# TECHNICAL APPENDIX
# (AVAILABLE SEPARATELY FROM AUTHORS)

## "Accounting for Primary and Secondary Demand Effects with Aggregate Data"

**Part (A):** ***Inversion of demand function, MSM estimation and Monte Carlo study***

**Part (B):** ***Other studies (for reviewer responses)***

# PART (A)

## A1. Inversion of the demand function

In this section of the appendix, we show that our proposed procedure (equation 12) for inverting the expected demand function (equation 11) to recover $\delta$ is a valid contraction mapping. That is, iteratively solving (12) converges to a vector $\delta$ that uniquely reconciles $\widetilde{Q}_{jst}$ in equation (11) with the average quantity per customer in the data, $q_{jst}$. Once we recover the vector of $\delta$-s implied by (12), we are able to control for the potential endogeneity of prices using instrumental variables, and estimate all parameters by the method of simulated moments.

The expected demand function (equations 10/11) implied by the model is:

$$\begin{aligned}\widetilde{Q}_{jst}(\delta;\Theta) &= \int -\frac{y_{st}}{\alpha_s p_{jst}} \Pr\left(D_{ijst}=1 \middle| I_{ist}=1\right) \ln\left[P\left(I_{ist}=0\right)\right]\phi(\Lambda)\partial\Lambda \\ &= \int \frac{y_{st}}{\alpha_s p_{jst}} \frac{e^{V_{ijst}}}{\sum_{k=1}^{J} e^{V_{ikst}}} \ln\left[1+\sum_{k=1}^{J} e^{V_{ikst}}\right]\phi(\Lambda)\partial\Lambda \\ &= \int \frac{y_{st}}{\alpha_s p_{jst}} \frac{e^{\delta_{jst}+\Omega_{ijst}}}{\sum_{k=1}^{J} e^{\delta_{kst}+\Omega_{ikst}}} \ln\left[1+\sum_{k=1}^{J} e^{\delta_{kst}+\Omega_{ikst}}\right]dF(\Omega)\end{aligned} \qquad \text{(A0)}$$

The iterative function $g(.):\mathbb{R}^J \rightarrow \mathbb{R}^J$ (equation 12) is defined as:

$$g(\delta) = \delta + \ln(q) - \ln\left[\widetilde{Q}(\delta)\right] \qquad \text{(A1)}$$

We show that our iterative procedure $g(.)$ is a valid contraction mapping by proving that it satisfies the conditions described in appendix 1 of BLP (1995). The subscripts *s* for 'store' and *t* for 'week' are dropped for clarity. The main conditions to prove are:

a) $g(.)$ is continuous in $\delta$

b) $\frac{\partial g_j(\delta)}{\partial \delta_r} \geq 0 \;\; \forall\, r,j$

c) $\sum_{r=1}^{J} \frac{\partial g_i(\delta)}{\partial \delta_r} < 1$

The function is continuous by construction. We prove (b) in two parts. First, we show that $\frac{\partial g_j(\delta)}{\partial \delta_j} \geq 0$, and then that $\frac{\partial g_j(\delta)}{\partial \delta_r} \geq 0, \forall r \neq j$.

(b.1) To show that $\frac{\partial g_j(\delta)}{\partial \delta_j} \geq 0$, note that $\frac{\partial}{\partial \delta_j}\left(e^{\delta_j+\Omega_{ij}} \Big/ \sum_{k=1}^{J} e^{\delta_k+\Omega_{ik}}\right) = \Pr\left(C_i=j \middle| I_i=1\right)$ $*\left[1-\Pr\left(C_i=j \middle| I_i=1\right)\right]$, and that, $\frac{\partial}{\partial \delta_j}\left(\ln\left[1+\sum_{k=1}^{J} e^{\delta_k+\Omega_{ik}}\right]\right) = \Pr\left(C_i=j, I_i=1\right)$. Also note that at a given guess of the parameter vector, $\alpha$ is known and is fixed. Hence, $\frac{\partial g_j(\delta)}{\partial \delta_j}$

$$= 1 - \frac{1}{\widetilde{Q}_j(.)} \int \frac{y}{\alpha p_j} \Pr\left(C_i=j \middle| I_i=1\right)\left[\Pr\left(C_i=j, I_i=1\right) - \ln\left\{\Pr\left(I_i=0\right)\right\}\left\{1-\Pr\left(C_i=j \middle| I_i=1\right)\right\}\right]dF(\Omega) \qquad \text{(A2)}$$

Comparing the numerator of the second term above to line 1 of equation (A0), we can see that for $\frac{\partial g_j(\delta)}{\partial \delta_j} \geq 0$, it is equivalent to prove that:

$$\left[\Pr(C_i = j, I_i = 1) - \ln\{\Pr(I_i = 0)\}\{1 - \Pr(C_i = j | I_i = 1)\}\right] \leq -\ln\{\Pr(I_i = 0)\}$$

which is equivalent to showing that $\Pr(C_i = j | I_i = 1)\left[1 - \Pr(I_i = 0) + \ln\{\Pr(I_i = 0)\}\right] \leq 0$. This holds since $\Pr(I_i = 0) \in [0,1]$. *QED*.

(b.2) To show that $\frac{\partial g_j(\delta)}{\partial \delta_r} \geq 0$, $r \neq j$, note first that $\frac{\partial}{\partial \delta_r}\left( e^{\delta_j + \Omega_{ij}} \Big/ \sum_{k=1}^{J} e^{\delta_k + \Omega_{ik}} \right)$
$= -\Pr(C_i = j | I_i = 1)\Pr(C_i = r | I_i = 1)$. Hence,

$$\begin{aligned}\frac{\partial g_j(\delta)}{\partial \delta_r} &= -\frac{1}{\widetilde{Q}_j(.)}\int \frac{y}{\alpha p_j}\Pr(C_i = j | I_i = 1)\Pr(C_i = r | I_i = 1)\left[\Pr(I_i = 1) + \ln\{\Pr(I_i = 0)\}\right] dF(\Omega) \\ &= \frac{1}{\widetilde{Q}_j(.)}\int \frac{y}{\alpha p_j}\Pr(C_i = j | I_i = 1)\Pr(C_i = r | I_i = 1)\underbrace{\left(-\ln\left[\{1 - \Pr(I_i = 1)\}e^{\Pr(I_i = 1)}\right]\right)}_{\geq 0} dF(\Omega) \qquad \text{(A3)} \\ &\geq 0\end{aligned}$$

*QED*.

(c) To show that the sum of the derivatives is less that 1, note that:

$$\sum_{r=1}^{J} \frac{\partial g_i(\delta)}{\partial \delta_r} = 1 - \frac{1}{\widetilde{Q}_j(.)}\int \frac{y}{\alpha p_j}\Pr(C_i = j | I_i = 1) f_i dF(\Omega) \qquad \text{(A4)}$$

where,

$$f_i = \left[\Pr(C_i = j, I_i = 1) - \ln\{\Pr(I_i = 0)\}\{1 - \Pr(C_i = j | I_i = 1)\} + \{\Pr(I_i = 1) + \ln(\Pr(I_i = 0))\}\sum_{r \neq j}\Pr(C_i = r | I_i = 1)\right]$$

Comparing the numerator of the second term in (A4) to line 1 of equation (A0), we can see that for $\sum_{r=1}^{J} \frac{\partial g_i(\delta)}{\partial \delta_r} < 1$, it is equivalent to prove that: $f_i < -\ln\{\Pr(I_i = 0)\}$. This reduces to showing that $\Pr(I_i = 1)\left[\Pr(I_i = 1) + \ln\{1 - \Pr(I_i = 1)\}\right] < 0$. This holds since $\Pr(I_i = 1) \in [0,1]$. *QED*.

Hence, $g(.)$ is a valid contraction mapping, and therefore, iteratively solving (A1) will converge to a unique vector $\delta$.

## A2. Method of Simulated Moments procedure

In the above section, we have shown how, for a *given* guess of the parameters $\Theta$, we can recover the unique vector $\delta_{jst}(\Theta)$ that exactly solves (11). With $\delta_{jst}(\Theta)$ in hand, we now set up the method of simulated moments (MSM) procedure to estimate all the parameters. We outline the technical details of GMM below, referring the interested reader to Hansen (1982) for a formal discussion. First, note that $\delta_{jst}(\Theta) = X_{jst}B - \alpha_s \ln(p_{jst}) + \xi_{jst}$; hence, given $\delta_{jst}(\Theta)$, we can recover the structural error vector, $\xi_{jst}(\Theta) = \delta_{jst}(\Theta) - X_{jst}B + \alpha_s \ln(p_{jst})$ across all store-weeks as a function of parameters, $\Theta$. We use moments implied by $\xi_{jst}(\Theta) = \delta_{jst}(\Theta) - X_{jst}B + \alpha_s \ln(p_{jst})$ to construct orthogonality conditions. Parameters are estimated by making the sample analogue of the orthogonality conditions as close to zero as possible. Technically, we can construct orthogonality conditions using any set of covariates, $Z_{st}$, that are mean-independent of $\xi_{jst}$. The concern for endogeneity arises since prices are expected to be correlated with $\xi_{jst}(\Theta)$. To resolve this, we construct moment conditions using a vector of instruments that includes the exogenous brand variables as well as wholesale prices that are assumed to be independent of $\xi_{jst}$. Formally, we use the conditional mean-independence assumptions, $E\left[Z_{st}'\xi_{st} \mid Z_{st}\right] = 0$. We define the sample analogue of the moment vector:

$$\widehat{M}(\Theta) = Z'\hat{\xi}(\Theta) \tag{A5}$$

Let $\hat{W} = \left(E\left[\hat{M}(\Theta)\hat{M}(\Theta)'\right]\right)^{-1}$ be an estimate of the asymptotically optimal weighting matrix.[1] Given these moment conditions, we can now estimate all parameters by minimizing the GMM objective function $M(\Theta)'\hat{W}M(\Theta)$. The asymptotic properties of our estimates are based on the Method of Simulated Moments (Pakes & Pollard 1989).

## A3. Monte Carlo simulation study

The objective of the simulation study is to investigate the ability of the proposed model to recover the true model parameters from (simulated) aggregate data formed by aggregating over the choices of an underlying population of consumers not constrained to single-unit purchases. We first consider the case where there is no heterogeneity in tastes, and then consider the case where there exists heterogeneity in tastes for the brands. For the latter, we also evaluate the performance of our proposed model when the unobserved brand characteristics, $\xi_{jst}$, are non-zero but uncorrelated with price. For the DCM case with endogeneity see Berry (1994).

For simplicity, we analyze the case where there are two alternatives in the choice set. These two alternatives can either be interpreted as one brand with an outside good or as two brands in a model where one restricts attention to only the inside goods. Actual scanner data from a two-brand product category are used for prices and promotions. The data comprises of prices and promotions at the chain level for 90 weeks for the oats category, in which Quaker Oats and the Dominick's store brand are the only two major brands. Price and promotional variables for the model were created by taking the difference of the variables across the two brands. No other variables including demographics were included. Average expenditures of consumers for each of the 90 weeks were simulated as *Uniform(5,20)*. Average quantities per consumer for the 90 weeks were generated by integrating over the expected demands of consumers who are allowed to make multiple-unit purchases. Parameter values for the intrinsic preference, price and promotion sensitivities were chosen to reflect the range of values that one typically obtains in aggregate discrete-choice models for frequently purchased grocery categories. 30 replications are used for the simulation.

Table A1 reports the results of the estimation. The results reveal that for the range of parameter values considered, the proposed model does a good job of recovering all the parameters.

We now investigate the performance of the model in the presence of unobserved heterogeneity. Again, to keep the estimation simple, we allow for unobserved heterogeneity in just the intrinsic

[1] Correcting for both potential serial dependence and spatial dependence across stores using a non-parametric approach (Conley 1999) generated negligible changes in the currently reported standard errors.

preference parameter. The number of draws for simulating the integral was fixed at 100. We consider four separate cases, corresponding to the four combinations of no/some variance in the unobserved fixed effect $\xi_{jst}$, and low/high variance in the intrinsic preference heterogeneity. The results are reported in Table A2. and correspond to the means, standard deviations and mean absolute percentage deviations (MAPD-s) of the recovered parameter values for the four cases, across 30 replications. The top panel in Table A2 corresponds to the cases with low and high variance in the intrinsic preference heterogeneity with no variance in the unobserved fixed effect, and the bottom panel corresponds to the cases with low and high variance in the intrinsic preference heterogeneity with some variance in the unobserved fixed effect. The results reveal that for the range of parameter values considered, the proposed model does a good job of recovering the intrinsic preference and the price and promotion sensitivity parameters. The variation in the standard deviation in the intrinsic preference heterogeneity across replications is high, but is comparable to those of the single-unit logit (see Chintagunta 2000). This is expected to improve with access to more cross sectional data (at the store-level as in our empirical application), and using more draws and better simulation techniques (c.f. Bhat 2001).

We conclude that our proposed model can recover the true model parameters from the aggregate data in the presence of multiple-unit purchases. Finally, we also note that the proposed model can accommodate situations where consumers typically buy a single-unit of the chosen brand. To see this, suppose that we run the proposed model on aggregate data generated using individual-level parameters that imply an expected conditional quantity of 1. As shown above, the model will be able to recover those parameters from the aggregate data and will thus predict that individuals in that category typically buy 1 unit of the chosen good in expected terms.

| Variable | True and estimated parameter values for the discrete/continuous model | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | True | Model | True | Model | True | Model | True | Model | True | Model | True | Model |
| Intrinsic preference | -4.5 | -4.4659 | -4.5 | -4.4575 | -4.5 | -4.4733 | -4.5 | -4.4803 | -4.5 | -4.4800 | -4.5 | -4.4831 |
| Price sensitivity | 1.5 | 1.5448 | 2.0 | 2.0499 | 2.5 | 2.5386 | 3.0 | 3.0320 | 3.5 | 3.5471 | 4.0 | 4.0366 |
| Promotion sensitivity | 0.5 | 0.4177 | 0.5 | 0.4129 | 0.5 | 0.4182 | 0.5 | 0.4024 | 0.5 | 0.4252 | 0.5 | 0.4282 |

**Table A1.** *Monte Carlo Results with no heterogeneity*

| | $Var(\xi_{jst})=0$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Low std. deviation in intrinsic preference | | | | High std. deviation in intrinsic preference | | | |
| | True | Mean | Std. Dev | MAPD | True | Mean | Std. Dev | MAPD |
| Intrinsic preference | -6.000 | -5.883 | 0.199 | 1.539 | -6.000 | -5.529 | 0.302 | 3.684 |
| Price sensitivity | 2.000 | 1.998 | 0.139 | 2.069 | 2.000 | 1.983 | 0.136 | 4.305 |
| Promotion sensitivity | 0.500 | 0.505 | 0.195 | 2.864 | 0.500 | 0.490 | 0.211 | 5.882 |
| Std. Deviation in intrinsic preference | 0.500 | 0.010 | 0.607 | 31.231 | 1.000 | 0.028 | 1.185 | 43.671 |
| | $Var(\xi_{jst})=1$ | | | | | | | |
| | Low std. deviation in intrinsic preference | | | | High std. deviation in intrinsic preference | | | |
| | True | Mean | Std. Dev | MAPD | True | Mean | Std. Dev | MAPD |
| Intrinsic preference | -6.000 | -6.536 | 1.200 | 24.637 | -6.000 | -5.665 | 1.297 | 28.650 |
| Price sensitivity | 2.000 | 1.897 | 0.609 | 14.430 | 2.000 | 1.637 | 0.626 | 19.274 |
| Promotion sensitivity | 0.500 | 0.652 | 0.382 | 18.508 | 0.500 | 0.563 | 0.671 | 21.641 |
| Std. Deviation in intrinsic preference | 0.500 | 0.303 | 0.811 | 67.498 | 1.000 | 0.305 | 1.298 | 84.750 |

**Table A2.** *Monte Carlo Results with heterogeneity in intrinsic preferences*

## PART (B)

### B1. Monte Carlo: Brand-choice, then quantity Poisson model

In this study, we generated data from a brand-choice-then-quantity model that is analogous to Dillon and Gupta (1996). In this model, the probability that a consumer will purchase the product is given by a logit. And conditional on purchase, the quantity chosen is given by a truncated Poisson. The goal of the study was to explore the predictions of the proposed model for choice probabilities and quantities, if the data were generated by the above model. For simplicity, we consider the case in which a homogenous set of consumers choose between 1 product and an outside good. In this model, the probability that the product is purchased, $d = 1$, is:

$$\Pr(d=1) = \frac{e^{\alpha-\beta p}}{1+e^{\alpha-\beta p}}.$$

Conditional on purchase, the quantity chosen is distributed a truncated Poisson:

$$\Pr(Q=q \mid d=1, q>0) = \frac{e^{-\lambda}\lambda^{q}}{\left(1-e^{-\lambda}\right)q!}.$$

The expected conditional quantity is therefore,

$$E(Q \mid d=1, Q>0) = \frac{e^{-\lambda}}{\left(1-e^{-\lambda}\right)}$$

Then, the expected per consumer demand is:

$$E[Q] = \Pr(d=1)\,E(Q \mid d=1, Q>0) = \frac{e^{\alpha-\beta p}}{1+e^{\alpha-\beta p}}\frac{e^{-\lambda}}{\left(1-e^{-\lambda}\right)}.$$

Let $y$ denote expenditures. To capture the dependence of expected quantities on prices and expenditures, we pick the specification $\lambda = \exp(\ln(y) - p)$. We first generate $N = 200$ expenditures ($y$) as uniform(5,20) and prices as uniform(2,5). We then generated expected per-consumer demands from the above brand-choice-then-quantity model for various values of the parameters, and then ran the proposed model on that data. Table B1 shows the results averaged across 100 replications. In the table, $\Pr(d=1)$ refers to the true and predicted mean probability of purchase and $E(Q \mid d=1, Q>0)$ refers to the true and predicted expected conditional quantity.

As we see from the results in table B1, the proposed model does a reasonable job of recovering the choice probabilities and conditional quantities from the data. We take the performance of the model in the above situation as an indication that the model does not strongly a priori restrict choice probabilities and conditional quantities.

### B2. Monte Carlo: Model that implies unitary expected quantities

In this study we generate data from the proposed model with parameters that imply an expected quantity of one, conditional on purchase. The goal is to demonstrate that the model is flexible enough to accommodate situations in which consumers have only unitary conditional demands. In particular, we demonstrate that the estimated results from the model will not be "wrong" we ran it on aggregate demand data generated by consumers that buy a single-unit of their chosen alternative.

As before, we generate $N = 200$ expenditures ($y$) as uniform(5,20) and prices as uniform(2,5). We consider the homogenous 1 product case with two parameters, viz., an intercept and prices. The intercept and price sensitivity were varied so that the true model predicts an expected conditional quantity = 1. The results from running the proposed model on this data are shown in Table B2. The columns under "model" refer to the estimated intercept, price sensitivity and implied mean quantity averaged across 100 replications.

We again see that the model does a very good job of recovering the true parameters and also predicts that the conditional quantities are unitary. This is not surprising since we have already showed in our previous Monte Carlo studies (table A1) that the model does an excellent job of recovering the true parameters from the data if the data are indeed generated by the discrete/continuous rule.

### B3. Simulation: Divisibility of quantities

Here we explore the implications of the control for the discreetness of purchase quantities as in Arora, Allenby and Ginter (1998) (henceforth AAG) for our aggregate demand system. The goal of the study is to evaluate the impact of the divisibility assumption on purchase quantities on our aggregate demand system. The AAG framework is analogous to the Hanemann (1984) and Chiang (1991) model. Our model can be viewed as the aggregate analog of the AAG framework in which quantities as treated as being divisible. For simplicity, here we consider the 1 alternative case in the AAG framework.

AAG allow for discrete quantities and have the following set up for choice probabilities,

$$\Pr(j) = e^{(\alpha/\gamma - \ln(p))/\mu} \Big/ \left(1 + e^{(\alpha/\gamma - \ln(p))/\mu}\right) \qquad \text{(Equation 10, pg. 33 in AAG)}$$

and, conditional quantities,

$$p_0(q) = \Pr(Q_j = q) = F\left[(q+0.5)\,p_j/\gamma\right] - F\left[(q-0.5)\,p_j/\gamma\right] \qquad \text{(Equation 11, pg. 33 in AAG)}$$

where, $F(.)$ is the cdf of an extreme value distribution with location parameter $\alpha/\gamma - \ln(p)$ $+\mu\ln\left(1/\Pr(j)\right)$ and scale parameter $\mu$. This explicitly takes care of the discrete nature of conditional quantities.

The corresponding *expected* conditional quantity to this model is:

$$E[Q]_{\text{discrete}} = \sum_{q=1}^{\infty}\left[q * p_0(q)\right].$$

It is this expected conditional quantity that enters the aggregate demand function corresponding to this model. The only difference between this and our approach is that rather than use $E[Q]_{\text{discrete}}$ in the aggregate demand equation, we use the analytically computed expected conditional quantity function:

$$E[Q]_{\text{continuous}} = \frac{\gamma}{p_j}\left[\alpha/\gamma - \ln(p_j) + \mu\left(\Gamma + \ln\{1/\Pr(j)\}\right)\right],$$ where $\Gamma$ is Euler's constant.

The choice probabilities are not different between this case and ours. So differences in the aggregate expected demand function between discrete and continuous quantity cases would arise only if there are large differences between $E[Q]_{\text{discrete}}$ and $E[Q]_{\text{continuous}}$. We can check this by simulation.

We first generated $N$ (=100) prices from uniform(2,4), and computed $E[Q]_{\text{discrete}}$ and $E[Q]_{\text{continuous}}$ for various values of $\alpha/\gamma$ and $\gamma$. For all computations, $\mu = 1$, and $"\infty" = 50$ (in the summation on $E[Q]_{\text{discrete}}$). The results are given in Table B3. We see that the difference in expected conditional quantity between the two cases is quite small. And since it is the expected quantity that impacts the aggregate demand functions, the difference between the two cases is likely to be very small.

| | TRUE | MODEL | TRUE | MODEL | TRUE | MODEL | TRUE | MODEL |
|---|---|---|---|---|---|---|---|---|
| $\beta = 3.5$ | $\alpha$ = -3 | | $\alpha$ = -4 | | $\alpha$ = -5 | | $\alpha$ = -6 | |
| $\Pr(d=1)$*1e3 | 0.0011 | 0.0012 | 0.3435 | 0.3814 | 0.1477 | 0.1802 | 0.4992 | 0.5421 |
| $E(Q \mid d=1, Q>0)$ | 1.2966 | 1.1702 | 1.2867 | 1.1998 | 1.3116 | 1.1449 | 1.3084 | 1.2542 |
| $\alpha$ = -5 | $\beta = 2.5$ | | $\beta = 3$ | | $\beta = 3.5$ | | $\beta = 4$ | |
| $\Pr(d=1)$*1e3 | 0.3932 | 0.3461 | 0.2325 | 0.2211 | 0.1637 | 0.1949 | 0.0823 | 0.1122 |
| $E(Q \mid d=1, Q>0)$ | 1.2880 | 1.5976 | 1.3169 | 1.4720 | 1.3481 | 1.1976 | 1.2894 | 1.0103 |

**Table B1**: *Simulation results from running the proposed model on data from brand-choice-Poisson model*

| | TRUE | MODEL | TRUE | MODEL | TRUE | MODEL | TRUE | MODEL |
|---|---|---|---|---|---|---|---|---|
| Intercept | -5.00 | -4.98 | -4.00 | -3.99 | -3.00 | -2.98 | -2.00 | -2.00 |
| Log(Price) | 3.50 | 3.51 | 3.60 | 3.62 | 3.70 | 3.72 | 3.80 | 3.81 |
| Mean conditional quantity | 1.0642 | 1.0599 | 1.0631 | 1.0580 | 1.0348 | 1.0287 | 1.0084 | 1.0101 |

**Table B2**: *Simulation results from running proposed model on data from model with expected quantity* = 1

| $\alpha/\gamma = 3.0$ | | | $\gamma = 1.0$ | | |
|---|---|---|---|---|---|
| $\gamma$ | $E[Q]_{\text{continuous}}$ | $E[Q]_{\text{discrete}}$ | $\alpha/\gamma$ | $E[Q]_{\text{continuous}}$ | $E[Q]_{\text{discrete}}$ |
| 1.0 | 0.9677 | 0.9833 | 3.0 | 0.8960 | 0.9150 |
| 1.1 | 1.0644 | 1.0807 | 3.1 | 0.9257 | 0.9492 |
| 1.2 | 1.1612 | 1.1767 | 3.2 | 0.9558 | 0.9831 |
| 1.3 | 1.2580 | 1.2720 | 3.3 | 0.9862 | 1.0166 |
| 1.4 | 1.3547 | 1.3671 | 3.4 | 1.0169 | 1.0496 |
| 1.5 | 1.4515 | 1.4623 | 3.5 | 1.0479 | 1.0820 |
| 1.6 | 1.5483 | 1.5577 | 3.6 | 1.0791 | 1.1137 |
| 1.7 | 1.6450 | 1.6533 | 3.7 | 1.1106 | 1.1447 |
| 1.8 | 1.7418 | 1.7491 | 3.8 | 1.1422 | 1.1750 |
| 1.9 | 1.8386 | 1.8451 | 3.9 | 1.1741 | 1.2047 |
| 2.0 | 1.9353 | 1.9413 | 4.0 | 1.2062 | 1.2339 |

**Table B3**: *Comparison of expected conditional quantity under discrete and continuous cases*

| | | Share of market | |
|---|---|---|---|
| | | Parameter | Std. Error. |
| *Store characteristics* | Holiday | -0.38 | 0.09 |
| | Average income | -1.69 | 0.36 |
| | Mean residential value | 0.02 | 0.00 |
| | Proportion of population of age > 60 | -1.11 | 0.68 |
| | Proportion of ethnic population | 0.23 | 0.46 |
| | Shopping index | 2.09 | 0.37 |
| | Distance to nearest Jewel | 0.31 | 0.03 |
| | EDLP | -0.04 | 0.01 |
| *Log Price Index* | Analgesics | 1.95 | 0.33 |
| | Bath soap | 0.58 | 0.13 |
| | Beer | -5.12 | 0.79 |
| | Bottled juices | -3.57 | 0.86 |
| | Canned cooking soups | 2.26 | 0.55 |
| | Cereals | 0.62 | 0.42 |
| | Cigarettes | -1.66 | 0.80 |
| | Cookies | -1.92 | 0.54 |
| | Crackers | -0.13 | 0.77 |
| | Dish detergent (liquid) | 0.48 | 0.72 |
| | Canned eating soups | -1.87 | 0.50 |
| | Front-end candies | 0.50 | 0.52 |
| | Frozen dinners | 1.51 | 0.39 |
| | Frozen entrees | 1.73 | 0.34 |
| | Frozen juices | -0.74 | 0.34 |
| | Fabric softeners | -1.24 | 0.96 |
| | Grooming products | -1.15 | 0.42 |
| | Laundry detergents | -1.10 | 0.82 |
| | Non-sliced cheeses (shredded, party, etc.) | 1.50 | 0.49 |
| | Dish detergent (powder) | -0.92 | 0.50 |
| | Canned salmon, crabs, etc | 1.06 | 0.36 |
| | Oatmeal | -2.83 | 0.66 |
| | Paper towels | -3.11 | 1.24 |
| | **Refrigerated juices** | **-0.20** | **0.16** |
| | Sliced cheeses | 3.09 | 0.66 |
| | Soft drinks | 2.05 | 0.64 |
| | Shampoos | 0.70 | 0.79 |
| | Snack crackers | 1.65 | 0.59 |
| | Soaps | 1.38 | 1.39 |
| | Toothbrushes | -4.60 | 1.09 |
| | Canned tuna | -0.43 | 0.26 |
| | Toothpastes | -0.83 | 0.50 |
| | Bathroom tissues | -3.89 | 0.85 |
| Constant | | 44.14 | 5.15 |
| Number of observations | | 1486 | |
| $R^2$ | | 0.37 | |

**Table B4:** *Regression of share of traffic on store characteristics and category price indices*

| Variable | With Display as instrument | | Without Display as an instrument | |
|---|---|---|---|---|
| | Parameter | *t*-stat | Parameter | *t*-stat |
| MinuteMaid 64 oz | -3.147 | -30.863 | -3.124 | -29.087 |
| MinuteMaid 96 oz | -3.892 | -35.535 | -3.915 | -33.772 |
| Dominicks 64 oz | -3.874 | -42.220 | -3.859 | -41.116 |
| Tropicana Prm 64 oz | -2.464 | -22.499 | -2.480 | -21.411 |
| Tropicana SB 64 oz | -3.432 | -32.955 | -3.401 | -31.218 |
| Tropicana Prm 96 oz | -2.973 | -25.989 | -3.010 | -24.709 |
| Florida 96 oz | -6.075 | -69.179 | -6.040 | -67.214 |
| -Log(price) | 2.614 | 28.767 | 2.531 | 25.646 |
| Display | 0.467 | 17.945 | 0.486 | 17.456 |
| -Log(price)*ethnic | 3.963 | 11.394 | 3.957 | 11.103 |
| -Log(price)*hval150 | -2.737 | -25.379 | -2.705 | -24.915 |
| Ethnic | 5.116 | 15.362 | 5.023 | 14.745 |
| Hval150 | -1.809 | -17.721 | -1.815 | -17.676 |
| Drvtm | -0.039 | -6.072 | -0.040 | -6.296 |
| Age60 | 0.422 | 3.499 | 0.500 | 4.124 |
| Hhlarge | 0.064 | 2.140 | 0.074 | -2.460 |
| Objective function | 1026.67 | | 1053.69 | |

**Table B5**: *Parameter estimates (homogenous case) with and without display as instruments*

| | MODEL 1 | | MODEL 2 | |
|---|---|---|---|---|
| | *Parameter* | *Std. error* | *Parameter* | *Std. error* |
| MM 64 | -0.991 | 0.212 | -1.322 | 0.170 |
| MM 96 | -1.024 | 0.211 | -1.332 | 0.176 |
| TR 64 | 0.314 | 0.245 | -0.038 | 0.201 |
| TR 96 | -1.240 | 0.274 | -1.587 | 0.224 |
| Price | -85.047 | 5.839 | -82.832 | 4.826 |
| Feature | 0.085 | 0.070 | 0.109 | 0.070 |
| Display | 1.977 | 0.380 | 2.104 | 0.403 |
| Inventory | | | 1.300E-03 | 1.000E-04 |

**Table B6**: *Results with and without including inventory*

| | OWN-PRICE ELASTICITY | | |
|---|---|---|---|
| Brand | Observed inventory | Zero inventory | Max: inventory |
| MM 64 | -2.88 | -2.89 | -2.85 |
| MM 96 | -2.88 | -2.89 | -2.86 |
| TR 64 | -3.32 | -3.34 | -3.26 |
| TR 96 | -3.72 | -3.72 | -3.71 |

**Table B7**: *Effects of including inventory on price elasticities*