Inferring Risk Preferences and Network Properties from Bids and Clearing Prices

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May 3, 2013
Solutions of optimization models are often observed while the relevant parameters are not.

Common examples include problems from network industries such as electricity.

When the data include constraint coefficients, the inverse optimization to discover the parameters is only partially identified.

Identification can be possible through norm minimization.
A Motivating Problem

Economics of Two-Stage Electricity Markets?
(Veit et al 2006; Sioshansi, Oren, O’Neill 2010; Botterud et al 2011)
Market Power In Electricity Markets?
(Cardell, Hitt, Hogan 1996; Jiang, Baldick 2005; Hogan 2012)

Results rely assumptions r.e. participant objectives:
⇒ Case Study on Wind Producer Objectives in Midwest ISO
  ▶ Two-stages: Day Ahead (DA), Real Time (RT) markets
  ▶ Forecast: Wind producers submit a DA production commitment
  ▶ Stochastic production: shortfall or surplus made up via RT prices

Wind DA Revenues:

\[ \text{Rev}(Q_{DA}) = Q_{DA} \times P_{DA} + (Q_{RT} - Q_{DA}) \times P_{RT} \]

Economic value of intermittent generation depends on forecast quality
(Gowrisankaran, Reynolds, Samano 2011; Skea, Anderson 2008 …)
Quick Look at the Data

Midwest ISO 2010 Data:
Forward Premium
\( E(P_{DA} - P_{RT}) > \$2.00 \)
But Under-commitment!

Why?
- Bad forecasting?
- Risk aversion?
- Exercise of Market Power?

Prerequisite for answer:
estimate \( E(\partial P_{DA}/\partial Q_{DA}) \) and \( E(\partial P_{RT}/\partial Q_{DA}) \)
Hard with standard econometrics due to endogeneity
Research Problem:

- Ideal solution to endogeneity question:
  
  *An accurate model of price determination process*

  - Zonal prices
    - uniform price multi-unit auction
      (c.f. Reguant 2012; Hortaçsu, Puller 2008; Wolak 2004)

- However, Locational Marginal Pricing dominate North American Markets
  (e.g. PJM, Midwest ISO, CAISO, ERCOT)

  - Prices depend on entire network structure
  - Network structure not directly observable
    
    *Critical Infrastructure Information Act (2002)*

- However, Available information:

  - Midwest ISO: prices, quantities, bids, active transmission constraints

Can a useful model of the network be inferred from this information? (Useful for market researchers, participants, *designers of CIIA*)
Roadmap

- Locational Marginal Price (LMP) Market: *Linear model*
- The Estimation Problem... *via Inverse Optimization*
- An Algorithm: *A sufficient explanation*
- Application to Data: *Midwest ISO*
Electricity Dispatch Model

Relaxation of the unit commitment problem:

Market Participant $i \in \{1..N\}$

- Produces $x_i$ MWh at an announced cost of $c_i$/MWh

**Lossless** network with links $(i,j)$

- Transmission between $i$ and $j$ of $y_{(i,j)}$ MWh

- Topology defined by matrices $E$, $A$, and $D$
  - Network Flow Constraints $Ey = x$
  - $R$ Physical Constraints $Ay = 0$
  - $L$ Transmission Constraints $Dy \leq d$

$$\begin{align*}
\text{min } c^T x \\
\text{s.t. } Ey &= x \\
Ay &= 0 \\
Dy &\leq d \\
u &\geq x \geq l \\
y &\geq 0
\end{align*}$$
Locational Marginal Prices

Definition:

1. The Locational Marginal Price (LMP) is the immediate cost of supplying one additional MW of power at a particular node.
2. The LMP is the shadow price of the flow constraint

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad Ey = x \\
& \quad Ay = 0 \\
& \quad Dy \leq d \\
& \quad u \geq x \geq l \\
& \quad y \geq 0 \\
\end{align*}
\]

dual variables

\begin{align*}
\pi \\
\sigma \\
\rho \\
\alpha, \gamma
\end{align*}
LMP Example

(Louie, Strunz 2008)

$C_A$: $20/\text{MWh}$  
$C_B$: $30/\text{MWh}$

Corresponding LP:

$$\text{min } 20x_A + 20x_B + 20x_C$$

s.t.  
$$E y = x$$
$$2y_{AB} - y_{AC} = 0$$
$$y_{BC} \leq 50$$
LMP Example

Corresponding LP:

\[
\begin{align*}
\text{min } & \quad 20x_A + 20x_B + 20x_C \\
\text{s.t.} & \quad Ey = x \\
& \quad 2y_{AB} - y_{AC} = 0 \\
& \quad y_{BC} \leq 50 \\
& \quad 0 \leq x_A \leq 500 \\
& \quad 0 \leq x_B \leq 100 \\
& \quad 0 \leq x_C \leq 500 \\
& \quad -300 \leq x_D \leq -300 \\
& \quad y \geq 0
\end{align*}
\]
LMP Example

Solution:

\[
\begin{align*}
\pi_A &= 20 \\
\pi_B &= 15 \\
\pi_C &= 25 \\
\pi_D &= 25 \\
x_A &= 150 \\
x_B &= 0 \\
x_C &= 150 \\
x_D &= -300 \\
y_{AB} &= 50 \\
y_{AC} &= 100 \\
y_{BC} &= 50 \\
y_{CD} &= 300 \\
\rho_{BC} &= -15
\end{align*}
\]
Estimation Problem (Single Sample)

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad E y = x \\
& \quad A y = 0 \\
& \quad D y \leq d \\
& \quad u \geq x \geq l \\
& \quad y \geq 0
\end{align*}
\]

dual variables

- Given data:
  - \( c, x, u, l, \pi, \rho \)

- Generate a network model: \( \bar{E}, \bar{A}, \bar{D}, \bar{d} \)

- Explaining shadow prices \( \pi \) and \( \rho \)
General Inverse Optimization

Given:

- a partial specification of an optimization model
- a (partial) specification of an optimal solution

Infer missing model parameters such that:

- Consistency: Known Parameters consistent with optimality
- Simplicity: Missing parameters minimize a norm

Standard form Zhang, Liu 1996, 1999 (linear); Ahuja, Orlin 2001 (general):

- Feasible set known
- Opt. solution known
- Cost parameters unknown
- Minimize $L_1$ or $L_\infty$ norm


\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad E y = x \\
& \quad A y = 0 \\
& \quad D y \leq d \\
& \quad \pi \\
& \quad \sigma \\
& \quad \rho
\end{align*}
\]
Our Inverse Optimization Problem

**Standard form:**
- Feasible set known
- Opt. solution known
- Cost parameters unknown
- Minimize L1 norm

**Electricity Market Problem:**
- Feasible set unknown
- Opt. solution partially known
- Cost parameters known
- Minimize L1 norm

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad Ey = x \\
& \quad Ay = 0 \\
& \quad Dy \leq d \\
& \quad u \geq x \geq l \\
& \quad y \geq 0
\end{align*}
\]

\[
\begin{align*}
\min & \quad c^T x \\
\text{s.t.} & \quad Ey = x \\
& \quad Ay = 0 \\
& \quad Dy \leq d \\
& \quad u \geq x \geq l \\
& \quad y \geq 0
\end{align*}
\]
Our Inverse Optimization Problem

- Find a “simplest” \( \bar{A}, \bar{D}, \bar{d}, \bar{E} \) satisfying optimality conditions
- Minimize 1-Norm
- Regularize \( \bar{A} \) s.t. \( \sigma_r = 1 \)

Resulting Optimization Problem:

\[
\min ||\bar{A}||_1 + ||\bar{D}||_1 + ||\bar{d}||_1 \\
\sum_{r<R} \bar{A}_{r(i,j)} + \sum_{\ell<L} \bar{D}_{\ell(i,j)} \rho(i,j) = \pi_j - \pi_i \\
\forall (i,j) \bar{y}_{ij} > 0
\]

\[
\bar{E} \bar{y} = x \\
\bar{A} \bar{y} = 0 \\
\bar{D} \bar{y} \leq \bar{d}
\]

2-Step Algorithm:

1. Find \( \bar{E} \) and \( \bar{y} \):
2. Find \( \bar{A}, \bar{D}, \bar{d} \):
Step 1: Determine $\bar{E}$ and $\bar{y}$

- Assume no loss
  $\bar{E}_{k(ij)} \in \{-1, 0, 1\}$

- Limit to flows between sources and sinks
  $\bar{E}_{i(ij)} = 1$ only if $x_i > 0$
  $\bar{E}_{j(ij)} = -1$ only if $x_j < 0$
  $\bar{E}_{k(ij)} = 0$ otherwise

- Minimize requirements on $\bar{A}$ and $\bar{D}$ to satisfy
  \[ \sum_r \bar{A}_{r(i,j)} + \sum_\ell \bar{D}_{\ell(i,j)} \rho(i,j) = \pi_j - \pi_i \quad \forall \bar{y}_{ij} > 0 \]

- By solving:
  \[ \min \sum_{ij} \bar{y}_{ij} \cdot \max\{\pi_i - \pi_j, 0\} \]
  \[ \text{s.t. } \bar{E}\bar{y} = x \]
  \[ \bar{y}_{ij} \geq 0 \]
Step 1: Determine \( \bar{E} \) and \( \bar{y} \)

- Assume no loss
  \( \bar{E}_{k(ij)} \in \{-1, 0, 1\} \)

- Limit to paths between sources and sinks
  \( \bar{E}_{i(ij)} = 1 \) only if \( x_i > 0 \),
  \( \bar{E}_{j(ij)} = -1 \) only if \( x_j < 0 \),
  \( \bar{E}_{k(ij)} = 0 \) otherwise

- Minimize requirements on \( \bar{A} \) and \( \bar{D} \) to satisfy

\[
\sum_r \bar{A}_{r(ij)} + \sum_{\ell} \bar{D}_{\ell(ij)} \rho(i,j) = \pi_j - \pi_i \quad \forall \bar{y}_{ij} > 0
\]

- By solving:

\[
\min y_{ij} \cdot \max\{\pi_i - \pi_j, 0\}
\]

s.t. \( \bar{E}\bar{y} = x \)

\( \bar{y}_{ij} \geq 0 \)
Step 2: Determine $\bar{A}$, $\bar{D}$, $\bar{d}$

Minimize sum of 1-norms subject to optimality constraints
Solving:

$$
\min \sum_{r,(i,j)} |\bar{A}_{r(ij)}| + \sum_{r,(i,j)} |\bar{D}_{\ell(ij)}| + \sum_{\ell} \bar{d}_{\ell}
$$

s.t. $\bar{A}\bar{y} = 0$

$$
\sum_{r} \bar{A}_{r(i,j)} + \sum_{\ell} \bar{D}_{\ell(i,j)} \rho(i,j) = \pi_j - \pi_i \quad \forall \bar{y}_{ij} > 0
$$

$d_{\ell} \geq 0$

A and D Constraints:

$$
\frac{1}{3} \bar{Y}_{AD} \leq 50
$$
Multiple Samples

For set of samples 1..S

1. Calculate $\bar{y}^s$ independently
2. Add constraints for each sample

$$\min \sum_{r, (i, j)} |\bar{A}_{r(ij)}| + \sum_{r, (i, j)} |\bar{D}_{\ell(ij)}| + \sum_{\ell} \bar{d}_\ell$$

s.t. $A\bar{y}^s = 0 \quad \forall s$

$$\sum_{r} \bar{A}_{r(i,j)} + \sum_{\ell} \bar{D}_{\ell(i,j)}\rho_{(i,j)}^s = \pi_j^s - \pi_i^s \quad \forall \bar{y}_{ij}^s > 0$$

$$\bar{d}_\ell \geq 0$$

A and D Constraints:

$$\frac{1}{3}(\bar{y}_{AD} - 2\bar{y}_{AB}) \leq 50$$
General Implementation

- Observations for each day and hour
  - \( x, \pi, \rho, c \) vary
  - \( E, D, A, d \) constant (approximately)

- Transmission and generation outages may change \( d \)
- Transmission losses not included
- Original “optimization" may be adjusted for other reasons (e.g., frequency, reliability)
Algorithm Observations

- Extension with multiple observations
  - Step 1 performed independently
  - Step 2 single optimization adding all constraints
- Algorithm feasible if rows in $A$ greater number of samples
- Polynomial approximation
  - Step 2: LP standard transformation
  - Each step presents $O(n^2)$ variables
Results: Application to Midwest ISO

2010/01/01 00:00:00

- 1403 Nodes
- 768 Aggregated Nodes
- 772 Active Links
- Transmission bounds per hour
- Imperfect data
- Naive implementation
  20 ARows → 40min
Additional Examples

- Macro-economic models
  - Prices observed in different regions
  - Purchase quantities observed
  - Equilibrium model set up as potential optimization
  - Unknown transportation routes and costs to discover

- Supply chain interactions
  - Prices and quantities observed
  - Unobserved relationships between suppliers and customers
  - Discover relationships and transactions
Future Directions

- Discussed modelling price determination process in LMP based Electricity Markets
  - Inverse optimization based formulation/algorithm consistent with dual interpretation of LMPs

Next Steps

- Predicting market characteristics: *price response, congestion costs...*
- Structural estimation: *model participant decision making*
- Solution quality: *solution robustness to data imperfections*
- Econometrics of general competitive markets: *extend to linear market models*
Conclusions

- Inverse optimization to discover constraints
  - Needed to determine objective of market participants
  - Many markets include price and quantity observations but not constraints
  - Difficulty from bilinear form with constraint and unknown variable values

- Solution method
  - Two-step process
  - Determine consistent primal variables first
  - Choose constraint coefficients with minimum 1-norm

- Results
  - Possible to discover simple network configurations
  - Reasonable results with multiple data observations
  - Possible inconsistencies from unknown parameter changes