Investment and Production Decisions with Exchange Risk

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Motivation

- Operations (e.g., flexible production, foreign production) can mitigate the effects of demand, price, and currency exchange risks
- Financial instruments also can reduce risks (but should have zero NPVs)
- Questions: what is the value of operational methods and how do they interact with financial methods?
Outline

• Preliminary discussion of “hedging”
• Specific case in foreign exchange
• Value calculations
• Investment problem solutions
• Operational policies
• Conclusions
Preliminary Discussion: Hedging

• Definition here: reducing risk (volatility)
• Alternative interpretations:
  – Only reducing risk without affecting mean values
  – Using “hedging” instruments (e.g., derivatives): financial hedging
• Some results (e.g., Chowdhry and Howe 1999):
  – Operational hedging has value over financial hedging because of flexibility in output and correlation between demand and prices (examples later)
Risk Management and Hedging

• What is a hedge?
  – Action designed to reduce risk of future outcome
  – In finance, perfect hedge leads to no risk (riskfree return)

• Use of hedges
  – Allow pricing of financial derivatives
  – Lead to markets in derivatives
  – Also possible with operations (operational hedges)
    • Quantity - flexible production
    • Timing
Who Should Hedge?

• Farmers?
• Situation:
  – Suppose either high-yield low-yield years for crops
  – Prices down in high years and up in the low years
Farmer’s Example

• Suppose yield of corn is either 200 k-bushels (high) or 100 k-bushels (low)
• Suppose price with high yield is $1 and price with low yield is $2
• Should the farmer use financial hedge? i.e., sell a future?
  – If so, how much?
Futures Contracts as Hedges

- *Futures contract*: an agreement to buy or sell a fixed quantity at given price at fixed time in future (marked to market every day)
- Example: can agree to sell 100 k-bushels at $1.50/bushel on October 15
- On October 15, we receive $150K and must deliver 100 k-bushels
Futures for the Farmer

• Advantages
  – Can accept the expected price now
  – No risk in the price for the amount we sell

• Potential problems
  – Risk on amount we can produce
  – May have to go into market

• Analysis: Hedge our expected yield (150 k-bushels)

  Guaranteed (all the time)          $225K
  High yield – can sell 50 more      + $50K (probability ½)
  Low yield – must buy 50           -$100K (probability ½)

  Expectation=225+50/2-100/2= $200k (same as no hedge)
  BUT variance (risk) is up (either $275k or $125 instead of $200k all the time)

• RESULT: should not use futures (alone)
Farmer’s Operational Hedge for Risk Management

• What else does the farmer have?
• **SILO!!**
  – *Operational hedge*
  – *Keep corn from high yield to sell at low yield*

• Now, suppose we keep 50 k-bushels in silo from high to low yield years
Farmer’s Silo “Hedge”

• Expected returns
  – High-yield years (prob. ½) $150 k
  – Low-yield years (prob. ½) $300 k
  – Expectation: ½(150+300)= $225k
  – Worth $225k-200k =$25k to use the silo
  – Value of the operational instrument (*option value of silo storage*)

• Combine with future?
  – Now, sell 150 k-bushels for $1.50 in October
  – Now, have the return guaranteed $225K

• Moral: Financial instrument only has value if farmer uses operational instrument
Copper Miner’s Example

- Should a copper mine hedge its output with futures?
- What is the nature of copper price differences?
- Demand versus supply curve change means high price-high quantity and low price-low quantity
Copper Hedging

• Suppose high demand leads to 200 k-pounds at $2/pound and low demand leads to 100 k-pounds at $1/pound
• Earn $400k (prob. ½) or $100k (prob. ½)
• Expected value of $250k
• Operational hedge? (save 50 k-lbs from high to low years – sell forward to customers)
  – High years: earn $300k (prob. ½)
  – Low years: earn $150k (prob. ½)
  – Expectation: $225k (lower value!)
Copper Futures?

• Suppose we sell all our customers forward contracts for 150 k-lbs at $1.50 in future

• Result now:
  – Guaranteed return: $225k
  – Risk reduced to 0

• Here: financial derivatives reduce risk but return is still down
Model for Single Period

• Suppose:
  – Price: \( p(\omega) \)
  – Cost: \( c \)
  – Max production: \( l+kp(\omega) \) \((k>0 \text{ or } <0)\)
  – Decision: \( \alpha \) (fraction of customers to sell forward)

• Objective
  \[
  \text{Max}_{0 \leq \alpha \leq 1} \left( \alpha(E(p)-c)(E(l+kp)) + (1-\alpha)E[(p-c)(l+kp)] \right) \\
  \iff \text{Max}_{0 \leq \alpha \leq 1} \left( \alpha k[E(p)^2-E(p^2)]+E(p)l+kE(p^2)-c(1+kE(p)) \right) \\
  = k[E(p)^2-E(p^2)]+E(p)l+kE(p^2)-c(1+kE(p))n \text{ if } k \leq 0 \text{ and} \\
  E(p)l+kE(p^2)-c(1+kE(p))n \text{ if } k \geq 0
  \]
  So, sell all forward if negative correlation and sell none forward if positive correlation (assuming this is possible).
Overall Observations

• Farmer:
  – Sell forward using operational instrument (storage)

• Miner:
  – Sell at spot price

• Financial instruments can reduce risk (but careful on use)

• Next: dynamic model; focus on the value addition over hedging
Foreign Currency Exchange Issues

• Foreign exchange risk costly (e.g., Laker/LVMH)
• Operational hedges may be valuable
• Supplier risk also important

Questions:
  – What is the value of foreign capacity?
  – What are alternatives for optimal capacity configurations?
  – What is an optimal operating policy?
  – How can values inform decisions without complete distribution information?
Capacity Alternatives

Domestic Investment
No flexibility

Domestic Parent Plant,
Foreign Subsidiary
Semi flexible

Two Plant Model
Fully flexible
**Notation**

- $i =$ index of the originating market, 1 domestic, 2 foreign;
- $j =$ index of the destination market, 1 domestic, 2 foreign;
- $r =$ domestic risk free rate;
- $r_f =$ foreign risk free rate;
- $s_i =$ sales price (in respective currency) of the product in country $i$;
- $c_i =$ production cost of the product in country $i$;
- $d_i =$ demand for the product in country $i$;
- $k_i =$ production capacity (in units) in country $i$, $\vec{k} = (k_1, k_2)$ is the investment vector;
- $Y_t =$ Foreign exchange rate given by (dom. currency)/(for. currency);
- $X_{ij}(t), i,j = 1,2$ - the amount of product produced in market $i$ and sold at market $j$ at time $t$.
- $Z_i^i = 1$ if capacity in market $i$ is available, $= 0$ if not available.
Assumptions

- Continuous monitoring and instantaneous shifts (if not, put in switching costs)
- No inventory holding (contracted or spot market)
- Focus on stochastic exchange rate - other data known
- Fixed same-country margin (contract prices and costs, but can include transportation costs)
- Positive margins if in same country
- No shutdown or abandonment
Valuation

• No flexibility
  – Stream of cash flows in two currencies

• Semi-flexibility
  – Continuous option to use lowest-cost source
  – Integral of option value over time gives value of flexibility

• Full flexibility
  – Option value over solution of transportation problem at each point in time
  – Integration over time of parametric LP solution
Value Calculations

No Flexibility:

\[
V_{\text{noflex}} = \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( (s_1 - c_1)d_1 + (s_2 Y_t - c_1)d_2 \right) Z_t^1 dt \right]
\]

Semi-flexibility \((k_1 \geq d_1 + d_2)\):

Instantaneous cash flow:

\[
p(t) = (s_1 - c_2)d_1 Z_t^1 + \max (d_2(s_2 Y_t - c_1)Z_t^1, (d_2 - k_2)(s_2 Y_t - c_1)Z_t^1 + k_2 Y_t(s_2 - c_2)Z_t^2)
\]

\[
V_{\text{semi-flex}}(\bar{k}) = \mathbb{E} \left[ \int_0^\infty e^{-rt} p(t) dt \right]
= \mathbb{E} \left[ \int_0^\infty e^{-rt} \left( (s_1 - c_1)d_1 + d_2(s_2 Y_t - c_1) \right) Z_t^1 + k_2c_2 \max (0, \frac{c_1}{c_2} - Y_t) Z_t^2 \right] dt
\]
**Full-Flexible Capacity Valuation**

\[ \text{Max} \quad (s_1 - c_1)X_{11} + (s_2Y_t - c_1)X_{12} + (s_1 - c_2Y_t)X_{21} + (s_2 - c_2)Y_tX_{22} \]

\[ \text{s.t.} \quad X_{11} + X_{12} \leq k_1; \quad X_{21} + X_{22} \leq k_2; \]

\[ X_{11} + X_{21} = d_1; \quad X_{12} + X_{22} = d_2; \]

\[ X_{ij} \geq 0, \text{ for all } i \text{ and } j. \]

**Solution:**

- Ordering of margins determines possible set of optimal bases; Maximum-margin allocation optimal
- Each ordering corresponds to 3 potential optimal bases; at most 5 optimal bases for given \( c_i, s_i \)
- Can compute analytically for each of the 5 bases as time-integrated continuum of options
- Can include transportation costs (with potentially more breakpoints of optimal bases)
- Random demand also possible with additional integration
Volatility Effect

\[ \sigma \]
\[ \text{Net Worth} \]
\[ \sigma' \]
\[ \max(\bar{V}_{\text{semi-flex}} - C \text{ for } (d_1 + d_2, 0)) \]
\[ \bar{V}_{\text{semi-flex}} - C \text{ for } (d_1, d_2) \]
\[ \bar{V}_{\text{semi-flex}} - C \text{ for } (d_1 + d_2, d_2) \]
Volatility Effect Observations

Either $\exists \sigma^*$ such that foreign capacity investment is optimal for $\sigma > \sigma^*$

or

$\nexists \sigma$ such that foreign capacity is optimal

With transaction (and other) costs, the critical value $\sigma^*$ declines

Allows for evaluation with bounds on costs and selling prices

Assumes stable (zero drift) exchange rate, costs, and selling prices
Optimal Capacity Investments

- Observations on value of capacity
  - Value of capacity is linear in capacity levels $k = (k_1, k_2)$ for semi-flexible case
  - In fully flexible case, $B^{-1}(h(k, d))$, where $B$ is an optimal basis at time $t$
  - Overall, integral over $t$ for value is piecewise-linear in $k$ with breakpoints at $d_1, d_2, d_1 + d_2$

- Assumptions on cost of capacity
  - Concave in $k$, e.g., fixed plus proportional

- Result:
  - Optimal capacity investments at extreme points of the demand possibilities with fixed demand
  - Alternatives for random demand (varying breakpoints)
Optimal Plant Configurations (No Switch Cost/Fixed Demand)

Domestic parent:
\[ \vec{k} \in \{(0, 0), (d_1 + d_2, 0), (d_1, d_2), (d_1 + d_2, d_2)\} \]

Two plant cases:
\[ \vec{k}^* \text{ is one of:} \]
- \( (0, 0) \)
- \( (d_1 + d_2, 0) \)
- \( (0, d_1 + d_2) \)
- \( (d_1, d_2) \)
- \( (d_2, d_1) \)
- \( (d_2, d_1 + d_2) \)
- \( (d_1 + d_2, d_1) \)
- \( (d_1 + d_2, d_1 + d_2) \).
Operational Model ($k_2 = d_2$)

Assumptions:

- Switching times $\tau_i$
- Switching amounts $\xi_i$
- Impulse control $u = (\tau, \xi)$
- $X(t)$ amount at 2
- Dynamics: $dX(t) = 0$, $\tau_i \leq t < \tau_{i+1}$
- Forced switches at times $\eta_i^1, \eta_i^2$

Objective: maximize

$$J^u(s, x, y) = E^{s,x,y} \left[ \int_0^\infty e^{-r(s+t)} \left[ (-c_1 + c_2Y_t) X_t dt + \sum_{i=1}^{\sup\{i: \tau_i < \infty\}} e^{-r(s+\tau_i)} (\alpha + \beta |\xi_i|) \right] \right]$$
Optimal Policies

Characterization:

- The continuous-time problem has a unique viscosity solution
- The Markov chain discretization converges to the unique viscosity solution
- The discretization provides characterization of optimal policies

Policies characterized by continuation region $D = (X^*, Y^*)$

- Within $D$, no un-forced switches
- When $Y_t$ reaches boundary of $Y^*$ given $X_t$, switch to $X_t$ in $X^*$
- Switches force $X_t = 0$ or $d_2$ (and then alternating)
- Region includes $(0, y), y \leq Y_H$ and $(d_2, y), y \geq Y_L$
- Can use policy result to compute analytical value of expectation at a renewal $X_t = 0$ or $d_2$ for efficient optimization
- Can extend to fully flexible case directly and random demand (with some assumption of costs to follow demand)
Example Results

Observation:

- Continuation region expands with higher volatility and higher transaction costs

| Base | 0.3704 | 0.3817 | 0.3933 | 0.4053 | 0.4176 | 0.4304 | 0.4435 | 0.4570 | 0.4709 | 0.4852 | 0.5 | 0.5152 | 0.5309 | 0.5471 | 0.5637 | 0.5808 | 0.5986 | 0.6168 |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----|--------|--------|--------|--------|--------|--------|--------|--------|
| 0    | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.05 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.1  | -3     | -3     | -3     | -3     | -3     | 0      | 0      | 0      | 0      | 0      | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.15 | -4     | -4     | -4     | -4     | -4     | -4     | 0      | 0      | 0      | 0      | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.2  | -5     | -5     | -5     | -5     | -5     | -5     | -5     | 0      | 0      | 0      | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.25 | -6     | -6     | -6     | -6     | -6     | -6     | -6     | -6     | 0      | 0      | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.3  | -7     | -7     | -7     | -7     | -7     | -7     | -7     | -7     | -7     | 0      | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.35 | -8     | -8     | -8     | -8     | -8     | -8     | -8     | -8     | -8     | -8     | 0   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.4  | -9     | -9     | -9     | -9     | -9     | -9     | -9     | -9     | -9     | -9     | -9   | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.45 | -10    | -10    | -10    | -10    | -10    | -10    | -10    | -10    | -10    | -10    | -10  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.5  | -11    | -11    | -11    | -11    | -11    | -11    | -11    | -11    | -11    | -11    | -11  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.55 | -12    | -12    | -12    | -12    | -12    | -12    | -12    | -12    | -12    | -12    | -12  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.6  | -13    | -13    | -13    | -13    | -13    | -13    | -13    | -13    | -13    | -13    | -13  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.65 | -14    | -14    | -14    | -14    | -14    | -14    | -14    | -14    | -14    | -14    | -14  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.75 | -16    | -16    | -16    | -16    | -16    | -16    | -16    | -16    | -16    | -16    | -16  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.8  | -17    | -17    | -17    | -17    | -17    | -17    | -17    | -17    | -17    | -17    | -17  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.85 | -18    | -18    | -18    | -18    | -18    | -18    | -18    | -18    | -18    | -18    | -18  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.9  | -19    | -19    | -19    | -19    | -19    | -19    | -19    | -19    | -19    | -19    | -19  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 0.95 | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
| 1    | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20    | -20  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      |
Conclusions and Next Steps

Results:

- Valuation of flexible resources with unreliable suppliers
- Characterization of optimal capacity configurations
- Analytical form for calculating value of different configurations
- Discrete convergence to obtain general operational policy characterization
- Maximum value on foreign capacity over all exchange rate volatilities

Result: with a small number of computations, can determine maximum values on configurations over multiple parameter ranges and critical levels of relevant parameters

Next steps:

- Demand uncertainty with switching costs
- Supplier reliability (e.g., delay/quality) and foreign investment
- Empirical verification of implications (e.g., effect of exchange rate volatility on investment)
- Extensions to additional markets