Risk Metrics in Supply Chain Management

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Themes

• Supply chain risk considerations allow for many interpretations
• Different approaches may lead to the same outcomes
• Consistency is in the eye of the beholder (but not always in the model)
• Consistency can appear in different ways
• Actual preferences may support “consistent” models that violate reasonable “axioms”
Outline

• Probabilistic (“robust”) and expected utility measures
• Consistent views of alternatives
• Axioms and principles for model selection and solution
• Behavioral violations and their implications
The Debates

Old World

• Expected Utility (Recourse Model)

• Probabilistic (Chance-constrained) Model

New World

• Coherent Risk Models

• Value-at-Risk and Robust Optimization Models
Mathematical Forms

Expected utility:
\[ \max_x E(f(x, \xi)) \]

(Coherent) Risk:
\[ \max_x (-)R(g(x, \xi)) \]

Prob. constraints:
\[ \max_x g(x) \quad s.t. \quad P(h(x, \xi) \geq 0) \geq \alpha \]

Robust opt.:
\[ \max_x g(x) \quad s.t. \quad h(x, \xi) \geq 0 \quad \forall \xi \in \Xi \]
Issues in Formulations

• What are the parameters? \((f, g, h, R, \Xi)\)

• Can all the models be consistent? (all “right”?)

• Is there a “best”? How to choose?
Misinterpretations

• Objective functions:
  – \( f, g (h) \) are the same in each model

• Probability distribution:
  – \( P, \Xi \) must be known with certainty

• Results of Model X are inconsistent with each rationality or behavior
Forms of Resolution

Make models look the same:

**Robust/Prob => Risk:** Let \( R(g'(x, \xi)) = g(x) + \delta(x|h(x, \xi)) \leq 0 \forall \xi \in \Xi \)

\[
\max_{x \in X} R(g'(x, \xi)) \iff \max_{x \in X} g(x) | h(x, \xi) \leq 0 \forall \xi \in \Xi
\]

**Risk=>Robust:** If \( R \) is coherent, \( \exists \mathcal{P}(R) \) domain of \( P \),

\[
h(x,g,P) = g - E_P(g(x, \xi)) \Rightarrow g(x, P) = E_P[f(x, \xi)]
\]

\[
\max_{x \in X} R(g(x, \xi)) \iff \max_{x \in X} g | h(x,g,P) \leq 0 \forall P \in \mathcal{P}
\]

Others: equivalence of solutions (almost always possible)
How to Choose?

• Potential criteria:
  – Self-consistency
  – Preference principles (axioms)
  – Tractability
  – Empirical evidence

• Consensus possible?
Self-Consistency

• Principle: the model should have self-consistent preference rankings
• Examples: if criteria are expected utilities, then rankings of equivalent outcomes should not vary
• Market interpretation: prices in the model should be consistent with criteria (e.g., no arbitrage => risk-neutral equivalence)
• In supply chains, price effects are consistent with equilibrium, costs are consistent with market, and incentives are compatible
Principles/Axioms: Coherent and Rational Risk Measures

• $R$ is a coherent (negative) risk measure if
  – $R$ is concave and increasing (monotonic)
  – $R(g(x, \xi) + a) = R(g(x, \xi)) + a$, $a \in \mathbb{R}$ (translation invariant)
  – $R(\lambda g(x, \xi)) = \lambda R(g(x, \xi)) \forall \lambda \geq 0$ (positive homogeneous)

Von Neumann-Morgenstern (rational) utility:

Complete, Transitive, Continuous, Independent
Axiom Implications

• Coherent risk measures:

\[ R(g(x, \xi)) = \min_{P \in \mathcal{P}} \mathbb{E}_P[g(x, \xi)] \]

Similar to Maxmin Expected Utility

May preserve higher orders of stochastic dominance.

• vNM utility:

\[ R(g(x, \xi)) = \mathbb{E}_P[f(x, \xi)] \]
What does not fit axioms?

- **Mean-variance or mean-standard deviation**
  - Not monotonic
  \[ E(g(x, \xi)) - \gamma (\text{Var}(g(x, \xi)))^{0.5} \]

- **Value-at-Risk**
  - Not convex
  \[ \alpha \text{-VaR}(g(x, \xi)) = \min \{ t \mid P(g(x, \xi) \leq t) \geq \alpha \} \]
  (e.g., \( X/Y = 0 \) w.p. 0.95, -1 w.p. 0.025, \( P(X=Y=-1)=0 \)
  \( 0.05-\text{Var}(X)=0.05-\text{VaR}(Y)=0; 0.05-\text{VaR}(0.5(X+Y))=-0.5 \))
What does fit?

- Coherent objectives:
  - Semi-deviations: $R(X) = E[((t-E(X))^p)^(1/p)]$
  - Conditional tail expectation/CVaR:
    $R(X) = E[t | t \leq \alpha - \text{Var}(X)]$
    LP form: $R(X) = \min_t \{t - (1/\alpha) E[(t - X)^-]\}$
- Other tail expectations
Inconsistent Information Issues with VaR Forms?

Example: Value of Information in “Blau’s dilemma”
Suppose demand $b=0$ w.p. 0.9 and $l$ w.p. 0.1
Problem:

$$\min x \text{ s.t. } 0.9 - \text{VaR}(b) \leq x \text{ or } P[x \geq b] \geq 0.9$$

Solution: $x^* = 0$

With perfect information: $x^P = 0$ w.p. 0.9 and $l$ w.p. 0.1
EVPI = Exp. Value without Perfect Information – Exp. Value with Perfect Information

$$= 0 - 0.1 = -0.1 < 0$$

(Same may be true with EVSampleInformation)

For RO, let $\Xi = \{b \mid P[b] \geq 0.9\} = \{0\}$
Problems with “Paradox”

• Utility may depend on information level
  – With no information, 0.9 may be acceptable but not the same with more information
  – Cannot make direct comparisons in information value

• Not including role of competitor
  – Competitor may gain information as well
  – In this case, more information may not always be beneficial
Reasons for Choice: Tractability

• Choose model so that we can solve it
• Computational problems:
  – Probabilistic constraints may give non-convexity but RO formulations allow convex problems
  – Non-coherent risk measures may be non-convex (but may be more robust to estimation problems)
  – Non-convexity makes solutions difficult
Toward a Consistent View: Competition

- Suppose (a) competitor(s) choose(s) $y(x, \xi)$ to maximize $c(x, y, \xi)$
- Formulation:
  $$\min_{x \in X} E_P [f(x, y, \xi) | y \in \arg\max c(x, y, \xi)]$$
- $y$ fixed (or $f$ independent of $y$) $\Rightarrow$ EU
- $y = \xi \in \Xi, f(x, y, \xi) = c(x, y, \xi) = g(x, \xi) \Rightarrow$ RO
- EU assumes irrelevant adversary
- RO assumes perfect adversary
- Coherent risk (MEU) assume constrained adversary
Evidence on Preferences?

- *Expected utility* maximizing? (Maybe)
- Stochastic order on outcomes? (Usually)
- Probabilistic measure on outcomes (Maybe)
- Worst-case (or worst with some probability) measure on outcomes (*robust measures*) (Maybe)
All True?

- What is observed? (e.g., Kahnemann-Tversky prospect theory)
  - Targets define utility
  - Preference depends on closeness to targets
  - Local properties as in models

Form for RO/Prob constrained

Small prob.
Combining: When to Use What?

• Risk-neutral expectation
  – Complete markets (after transformation) and discounting

• Traditional expected utility
  – Can define function, incomplete market (keep consistent)

• “Worst-case” robust or given probability
  – Little information, only survivability counts

• Competition and distribution domains
  – Allows consistent view from risk-neutral to “worst case”; can be tractable
Further Implications and Limitations

• In models:
  – Adjust all probabilities to consistency with market
  – Optimize with risk-neutral expectations

• What if market is not complete?
  – Choice of probabilities
  – Alternative decisions and values consistent with market (in some range)
  – Which one to pick?
    • Be consistent with preference of decision maker
    • Generate alternatives
    • Find range of consistent decisions and objectives