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In this review session, we will use the daily S&P 500 data.

1 Historical Volatility

We are interested in computing the annualized daily volatility using a moving average window of 30 days for the squared returns. In other words, we use the 30 most recent observations to get an estimate of volatility at a point in time. From the File menu, select New and then Program. Enter the following code in the program file, save it and click Run:

```
series retsq=sprtrn^2
smpl @first+30 @last
series vol30=0
for !i=0 to 2117
  smpl @first+!i @first+!i+30
  vol30(!i+31)=@sqrt(252*@mean(retsq))
next
smpl @all
```

**Question 1.** What is the above code achieving? Formally, how are we computing volatility at a given point in time?

**Question 2.** What does this suggest about our choice of days in the moving average window?

Consider now computing the annualized daily volatility using a moving average window of 250 days for the squared returns. In other words, we use the 250 most recent observations to get an estimate of volatility at a point in time.

```
smpl @first+250 @last
series vol250=0
for !i=0 to 1897
  smpl @first+!i @first+!i+250
  vol30(!i+251)=@sqrt(252*@mean(retsq))
next
smpl @all
```

**Question 3.** What is the above code achieving? Formally, how are we computing volatility at a given point in time?

**Question 4.** How does this compare to the previous plot?
2 Exponential Smoothing

We are interested in constructing the risk metrics exponential smoother for the daily S&P500 return series. In other words, we consider a specific updating rule to compute the annualized daily volatility series. Consider the case of $\lambda = 0.9$. Use the following code to compute the smoothed estimate:

```plaintext
series expsm1=0
expsm1(1)=@var(sprtrn)
smpl @first+1 @last
expsm1=0.9*retsq(-1)+0.1*expsm1(-1)
smpl @all
expsm1=@sqrt(252)*@sqrt(expsm1)
```

**Question 5.** What is the above code achieving? How are we initializing the first variance (denoted $\sigma_1^2$)?

**Question 6.** What is the Risk Metrics updating rule formally given by?

**Question 7.** How is our choice of $\lambda$ seen in the plot?

Consider now the case of $\lambda = 0.5$. Use the following code to compute the smoothed estimate:
Figure 2: 250 day average volatility

```
series expsm2=0
expsm2(1)=@var(sprtrn)
smpl @first+1 @last
expsm2=0.5*retsq(-1)+0.5*expsm2(-1)
smpl @all
expsm2=@sqrt(252)*@sqrt(expsm2)
```

**Question 8.** What is the Risk Metrics updating rule formally given by in this case?

**Question 9.** How does this plot compare with the previous plot?

## 3 ARCH and GARCH models

GARCH models have proven to be a remarkable success in modeling conditional variance dynamics. Specifically, GARCH models have been quite popular in modeling the volatility of stock returns. Let’s begin with motivating an ARCH(1) model via an applied example.

To estimate an ARCH(1) model for the S&P500 returns, use:

```
arch(1,0) sprtrn
```
Notice that, by default, EViews estimates a GARCH model, so, to estimate an ARCH model, we need to specify the GARCH order to be 0. Suppose that we want to plot the estimated time series of volatilities. Open the equation object corresponding to the ARCH(1) model and, under the View tab, select the GARCH graph... option. The estimated volatilities are plotted in Fig. 5.

**Question 10.** Is the lagged squared return significant?

**Question 11.** What is the unconditional variance given by?

**Question 12.** What is the conditional variance given by?

**Question 13.** How do we write out the estimated model?

To evaluate the goodness of fit, consider plotting the original data series, together with the ±2 standard deviations. To do this, first save the conditional variances by selecting the Proc tab in the equation object and choosing Make Garch Variance Series.... Then, to calculate the ±2 standard deviations, use:

```plaintext
series archbp=2*@sqrt(arch1var)
series archbm=-2*@sqrt(arch1var)
```

The plot is presented in Fig. 6.

Consider now estimating a GARCH(1,1) model. Use:
Suppose that we want to plot the estimated time series of volatilities. Open the equation object corresponding to the ARCH(1) model and, under the **View** tab, select the **GARCH graph...** option. The estimated volatilities are plotted in Fig. 5.

**Question 14.** *Is the lagged squared return significant?*

**Question 15.** *What is the unconditional variance given by?*

**Question 16.** *What is the conditional variance given by?*

**Question 17.** *How do we write out the estimated model?*

To evaluate the goodness of fit, consider plotting the original data series, together with the ±2 standard deviations. To do this, first save the conditional variances by selecting the **Proc** tab in the equation object and choosing **Make Garch Variance Series**.... Then, to calculate the ±2 standard deviations, use:

```
series archbp=2*@sqrt(garch11var)
series archbm=-2*@sqrt(garch11var)
```

The plot is presented in Fig. 8.

**Question 18.** *Does the GARCH(1,1) provide a better fit than the ARCH(1) model?*
Figure 5: Condition volatility, ARCH(1) model
Figure 6: Data ± 2 standard deviations, ARCH(1) model
Figure 7: Condition volatility, GARCH(1,1) model
Figure 8: Data ± 2 standard deviations, GARCH(1,1) model