You have 3 hours to complete the exam. Use can use a calculator. Try to fit all your work in the space provided. If you find you need more space continue on the back of the page. Don’t panic. The last page contains a set of formulas that might be useful on the exam. No other notes or texts are permitted. When possible, I expect numerical answers, not just formulas.

Students in my class are required to adhere to the standards of conduct in the GSB Honor Code and the GSB Standards of Scholarship. I understand that discussion of the contents of this exam prior to the completion by all students is a violation of the Honor Code. The GSB Honor Code also require students to sign the following GSB Honor pledge,

"I pledge my honor that I have not violated the Honor Code during this examination.”

Please sign here to acknowledge _______________________________
1. Consider building a model for quarterly log GDP. We denote log GDP by $y_t$. There are 243 observations so that $T=243$.

The first model is a trend stationary model. $y_t - trend_t = \tilde{y}_t$

where $\tilde{y}_t = .973\tilde{y}_{t-1} + \epsilon_t, \quad \epsilon_t \sim iid \ N\left(0,.0014^2\right)$, $trend_t = 7.41+.00825^*t$. $t$ takes the values 1 in the first period through 243 in the last period.

a. For the last observation in the sample, the difference between the log GDP and the trend at the end of the sample is -.0611 i.e. $\tilde{y}_T = -.0611$. Write the expression for the $k$-step ahead forecast for log GDP as a function of $k$.

b. Construct the forecast error standard deviation for this $k$-step ahead forecast. Be specific.
A second model for log GDP is a random walk model. \( y_t = .0082 + y_{t-1} + \eta_t \), where \( \eta_t \sim iid \ N\left(0, .0097^2\right) \).

c. The last observation in the sample has a log of GDP given by 9.36. Write the expression for the k-step ahead forecast of log GDP as a function of k.

d. Construct the forecast error standard deviation for this k-step ahead forecast. Be specific.

e. When k gets very large what value does the forecast error standard deviation get close to in the trend stationary model?

f. Compare this forecast error standard deviation of the unit root to the forecast error standard deviation trend stationary model. Explain how the standard deviations differ and how this affects a 95% prediction interval for large k.
2. Consider the following GARCH(1,1) model estimated for S&P500 daily returns data.

Dependent Variable: SP500  
Method: ML - ARCH (Marquardt)  
Date: 03/17/08   Time: 11:13  
Sample(adjusted): 1/02/1996 2/14/2006  
Included observations: 2641 after adjusting endpoints  
Convergence achieved after 12 iterations  
Variance backcast: ON

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td>Variance Equation</td>
<td></td>
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<tr>
<td>C</td>
<td>8.83E-07</td>
<td>2.89E-07</td>
<td>3.056034</td>
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<tr>
<td>ARCH(1)</td>
<td>0.075411</td>
<td>0.007040</td>
<td>10.71117</td>
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<tr>
<td>GARCH(1)</td>
<td>0.919688</td>
<td>0.007752</td>
<td>118.6390</td>
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</tbody>
</table>

R-squared -0.001340   Mean dependent var 0.000415  
Adjusted R-squared -0.002099   S.D. dependent var 0.011334  
S.E. of regression 0.011346   Akaike info criterion -6.349373  
Sum squared resid 0.339602   Schwarz criterion -6.342695  
Log likelihood 8387.347   Durbin-Watson stat 2.026833

a. The last return in the sample is -.0044 and the value of \( h_t \) associated with the last observation in the sample is .000032. Write the one-step ahead forecast of the variance for time T+1 (i.e. one-step ahead out of sample forecast).

b. If returns are conditionally Normal, find the one day ahead, 1% Value at Risk on a million dollar investment in the S&P500 using the forecast from part a. Recall \( Pr(Z<-2.33)=.01 \) for a standard Normal random variable Z.
The figures above are the autocorrelations associated with the square of the standardized residual and the histogram of the standardized residual.
c. What do these plots tell you about whether or not the GARCH(1,1) model appears to fit the volatility dynamics well? Be specific and explain why.

d. What do these plots tell you about the validity of the conditional Normality assumed in part b? Be specific.

e. Explain whether the Value at Risk is too large or too small in part b. Be specific and back up your argument with facts (numbers).

f. Suppose that you estimate a GARCH model with a t-distribution for the errors, $\varepsilon_t$. The estimated parameter for the t-distribution is 8.1. For a t-distribution with 8.1 degrees of freedom, the $\Pr(t<-3.35)=.01$. Find the 1% VaR for this model using the forecast from part a.

g. Explain how you would improve the Value at Risk using the information in the empirical distribution of $z$. Be specific.
The following is output for an asymmetric GARCH model.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.58E-06</td>
<td>2.48E-07</td>
<td>6.364375</td>
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<tr>
<td>ARCH(1)</td>
<td>-0.008388</td>
<td>0.007530</td>
<td>-1.113962</td>
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<tr>
<td>(RESID&lt;0)*ARCH(1)</td>
<td>0.158472</td>
<td>0.011187</td>
<td>14.16568</td>
</tr>
<tr>
<td>GARCH(1)</td>
<td>0.921829</td>
<td>0.007248</td>
<td>127.1870</td>
</tr>
</tbody>
</table>

R-squared: -0.001340, Mean dependent var: 0.000415, Adjusted R-squared: -0.002479, S.D. dependent var: 0.011334, S.E. of regression: 0.011348, Akaike info criterion: -6.394901, Schwarz criterion: -6.385996, Log likelihood: 8448.466, Durbin-Watson stat: 2.026833.

h. The Schwarz criterion in the bottom of the output is another name for the BIC criterion discussed in class. Does the asymmetric GARCH model dominate the symmetric GARCH model according to the BIC? Explain.

i. Do the estimated parameters of the asymmetric model “look OK” or is there something to be concerned about?

j. Build the one-step ahead forecast for the variance at time T+1 (i.e. one-step ahead out of sample forecast) using this asymmetric model given the last return in the sample is -.0044 and the value of $h_t$ associated with the last observation in the sample is .000032.

k. Explain how and why the distribution of the return over the next 10 days will be different from the 10 day return constructed using the symmetric GARCH model. Be specific.
3. The following figure plots the national average home sales price and the national average home rental price from 1975 through 2007. Prices are normalized so that both averages are 100 in 1975.

![GAP YEARS](image)

In the boom, prices grew faster than rents.

- **Home Price**
- **Rent**

a. Explain why these prices might be cointegrated. You should consider economic arguments.

b. Explain in detail, step by step, how you would test the hypothesis that the prices are cointegrated assuming you know the cointegrating relationship.
Suppose that you determine the prices are cointegrated and estimate the following Error Correction Model (ECM):

$$
\Delta y_t = \begin{bmatrix} 6.2 \\ 1.2 \end{bmatrix} + \begin{bmatrix} .43 & .11 \\ .08 & .54 \end{bmatrix} \Delta y_{t-1} + \begin{bmatrix} -.12 \\ .08 \end{bmatrix} z_{t-1} + \begin{bmatrix} \epsilon_t \\ \eta_t \end{bmatrix}
$$

where $y_t = \begin{bmatrix} sales_t \\ rent_t \end{bmatrix}$ and $z_t = sales_t - rent_t$.

c. Provide an interpretation of the coefficients on $z_{t-1}$. Do they make sense?

d. In the last period, $sales=160$, $Rent=97$ and $\Delta y_t = \begin{bmatrix} -6.1 \\ 3.2 \end{bmatrix}$. Find the one-step ahead forecast for the change in the home price index and the change in the rental price index. I expect numbers here.
e. Again, in the last period, $sales=160$, $Rent=97$ and $Δy_t = \begin{bmatrix} -6.1 \\ 3.2 \end{bmatrix}$. Find the two step ahead forecast for the change in the home price index and the change in the rental price index. I expect numbers here.

f. Suppose instead that both sales and rent appear to follow a random walk, but you conclude that the two series are not cointegrated. Explain how you would model the series in this case. Be specific.
4. Consider the following 1 factor model for annual returns. \( r_{1t} = \beta_1 r_{mt} + \varepsilon_{1t} \) and 
\( r_{2t} = \beta_2 r_{mt} + \varepsilon_{2t} \) where \( \varepsilon_{1t} \) and \( \varepsilon_{2t} \) are independent of each other and iid. They are also uncorrelated with \( r_{mt} \). The standard deviation of \( \varepsilon_1 \) and \( \varepsilon_2 \) are given by 
\( \sigma_1 = .07 \) and \( \sigma_2 = .04 \). \( r_{mt} \) is the monthly return on the S&P500. The mean monthly S&P500 return is estimated to be .007 and we estimate the a GARCH(1,1) model for the volatility process. Suppose that the betas for the two stocks are \( \beta_1 = .82 \) and \( \beta_2 = 1.1 \). I expect numerical answers in what follows.

a. What is the expected monthly return on assets 1 and 2?

To keep the math easy, treat the mean of \( r_1 \) and \( r_2 \) as zero in what follows.

b. When the monthly volatility (\( \sigma_{sp} \)) of the market portfolio (S&P500) is .034. Find the variance of asset 1 and asset 2, the covariance between asset 1 and asset 2 and their correlation.

c. When the volatility \( \sigma_{sp} \) of the market portfolio (S&P500) is .092. Find the variance of asset 1 and asset 2, the covariance between asset 1 and asset 2 and their correlation.
d. Find the variance of a portfolio that allocates .5 of the wealth in asset 1 and .5 wealth in asset 2 when the market volatility is .034.

e. Find the variance of a portfolio that allocates .5 of the wealth in asset 1 and .5 wealth in asset 2 when the market volatility is .092.

f. Use the 1 factor model to discuss the relationship between default probabilities for corporate debt across different companies. You may assume that default probabilities are closely tracked by movements in the stock price (increase in stock price corresponds to a decrease in default probability). I expect a paragraph here.

g. If the market portfolio follows an asymmetric GARCH model explain how this would affect default probabilities in part f. I expect a short paragraph here.
Forecasting a Trend Stationary Model:

\[
E(\tilde{Y}_{t+k} | Y_t) = \tilde{Y}_t + \beta_0 + \delta (t + k)
\]

If \( t_v \) has a t-distribution with \( v \) degrees of freedom then \( Var(t_v) = \left( \frac{v}{v-2} \right) \sigma^2 \)