Financial Engineering and the Effects of the Credit Crisis

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Introduction

• History of financial engineering
  – Rapid expansion of derivative market (total now greater than global equity)
  – Rise in successful quantitative investors (e.g., hedge funds)
  – Applications in asset management and risk management
  – Dot-com boom market
  – Securitization, housing bubble, and current crisis

• Current situation
  – Overall consolidation in the industry
  – Maintaining asset management and risk management interest
Presentation Outline

• Selected applications
  • Option pricing
  • Portfolio/asset-liability models
  • Tracking and trading
  • Securitization and its role in the crisis
  • Risk management/real options and going forward

• Future potential
Option Models

- “Derivative” securities
  - Example: Call: Buy a share at a given price at a specific time (European)
    - If by a specific time - American
  - Put: Sell; Straddle: Buy or sell

- Why?
  - Reduce risk (hedge)
  - Speculate
  - Arbitrage

- Original analysis - L. Bachelier (1900 - Brownian motion)
Results on European Options

• Black-Scholes-Merton formula

• Put-call parity for exercise price $K$ and expiration $T$
  
  $C_t - P_t = S_t - e^{-r(T-t)}K$

American options:

• Can exercise before $T$

• No parity

• Calls not exercised early if no dividend

• Puts have value of early exercise
American Option Complications

• American options
  – Decision at all t - exercise or not?

• Find best time to exercise (optimize!)

![Graph showing price over time with decision points for exercise.]
American Options

• Difficult to value because:
  – Option can be exercised at any time
  – Value depends on entire sample path not just state (current price)

• Model (stopping problem):
  \[
  \max_{0 \leq t \leq T} e^{-rt} V_t(S_{0t})
  \]

• Approaches:
  – Linear programming, linear complementarity, dynamic programming
Formulating as Linear Program

- At each stage, can either exercise or not

\[ V_t(S) \geq K - S \text{ and } e^{-r\delta} (pV_{t+\delta} uS + (1-p)V_{t+\delta} dS) \]

If minimize over all \( V_t(S) \) subject to these bounds, then find the optimal value.

- Linear program formulation (binomial model)

\[
\begin{align*}
\text{min } & \sum_t \sum_{kt} V_{t,kt} \\
\text{s. t. } & V_{t,kt} \geq K - S_{t,kt}, \quad t=0,\delta,2\delta,\ldots,T; \quad V_{T,kT} \geq 0 \\
& V_{t,kt} \geq e^{-r\delta} (pV_{t+\delta,u(kt)} + (1-p)V_{t+\delta,d(kt)}) \\
& t=0,\delta,2\delta,\ldots,T-1; \quad kt=1,\ldots,t+1; \\
& S_{t+\delta}(U(kt)) = uS(kt); \\
& S_{t+\delta}(D(kt)) = dS(kt); \quad S_{0,1} = S(0). 
\end{align*}
\]

Result: can find the value in a single linear program
Extensions of LP Formulation

• General model:
  – Find a value function $v$ to
    $$\min \langle C, V \rangle \text{ s.t. } V_t(S_t) \geq (K-S_t)^+, \quad \mathcal{L}V + (\partial V/\partial t) \geq 0, \quad V_T(S_T) = (K-S_T)^+$$
    where $C>0$ and $\mathcal{L}$ denotes the Black-Scholes operator for price changes on a European option.

• Can consider in linear complementarity framework

• Solve with various discretizations
  – Finite differences
  – Finite element methods
General Option Pricing
Applications: Implied Trees

• Basic Idea:
  – Assume a discrete representation of the price dynamics (often binomial) but not with associated probabilities
  – Observe prices of all assets associated with this tree of sample paths (and imply probabilities)
  – Find price for new claim (or check on consistency of option in market)

• Methodology:
  – Minimize deviations in prices or maximize/minimize price subject to fitting different set of prices (linear programming)
Finding Implied Trees

• Given call prices \((\text{Call}(K_i, T_i))\) at exercise prices \(K_i\) and maturities \(T_i\) (assuming risk-neutral pricing)

• Find probabilities \(P_j\) on branches \(j\) to:

\[
\min \sum_i (u_i^+ + u_i^-)
\]

s.t.
\[
\sum_j P_j (S_j - K_i) + u_i^+ - u_i^- = \text{FV(Call}(K_i, T_i))
\]

\[
\sum_j P_j S_j = \text{FV}(S_t)
\]

\[
\sum_j P_j = 1, P_j \geq 0.
\]
OUTLINE

• Applications
  • Option pricing
    • Portfolio/asset-liability models
  • Tracking and trading
  • Securitization
  • Risk management/Real options
• Future Potential
Overview of Approaches

• General problem
  – How to allocate assets (and accept liabilities) over time?
  – Uses: financial institutions, pensions, endowments

• Methods
  – Static methods and extensions:
  – Dynamic extensions of static
  – Portfolio replication (duration matching)
  – DP policy based
  – Stochastic program based
Static Portfolio Model

Traditional model

– Choose portfolio to minimize risk for a given return

– Find the efficient frontier

**Quadratic program** (Markowitz):

find investments \( x=(x(1),...,x(n)) \) to

\[
\min x^T Q x \\
\text{s.t. } r^T x = \text{target}, \ e^T x=1, \ x\geq0.
\]
Static Model Results

For a given set of assets, find

- fixed percentages to invest in each asset
- maintain same percentage over time
- implies trading but gains over “buy-and-hold”

Needs

- rebalance as returns vary
- cash to meet obligations

Problems

- transaction costs
- cannot lock in gains
- tax effects
Static Asset and Liability Matching: Duration +

- Idea: Find a set of assets to match liabilities (often WRT interest rate changes)
  - Duration (first derivative) and convexity (second derivative) matching

- Formulation:
  Given duration \( d \), convexity \( v \) and maturity \( m \) of target security or liability pool, find investment levels \( x_i \) in assets of cost \( c_i \) to:

\[
\min \sum_i c_i \ x_i \\
\text{s.t. } \sum_i d_i \ x_i = d; \sum_i v_i \ x_i = v; \sum_i m_i \ x_i = m; \ x_i \geq 0, \ i = 1 \ldots n
\]

- Extensions:
  - Put in scenarios for the durations.. extend their application

- Problems:
  - Maintaining position over time
  - Asymmetry in reactions to changing (non-parallel yield curve shifts)
  - Assumes assets and liabilities face same risk
Extension to Liability Matching

• Idea (Black et al.)
  – Best thing is to match each liability with asset
  – Implies bonds for matching pension liabilities

• Formulation:
  Suppose liabilities are \( l_t \) at time and asset \( i \) has cash flow \( f_{it} \) at time, then the problem is:
  \[
  \min \sum_i c_i \, x_i \\
  \text{s.t. } \sum_i f_{it} \, x_i = l_t \text{ all } t; \ x_i \geq 0, \ i = 1\ldots n
  \]

• Advantages:
  – Liabilities matched over time
  – Can respond to changing yield curve

• Disadvantages
  – Still assumes same risk exposure
  – Does not allow for mix changes over time
Further Extensions to Liability Matching

• Include scenarios s for possible future liabilities and asset returns

• Formulation:

\[ \min \sum_i c_i \ x_i \]

\[ \text{s.t. } \sum_i f_{its} \ x_i = l_{ts} \text{ all } t \text{ and } s; \ x_i \geq 0, \ i = 1...n \]

• If not possible to match exactly then include some error that is minimized.

• Allows more possibilities in the future, but still not dealing with changing mixes over time.

• Also, does not consider possible gains relative to liabilities which can be realized by rebalancing and locking in
Extended Policies – Dynamic Programming Approaches

• Policy in static approaches
  – Fixed mix or fixed set of assets
  – Trading not explicit

• DP allows broader set of policies

• Problems: Dimensionality, Explosion in time

• Remedies: Approximate (Neuro-) DP

• Idea: approximate a value-to-go function and possibly consider a limited set of policies
Dynamic Programming Approach

- **State:** $x_t$ corresponding to positions in each asset (and possibly price, economic, other factors)
- **Value function:** $V_t (x_t)$
- **Actions:** $u_t$
- **Possible events $s_t$, probability $p_{st}$**
- **Find:**
  \[
  V_t (x_t) = \max -c_t u_t + \sum_{s_t} p_{st} V_{t+1} (x_{t+1}(x_t, u_t, s_t))
  \]

**Advantages:** general, dynamic, can limit types of policies

**Disadvantages:** Dimensionality, approximation of $V$ at some point needed, limited policy set may be needed, accuracy hard to judge
General Methods

• Basic Framework: Stochastic Programming
  – Allows general policies

• Model Formulation:

\[
\begin{align*}
\text{max} & \quad \sum_{\sigma} p(\sigma) \left( U(W(\sigma, T)) \right) \\
\text{s.t.} (\text{for all } \sigma): & \quad \sum_k x(k, 1, \sigma) = W(o) \\
\text{(initial)} & \\
\sum_k r(k, t-1, \sigma) x(k, t-1, \sigma) - \sum_k x(k, t, \sigma) = 0, \text{ all } t > 1; \\
\sum_k r(k, T-1, \sigma) x(k, T-1, \sigma) - W(\sigma, T) = 0, \text{ (final);} \\
x(k, t, \sigma) & \geq 0, \text{ all } k, t;
\end{align*}
\]

Nonanticipativity:
\[
x(k, t, \sigma') - x(k, t, \sigma) = 0 \text{ if } \sigma', \sigma \in S_t \text{ for all } t, i, \sigma', \sigma
\]

This says decision cannot depend on future.

Advantages: General model, can handle transaction costs, include tax lots, etc.

Disadvantages: Size of model, computational capabilities, insight into policies.
General Model Properties

- Assume possible outcomes over time
  - discretize generally
- In each period, choose mix of assets
- Can include transaction costs and taxes
- Can include liabilities over time
- Can include different measures of risk aversion
Example: Investment to Meet Goal

- Proportion in stock versus bonds depends on success of market (no fixed fraction)

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After 5 years

After 10 years

Stock Fraction
Bond Fraction
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  • Securitization
  • Risk management/Real options

• Future Potential
Tracking a Security/Index

• **GOAL**: Create a portfolio of assets that follows another security or index with maximum deviation above the underlying asset.
Asset Tracking Decisions

• Pool of Assets:
  – TBills
  – GNMAs, Other mortgage-backed securities
  – Equity issues

• Underlying Security:
  – Mortgage index
  – Equity index
  – Bond index

• Decisions:
  – How much to hold of each asset at each point in time?
Traditional Approach

- **MODEL**: variant of Markowitz model
- **SOLUTION**: Nonlinear optimization
- **PROBLEMS**:
  - Must rebalance each period
  - Must pay transaction costs
  - May pay taxes
  - Reward on beating target?
- **RESOLUTION**:
  - Make transaction costs explicit
  - Include in dynamic model
Trading and Pricing

• Situation:
  – A can borrow 7% fixed or LIBOR+3%
  – B can borrow 6.5% fixed or LIBOR+2%
  – Dealer offers a swap of fixed interest rate for floating (LIBOR)

• Questions
  – How to price? Who pays what?
  – How to trade? How to identify partners?

Counterparty A
(Net: LIBOR+2.8%)

Counterparty B
(Net: 6.30% fixed)

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Dynamic Trading Formulation

- **PRICES**: $p(i)$ for asset $i$ with future cash flows $c(i,t,s)$ under scenario $s$; required cash flow of $b(t,s)$;
- Pay $x(i)$ now (and perhaps in future)
- **PRICING MODEL** (like liability matching):

$$\min \sum_i p(i) x(i)$$

s.t. (for all $s$): $\sum_i c(i,t,s) x(i) = b(t,s)$ all $t,s$.

**Extensions**
- Different maturity on the securities
- Maintain hedge over time
- Trade securities and match as closely as possible
- Again, can include transaction costs.
Real-time Trading

• Arbitrage searching:
  – Assume a set of prices $p_{ijk}$ for asset i to asset j trade in market k (e.g., currency)
  – Start with initial holdings $x(i)$ and maximize output $z$ from asset 1 over trades $y$

$$\max z(1)$$

$$\text{s.t. } x(i) - \sum_{jk} p_{ijk} y_{ijk} + \sum_{jk} p_{jik} y_{jik} = z(i)$$

$y \geq 0, z \geq 0$

(Generalized network: want to find negative cycles)
Trading and Market Impact

• Suppose goal is to purchase $Q$ shares.
• The transaction cost of trading increases in the amount of each trade by going through order book.
• Objective: break $Q$ into $q_1, \ldots, q_N$ to minimize transaction cost.

Order book: list of limit orders to buy or sell at a given price.
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Securitization

- Suppose you hold a collection of assets (loans, royalties, real properties) with different credit worthiness, maturities, and chance for early return of principal
- Idea: divide cash flows into marketable slices with different ratings, maturities
- Maximize value of division of asset cash flows:
\[
\max \sum_i p(i) x(i)
\]
s.t. (for all s): \[
\sum_i c(i,t,s) x(i) = b(t,s) \text{ all } t,s.
\]
Securitized Products

- Collateralized Debt Obligations (CDOs):
  Re-organize debt by losses due to default

Some may default, then collect collateral.

CDO Tranches:
- First 3% of losses: Equity
- 3-7% of losses: 1st Mezzanine
- 7-10% of losses: 2nd Mezzanine
- 10-15% of losses: Senior
- 15-30% of losses: Super Senior
Extensions and Implications of CDOs

• Synthetic CDOs: Instead of actual loans, make payments based on other party’s credit quality (or an index)
  – Funding requirement: Issuer buys credit default swap (CDS) to insure payments on the CDO
  – Requires credit worthiness of CDS counter-party

• CDO-squared: CDO composed of other CDOs
Key Assumptions for Valuing CDOs

• Known credit quality of original loans (often assumed homogeneous)
• Correlation structure of defaults
• Valuation of collateral
• Credit quality of counterparty for CDO (and their CDS counterparty)
Implications of Models: Multiple Interconnections

CDO Issuer

CDS Issuer

CDO Issuer

CDS Issuer

CDO Issuer

CDS Issuer

CDO Issuer

CDS Issuer

CDO Issuer

CDS Issuer

Loan obligors

Loan obligors

Loan obligors
Sequence of Events

• Interest rate rise $\Rightarrow$ Defaults $\uparrow$
  Collateral value $\downarrow$
  High correlation
• Defaults$\uparrow$/Collateral $\downarrow$ $\Rightarrow$ Multiple CDO tranches $\downarrow$
  $\Rightarrow$CDS counterparty stretched
  $\Rightarrow$Liquidations to meet obligations
  $\Rightarrow$More defaults/counterparty defaults and repetition
  $\Rightarrow$No confidence in prices and credit quality
Problems for Models

• How to assess the credit worthiness of multiple inter-connected obligations?
• What is the impact of multiple guarantees on a single asset?
• What happens with “agency issues”?
• How to structure products that can be properly valued and restore liquidity?
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Real Options for Comprehensive Risk Management

• Use real option approach to risks of the firm
• Combine operational and financial decisions
• Set levels for risk (insurance from buy and sell sides)
• Use of stochastic models on several levels and distributed optimization
Future Possibilities and Needs

• Better discretization methods (FEM v. finite differences)
• On-line (continual) optimization for real-time applications
• Inclusion of incomplete markets – distributed optimization
• Consideration of taxes – nonconvex and discrete optimization
• Integration of stochastic model/simulation and optimization
Conclusions

• Analysis and optimization bring value to financial engineering
• Existing implementations in multiple areas of financial industry
• Current crisis partly caused by inability to assess higher-level complexity of interactions
• Potential for resolution with comprehensive risk management models requiring research, theory, methodology, and implementation in real options, incomplete markets, and broader pricing issues