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The Value of Operational Hedges in Enterprise Risk Management

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Motivation

Operations (e.g., flexible production, foreign production) can mitigate risk across an enterprise from the effects of demand, price, and currency exchange volatility.

Financial instruments also can reduce risks (but should have zero NPV’s).

Questions: what is the value of operational methods and how do they interact with financial methods?
Outline

Preliminary discussion of “hedging”
Specific case in foreign exchange
Value calculations
Investment problem solutions
Operational policies
Conclusions
Preliminary Discussion: Hedging

Definition here: *reducing risk (volatility)*

Alternative interpretations:

Only reducing risk without affecting mean values

Using “hedging” instruments (e.g., derivatives): *financial hedging*

Some results (e.g., Chowdhry and Howe 1999):

Operational hedging has value over financial hedging because of flexibility in output and correlation between demand and prices (examples later)
Risk Management and Hedging

What is a hedge?

Action designed to reduce risk of future outcome
In finance, perfect hedge leads to no risk (riskfree return)

Use of hedges

Allow pricing of financial derivatives
Lead to markets in derivatives
Also possible with operations (operational hedges)
  Quantity - flexible production
  Timing
Who Should Hedge?

Farmers?

Situation:

Suppose either high-yield or low-yield years for crops

Prices down in high years and up in the low years
Farmer’s Example

Suppose yield of corn is either 200 k-bushels (high) or 100 k-bushels (low).

Suppose price with high yield is $1 and price with low yield is $2.

Should the farmer use financial hedge? i.e., sell a future?

If so, how much?
Futures Contracts as Hedges

*Futures contract*: an agreement to buy or sell a fixed quantity at given price at fixed time in future (marked to market every day)

Example: can agree to sell 100 k-bushels at $1.50/bushel on October 15

On October 15, we receive $150K and must deliver 100 k-bushels

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Futures for the Farmer

Advantages

- Can accept the expected price now
- No risk in the price for the amount we sell

Potential problems

- Risk on amount we can produce
- May have to go into market

Analysis: Hedge our expected yield (150 k-bushels)

Guaranteed (all the time) $225K

High yield – can sell 50 more + $50K (probability ½)

Low yield – must buy 50 -$100K (probability ½)

Expectation=225+50/2-100/2= $200k (same as no hedge)

BUT variance (risk) is up (either $275k or $125 instead of $200k all the time)

RESULT: should not use futures (alone)
Farmer’s Operational Hedge for Risk Management

What else does the farmer have?

**SILO!!**

*Operational hedge*

*Keep corn from high yield to sell at low yield*

Now, suppose we keep 50 k-bushels in silo from high to low yield years
Farmer’s Silo Hedge

Expected returns

High-yield years (prob. ½) $150 k
Low-yield years (prob. ½) $300 k
Expectation: ½(150+300)= $225k
Worth $225k-200k = $25k to use the silo
Value of the operational hedge (option value of silo)

Combine with future?

Now, sell 150 k-bushels for $1.50 in October
Now, have the return guaranteed $225K

Moral: Financial instrument only has value if farmer uses operational hedge
Copper Miner’s Example

Should a copper mine hedge its output with futures?

What is the nature of copper price differences?

Demand versus supply curve change means high price-high quantity and low price-low quantity
Copper Hedging

Suppose high demand leads to 200 k-pounds at $2/pound and low demand leads to 100 k-pounds at $1/pound

Earn $400k (prob. ½) or $100k (prob. ½)

Expected value of $250k

Operational hedge? (save 50 k-lbs from high to low years?)

High years: earn $300k (prob. ½)

Low years: earn $150k (prob. ½)

Expectation: $225k (lower value!)
Copper Futures?

Suppose we sell 200 k-lbs at $1.50 in future

Result now:

- **Futures return:** $300k (all the time)
- **High demand:** + $0k (with probability $\frac{1}{2}$)
- **Low demand:** - $100k (with probability $\frac{1}{2}$)
- **Expectation:** $250k
- **Risk reduced** ($300 or $200 v. $400 or $100)

Here: financial derivatives give value (how much? present value?)
Model for Single Period

Suppose:

Price: $p(\omega)$
Cost: $c$
Max sales: $l+kp(\omega)$ ($k>0$ or $<0$)
Decision: $x$ (amount to hedge, i.e., sell forward)

Objective

\[
\max (E(p)-c)x + E[(p-c)^+(l+kp-x)^+] + (c-p)(l+kp-x)^-]
\]
Single Period Results

When does hedging add value?

For $k < k^*$, hedge.
For $k \geq k^*$, do not hedge.

When prices are supply-driven, hedging can be beneficial in securing higher prices when demand is high.

When prices are demand-driven, hedging can negate the value of potential cost advantage over the market.
Overall Observations

Farmer:
   Financial and operational together

 Miner:
   Financial alone (but only for risk reduction)

One-period model
   Hedging when correlation of price and quantity is below a threshold

Next: dynamic model with currency
Operational Flexibility and ForEx Risk

- Mis-matched operations leads to FOREX risk
- Flexible operations can be valuable in shifting costs to balance risk exposures
- Optimal policies involving operations in different regions and can be valued effectively
Alternative Operations

Production Distribution

Domestic

Foreign
Valuation

- No flexibility
  - Stream of cash flows in two currencies

- Semi-flexibility
  - Continuous option to use lowest-cost source
  - Integral of option value over time gives value of flexibility

- Full flexibility
  - Option value over solution of transportation problem at each point in time
  - Integration over time of parametric LP solution
Value Calculations

No Flexibility:

\[ V_{\text{noflex}} = E\left[ \int_0^\infty e^{-rt}((s_1 - c_1)d_1 + (s_2Y_t - c_1)d_2)Z^1_t dt \right] \]

Semi-flexibility \((k_1 \geq d_1 + d_2)\):

Instantaneous cash flow:

\[ p(t) = (s_1 - c_2)d_1Z^1_t + \max(d_2(s_2Y_t - c_1)Z^1_t, (d_2 - k_2)(s_2Y_t - c_1)Z^1_t + k_2Y_t(s_2 - c_2)Z^2_t) \]

\[ V_{\text{semi-flex}}(\vec{k}) = E\left[ \int_0^\infty e^{-rt}p(t)dt \right] \]

\[ = E\left[ \int_0^\infty e^{-rt}((s_1 - c_1)d_1 + d_2(s_2Y_t - c_1))Z^1_t \right. \]

\[ + k_2c_2\max(0, (\frac{c_1}{c_2} - Y_t)Z^2_t))dt \]
Full-Flexible Capacity Valuation

\[ \text{Max } (s_1 - c_1)X_{11} + (s_2 Y_t - c_1)X_{12} + (s_1 - c_2 Y_t)X_{21} + (s_2 - c_2)Y_tX_{22} \]  
\[ \text{s.t. } X_{11} + X_{12} \leq k_1; X_{21} + X_{22} \leq k_2; \]
\[ X_{11} + X_{21} = d_1; X_{12} + X_{22} = d_2; \]
\[ X_{ij} \geq 0, \text{ for all } i \text{ and } j. \]  

Solution:

- Ordering of margins determines possible set of optimal bases; Maximum-margin allocation optimal
- Each ordering corresponds to 3 potential optimal bases; at most 5 optimal bases for given \( c_i, s_i \)
- Can compute analytically for each of the 5 bases as time-integrated continuum of options
- Can include transportation costs (with potentially more breakpoints of optimal bases)
- Random demand also possible with additional integration
Volatility Effect

\[
\sigma\text{Net Worth} \quad \sigma'\text{Net Worth} \\
\overline{V}_{\text{semi-flex}}\text{-C}(d_1 + d_2, d_2) \\
\overline{V}_{\text{semi-flex}}\text{-C}(d_1, d_2) \\
\max(\overline{V}_{\text{semi-flex}}\text{-C for (d_1+d_2,0)} \\
\overline{V}_{\text{semi-flex}}\text{-C for (d_1,d_2)}) \\
\overline{V}_{\text{semi-flex}}\text{-C for (d_1+d_2,d_2)}
\]
Volatility Effect Observations

Either $\exists \sigma^*$ such that foreign capacity investment is optimal for $\sigma > \sigma^*$ or

$\not\exists \sigma$ such that foreign capacity is optimal

With transaction (and other) costs, the critical value $\sigma^*$ declines

Allows for evaluation with bounds on costs and selling prices

Assumes stable (zero drift) exchange rate, costs, and selling prices
Optimal Capacity Investments

- Observations on value of capacity
  - Value of capacity is linear in capacity levels $k = (k_1, k_2)$ for semi-flexible case
  - In fully flexible case, $B^{-1}(h(k, d))$, where $B$ is an optimal basis at time $t$
  - Overall, integral over $t$ for value is piecewise-linear in $k$ with breakpoints at $d_1, d_2, d_1 + d_2$

- Assumptions on cost of capacity
  - Concave in $k$, e.g., fixed plus proportional

- Result:
  - Optimal capacity investments at extreme points of the demand possibilities with fixed demand
  - Alternatives for random demand (varying breakpoints)
Optimal Plant Configurations (No Switch Cost/Fixed Demand)

Domestic parent:
\[ \vec{k} \in \{(0, 0), (d_1 + d_2, 0), (d_1, d_2), (d_1 + d_2, d_2)\} \]

Two plant cases:
\[ \vec{k}^* \text{ is one of:} \]
- (0, 0)
- (\( d_1 + d_2, 0 \))
- (0, \( d_1 + d_2 \))
- (\( d_1, d_2 \))
- (\( d_2, d_1 \))
- (\( d_2, d_1 + d_2 \))
- (\( d_1 + d_2, d_1 \))
- (\( d_1 + d_2, d_1 + d_2 \)).
Operational Model \((k_2 = d_2)\)

Assumptions:

- Switching times \(\tau_i\)
- Switching amounts \(\xi_i\)
- Impulse control \(u = (\tau, \xi)\)
- \(X(t)\) amount at 2
- Dynamics: \(dX(t) = 0, \ \tau_i \leq t < \tau_{i+1}\)
- Forced switches at times \(\eta_1^i, \eta_2^i\)

Objective: maximize

\[
J^u(s, x, y) = E^{s,x,y}\left[ \int_0^\infty e^{-r(s+t)} [(-c_1 + c_2 Y_t)] X_t \, dt + \sum_{i=1}^{\sup\{i: \tau_i < \infty\}} e^{-r(s+\tau_i)} (\alpha + \beta |\xi_i|) \right]
\]
Optimal Policies

Characterization:

- The continuous-time problem has a unique viscosity solution
- The Markov chain discretization converges to the unique viscosity solution
- The discretization provides characterization of optimal policies

Policies characterized by continuation region \( D = (X^*, Y^*) \)

- Within \( D \), no un-forced switches
- When \( Y_t \) reaches boundary of \( Y^* \) given \( X_t \), switch to \( X_t \) in \( X^* \)
- Switches force \( X_t = 0 \) or \( d_2 \) (and then alternating)
- Region includes \( (0, y), y \leq Y_H \) and \( (d_2, y), y \geq Y_L \)
- Can use policy result to compute analytical value of expectation at a renewal \( X_t = 0 \) or \( d_2 \) for efficient optimization
- Can extend to fully flexible case directly and random demand (with some assumption of costs to follow demand)
Proposition 0.1 Starting at $X_0 = 0$, $0 < Y_0 < h$, the value function under policy $u(h, l)$ is given by:

$$J^{u(h,l)}(0, 0, y) = (h/y)^{-\gamma^*}(-K(d_2) + c(d_2, h) - (c(d_2, l) + K(d_2))(h/l)^{-\gamma^*})/(1 - (h/l)^{-(\gamma^* + \gamma^*)});$$

the optimal value $V(0, y)$ is given by

$$V(0, y) = \sup_{0 \leq l, y \leq h} J^{u(h,l)}(0, 0, y),$$

whenever $\sup_{0 \leq l, y \leq h} J^{u(h,l)}(0, 0, y) > \sup_{0 \leq l} J^{u(y,l)}(0, 0, y),$

where $\gamma^*$ and $\gamma_*$ are functions of rates and volatility, optimal value if attained is unique, and sensitivity to changes in costs, rates, and volatility can be explicitly obtained.
Example Results

Observation:

- Continuation region expands with higher volatility and higher transaction costs
Valuing the Alternatives

• If sufficient flexible capacity, produce in the market with favorable exchange rate
• Set thresholds for production shifts to overcome setup and changeover costs
• Shift production when limits are exceeded
• Gain: natural balance
• Cost: additional capacity and transaction
Flexible Capacity Results

- Operational flexible can result in gains from FOREX exposures
- Additional flexibility can be valued on the basis of rate volatility and changeover costs
- Risk can be reduced without relying on financial instruments (although they can be added)
Conclusions

• Risk management should include operational flexibility
• Operations can reduce risk and improve contributions
• The nature of price, demand, and exchange risks may change the value of operational risk management
• Valuations possible with respect to many types of exposures
Thank you!