Financial Econometrics: Proposed Midterm Solutions

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February 20, 2019

1
(2 points each)
a. x4
b. x3
c. x2
d. x1

2
(2 points each)
a. Y2
b. Y1
c. Y5
d. Y3
e. Y4

3
(3 points each)
a. ARMA likely, MA possible. Observe that both the ACF and the PACF becomes insignificant after the second lag. I would start with ARMA(2,2). Remember that for an AR process, the PACF cuts off and the ACF decays slowly. Therefore, AR is the only model that we can eliminate from contention. Additionally, recall that the PACF significant lags indicates the number of AR lags while the number of significant ACF lags are related to the MA terms.

b. The residual correlogram does not indicate that many problems for the model. Many of you noted that there are some significant lags at 5 and 12 which is true. We may want to try another model but overall, these residuals don’t look terrible.

The coefficient for the MA term isn’t significant. This might indicate that we want to try additional AR lags but again, it’s not hugely problematic that there’s a term in there that isn’t statistically different from zero. Should we try to improve the model, we may want to try dropping the MA term but overall, the model doesn’t look terrible based on the residual diagnostics.
c. 

\[ G_{T+1} = b_0 + b_1 G_T + \theta \epsilon_T \]
\[ = .0038 + .511 \times 0.0083 - 0.1714 \times -0.000177 \]
\[ = 0.00807 \]

d. We have the conditional mean above. Remember that conditional on \( G_T, \epsilon_T \), the variance is due to the unconditional variance of \( \epsilon_t \) which we get from the output. Note that \( \text{SIGMASQ} \approx (\text{SEof regression})^2 \) so either is fine for this case.

\[ f(G_{T+1}|G_T) \sim N(.00807,.00874^2) \]

e. 

\[ .00807 \pm 2\sqrt{\sigma^2} = .00807 \pm 2 \times .00874 \]

4

(3 points each)
a. The series mean reverts to the unconditional mean of

\[ E[r_t] = \frac{\beta_0}{1 - \beta_1} = \frac{.01}{1 - .85} = .0667 \]

b. \( r_{T+1} = .01 + 0.85 \times 0.04 = 0.044 \)

c. 

\[ \nu(\epsilon^1_t) = \frac{(1 - 0.85^2)}{1 - 0.85^2} \times 0.003^2 = .003^2 \]

d. 

\[ f(r_{T+1}|r_T) = .01 + 0.85 \times 0.04 + f(\epsilon_T|r_T) \sim N(.044,.003^2) \]

e. 

\[ .044 \pm 2\sqrt{s^2} = .044 \pm 2 \times 0.003 \]

f. \( r_{T+12} = 0.85^2 \times 0.044 + (1 - 0.85^2) \times .0667 \approx 0.643 \)

g. 

\[ 0.0643 \pm 2 \sqrt{\frac{1 - 0.85^2}{1 - 0.85}} \times 0.003^2 = 0.0643 \pm 0.113 \]

h. It converges to the unconditional mean, 0.0667

i. 

\[ \frac{\sigma^2}{1 - \beta_1^2} = \frac{0.003^2}{1 - .85^2} = .0000324 \]
5

(3 points each)

a. \( V_{T+1} = 10 + 1000 = 1010 \)

b. \( \text{var}(\epsilon_t) = k \sigma^2 = 100^2 \)

c. \( f(10 + 1000 + \epsilon_t | V_T = 1000) \sim N(1010, 100^2) \)

d. This was a typo, it should’ve been \( V_{t+K} \). You got full credit for \( V_{t+1} \) but you needed to figure out \( V_{t+k} \) for the next problem anyway.

\[ f(V_{T+k} | V_T) \sim N(1000 + 10k, k100^2) \]

See homework 3 for derivation.

e. In 4 years, the distribution of stock prices is given by

\( \sim N(1040, 4 \times 100^2) \)

The question asks for probability of rising above 1240. Notice that 1240 – 1040 = 200 which is precisely one standard deviation above the mean. This implies

\[ F(1240) = F(1040) + (F(1240) – F(1040)) \]

The first term is just the mean mark which is 0.5. Recalling that 68\% of the weight lies between one standard deviation on either side of the mean for the normal distribution, this gives

\[ F(1240) – F(1040) = \frac{0.68}{2} = 0.34 \]

The probability of rising above 1240 is then,

\[ 1 – (0.5 + 0.34) = 0.16 \]

so the probability is 16\%. You just had to follow this logic, even if you didn’t know the exact 68\% number.

f. The forecast is 1000 + 10 * k so this goes to \( \infty \) as \( k \rightarrow \infty \)

g. Forecast error variance is \( k \sigma^2 \rightarrow \infty \) as \( k \rightarrow \infty \)

6

a. (6 points). The process of model selection involves testing out multiple models. One of the problems is that you may find one that happens to fit that particular dataset, just by random chance. This is especially problematic the more models you try. By splitting the data and reserving an out of sample set for testing the predictions of the selected model, we will make it harder to overfit a model by seeing if it works out of sample. If you had just randomly selected a model that works, there’s no reason to think it will work well out of sample since we managed to fit this one by random chance.

b. (6 points)

- Split the sample into two, a \( \frac{2}{3} \) split is reasonable though there’s no right answer.
- Using the in-sample dataset, estimate and select amongst models using tools we’ve discussed in class like AIC, BIC, \( R^2 \)
- Given the model selected, forecast values into the out of sample dataset
- Using the forecasted values, evaluate against the actual out of sample data using some well defined loss function.
A reasonable loss function to use might be based on the trading strategy described in the problem. We may care about average profit or some risk adjusted profit based on this loss function. Simpler loss functions like MSE might work as well if we’re simply evaluating the model.

c. (7 points) Importantly, statistical tests like AIC, BIC are not what we’re looking for here. A straightforward application of section b responses is to design a test around

\[ H_0 : \bar{r} > 0 \]
\[ H_1 : \bar{r} \leq 0 \]

where \( \bar{r} \) are the mean returns from the trading strategy. Other things might revolve around profits or potential losses from a different trading strategy. An oil firm might care about how well they’ve hedged against potential oil price volatility in the future. The point is that we care about how well our models work in a direct application.