policy that sets the money growth, \( N_{i,t+1} \), to be monotone decreasing in \( D_{i,t,t+1}^H \) naturally dampens volatility of the stock prices.\(^{35}\) Following Hassan et al. (2016), we will use the term ”stabilization

\(^{35}\)Note that, importantly, the fact that \( N_{i,t+1} \) is monotone decreasing in \( D_{i,t,t+1}^H \) does not mean that the monetary policy directly reacts to the stock market. What it actually means is that the monetary policy reacts to the economic conditions (ac captured by the shocks to \( \Psi_{i,t}^H \)) which are also reflected in \( D_{i,t,t+1}^H \). Since both \( N_{i,t+1} \) and \( D_{i,t,t+1}^H \) react to the same macroeconomic shocks, a stabilization policy will always
policy” to describe such state contingent monetary injections. The following result is a direct consequence of Proposition 8.

**Proposition 11** Suppose that the quotient \( f_i(\omega_{i,t+1})/f_i^H(\omega_{i,t+1}) \) is monotone increasing in \( \omega_{i,t+1} \). Suppose also that \( N_{i,t+1} = F_t(D_{i,t,t+1}^H)N_{i,t+1}^{\text{shock}} \), where \( F_t \) is a monotone function of \( D_{i,t,t+1}^H \), while \( N_{i,t+1}^{\text{shock}} \) is a small, independent noise. Then, the following is true.

- If the stabilization policy is mild or absent (i.e., \( \partial \log N_{i,t+1} / \partial \log D_{i,t,t+1}^H > -1 \)), then \( \mu_{i,t} > 0 \).
- If the stabilization policy is strong (i.e., \( \partial \log N_{i,t+1} / \partial \log D_{i,t,t+1}^H < -1 \)), then \( \mu_{i,t} < 0 \).

Proposition 11 shows that stabilization policies can lead to instabilities if the policy maker “overdoes” the job: if money supply “overshoots”, the signs of the Lagrange multiplier \( \mu_{i,t} \) flips (\( 0 > \mu_{i,t} \)), pushing customers into a “risk-on” regime. Intermediaries respond by charging high markups for the access to the “upside” (e.g., through out-of-the-money call options) corresponding to states with high realizations of \( D_{i,t,t+1}^H \). In this case, \( S_{i,t+1}^*/S_{i,t}^* = N_{i,t+1} (D_{i,t,t+1}^H) \) is monotone decreasing in \( D_{i,t,t+1}^H \): That is, the monetary policy is so strong that it reverses the sign of the reaction of the stock market to fundamental shocks.

Formula (22) and the results of Proposition 11 imply that the nature of the response of exchange rates to monetary shocks depends crucially on the nature of the stabilization policy pursued by a given country. The following is true.

**Proposition 12** [*Dornbusch Overshooting Effect*] If the stabilization policy in country \( i \) is mild, then the real exchange rate \( E_{i,t} \) “overshoots” in response to a monetary shock \( N_{i,t+1}^{\text{shock}} \).

By contrast, if the stabilization policy in country \( i \) is strong, then the real exchange rate “undershoots” in response to unexpected money supply shocks.

---

36Reinhart and Rogoff (2004) show that 88% of countries (representing 47% of world GDP) stabilize their currency relative to some target country. As Hassan et al. (2016) argue, adjusting money supply in response to shocks can be interpreted as direct (nominal) currency interventions.

37As Law et al. (2017) show, such scenarios are not uncommon and have occurred multiple times in the history of the US monetary policy.
It is instructive to discuss the relationship between the mechanisms underlying the result of Proposition 12 and the classical Dornbusch (1976) overshooting model. According to that model, after a monetary policy shock, the market adjusts to a new equilibrium between prices and quantities whereby nominal price stickiness of tradable goods “stickiness” of prices of goods implies that the short run equilibrium level will first be achieved through shifts in financial market prices; as a result, exchange rates overreact to a monetary shock in the short run. In contrast to Dornbusch (1976), goods prices are fully flexible in our model. Yet, exchange rates may over- or under-react to monetary shocks due to an endogenous “markup stickiness” that arises because the contracts between customers and intermediaries are signed ex-ante, before the monetary shock is realized and hence naturally “sticky”. An unanticipated shock hits intermediaries’ balance sheets and their risk bearing capacity, leading to a repricing in the foreign exchange market.38

5.3 Safe haven currencies and monetary policy uncertainty

There is a large literature investigating differences in the stochastic properties of exchange rates across countries and, in particular, the tendency of some currencies to appreciate in bad times. Many explanations for this behaviour have been proposed, including differences in intermediaries’ risk bearing capacity (Gourinchas et al. (2010) and Maggiori (2013)), country size (Martin (2012), Hassan (2013)), factor endowments (Ready et al. (2017), Powers (2015)), sensitivity to disaster risk (Farhi and Gabaix (2016)), trade centrality (Richmond (2015)), and exposure to long run risk (Colacito et al. (2017)). In this section, we propose a new driver of stochastic properties of exchange rates: the monetary policy uncertainty. Our goal is to characterize the so-called “safe haven currencies” that appreciate in times of a global stock market crash.

38The fact that, in the presence of a strong stabilization policy, exchange rates may under-react to shocks implies that such strong policies may generate “momentum”-type effects that may also spillover onto other asset classes such as stocks.
**Definition 13** We say that currency $i$ is safe haven relative to currency $j$ if, conditional on time–t information and absent monetary shocks, the relative exchange rate $E_{i,t+1}/E_{j,t+1}$ is monotone decreasing in $D_{t+1}^H$.

For simplicity, we assume that discount factors $\Psi_{t,t}^J = \Psi_{t,t}^J$, $J = H, I$, and and $C_{i,0}^I = C_{s,0}^I$ are independent of $i = 1, \cdots, N$, and hence countries only differ in the paths of money supply. In this case, stock prices are given by $S_{i,t}^* = M_{i,t} D_{t,t+1}^H$. A fall in the global discount factor $D_{t,t+1}^H = D_{t,t+1}^H$ can thus be interpreted as a global crisis state whereby stock prices fall across all countries. In this case, formula (22) implies that

$$\frac{E_{i,t+1}}{E_{s,t+1}} \approx \left( 1 + (\lambda_{i,t} - \lambda_{s,t} + (D_{t,t+1}^H - 1)(\mu_{i,t} N_{i,t+1} - \mu_{s,s} N_{s,s+1})) \right),$$

Thus, absent monetary shocks, we have

$$\frac{E_{i,t+1}}{E_{s,t+1}} \approx \left( 1 + (\lambda_{i,t} - \lambda_{s,t} + (D_{t,t+1}^H - 1)(\mu_{i,t} - \mu_{s,s})) \right), \quad (23)$$

and hence US Dollar is a safe haven relative to currency $i$ if and only if $\mu_{i,t} < \mu_{s,t}$: Indeed, in this case formula (23) implies that $\frac{E_{i,t+1}}{E_{s,t+1}}$ is monotone increasing in $D_{t,t+1}^H$, implying that US Dollar value is decreasing in $D_{t,t+1}^H$. The following result follows then from Proposition 8 by direct calculation.

**Proposition 14** Suppose that both $N_{i,t+1}$ and $N_{s,t+1}$ are independent of $\Psi_{t,t}^H$, $D_{t,t+1}^H$, $W_{t+1}^H/I$, and $C_{i,0}^I = C_{s,0}^I$. Then, the US dollar is a safe haven currency relative to country $i$ if and only if there is less uncertainty in US monetary policy, so that

$$\text{Var}_t[N_{s,t+1}^{-1}] < \text{Var}_t[N_{i,t+1}^{-1}].$$

In the frictionless economy, monetary policy uncertainty is irrelevant in the setup of Proposition 14: Exchange rates satisfy $\frac{E_{i,t+1}}{E_{s,t+1}} = \frac{N_{s,t+1}}{N_{i,t+1}}$, and hence they do not correlate at
all with macroeconomic shocks. Proposition 14 shows that intermediation frictions break this neutrality result; in fact, monetary policy uncertainty is bad for economic stability.\textsuperscript{39}

The mechanism underlying the result of Proposition 14 relies on the expectations channel. Customers anticipating a rise in monetary policy uncertainty contact intermediaries to buy insurance against future shocks. Intermediaries charge markups for providing this insurance, which limits customers’ ability to efficiently allocate consumption across future states and limits their ability to buy insurance against global stock market crashes. Proposition 14 implies that these distortions increase with the amount of monetary policy uncertainty. That is, customers in countries with greater monetary policy uncertainty are less able to insure against global shocks, and their consumption is more sensitive to these shocks.\textsuperscript{40} When an adverse shock hits global markets, intermediaries in all countries suffer and see their balance sheets shrink, while marginal utilities go up. However, intermediaries in countries with greater policy uncertainty suffer less that those in the US because they have sold less insurance against those states. This means that the exchange rate – given by the ratio of intermediaries’ marginal utilities – depreciates relative to the safer US dollar. Thus, as in Gourinchas et al. (2010) and Maggiori (2013), US dollar is special in our model because US intermediaries act as global insurance providers. However, the underlying mechanism is different; namely, while Gourinchas et al. (2010) and Maggiori (2013) assume that US intermediaries are special because they have a higher risk bearing capacity, in our model intermediaries in both countries are identical ex-ante, but behave differently ex-post because of different expectations about future domestic monetary policy.\textsuperscript{41}

\textsuperscript{39}There is ample empirical evidence suggesting that policy uncertainty is important for the transmission of monetary shocks. See, for example, Arbatli et al. (2017) for recent evidence for Japan.

\textsuperscript{40}The result of Proposition 14 is also broadly consistent with international evidence. For example, in emerging markets monetary policy tends to be less predictable, while the interest rates are high. Of course, there are many other factors distinguishing emerging market economies, such as higher inflation and greater marginal productivity of capital.

\textsuperscript{41}In the Appendix A.3, we derive technical conditions under which the currency of the country with with a larger intermediation sector endogenously arises as a safe haven.
5.4 Violations of covered interest parity

In the real world, customers in a foreign country do not have access to direct dollar borrowing and lending. Instead, they have to do it through intermediaries by lending/borrowing in the local currency and then swapping their position into dollar. In a fictitious perfect market, no-arbitrage conditions imply that the corresponding FX swap-implied dollar rate should be equal to the direct dollar rate. This arbitrage relationship is known as the covered interest parity (CIP) condition. However, a growing empirical literature (see, for example, Du et al. (2016), Avdjiev et al. (2016), Borio et al. (2016), and Rime et al. (2017)) provides strong evidence for large and persistent CIP deviations across a multitude of currencies. In our model, market fragmentation naturally leads to a violation of the CIP relationship because customers willing to enter the FX swap position need to do this through intermediaries in an OTC market with non-competitive prices. The goal of this section is study the potential macroeconomic drivers of such deviations. We will need the following definition.

Definition 15 We denote by $r_{H,i}^{H,i,t}$ the “synthetic” nominal US dollar interest rate quoted by intermediaries to customers of country $i$. By definition,

$$e^{-r_{H,i}^{H,i,t}} = E_t[M_{i,t+1}^H(E_{i,t}/E_{i,t+1})].$$

We also define the cross-currency basis against the US dollar as

$$\text{Basis}_{i,t}^S = e^{-r_{S,t}} - e^{-r_{H,i}^{H,i,t}} \approx r_{H,i}^{H,i,t} - r_{S,t}.$$  

That is, the basis is given by the difference between US dollar rate $r_{S,t}^H$ that country $i$ customers can back out from intermediaries’ quotes and the direct dollar rate $r_{S,t}$, is generally non-zero.

Recall that $\widetilde{\text{Cov}}_t^{I,i}$ denotes the conditional covariance under the country $i$ nominally risk.
neutral measure with the density $M_{i,t,t+1}e^r_{i,t}$. By Proposition 5,

$$\text{Basis}^s_{i,t} = -e^{-r_{i,t}} \text{Cov}_{i,t}^{I_i} (\Gamma_{i,t,t+1} \cdot (\mathcal{E}_{i,t}/\mathcal{E}_{i,t+1})).$$ (24)

Formula (24) shows how the Basis arises in our model as a form of risk premium, whereby intermediaries charge markups to customers for having access to the US dollar bond market depending on the interaction between state-contingent markups, $\Gamma_{i,t,t+1}$, and exchange rates, $\mathcal{E}_{i,t}/\mathcal{E}_{i,t+1}$. The goal of this section is to understand the nature of this interaction.

As above, we assume that the monetary policy rule pursued by the central bank is given by

$$N_{i,t+1} = (D^H_{i,t,t+1})^{-\alpha^N_{i,t}}, \ i = k,.$$

(25)

Here, $\alpha^N_{i,t}$ captures the sensitivity of monetary policy go local demand shocks summarized by $D^H_{i,t,t+1}$. For simplicity, we will also assume that $\Psi^I_{i,t} = \Psi^I_{t}$, and we will denote

$$\alpha^I_{i,t} = \frac{\partial \log W^{*I,H}_{i,t+1}}{\partial \log D^H_{i,t+1}}|_{D^H_{i,t+1}=1}, \ i = k,.$$

As we explain above (see Lemma 10 and the discussion thereafter), $\alpha^I_{i,t}$ can be viewed as a proxy for intermediary leverage.

**Proposition 16** The absolute size of the basis, $|\text{Basis}^s_{i,t}|$, is proportional to $W^{*I,H}_{i,t}$. Furthermore, if $D^H_{i,t,t+1}, \ i = k,,$ have small variance and the monetary policy is given by (25), then the Basis$^s_{i,t}$ is monotone increasing in and is proportional to

- $\alpha^N_{i,t} - \alpha^N_{k,t}$ if $D^H_{k,t,t+1} - D^H_{i,t,t+1} = C^I_{k,0} - C^I_{i,0} = 0$;
- $\frac{\alpha_{i,t+1} - \alpha^N_{i,t}}{\alpha^I_{i,t}} (C^I_{k,0} - C^I_{i,0})$ if $\alpha_{i,t} - \alpha_{k,t} = D^H_{k,t,t+1} - D^H_{i,t,t+1} = 0$. 

40
In our model, *CIP deviations arise because of the price pressure created by the demand of customers in country* \(i\) *for exposure to the US dollar.* This price pressure is in turn determined by the level and the risk properties of the exchange rate and the ability of US dollar to serve as a hedge against times when the customers need cash the most. These properties depend on the behaviour of the ratio of marginal utilities of intermediaries. The latter, in turn, depend on the intermediary wealth relative to that of customers. There are two factors that drive this relative wealth: the quotient of discount factors, \(W_{i,t+1}^{*,I/H} \), and the relative shadow cost of intermediation,

\[
(\lambda_{i,t}(S_{i,t+1}^*/S_{i,t}^*) + \mu_{i,t})/(S_{i,t+1}^*/S_{i,t}^*) = \lambda_{i,t} + \mu_{i,t}(D_{i,t,t+1}^H)^{\alpha_{i,t}^N} - 1.
\]

The first important result that we prove in the Appendix is: the relative shadow cost, \(\lambda_{i,t} + \mu_{i,t}(D_{i,t,t+1}^H)^{\alpha_{i,t}^N} - 1\), is monotone decreasing in \(\alpha_{i,t}^N\) when \(D_{i,t,t+1}^H\) is close to one. That is, *stronger stabilization policies facilitate rent extraction* by lowering the shadow cost of intermediation because such policies stimulate risk taking by customers, increasing their demand for levered bets in the D2C market. When US pursues an aggressive monetary policy, this pushes down US intermediaries shadow costs, leading to a drop in their marginal utility of wealth, making US dollar depreciate. As a result, country \(i\) customers find it attractive to borrow in US dollar, and intermediaries exploit this by offering this a dollar rate \(r_{H,i,t}^H\) above the spot rate \(r_{S,t}^S\); hence, a positive basis emerges.\(^{43}\) While arbitrage forces (price pressures) are naturally present in our model, they are limited by the effective market incompleteness that prevents Japanese investors and intermediaries from fully arbitraging the currency basis away. Naturally, these effects become stronger when intermediaries have

\(^{42}\)Indeed, in the frictionless limit, customer wealth coincides with the aggregate stock market wealth.

\(^{43}\)See, for example, Cieslak et al. (2016) and Cieslak and Vissing-Jorgensen (2017) who provide evidence for a strong “Fed Put”: the tendency of the Fed to ease monetary policy in response to low stock market returns. As we explain above, stabilization policies may involve both counter-cyclical monetary policies and actual currency stabilization policies in the sense of Reinhart and Rogoff (2004) and Hassan et al. (2016).
higher (domestic) market power (proportional to their net worth) and when monetary policy divergence, as measured by the difference $\alpha_{t,s} - \alpha_{t,k}$, is stronger.

The result in the second item of Proposition 16 is more subtle. It shows how the sign of the basis is determined by the interaction between intermediary leverage and the aggressiveness of stabilization policies. The intuition is as follows. As above, the attractiveness of US bonds is determined by the exchange rate properties, which, in turn, are determined by the dynamics of intermediary wealth. When $\alpha_{i,t}^I$ is high, the effect from the change in the demand for the local currency originating from $W_{i,t+1}$ is stronger than that originating from intermediation markups; as a result, a positive basis emerges. Consistent with this narrative, the cross-currency basis post-crisis has been positive for currencies of countries with a very large (relative to the domestic wealth) intermediation sector, such as Japanese Yen (JPY) and the Swiss Franc (CHF) (see, for example, Du et al. (2016), Avdjiev et al. (2016), Borio et al. (2016), and Rime et al. (2017)). At the same time in countries with a smaller intermediation sector markups may also lead to a negative basis. A typical example may be the Australian Dollar, for which the basis post-crisis has often been negative.

6 International Trade and Monetary Transmission

Having investigated the autarky economy, we now proceed with studying the economy with trade. Writing down market clearing equations (13), we arrive at the following system of equilibrium equations:

\[
\begin{align*}
\Omega_{k,t,\tau}(M_{I,t,\tau})^{1/2} + \Psi_{k,t,\tau}(M_{I,t,\tau})^{-1} &= \mathcal{M}_{k,\tau} \mathcal{E}_{k,\tau}, \quad k > 1 \\
\Omega_{S,t,\tau}(M_{I,t,\tau})^{1/2} + \Psi_{S,t,\tau}(M_{I,t,\tau})^{-1} &= \mathcal{M}_{S,\tau}.
\end{align*}
\]
where we have defined

\[
\bar{\Lambda}^H_{k,t,\tau} \equiv \sum_i (E_{i,t}C^H_{i,t}) \Lambda_{i,t,\tau}(E_{i,t+t/\tau}/E_{i,t})^{1/2} \theta_{i,k,\tau}
\]

\[
\bar{\Psi}^J_{k,t,\tau} \equiv \sum_i E_{i,t}C^I_{i,t} \Psi_{i,t,\tau} \theta_{i,k,\tau}, \ J = H, I.
\]

Here, \(\Psi^I_{k,t,\tau}\) and \(\Lambda^H_{k,t,\tau}\) are the effective time discount factors of intermediaries and customers respectively, weighted with their preference for country-\(k\) goods. The aggregate consumption of the corresponding agents’ class is proportional to the corresponding discount factor, and is inversely related to the D2D stochastic discount factor. Solving second equation in (26) for \(M^I_{S,t,\tau}\), and then substituting in the first equation, we arrive at the following result.

**Theorem 17** *In the economy with intermediation, we have*

\[
M^I_{S,t,\tau} = \left( \frac{\bar{\Lambda}^H_{S,t,\tau} + \left( \bar{\Lambda}^H_{S,t,\tau} \right)^2 + 4\bar{\Psi}^I_{S,t,\tau}M_{S,\tau}}{2M_{S,\tau}} \right)^{1/2}
\]

*and*

\[
\mathcal{M}_{k,\tau} E_{k,\tau} = \mathcal{M}_{S,\tau} \frac{\bar{\Psi}^I_{k,t,\tau}}{\bar{\Psi}^I_{S,t,\tau}} + \bar{\Psi}^I_{k,t,\tau} \left( \frac{\bar{\Lambda}^H_{S,t,\tau}}{\bar{\Psi}^I_{S,t,\tau}} - \frac{\bar{\Lambda}^H_{S,t,\tau}}{\bar{\Psi}^I_{S,t,\tau}} \right) (M^I_{t,\tau})^{-1/2}
\]

Formulas (27) and (28) show how the global equilibrium pricing kernel as well as exchange rates are directly linked to the international shadow cost of financial intermediation, weighted by customers’ dollar net worth. The wealthier country \(i\) is, the larger its impact on global financial markets. Furthermore, the size of this impact is determined by the net worth (or, equivalently, the risk bearing capacity) of the intermediation sector, as captured by \(\bar{\Psi}^I_{S,t,\tau}, \bar{\Psi}^I_{i,t,\tau}\). When this net worth is large, the economy converges to the frictionless limit of Proposition 21.
In general, the interaction of these effects may be quite complex, as it depends in a non-trivial way on the distribution of preference parameters $\theta_{i,k,t}$ across countries. To isolate the domestic demand/supply effects from those of global demand and supply, we will assume that consumption demand exhibits a single factor structure, so that

$$\theta_{i,k,t} = \bar{\theta}_k \beta_i + (1 - \beta_i) \delta_{i,k},$$

where $\delta_{i,k}$ is the Kronecker delta, and $\sum_{k=1}^{N} \bar{\theta}_k = 1$. Here, $1 - \beta_i$ measures the degree of consumption home bias in country $i$, while $\bar{\theta}_k$ reflects the global demand for country $k$ goods.

In order to maintain analytical tractability, we will follow the approach of Itskhoki and Mukhin (2017) and study the behaviour of prices and exchange rates in the limit of a substantial home bias, corresponding to the case when $\beta_i$ is small for all $i$.\footnote{As Itskhoki and Mukhin (2017) argue, many countries exhibit significant home bias in consumption. We have also solved the opposite limit of vanishing home bias and most of our results qualitatively hold in this environment. We therefore expect that our results are robust to the degree of the home bias.}

Recall that absent intermediation frictions and international trade, exchange rates are given by

$$\frac{E^{*}_{j,t}}{E^{*}_{j,0}} = M^{-1}_{j,t} \hat{E}^{*}_{j,t},$$

where we have defined

$$\hat{E}_{j,t} = M_{s,t} \frac{\Psi^{H}_{j,0,t}}{\Psi_{s,0,t}}.$$

We will also define

$$\hat{\bar{E}}_{j,t} \equiv \frac{E_{t} \left[ \sum_{\tau=1}^{T} \hat{E}_{j,\tau} M_{s,\tau}^{I,s} \right]}{S_{s,t}}, \quad j = 1, \cdots, N,$$
to be the (normalized) present value of future exchange rates.

Recall that \(1 - \beta_j\) is the degree of country \(j\) home bias, while \(\beta_j\) captures the demand for foreign goods by country \(j\) customers. We can therefore define the trade-weighted US Dollar index as follows:

\[
Dollar_t \equiv - \sum_j \beta_j M_{j,t} E_{j,t}^*.
\]

This factor captures the total consumption power of foreign customers in purchasing US goods. We will also define

\[
\hat{Dollar}_t \equiv E_t \left[ \sum_{\tau=t}^{T} Dollar_{\tau} M_{i,t,\tau} \right] / S_{i,t}^*.
\]

The numerator of \(\hat{Dollar}_t\) is the value of a portfolio of Dollar forwards with maturities \(\tau = t, \cdots, T\). Hence, \(\hat{Dollar}_t\) is the forward value or the dollar relative to the value of the US stock market. Since the frictionless exchange rates, \(E_{i,t}^*\) are given by ratio of discount factors, \(\hat{Dollar}_t\) effectively equals the present discounted value of future interest rate differentials between US and the rest of the world. We also define

\[
Dollar_{t,t+1} \equiv \frac{Dollar_{t+1}}{Dollar_t}, \quad \hat{Dollar}_{t,t+1} \equiv \frac{\hat{Dollar}_{t+1}}{\hat{Dollar}_t}.
\]

Recall formulas (18) and (21). The following is true.
Theorem 18  The country $i$ D2D pricing kernel is given by

$$
M_{i,t,t+1}^I \approx N_{i,t+1}^{-1} \Psi_{i,t,t+1}^H
\times \left(1 + \left(\Delta C_{i,t,t+1}^{I/H^*} + \Delta W_{i,t,t+1}^{I/H^*} + (\lambda_{i,t} - 1) + \mu_{i,t}N_{i,t+1}D_{i,t,t+1}^H\right)^{-1}\right)
+ \bar{\theta}_i \left(-\frac{\text{Dollar}_t}{\bar{\varepsilon}_{i,t}} \left(\frac{\text{Dollar}_{i,t,t+1}}{\bar{\varepsilon}_{i,t,t+1}} - 1\right) - \frac{\text{Dollar}_t}{\bar{\varepsilon}_{i,t}} \left(\frac{\text{Dollar}_{i,t,t+1}}{\bar{\varepsilon}_{i,t,t+1}} - 1\right)\right)
$$

while exchange rates are given by

$$
\frac{\varepsilon_{i,t+1}}{\varepsilon_{i,t}} = \frac{M_{i,t,t+1}^I}{M_{i,t,t+1}^I}.
$$

In our model, international transmission of shocks is determined exclusively by trade in real goods. As a result, the domestic pricing kernel depends crucially the (trade weighted) exchange rate index against the local currency. In particular, with the US dollar as the reference currency, the US dollar pricing kernel depends on the global dollar index. When this dollar index depreciates (i.e., dollar weakens), global demand for US goods goes up, while the US domestic consumption decreases.$^{45}$ As a result, insurance against dollar depreciation states becomes valuable, and the US dollar pricing kernel loads negatively on the dollar index. The fact that Dollar is a priced factor is supported by the empirical evidence. Verdelhan (2017) finds that US dollar is an important risk factor explaining a significant fraction of the cross-section of currency returns.$^{46}$ Theorem 18 shows that our model also generates a multi-factor model for the US dollar pricing kernel $M_{i,t,t+1}^I$, with key risk factors given by the local intermediaries’ discount factors and net worth relative to that of the local customers, as well as two dollar factors. The first dollar factor is just a trade-weighted dollar index; the second one accounts for the future expected discounted value of this dollar index. The

$^{45}$See also Gourinchas et al. (2017) who show that the US dollar index is negatively related to global trade when US Dollar is an invoicing currency.

$^{46}$See also Brusa et al. (2014).
latter is a net worth factor that arises purely due to intermediation markups because the
magnitude of these markups is determined by customers’ net worth, which, in turn, depends
on the anticipated strength of their currencies. Investigating the relationship between our
model-implied factors and other, classical risk factors in international finance (such as, e.g.,
the carry trade) is an important topic for future research.

It is well known (see, for example, Froot and Ramadorai (2005)) that exchange rates
are naturally related to the value of future interest rate differentials. However, in the
frictionless model this relationship is dominated by the instantaneous interest rate differential
(the “pure” UIP). Theorem 18 shows that intermediation frictions do lead to a non-trivial
relationship between exchange rates the present discounted value of future interest rate
differentials between US and the rest of the world, as captured by $\hat{\text{Dollar}}_t$.\footnote{Note also
that the appearance of $\hat{\text{Dollar}}_t$ in the exchange rates implies that exchange rates may forecast
fundamentals in our model, consistent with the findings of Engel and West (n.d.).} However, this
relationship is non-linear and may be time-varying, depending on the intermediation capacity.
In particular, shocks to this capacity will generate endogenous flows between customers and
intermediaries, changing the dynamics of exchange rates, in agreement with the findings of
Froot and Ramadorai (2005).

We complete this section with a discussion of the role of the US dollar factor for CIP
deviations. Absent international trade, Proposition 16 shows how such deviations arise solely
due to monetary policies divergence across countries because these policies determine the
risk profiles of exchange rates. In the presence of trade, these risk profiles are in turn closely
linked to the Dollar factor through equations (29)-(30). As a result, the Dollar factor also
influences the dynamics and the cross-section of currency basis. The following is true.

**Proposition 19** Suppose that countries $i$ and US have identical preferences and identical
monetary policies, so that $\Psi_{i,t}^J = \Psi_{\$t}^J$, $J = H, I$ and $N_{i,t+1} = N_{\$t+1}$. Then,

- The absolute size of the basis is proportional to $|\hat{\text{Dollar}}_t|$.

47
• If $\widehat{\text{Dollar}}_t$ is expected to appreciate between $t$ and $t+1$, then the sign of $\text{Basis}^\$_{i,t}$ coincides with that of $\theta_i - \theta_s$;

• If $\widehat{\text{Dollar}}_t$ is expected to depreciate between $t$ and $t+1$, then the sign of $\text{Basis}^\$_{i,t}$ coincides with that of $\theta_i - \theta_s$.

The result of Proposition 19 is a pure “flight-to-safety” phenomenon that can be explained as follows. The $\widehat{\text{Dollar}}_t$ factor is a net worth effect that arises due to intermediation markups. The parameter $\theta_i$ measures the sensitivity of the global demand for country $-i$ goods to the global economic conditions. That is, it captures how strongly country $i$ is integrated in the world economy. Under the hypothesis of Proposition 19, we have $\Psi^H_{i,t} = \Psi^H_{\$t}$ and hence the frictionless part of exchange rate equals one. Suppose that the global demand for country $-i$ goods is more sensitive than that for US goods, so that $\theta_i > \theta_s$. Suppose that country $i$ customers anticipate that the Dollar index is expected to depreciate against the rest of the world, meaning that the global demand for both US goods and country $i$ goods will rise. This leads to an expected drop in country $i$ customers’ purchasing power. Since $\theta_i > \theta_s$, country-$i$ customers consider dollar assets as a hedge against the loss of purchasing power because they suffer less from this global demand effect. Thus, they contact intermediaries and use US dollar as an insurance against future drop in purchasing power. This creates a price pressure in the D2C market, which intermediaries exploit by charging higher markups, giving rise to a positive basis. While the forces underlying the emergence of the basis are purely macroeconomic and trade-driven, those forces affect exchange rates through intermediaries’ balance sheet: A stronger dollar redistributes wealth across intermediaries in different countries, depending on their dollar exposures.

Interestingly enough, conditional on the fact that the basis is consistently positive, the result of Proposition 19 is consistent with recent empirical findings of Avdjiev et al. (2016), who find that the cross-currency basis exhibits a positive correlation with a US Dollar.

\footnote{Rest of the world here means all countries except $i$ and $\$.}
index. However, these effects compete with the domestic redistributive effects described in Proposition 16. Since the size of the latter effects is proportional to the intermediation capacity, the channels described in Proposition 16 dominate those from Proposition 19 when intermediation capacity is small. This result suggests that the recent strengthening of the link between the basis and the dollar may be partially driven by the post-crisis drop in intermediation capacity.

We complete this section with a discussion of the international transmission of monetary shocks. The following is true.

**Proposition 20 (Monetary Spillovers and Intermediation Capacity)** The absolute size of the impact of monetary shocks to a country \( k \) on the amount of trade and the exchange rate between countries \( i, j \neq k \) is proportional to the relative intermediation capacity \( C_{k,0}/(C_{i,0} + C_{j,0}) \).

The result of Proposition 20 stands in stark contrast with the classical Mundellian Trilemma: In the presence of intermediation frictions, exchange rates are not able to perfectly absorb foreign monetary policy shocks and these shocks have an impact on the real side of the economy. Importantly, consistent with Shin (2017), Proposition 20 shows how countries with a large intermediation sector (such as, e.g., US) emergence endogenously as a driver of the global matrix of monetary transmission.

## 7 Conclusions

We introduce an imperfectly competitive intermediation sector into a standard, international monetary model a-la Lucas (1982). We show that one simple friction, whereby intermediaries exploit their market power and charge endogenous markups for providing customers access to foreign securities, is able to generate a rich behaviour of risk premia and exchange rates. The simple intermediation friction helps account for some of the major anomalies
in foreign exchange and international capital markets including the safe haven properties of exchange rates, and the breakdown of covered interest parity (CIP). Crash risk and the shock absorption role of exchange rates are amplified whenever intermediation capacity drops. Our model shows explicitly how the nature of stabilization policies (including internal monetary policies and foreign exchange interventions) generate cross-sectional heterogeneity in a currency’s risk profile. Moreover, our model endogenously generates a multi-factor pricing kernel, with the US dollar being an important priced risk factor.
A Additional Material

A.1 Frictionless Economy

In this section, we solve for the equilibrium in the special case when there are no interme-
diation frictions and customers can freely trade with each other. This analysis serves as an
important benchmark for the analysis in the main text. In this case, market completeness
implies that all local nominal pricing pricing kernels are linked through the state-by-state
relationship with the US dollar pricing kernel:

\[ M_{i,0,t}^H = M_{s,0,t}^H \mathcal{E}_{i,t} / \mathcal{E}_{i,0}. \]

Furthermore, local nominal pricing kernels are determined by the cash-in-advance constraint,

\[ \sum_i C_{i,0}^H \Psi_{i,t}^H (M_{i,0,t}^H)^{-1} \Theta_{i,k,t} \mathcal{E}_{i,t} = \mathcal{M}_{k,t} \mathcal{E}_{k,t}. \]

so that

\[ M_{k,0,t}^H = (\mathcal{M}_{k,t})^{-1} \Theta_{k,t}^H, \quad (31) \]

while the exchange rates are then given by

\[ \mathcal{E}_{k,t} = \frac{M_{k,0,t}^H}{M_{s,0,t}^H} = \frac{\mathcal{M}_{s,t}}{\mathcal{M}_{k,t}} \frac{\Theta_{k,t}^H}{\Theta_{s,t}^H}, \quad (32) \]

where we have defined

\[ \Theta_{k,t}^J = \sum_i \mathcal{E}_{i,0} C_{i,0}^J \Psi_{i,t}^J \Theta_{i,k,t}, \quad k = 1, \ldots, N, \quad J = I, H \]

to be the international wealth-weighted discount factor for goods of country \( k \).
Money is super-neutral\textsuperscript{49} in the frictionless economy, and both goods prices and nominal stock prices are proportional to money supply. The fact the money super-neutrality holds in frictionless cash-in-advance economies is well known: Money simply serves as a numeraire, and has no impact on real asset prices. Similar arguments concern the other phenomena: Exchange rates exhibit a trivial behaviour and simply reflect preferences for local goods, with the parameters $\theta_{k,t}$ being the primitive drivers of exchange rates dynamics. Furthermore, exchange rates perfectly perform their role of shock absorbers: Flexible exchange rates and capital flows guarantee monetary policy independence, as in Obstfeld and Taylor (2004) and in complete agreement with the Mundellian trilemma.

These simplistic features of the benchmark frictionless model are very useful for the analysis of the model with intermediation frictions: Indeed, they immediately imply that any interesting dynamic properties of prices and exchange rates are due solely to the intermediation frictions. We summarize these observations in the following proposition.

**Proposition 21 (Frictionless economy)** The following is true in a frictionless economy in which customers can freely trade all securities with each other:

(1) Money is super-neutral: nominal pricing kernels \textsuperscript{(31)} are inversely proportional to money supply, while nominal prices of real goods as well as stock prices are proportional to money supply:

\[
P_{i,k,t} = \frac{M_{i,t}}{X_{k,t}} \frac{\Theta_{k,t}^H}{\Theta_{i,t}^H} \\
S_{i,t} = M_{i,t} E_t \left[ \sum_{\tau=t}^T \frac{\Theta_{i,\pi}^H}{\Theta_{i,t}^H} \right]
\]

In particular, domestic inflation, stock prices, and the domestic pricing kernel are independent of foreign monetary policy shocks.

\textsuperscript{49}Money is said to be super-neutral when neither the current money supply nor the expectations about the future monetary policy have any impact on real (inflation-adjusted) asset prices.
(3) Exchange rates are given by (32).

The following corollary summarizes basic properties of exchange rates in the frictionless economy.

**Corollary 22** In a frictionless economy,

- The exchange rate $E_{i,t}$ always scales inversely with relative money supply. In particular, if country $i$ expands the monetary base more than the US, then its currency always depreciates relative to US dollar.

- Expectations about future monetary policy (forward guidance) have no impact on exchange rates: They only depend on current money supply.

- Monetary shocks outside of US and country $i$ have no impact on $E_{i,t}$.

**A.2 The disconnect of exchange rates and consumption**

As in Gabaix and Maggiori (2015), in our model intermediaries are marginal investors in the international financial markets, and hence exchange rates are determined by their marginal utilities which can be quite different from those of households. Specifically, we have

$$M^I_{i,t,t+1} = \Psi^I_{i,t,t+1}(C^I_{i,t+1}/C^I_{i,t})^{-1} \neq \Psi^H_{i,t,t+1}(C^H_{i,t+1}/C^H_{i,t})^{-1},$$

and hence

$$E_{i,t+1}/E_{i,t} = \frac{M^I_{i,t,t+1}}{M^I_{\$,t,t+1}} = \frac{\Psi^I_{i,t,t+1}(C^I_{i,t+1}/C^I_{i,t})^{-1}}{\Psi^I_{\$,t,t+1}(C^I_{\$,t+1}/C^I_{\$,t})^{-1}} \neq \frac{\Psi^H_{i,t,t+1}(C^H_{i,t+1}/C^H_{i,t})^{-1}}{\Psi^H_{\$,t,t+1}(C^H_{\$,t+1}/C^H_{\$,t})^{-1}}.$$

Thus, our model is naturally able to generate deviations from the one-to-one relationship between exchange rates and consumption, known as the Backus and Smith (1993) puzzle.
Consider a simplified setup in which two countries, \( i \) and \( j \), have identical discount factors \( \Psi_{i,t}^H = \Psi_{j,t}^H \) and hence their only differences stem from monetary policies. By the cash in advance constraint, aggregate nominal consumption \( C_{i,t} = C_{i,t}^I + C_{i,t}^H \) coincides with the money supply, and hence \( C_{i,t+1}/C_{i,t} = N_{i,t+1} \). As a result, in the frictionless model, the correlation of exchange rates in (34) with relative consumption growth equals one, in stark contrast with the empirical evidence where this correlation is almost always negative (see, e.g., Backus and Kehoe (1992)). Here, we note that our model is also able to generate zero or negative correlation. For example, if the countries have identical monetary policies, so that \( N_{i,t+1} = N_{j,t+1} \), then \( (C_{i,t+1}/C_{i,t})/(C_{j,t+1}/C_{j,t}) = 1 \) and hence its correlation with exchange rates is zero. At the same time, if intermediaries in the two countries are different, then exchange rates will exhibit non-trivial dynamics, unrelated to relative consumption.

### A.3 Intermediation Capacity and Safe Haven Properties of Exchange Rates

As above, we will assume that \( \Psi_{i,t}^H = \Psi_{j,t}^H \), and we will assume that monetary policies are also homogeneous across countries, so that

\[
N_{i,t+1} = (D_{i,t+1}^H)^{-\alpha^N},
\]

where the parameter \( \alpha^N \) measures the strength of the stabilization policies.

As we explain above (see Assumption 3 and Lemma 10), the quotient \( \Psi_{i,t+1}^I/\Psi_{i,t+1}^H \) captures the degree of intermediary leverage: When this quotient is increasing, intermediaries’ consumption (which is proportional to \( \Psi_{i,t+1}^I \)) responds more than one-to-one to economic shocks. We will make the following assumption.

**Assumption 4** Assumption 3 holds, so that \( \Psi_{i,t}^J = \prod_{\tau=1}^t f_{i,\tau}^J(\omega_{i,\tau}), \ J = I, H \) for some monotone functions \( f_{i,\tau}^J \) of the Markov state \( \omega \), and intermediary leverage is asymmetric.
there exists a (sufficiently low) threshold $\omega_*$ such that

$$\frac{\partial \log(f_I(\omega_{i,t+1})/f_H(\omega_{i,t+1}))}{\partial \omega_{i,t+1}} = \begin{cases} 
\alpha^L_i, \; \omega_{i,t+1} < \omega_* \\
\alpha^H_i, \; \omega_{i,t+1} > \omega_* 
\end{cases}$$

with $\alpha^L_i < \alpha^H_i$.

Under Assumption 4, intermediary relative net worth, $W^{*,I/H}_{i,t+1}$ will also exhibit an asymmetry, strongly increasing in $\omega_{i,t+1}$ on the upside, when $\omega_{i,t+1}$ is large, while increasing is at a slower pace when $\omega_{i,t+1}$ is low. While we do not micro-found such asymmetries, there is strong evidence for their presence in real world, with the most important one originating from implicit government guarantees, such that the financial sector is “protected” in a case of global systemic crisis. As Kelly et al. (2016) show, such guarantees were strongly anticipated and priced in the cross-section of options financial sector stocks. We interpret $\partial \log W^{*,I}_{i,t+1}/\partial \log W^{*,H}_{i,t+1}$ as the financial intermediaries’ beta with respect to the stock market. While these betas are often above one for positive stock market returns, there is evidence for a large degree of asymmetry, especially at times of a crisis. See, for example, Miranda-Agrippino and Rey (2015). Our goal here is to understand the link between safe haven properties of exchange rates and differences between intermediation sectors across countries. Since the random variable $D^H_{t,t+1}$ represents a measure of a global sentiment, a drop in this measure corresponds to a global stock market crash. The next proposition studies the behavior of exchange rates during such a crash.

**Proposition 23 (Safe haven currencies and the strength of interventions)** Suppose that $\alpha^L_k$ is sufficiently small. Then, US Dollar appreciates during large drops in the global sentiment, $D^H_{t,t+1}$, if at least one of the following is true:\footnote{Note that, while we do focus on the US in our discussion of safe haven currencies, this discussion equally applies to other safe haven currencies such as Japanese Yen and the Swiss Franc.}
- **US intermediaries have a larger capacity**: $C_{s,0}^I > C_{i,0}^I$;

- **US intermediaries have a higher leverage**: $\alpha^H_s > \alpha^H_i$.

To understand the intuition behind Proposition 23, we need to go back to formula (22) and understand the impact of different terms in (22) on the behaviour of exchange rates. Suppose first that the only difference between the two countries is in the size (capacity) of the intermediation sectors: $C_{s,0}^I > C_{i,0}^I$. In this case, by Proposition 8, given that the countries only differ in the intermediation capacities, there exist $\lambda^*_t, \mu^*_t$ such that $\lambda_{i,t} \approx C_{i,0}^I \lambda^*_t, \mu_{i,t} \approx C_{i,0}^I \mu^*_t$, and hence formula (22) takes the form

$$\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{s,t}} \approx 1 + (C_{i,0}^I - C_{s,0}^I) \left( (\Delta(\Psi^I_{0,t+1}/\Psi^H_{0,t+1}) + \Delta((\Psi^I_{0,t+1}D^I_{0,t+1})/(\Psi^H_{0,t+1}D^H_{0,t+1})) \\
+ \lambda^*_t + \mu^*_t N^{-1}_t(D^H_{i,t+1})^{-1} \right).$$

(33)

In the presence of intermediary leverage, the first two terms on the right-hand side of (33) work in the intuitive direction. A country with a larger intermediation sector naturally suffers more in a global downturn because intermediaries’ discount factor matters more for domestic risk sharing and domestic pricing kernel: a drop in that discount factor means that money is less valuable for intermediaries, and, hence leads to a currency depreciation. This effect is similar to the “reserve currency paradox” highlighted in Maggiori (2013). He shows that a larger intermediation sector implies that US effectively acts as an insurer for the rest of the world against global economic downturns, forcing the US dollar to depreciate in bad times. However, as Proposition 23 shows, intermediation markups are able to flip the sign of this relationship.

As most of the novel effects in our model, the impact of intermediation markups on the US dollar safe haven property operates via a redistribution channel. Specifically, a drop in $D^H_{0,t+1}$ leads a redistribution of wealth between local customers and local intermediaries. By
Proposition 11, when the stabilization policy is mild, we have $\mu^*_t > 0$ and customers end up in a “risk-off” regime, whereby intermediaries effectively sell insurance to customers against states with poor stock returns. When those adverse states are realized, intermediaries pay the promised insurance to customers. In turn, their net worth drops, and their marginal utility spikes, leading to a USD appreciation. A similar logic applies when $\alpha^N$ is above one: in this case, $\mu^*_t < 0$ by Proposition 11, but $N^{-1}_{s+1}(D^H_{s,s+1})^1$ is monotone increasing in $D^H_{s,s+1}$. Thus, $\mu^*_t N^{-1}_{s+1}(D^H_{s,s+1})^1$ is always monotone decreasing in $D^H_{s,s+1}$.

As a result, US dollar behaves as a safe haven currency if and only if the first (discount factor) effect is weaker than the second (wealth) effect. The latter is guaranteed by the technical condition of low $\alpha^L_S = \alpha^L_i$. Indeed, when $\alpha^L_S = \alpha^L_i$ is sufficiently low, both $\Psi^I_{0,t}/\Psi^H_{0,t}$ and $(\Psi^I_{0,t} D^I_{0,t})/(\Psi^H_{0,t} D^H_{0,t})$ are insensitive to $D^H_{i,s,s+1}$ when the latter is small.

Suppose now that $C^I_{t,0} = C^I_{t,0}$, but that US monetary policy tends to stabilise more than elsewhere $\alpha^H_S > \alpha^H_i$. The term $(\Delta C^s_{i,t,\tau} - \Delta C^s_{i,t,\tau}) + (\Delta W^s_{i,t,\tau} - \Delta W^s_{i,t,\tau})$ is either monotone increasing in $\alpha^L_S > \alpha^L_i$, or is decreasing, but at a rate of at most $\alpha^L_S$. Thus, when $\alpha^L_S$ is sufficiently small, US Dollar appreciates at times of an economic downturns if the intermediation markups term, $(D^H_{i,t+1})^{-1} N^{-1}_{s,t+1}(\mu_{i,t} - \mu_{s,t})$ is monotone increasing in $D^H_{i,t+1}$. This is indeed the case when $\alpha^H_S > \alpha^H_i$. Summarizing, we conclude that, if the US is characterised by a large, leveraged intermediation sector, there are large gains from risk sharing between customers and intermediaries, allowing the latter to charge larger markups. At the same time, these markups become more sensitive to stock market downturns, implying that US intermediaries’ marginal utility is more sensitive to global stock returns. As a result, in a global stock market crash, this marginal utility spikes relative to that of a country with a smaller intermediation sector, leading to a US dollar appreciation. We believe that this markup channel, while not fully resolving Maggiori’s reserve currency paradox, offers a promising way of looking at this problem.

We complete this subsection with a discussion of the link between the strength of stabi-
lization policies and the response of exchange rates to global stock market shocks, $D_{t,t+1}^H$. In the frictionless model, exchange rates are given by (32) and hence

$$\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \frac{N_{i,t+1}}{N_{i,t+1}} = (D_{i,t+1}^H)\alpha_i^N - \alpha_s^N. \quad (34)$$

Since exchange rates scale inversely with the money supply in a frictionless model because money is neutral, a country $i$ whose monetary policy responds aggressively to global shocks (so that $\alpha_i^N$ is large and positive) will have its currency depreciating in times of a crisis simply because it increases the money supply too much relative to the rest of the world. In the presence of intermediation frictions, Propositions 14 and 23 show that this relationship may revert for nominal rates, while Proposition 12 illustrates a similar effect for real exchange rates. In particular, if US pursues an aggressive monetary policy, US dollar may still preserve its safe haven status if US monetary policy is less noisy and/or US intermediaries have a larger capacity and are more leveraged. Our results thus offer a different perspective on the role of global safe asset providers in the international monetary system, emphasized in Farhi and Maggiori (2017). As Gourinchas et al. (2010) argue, a country willing to preserve the globally safe status of its currency may need to face the “exorbitant duty” of accommodating the demand for its currency in global crisis states. While the two properties (accommodating demand for currency and keeping the safe haven status) are always inconsistent in a frictionless model (see (34)), they can be consistent in the presence of intermediation frictions in the country has a sufficiently developed intermediation sector.

\[51\] In the real world, safe haven properties of exchange rates depend crucially on local institutions and political systems that determine the functioning of local markets. While our model is too stylized to microfound these important features, it show how the interaction between the intermediation sector and local policies lies at the heart of risk properties of exchange rates.
A.4 Crash risk

As we explained above, the state-contingent intermediation markups (11) represent the cost of insurance in the D2C market segment: when this cost is high, customers reduce their consumption in those states, driving down the value of the local currency. This is evident from formula (15): Exchange rates are monotone increasing in the shadow cost $\Lambda_{i,t}$, while the intermediation markups $\Gamma_{i,t}$ in (11) are monotone decreasing in this shadow cost.

This fact has an important link with the empirical regularity known as the negative currency skew: That is, the fact that, for many currencies, implied volatilities for out of the money put options tend to be higher than those for out of the money calls (see, e.g., Farhi et al. (2015) and Chernov et al. (2017)), implying that the costs of insurance against currency depreciation are high relative to those for currency appreciation. Indeed, in our model, states with a low shadow costs $\Lambda_{i,t}$ are costly to insure against, and correspond to states with depressed exchange rates.

Thus, customers that have the desire to buy insurance against currency depreciation states using out of the money put options in the D2C markets will observe highly skewed quotes. Furthermore, the size of this effect depends on the ability of intermediaries to serve as shock-absorbers: When the intermediation capacity $C_{i,0}$ is large, the term in (15) with $\Psi_{i,t}$ dominates and intermediaries protect the currency from a crash. As we explain above, $\Lambda_{i,0,t}$ can also be interpreted as the effective (endogenous) discount factor of customers, generated by the frictions in the D2C market segment. Then, equilibrium in our model is equivalent to that in a frictionless market with customers having relative risk aversion of two and discounting future using $\Lambda_{i,0,t}$. In this interpretation, the importance of $\Lambda_{i,0,t}$ for exchange rates is clear: when $\Lambda_{i,0,t}$ drops, customers do not care about consumption in those states, and currency value also drops. This may happen even if the fundamental discount factor $\Psi_{i,0,t}$ is high in those states so that, absent intermediation frictions, currency value would be high. While the fundamental discount factor $\Psi_{i,0,t}$ can be assumed to be well behaved,
this is not the case for the endogenous discount factor $\Lambda_{i,0,t}$. In fact, by (12), we have that 
$\Lambda_{i,t,t+1} = (\Psi_{i,t,t+1}^H/D_{i,t,t+1}^H)^{1/2}(\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})^{1/2}$, and hence $\Lambda_{i,0,t}$ can fall all the way to zero if $\mu_{i,t} < 0$. This is what we call a currency crash. Recalling that a negative $\mu_{i,t}$ corresponds to a risk-on scenario whereby customers increase their risky asset positions, we arrive at the following result.

**Corollary 24** Suppose that time $t$ expectations lead customers into a risk-on regime. Then, a large enough drop in the stock market at time $t+1$ always leads to a currency crash.

Corollary 24 highlights an important boom and bust feature of currency crashes in the model. A “boom” that leads to a build up of optimistic expectations and drives customers into a “risk-on” regime leads to an endogenous build-up of risk in intermediaries’ balance sheets. In such episodes, strong drops of asset prices go hand in hand with currency crashes. This finding suggests that it may make sense to differentiate between “good” and “bad crashes”: A good crash (e.g., like the one following a dot com bubble) hits only customers, but has no systemic implications; a bad crash hits intermediaries and therefore comes with “systemic” implications.

**B  Proofs**

**Proof of Lemma 3.** The customer rationally anticipates that he will be consuming as in formula (7): given the time $t+1$ wealth $W_{i,t+1}$, the agent will consume according to

$$C_{i,t+\tau} = \frac{W_{i,t+1}}{D_{i,t+1}} \Psi_{i,t+1,t+\tau}^H M_{t+1,t+\tau}^{-1}, \quad \tau \in [1, \cdots, T-t].$$

Therefore, the agent’s future value function is given by

$$U_{t+1}(W_{i,t+1}) = E_{t+1} \left[ \sum_{\tau=1}^{T-t} \Psi_{i,t+1,t+\tau}^H \log C_{i,t+\tau} \right] = D_{i,t+1} \log W_{i,t+1} + \text{Const}_{i,t+1}.$$
Thus, the optimization problem of the customer as a function of the quoted pricing kernel \( M_{H,t,t+1} \) takes the form

\[
U_{i,t}(W_{i,t}, M_{H,t,t+1}) = \max_{W_{i,t+1}} \left( \log(W_{i,t} - E_t[M_{H,t,t+1}W_{i,t+1}]) + E_t[\Psi^H_{i,t,t+1}U_{i,t+1}(W_{i,t+1})] \right)
\]

and the first order condition implies

\[
C_{i,t}^{-1} M_{H,t,t+1} = \Psi^H_{i,t,t+1} D_{i,t+1} W_{i,t+1}^{-1}
\]

and hence

\[
W_{i,t+1} = \Psi^H_{i,t,t+1} D_{i,t+1} C_{i,t} M_{H,t,t+1}^{-1} = \Psi^H_{i,t,t+1} D_{i,t+1} W_{i,t} D_{i,t}^{-1} M_{H,t,t+1}^{-1}.
\]

Q.E.D.

In the autarky case, we just need to solve for the D2C and D2D pricing kernels in a given country. For this reason, to simply the notation, we will omit the country index everywhere in this appendix.

**Proof of Proposition 4.** Suppose first that \( \mu_t > 0 \). Define \( \lambda_t = \lambda_t / \mu_t \). Then, we need to solve the system

\[
\begin{aligned}
E_t \left[ \frac{(\Psi_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{\left(\lambda_t(S_{\tilde{t}+1}/S_t) + 1\right)^{1/2} \mu_t^{1/2}} \right] &= E_t[M_{I,t,t+1}]; \\
E_t \left[ \frac{(\Psi_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S^*_t}{\left(\lambda_t(S_{\tilde{t}+1}/S_t) + 1\right)^{1/2} \mu_t^{1/2}} \right] &= E_t[M_{I,t,t+1} S^*_t].
\end{aligned}
\]

The first equation gives \( \mu_t^{1/2} = E_t \left[ \frac{(\Psi_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{\left(\lambda_t(S_{\tilde{t}+1}/S_t) + 1\right)^{1/2}} \right] / E_t[M_{I,t,t+1}] \), and, substituting
into the second equation, we get

$$E_t \left[ \frac{(\tilde{\Psi}_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}^{1/2}}{\lambda_t (S_{t+1} / S_t) + 1} \right] = E_t[M_{I,t,t+1} S_{t+1}]$$ (36)

By direct calculation, the left-hand side of (36) is monotone decreasing in $\lambda_t$. When $\lambda_t = 0$, the left-hand side of (36) becomes

$$E_t \left[ \frac{(\tilde{\Psi}_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}^{1/2}}{(S_{t+1} / S_t)^{1/2}} \right]$$

When $\lambda_t$ converges to $+\infty$, the left-hand side of (36) converges to

$$E_t \left[ \frac{(\tilde{\Psi}_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}^{1/2}}{(\tilde{\Psi}_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2}} \right]$$

while when $\lambda_t$ converges to its minimal possible negative value, the left-hand side of (36) converges to $\max(S_{t+1}) E_t[M_{I,t,t+1}]$. Thus, we need to consider three scenarios: if $\max(S_{t+1}) > E_t[M_{I,t,t+1} S_{t+1}] > E_t[M_{I,t,t+1}] / E_t[(\tilde{\Psi}_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2}]$ then there exists $\lambda_t < 0$ satisfying (35). This is equivalent to

$$\tilde{E}_t[S_{t+1}] > E_t \left[ \frac{(\tilde{\Psi}_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}^{1/2}}{(\tilde{\Psi}_{H,t,t+1} \tilde{D}_{H,t,t+1})^{1/2} M_{I,t,t+1}^{1/2}} \right]$$
If however

\[
E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right] > E_t \frac{E_t [M_{I,t,t+1} S_{t+1}]}{E_t [M_{I,t,t+1}]} \]

then there exists a unique positive \( \hat{\lambda}_t \). Finally, if

\[
E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right] \frac{1}{E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right]} > E_t \frac{E_t [M_{I,t,t+1} S_{t+1}]}{E_t [M_{I,t,t+1}]} ,
\]

then \( \mu_t \) needs to be negative. In this case, slightly abusing the notation, we will use \( \mu_t \) to denote \( -\mu_t \), so that we can rewrite the system as

\[
E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right] = E_t [M_{I,t,t+1}];
\]

\[
E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right] = E_t [M_{I,t,t+1} S_{t+1}],
\]

and we need to show that there is a unique positive solution \( \hat{\lambda}_t \) to

\[
E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right] \frac{1}{E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right]} = E_t \frac{E_t [M_{I,t,t+1} S_{t+1}]}{E_t [M_{I,t,t+1}]} .
\]

When \( \hat{\lambda}_t \to +\infty \), the left-hand side converges to

\[
E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right] \frac{1}{E_t \left[ \left( \frac{\Psi_{H,t,t+1} \mathcal{D}_{H,t,t+1}}{(S_{t+1}/S_t)^{1/2}} \right)^{1/2} M_{I,t,t+1}^{1/2} \right]} ,
\]

63
while it converges to \( \min S_{t+1} \) when \( \hat{\lambda}_t \to S_t / \min S_{t+1} \), and hence there is always a positive solution \( \hat{\lambda}_t \). Q.E.D.

**Proof of Proposition 9.** We have

\[
M_{H,0,t} = \Psi_t^H \mathcal{M}_t^{-1} + C_0 \Psi_t^I \mathcal{M}_t^{-1} M_{H,0,t} \mathcal{M}_t^{-1}
\]

\[
\approx \Psi_t^H \mathcal{M}_t^{-1} + C_0 \Psi_t^I (M_{I,0,t}^*)^{-1} M_{H,0,t}^*(1 + M_{H,0,t}^{(1)} - M_{I,0,t}^{(1)}) \mathcal{M}_t^{-1}
\]

\[
= \Psi_t^H \mathcal{M}_t^{-1} + C_0 \Psi_t^\mathcal{M}_t^{-1}(1 + C_0 (M_{H,0,t}^{(1)} - M_{I,0,t}^{(1)})),
\]

where we have used the fact that \( M_{I,0,t}^* = M_{H,0,t}^* \). We thus get

\[
M_{I,t,\tau} \approx M_{H,t,\tau} \left( 1 + C_0 \left( 2(\Psi_{I,\tau} \Psi_t^{-1} \Psi_t^I (\Psi_t^{-1} - \Psi_H t)) + \sum_{s=t}^{\tau-1} \hat{\lambda}_s + (Z_{\tau} - Z_t) + \sum_{s=t}^{\tau-1} \hat{\mu}_s N_{s+1} \bar{D}_{H,s,s+1} \right) \right),
\]

and the expression for exchange rates follows from because they are given by the quotient of pricing kernels. Q.E.D.

**Proof of Proposition 14.** Since all expressions are homogeneous of degree zero in \( E_t[N_{i,t+1}^{-1}] \), we can impose the normalization \( E_t[N_{i,t+1}^{-1}] = 1 \). Under the independence assumption and the homogeneity of discount factors assumption, we have

\[
\mu_{i,t} = \frac{E_t[N_{i,t+1}^{-1} \Psi_{i,t+1}^H] E_t[W_{i,t+1}^H D_{i,t+1}^H] - E_t[N_{i,t+1}^{-1} \Psi_{i,t+1}^H W_{i,t+1}^H] E_t[D_{i,t+1}^H \Psi_{i,t+1}^H]}{E_t[\Psi_{i,t+1}^H D_{i,t+1}^H] E_t[\Psi_{i,t+1}^H (D_{i,t+1}^H)^{-1} N_{i,t+1}^{-2}] - (E[\Psi_{i,t+1}^H N_{i,t+1}^{-1}])^2}
\]

\[
= \frac{E_t[\Psi_{i,t+1}^H] E_t[W_{i,t+1}^H D_{i,t+1}^H] - E_t[\Psi_{i,t+1}^H W_{i,t+1}^H] E_t[D_{i,t+1}^H \Psi_{i,t+1}^H]}{E_t[N_{i,t+1}^{-2}] E_t[\Psi_{i,t+1}^H D_{i,t+1}^H] E_t[\Psi_{i,t+1}^H (D_{i,t+1}^H)^{-1}] - (E[\Psi_{i,t+1}^H])^2}
\]

\[
= \frac{\alpha}{\vartheta_t[N_{i,t+1}^{-1}] \beta + \gamma}
\]

for some constants \( \alpha, \beta, \gamma > 0 \) that are independent of the country identity. At the same
time,

\[ \lambda_{i,t} \approx 1 + W_{i,t}^{*,1/H} - \frac{\text{Cov}_t(S_{i,t+1}^*, W_{i,t+1}^{*,1/H}, 1/S_{i,t+1}^*)}{\text{Cov}_t(S_{i,t+1}^*, 1/S_{i,t+1}^*)} \]

\[ = 1 + W_{i,t}^{*,1/H} \]

\[ + \frac{E_t[N_{i,t+1}^{-1}] E_t[W_{i,t+1}^{*,1/H} N_{i,t+1}^{-1}] - E_t[D_{i,t+1} H_{i,t+1}] E_t[W_{i,t+1}^{*,1/H}]}{E_t[H_{i,t+1} D_{i,t+1}] E_t[H_{i,t+1} D_{i,t+1}^{-1} N_{i,t+1}^{-2}] - (E[H_{i,t+1} N_{i,t+1}^{-1}])^2} \]

\[ \approx 1 + W_{i,t}^{*,1/H} \delta - \eta \text{Var}_t[N_{i,t+1}^{-1}] + \text{Var}_t[N_{i,t+1}^{-1}] \beta + \gamma \]

where we have defined

\[ \delta = E_t[H_{i,t+1} D_{i,t+1}] E_t[W_{i,t+1}^{*,1/H} H_{i,t+1}] - E_t[D_{i,t+1} H_{i,t+1}] E_t[W_{i,t+1}^{*,1/H}] \]

\[ \eta = E_t[D_{i,t+1} H_{i,t+1}] E_t[W_{i,t+1}^{*,1/H}] - E_t[H_{i,t+1} D_{i,t+1}] E_t[W_{i,t+1}^{*,1/H} D_{i,t+1}^{-1}] \]

\[ \beta = E_t[H_{i,t+1} D_{i,t+1}] E_t[H_{i,t+1} D_{i,t+1}^{-1}] \]

\[ \gamma = E_t[H_{i,t+1} D_{i,t+1}] E_t[H_{i,t+1} D_{i,t+1}^{-1}] - (E[H_{i,t+1}])^2 \]

\[ \alpha = E_t[H_{i,t+1}] E_t[W_{i,t+1}^{*,1/H} D_{i,t+1} H_{i,t+1}^{-1}] - E_t[H_{i,t+1} W_{i,t+1}^{*,1/H}] E_t[D_{i,t+1} H_{i,t+1}] \]

The claim follows. Q.E.D.
Proof of Proposition 16. We have

\[-e^{-\psi_{s,t}} + e^{-\psi_{s,t}} = -E_t[M^{H,H}_{i,t,t+1}(\mathcal{E}_{i,t}/\mathcal{E}_{i,t+1})] + E_t[M^{H,H}_{s,t,t+1}]

= E_t \left[ N^{-1}_{s,t+1} \psi^{H}_{s,t+1} (1 + \Delta C^{s,I/H}_{s,t}) \right]

- E_t \left[ N^{-1}_{i,t+1} \psi^{H}_{i,t+1} (1 + \Delta C^{s,I/H}_{i,t}) \frac{N^{-1}_{s,t+1} \psi^{H}_{s,t+1}}{N^{-1}_{i,t+1} \psi^{H}_{i,t+1}} \right]

\left(1 - \left(\Delta C^{s,I/H}_{i,t,t+1} - \Delta C^{s,I/H}_{s,t,t+1}\right) + \left(\Delta W^{s,I/H}_{i,t,t+1} - \Delta W^{s,I/H}_{s,t,t+1}\right)

+ \left((\lambda_{i,t} - \lambda_{s,t}) + (\mu_{i,t} N^{-1}_{i,t+1}(D^{H}_{i,t,t+1})^{-1} - \mu_{s,t} N^{-1}_{s,t+1}(D^{H}_{s,t,t+1})^{-1}))\right)\right]\]

\approx \ E_t \left[ N^{-1}_{s,t+1} \psi^{H}_{s,t+1} (1 + \Delta C^{s,I/H}_{s,t}) \right]

- E_t \left[ N^{-1}_{i,t+1} \psi^{H}_{i,t+1} \right]

\left(1 - \left(-\Delta C^{s,I/H}_{s,t,t+1}\right) + \left(\Delta W^{s,I/H}_{i,t,t+1} - \Delta W^{s,I/H}_{s,t,t+1}\right)

+ \left((\lambda_{i,t} - \lambda_{s,t}) + (\mu_{i,t} N^{-1}_{i,t+1}(D^{H}_{i,t,t+1})^{-1} - \mu_{s,t} N^{-1}_{s,t+1}(D^{H}_{s,t,t+1})^{-1}))\right)\right]\]

= E_t \left[ N^{-1}_{s,t+1} \psi^{H}_{s,t+1} \left(\Delta W^{s,I/H}_{i,t,t+1} - \Delta W^{s,I/H}_{s,t,t+1}\right)

+ \left((\lambda_{i,t} - \lambda_{s,t}) + (\mu_{i,t} N^{-1}_{i,t+1}(D^{H}_{i,t,t+1})^{-1} - \mu_{s,t} N^{-1}_{s,t+1}(D^{H}_{s,t,t+1})^{-1}))\right)\right]\]

Suppose first that there is no noise in monetary policy. Using the approximation

\[E[X] = E[e^{\log X}] \approx e^{E[\log X] + 0.5\text{Var}[\log X]} \approx e^{E[\log X]}(1 + 0.5\text{Var}[\log X])\]
that holds in the limit of small variance, we get

\[
\mu_{i,t} = \frac{E_t[N_{i,t+1}^{-1} \Psi_{i,t+1}^H] E_t[W_{i,t+1}^{*,I/H} D_{i,t+1}^H \Psi_{i,t+1}^H] - E_t[N_{i,t+1}^{-1} \Psi_{i,t}^H W_{i,t+1}^{*,I/H}] E_t[D_{i,t+1}^H \Psi_{i,t+1}^H]}{E_t[\Psi_{i,t+1}^H D_{i,t+1}^H] E_t[\Psi_{i,t+1}^H (D_{i,t+1}^H)^{-1} N_{i,t+1}^2] - (E[\Psi_{i,t+1}^H N_{i,t+1}^2])^2}
\]

\[
= W_{i,t}^{*,I/H} \frac{E_t[e^{\psi+\alpha^N_i d}] E_t[e^{\psi+\alpha^l_i d+\psi}] - E_t[e^{\alpha^N_i d+\psi+\alpha^l_i d}] E_t[e^{\psi+\psi}]}{E_t[e^{\psi+d}] E_t[e^{\psi+2\alpha^N_i d}] - E_t[e^{\psi+\alpha^N_i d}]^2}
\]

\[
= W_{i,t}^{*,I/H} \frac{\text{Var}_t[\psi + \alpha^N_i d] + \text{Var}_t[(1 + \alpha^l_i) d + \psi] - \text{Var}_t[(\alpha^l_i + \alpha^l_i) d + \psi] - \text{Var}_t[d + \psi]}{\text{Var}_t[\psi+d] + \text{Var}_t[\psi+(\alpha^N_i - 1) d] - 2 \text{Var}_t[\psi + \alpha^N_i d]}
\]

\[
= W_{i,t}^{*,I/H} \frac{(\alpha^N_i)^2 + (1 + \alpha^l_i)^2 - (\alpha^N_i + \alpha^l_i)^2 - 1}{1 + (2\alpha^N_i - 1)^2 - 2(\alpha^N_i)^2} = W_{i,t}^{*,I/H} \frac{\alpha^l_i}{1 - \alpha^N_i}
\]

Similarly,

\[
\tilde{\text{Cov}}_t(S_{i,t+1}^*, W_{i,t+1}^{*,I/H}, 1/S_{i,t+1}^*)
\]

\[
= W_{i,t}^{*,I/H} \frac{\text{Var}_t[\psi + \alpha^N_i d] + \text{Var}_t[(\alpha^N_i + \alpha^l_i) d + \psi] - \text{Var}_t[(1 + \alpha^l_i) d + \psi] - \text{Var}_t[(2\alpha^N_i - 1) d + \psi]}{\text{Var}_t[\psi+d] + \text{Var}_t[\psi+(\alpha^N_i - 1) d] - 2 \text{Var}_t[\psi + \alpha^N_i d]}
\]

\[
= W_{i,t}^{*,I/H} \frac{(\alpha^N_i)^2 + (\alpha^N_i + \alpha^l_i)^2 - (1 + \alpha^l_i)^2 - (2\alpha^N_i - 1)^2}{2(\alpha^N_i - 1)^2} = W_{i,t}^{*,I/H} \frac{\alpha^l_i - \alpha^N_i + 1}{\alpha^N_i - 1}
\]

In the first three cases of the Proposition, the basis is given by

\[
-e^{-s_{H,t}} + e^{-r_{s,t}}
\]

\[
\approx E_t \left[ N_{s,t+1}^{-1/2} \Psi_{s,t+1}^H \left( (\Delta W_{i,t+1}^{*,I/H} - \Delta W_{s,t+1}^{*,I/H}) + W_{s,t}^{*,I/H} \frac{\alpha^l_i}{\alpha^N_i - 1} (1 - (D_{s,t+1}^H)_{\alpha^N_i - 1} - W_{s,t}^{*,I/H} \frac{\alpha^l_i}{\alpha^N_i - 1}) \right) \right].
\]
Thus, we get
\[
-e^{-r^{H,i}_{s,t}} + e^{-r^{s,t}} 
\approx
\begin{array}{c}
E_t \left[ \mathcal{N}^{-1}_{s,t+1} \Psi^H_{s,t,t+1} \left( W^{*,I/H}_{i,t} \alpha^l (F(\alpha^N_t, D^H_{t,t+1}) - F(\alpha^N, D^H)) \right) \right].
\end{array}
\]

with \( F(\alpha, x) = (1 - x^{\alpha - 1}) / (\alpha - 1) \) and the claim follows because \( F \) is monotone decreasing in \( \alpha \) for \( x \) close to one.\(^{52}\) In the second case, we get
\[
-e^{-r^{H,i}_{s,t}} + e^{-r^{s,t}} 
\approx
\begin{array}{c}
E_t \left[ \mathcal{N}^{-1}_{s,t+1} \Psi^H_{s,t,t+1} \right. \\
\left. \left( \Delta W^{*,I/H}_{i,t,t+1} - \Delta W^{*,I/H}_{s,t,t+1} \right) \right. \\
\left. + \left( W^{*,I/H}_{i,t} \frac{\alpha^l}{\alpha^N - 1} (1 - (D^H_{t,t+1})^{\alpha^N - 1}) - W^{*,I/H}_{s,t} \frac{\alpha^l}{\alpha^N - 1} (1 - (D^H_{t,t+1})^{\alpha^N - 1}) \right) \right].
\end{array}
\]

Furthermore,
\[
(\Delta W^{*,I/H}_{i,t,t+1} - \Delta W^{*,I/H}_{s,t,t+1}) \\
+ \left( W^{*,I/H}_{i,t} \frac{\alpha^l}{\alpha^N - 1} (1 - (D^H_{t,t+1})^{\alpha^N - 1}) - W^{*,I/H}_{s,t} \frac{\alpha^l}{\alpha^N - 1} (1 - (D^H_{t,t+1})^{\alpha^N - 1}) \right) \\
= W^{*,I/H}_{i,t} (C^l_{i,0} - C^l_{s,0}) \left( (D^H_{t,t+1})^{\alpha^l - 1} - 1 + \frac{\alpha^l}{\alpha^N - 1} (1 - (D^H_{t,t+1})^{\alpha^N - 1}) \right) \\
= \alpha^l W^{*,I/H}_{i,t} (C^l_{i,0} - C^l_{s,0}) \left( \frac{(D^H_{t,t+1})^{\alpha^l - 1} - 1}{\alpha^l} - \frac{1}{\alpha^N - 1} ((D^H_{t,t+1})^{\alpha^N - 1} - 1) \right)
\]

and the claim follows because the function \((x^{\alpha - 1})/\alpha\) is monotone increasing in \( \alpha \) for \( x \) close to one. Q.E.D.

\(^{52}\)Since \( D^H_{i,t,t+1} \) has a low variance, it is close to one with a probability close to one.
B.1 International Trade

Proof of Proposition 18. We can rewrite market clearing for dollars as

\[
((1 - \beta_k) C_{k,0}^H \Psi_{k,0,t}^H (M_{k,0,t}^H)^{-1} + (1 - \beta_k) C_{k,0}^I \Psi_{k,0,t}^I (M_{k,0,t}^I)^{-1}) \mathcal{E}_{k,t}
\]

Thus,

\[
M_{k,0,t}^H = (\mathcal{E}_{k,t} \mathcal{M}_{k,t})^{-1} M_{k,0,t}^H \left( ((1 - \beta_k) C_{k,0}^H \Psi_{k,0,t}^H (M_{k,0,t}^H)^{-1} + (1 - \beta_k) C_{k,0}^I \Psi_{k,0,t}^I (M_{k,0,t}^I)^{-1}) \mathcal{E}_{k,t} \right)
\]

\[
+ \bar{\theta}_k \sum_j \beta_j \left( C_{j,0}^H \Psi_{j,0,t}^H (M_{j,0,t}^H)^{-1} + C_{j,0}^I \Psi_{j,0,t}^I (M_{j,0,t}^I)^{-1} \right) \mathcal{E}_{j,t} = \mathcal{E}_{k,t} \mathcal{M}_{k,t}.
\]
where we have used that $\mathcal{E}_{j,t}^{(1)} - \mathcal{E}_{k,t}^{(1)} = M_{j,0,t}^{I,(1)} - M_{k,0,t}^{I,(1)}$. Thus, we now need to compute the first order expansions, which is equivalent to finding equilibrium derivatives with respect to $\beta$ and $C_0^I$ parameters. While this has already been done for $C_0^I$, we need to do it now for the $\beta$ parameters. So, let us set $C_0^I = 0$, and suppose that $\beta$'s are small. Then, equilibrium equations take the form

\[
M_{k,0,t}^{H} \approx M_{k,t}^{-1}(1 - \beta_k) C_{k,0}^{H} \Psi_{k,0,t}^{H} + M_{k,t}^{-1}(\mathcal{E}_{k,t}^{*})^{-1} \left( \bar{\theta}_k \sum_j \beta_j C_{j,0}^{H} \Psi_{j,0,t}^{H} \frac{M_{k,0,t}^{H,*}}{M_{j,0,t}^{H,*}} \mathcal{E}_{j,t}^{*} \right)
\]

\[
= M_{k,t}^{-1}(1 - \beta_k) C_{k,0}^{H} \Psi_{k,0,t}^{H} + M_{k,t}^{-1} \bar{\theta}_k \sum_j \beta_j C_{j,0}^{H} \frac{\mathcal{E}_{j,0}^{*} \Psi_{k,0,t}^{H}}{\mathcal{E}_{j,0}^{*} \Psi_{j,0,t}^{H}}
\]

\[
= \mathcal{E}_{k,0}^{-1} M_{k,t}^{-1}(\bar{\theta}_k \Psi_{0,t}^{H} + (1 - \beta_k) C_{k,0}^{H} \Psi_{k,0,t}^{H})
\]

Thus, using the results from the proof of Proposition 9, we get

\[
M_{k,t,t+1}^{H} \approx N_{k,t+1}^{-1} \Psi_{k,t,t+1}^{H} (1 + M_{k,t,t+1}^{(1)})
\]

with

\[
M_{k,t,t+1}^{H,(1)} = \Delta C_{k,t,t+1}^{I/H,*} + \bar{\theta}_k (\tilde{\Psi}_{t+1}^{H} / \Psi_{k,0,t+1}^{H} - \tilde{\Psi}_{t}^{H} / \Psi_{k,0,t}^{H}) = \Delta C_{k,t,t+1}^{I/H,*} + \bar{\theta}_k \Delta C_{t,t+1}^{H/k,*},
\]

where we have defined the global customers' discount factor,

\[
\tilde{\Psi}_{t}^{H} \equiv \sum_j \beta_j \mathcal{E}_{j,0}^{*} \Psi_{j,0,t}^{H}.
\]
Note that we have $Dollar_t = -\frac{\Psi_t^H}{\Psi_{k,0,t}^H}$. Furthermore,

\[
M_{k,t,t+1}^I = (M_{k,t,t+1}^H)^2 (\Psi_{k,t,t+1}^H D_{k,t,t+1}^H)^{-1} (\lambda_{k,t}(S_{k,t+1}/S_{k,t}) + \mu_{k,t})
\]

\[
\approx \mathcal{N}_{k,t+1}^{-2} (1 + 2M_{k,t,t+1}^H) \Psi_{k,t,t+1}^H (D_{k,t,t+1}^H)^{-1} (\lambda_{k,t}(S_{k,t+1}/S_{k,t}) + \mu_{k,t}).
\]

Thus,

\[
S_{k,t} = E_t \left[ \sum_{\tau=t}^T M_{k,t,\tau}^H \mathcal{M}_{k,\tau} \right] = \mathcal{M}_{k,t} \left( D_{k,t}^H \right)
\]

\[
+ E_t \left[ \sum_{\tau=t}^T \Psi_{k,t,\tau}^H \Delta C_{k,t,\tau} + \Psi_{k,t,\tau}^H \tilde{\theta}_k (\tilde{\Psi}_{k,\tau}^H/\Psi_{k,\tau}^H - \tilde{\Psi}_t^H/\Psi_{k,t}^H) \right]
\]

\[
= \mathcal{M}_{k,t} \left( D_{k,t}^H + (\Psi_{k,t}^H/\Psi_{k,t}^H)(D_{k,t}^I - D_{k,t}^H) + \tilde{\theta}_k \sum_j \beta_j C_{j,0}^H \mathcal{E}_{j,0} (\Psi_{k,t}^H/\Psi_{k,t}^H)(D_{j,t}^H - D_{k,t}^H) \right)
\]

\[
= \mathcal{M}_{k,t} D_{k,t}^H (1 + (W_{k,t}^{I/H,*} - (\Psi_{k,t}^I/\Psi_{k,t}^H)) + \tilde{\theta}_k (\tilde{W}_t^{H/k,*} - (\tilde{\Psi}_t^H/\Psi_{k,t}^H)) ),
\]

where we have defined

\[
\tilde{W}_t^{H/k,*} \equiv \sum_j \beta_j C_{j,0}^H \mathcal{E}_{j,0} \Psi_{j,0,t}^H D_{j,t}^H
\]

and

\[
\tilde{W}_t^{H/k,*} \equiv \frac{\tilde{W}_t^H}{\Psi_{k,0,t}^H D_{k,t}^H}
\]

Importantly, we have

\[
\overline{Dollar_t} = \frac{E_t \left[ \sum_{\tau=t}^T Dollar_{\tau} \Psi_{k,\tau,t} \right]}{E_t \left[ \sum_{\tau=t}^T \Psi_{k,\tau,t} \right]} = - \frac{E_t \left[ \sum_{\tau=t}^T \Psi_{k,0,\tau}^H \Psi_{k,\tau,t} \right]}{E_t \left[ \sum_{\tau=t}^T \Psi_{k,\tau,t} \right]} = - \frac{\tilde{W}_t^H}{\Psi_{k,0,t}^H D_{k,t}^H}.
\]

71
Thus,

$$\bar{W}^{H/k,*}_t = \frac{\text{Dollar}_t}{\mathcal{E}_{k,t}}.$$

Hence,

$$M^{I}_{k,t,t+1} \approx \mathcal{N}_{k,t+1}^{-1} \Psi_{k,t,t+1}^H$$

$$\times \left(1 + 2M^{H,(1)}_{k,t,t+1} + \Delta W^{I/H,*}_{k,t,t+1} - \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} - \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D^{H}_{k,t,t+1})^{-1} \right)$$

$$= \mathcal{N}_{k,t+1}^{-1} \Psi_{k,t,t+1}^H$$

$$\times \left(1 + \Delta C^{I/H,*}_{k,t,t+1} + \bar{\theta}_k \Delta \bar{C}^{H/k,*}_{t,t+1} + \Delta W^{I/H,*}_{k,t,t+1} + \bar{\theta}_k \Delta \bar{W}^{H/k,*}_{t,t+1} + \lambda^{(1)}_{k,t} + \mu_{k,t} (\mathcal{N}_{k,t+1} D^{H}_{k,t,t+1})^{-1} \right)$$

and hence

$$\mathcal{E}_{k,t,t+1} = \mathcal{E}_{k,t,t+1}^*(1 + \mathcal{E}^{(1)}_{k,t,t+1})$$

with

$$\mathcal{E}^{(1)}_{k,t,t+1} = M^{I,(1)}_{k,t,t+1} - M^{I,(1)}_{S,t,t+1}.$$

Finally, Lagrange multipliers are determined by the system

$$E_t[M^{I}_{k,t,t+1}] = E_t[M^{H}_{k,t,t+1}], \quad E_t[M^{I}_{k,t,t+1}(S_{k,t+1}/S_{k,t})] = E_t[M^{H}_{k,t,t+1}(S_{k,t+1}/S_{k,t})].$$

Q.E.D.
Proof of Proposition 19. In complete analogy with Proposition 16, we get

\[-e^{-r^H_{i,t}} + e^{-r^s,t} \approx \mathbb{E}_t \left[ \mathcal{N}^{-1}_{s,t+1} \Psi^H_{s,t,t+1} \left( (\Delta W^{*,I/H}_{i,t,t+1} - \Delta W^{*,J/H}_{s,t,t+1}) + \theta_i \Delta W^{H/*,i,t} - \theta_s \Delta W^{H/s,*} \\
+ \left( (\lambda_i,t - \lambda_{s,t}) + (\mu_i,t \mathcal{N}^{-1}_{i,t,t+1}(D^H_{i,t,t+1}) - \mu_{s,t} \mathcal{N}^{-1}_{s,t,t+1}(D^H_{s,t,t+1})^{-1}) \right) \right] \]

and the claim follows. Q.E.D.

References


