Unless otherwise stated, \( \epsilon_i \sim iid \ N \left( 0, \sigma^2 \right) \).

1. Consider the following series and explain if they are weakly stationary.

   a. \( y_t = \begin{cases} 
   \beta_0 + \epsilon_t & \text{if } t < t' \text{ and } \epsilon_t \sim iidN \left( 0, \sigma^2_1 \right) \\
   \beta_0 + \eta_t & \text{if } t \geq t' \text{ where } \eta_t \sim iidN \left( 0, \sigma^2_2 \right) 
   \end{cases} \) for \( \sigma^2_1 \neq \sigma^2_2 \)

   b. \( y_t = \beta_1^0 t + \epsilon_t \)

   c. \( y'_i = \epsilon'_i \) where \( i \) denotes the \( i \)th realization of the series, \( \epsilon'_i \sim iidN \left( 0, \sigma^2_i \right) \), and \( \sigma^2_i = \eta_i^2 \) where \( \eta_i \sim iidN \left( 0, 1 \right) \)

2. In the following problems \( \epsilon \sim iid \ N \left( 0, \sigma^2 \right) \). In Excel, Eviews, or any other software package, simulate 200 data points from the following AR(1) models where \( y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t \) and \( \epsilon_t \sim iidN \left( 0, \sigma^2 \right) \). For the simulations, fix the first value of the series at the unconditional mean of \( y \) \(( \mu = \frac{\beta_0}{1-\beta_1} )\). Then sequentially update the series using the previous value of the series and a random draw from a standard normal. In Eviews, you can generate random draws from a standard normal distribution (let’s call the variable eps) using the generate series: \( eps=nrnd \) function.

   i. \( \beta_0 = 0, \beta_1 = 0, \sigma^2 = 1 \)

   ii. \( \beta_0 = 0, \beta_1 = .9, \sigma^2 = 1 \)

   iii. \( \beta_0 = 0, \beta_1 = .1, \sigma^2 = 1 \)

   iv. \( \beta_0 = 0, \beta_1 = -.5, \sigma^2 = 1 \)

   v. \( \beta_0 = 1, \beta_1 = .9, \sigma^2 = 1 \)

   a. First, find the unconditional mean and variance for each series.

   b. For each series find an expression for the \( j \)th autocorrelation.
c. Plot the simulated series. Do they look like you expect? What is the difference between series with large values of beta (near 1) and small values of beta?

d. Using the last observation in your sample (T=200), write down the one step ahead forecast of the series (i.e. find $\mu_{T+1}$). Recall that the AR(1) model is a regression model, so you can answer this prediction problem as you would in any regression model.

e. For your simulated data, find the one step ahead conditional distribution for the 201st observation. Find $f(y_{T+1} | y_T)$.

f. Write down a 95% predictive interval for this one step ahead forecast.

g. Estimate an AR(1) model for each set of simulated data. Examine the residuals from the estimated models (look at the autocorrelations). Do they look like you expect? Explain.

3. Check out the Federal reserve economic data page (FRED) at: https://fred.stlouisfed.org/. Perform a search for Case-Shiller (in the search window at near the top of the page). Find the Case Shiller Home Price Index. There are two options, the seasonally adjusted and the non-seasonally adjusted. By clicking on each of these series you are taken to a page where you can download the data. Above the graph is a tab to download the data. Click this link and then select “Excel” for the file type. Download both datasets and read them in to whatever stat package you are using. We will focus on growth rates for this problem. To compute continuously compounded growth rates, you create a new series $r_t = \ln(y_t) - \ln(y_{t-1})$ where $\ln$ is the natural logarithm. Create the growth rate series for both the seasonally adjusted series and the unadjusted series. The seasonally adjusted series removes the cyclical variation that repeats itself year after year (i.e. summer tends to have higher prices than the winter).

a. Examine the ACF of the raw growth rates out through lag 48. Do you see a pattern in the autocorrelations? Explain.

b. Do you think an AR(1) model would work for the raw data? Explain.

c. Examine the ACF of the adjusted rates. Do you see the same pattern? Explain.

d. Fit an AR(1) model to this data.

e. Examine the residual autocorrelations.

f. Does the AR(1) model fit the data?

g. Using residual diagnostics, find an AR(p) model that fits the data well.
h. Build the one step ahead forecast for every observation in the sample using your fitted AR(p) model. Plot the forecast and the real data on the same plot.

i. What is one step ahead out of sample forecast (i.e. the forecast of T+1) and find the 95% prediction interval.

4. Consider the AR(2) model \( y_t = 1.1 + 0.95 y_{t-1} - 0.1 y_{t-2} + \epsilon_t \). Let \( \sigma^2 = 4 \).
   a. Assuming stationarity. Find the unconditional mean and variance.
   b. Assuming stationarity find the autocorrelations. Multiply both sides by \( y_{t-1}, y_{t-2} \) and so on. You will have a system of equations with two unknowns (these are called the Yule Walker equations). Higher order autocorrelations can be found in a recursive way.

5. Use the data set tbill.xls for this problem.
   a. Estimate the AR(1) model. If you use ar(1) in the equation, Eviews outputs the mean for the intercept. If you run OLS, Eviews outputs the estimated intercept. You can go back and forth using the formulas we derived in class: 
      \[ \mu = \frac{\beta_0}{1 - \beta_1} \] 

   b. Construct the in sample errors and find the estimate for the standard deviation of the errors.
   c. Simulate a series of length 81 (the same as the original data) using the intercept, slope, and standard deviation you estimated for the tbills data set (be sure to use the right intercept). Take a draw from the unconditional distribution to start the simulation (\( y_0 \)). The unconditional distribution will be Normal with mean and variance equal to the unconditional mean and variance implied by the model. Next, simulate a series of length 81. Estimate an AR(1) model on the simulated data and report the estimated slope coefficient. Repeat this simulation 1000 times and each time record the estimate of the slope parameter. Plot the empirical distribution of the slope parameters you obtained from your simulated data. What does this suggest about the finite sample bias in AR(1) estimates?