Operational Hedging for Exchange Risk (and Other Uncertainties)

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Motivation

• Operations (e.g., flexible production, foreign production) can mitigate the effects of demand, price, and currency exchange risks
• Financial instruments also can reduce risks (but should have zero NPVs)
• Questions: what is the value of operational methods and how do they interact with financial methods?
Outline

• Preliminary discussion of “hedging”
• Specific case in foreign exchange
• Value calculations
• Investment problem solutions
• Operational policies
• Conclusions
Preliminary Discussion: Hedging

• Definition here: *reducing risk (volatility)*

• Alternative interpretations:
  – Only reducing risk without affecting mean values
  – Using “hedging” instruments (e.g., derivatives): financial hedging

• Some results (e.g., Chowdhry and Howe 1999):
  – Operational hedging has value over financial hedging because of flexibility in output and correlation between demand and prices (examples later)
Risk Management and Hedging

• What is a hedge?
  – Action designed to reduce risk of future outcome
  – In finance, perfect hedge leads to no risk (riskfree return)

• Use of hedges
  – Allow pricing of financial derivatives
  – Lead to markets in derivatives
  – Also possible with operations (operational hedges)
    • Quantity - flexible production
    • Timing
Who Should Hedge?

- Farmers?
- Situation:
  - Suppose either high-yield low-yield years for crops
  - Prices down in high years and up in the low years
Farmer’s Example

- Suppose yield of corn is either 200 k-bushels (high) or 100 k-bushels (low)
- Suppose price with high yield is $1 and price with low yield is $2
- Should the farmer use financial hedge? i.e., sell a future?
  - If so, how much?
Futures Contracts as Hedges

- *Futures contract*: an agreement to buy or sell a fixed quantity at given price at fixed time in future (marked to market every day)
- Example: can agree to sell 100 k-bushels at $1.50/bushel on October 15
- On October 15, we receive $150K and must deliver 100 k-bushels
Futures for the Farmer

• Advantages
  – Can accept the expected price now
  – No risk in the price for the amount we sell

• Potential problems
  – Risk on amount we can produce
  – May have to go into market

• Analysis: Hedge our expected yield (150 k-bushels)
  Guaranteed (all the time)    $225K
  High yield – can sell 50 more + $50K (probability ½)
  Low yield – must buy 50      -$100K (probability ½)
  Expectation=225+50/2-100/2= $200k (same as no hedge)
  BUT variance (risk) is up (either $275k or $125 instead of $200k all the time)

• RESULT: should not use futures (alone)
Farmer’s Operational Hedge for Risk Management

- What else does the farmer have?
- **SILO!!**
  - *Operational hedge*
  - *Keep corn from high yield to sell at low yield*

- Now, suppose we keep 50 k-bushels in silo from high to low yield years
Farmer’s Silo Hedge

• Expected returns
  – High-yield years (prob. ½) $150 k
  – Low-yield years (prob. ½) $300 k
  – Expectation: ½(150+300)= $225k
  – Worth $225k-200k =$25k to use the silo
  – Value of the operational hedge (*option value of silo*)

• Combine with future?
  – Now, sell 150 k-bushels for $1.50 in October
  – Now, have the return guaranteed $225K

• Moral: Financial instrument only has value if farmer uses operational hedge
Copper Miner’s Example

• Should a copper mine hedge its output with futures?
• What is the nature of copper price differences?
• Demand versus supply curve change means high price-high quantity and low price-low quantity
Copper Hedging

• Suppose high demand leads to 200 k-pounds at $2/pound and low demand leads to 100 k-pounds at $1/pound
• Earn $400k (prob. ½) or $100k (prob. ¼)
• Expected value of $250k
• Operational hedge? (save 50 k-lbs from high to low years?)
  – High years: earn $300k (prob. ½)
  – Low years: earn $150k (prob. ¼)
  – Expectation: $225k (lower value!)
Copper Futures?

- Suppose we sell 200 k-lbs at $1.50 in future

- Result now:
  - Futures return: $300k (all the time)
  - High demand: + $0k (with probability $\frac{1}{2}$)
  - Low demand: - $100k (with probability $\frac{1}{2}$)
  - Expectation: $250k
  - Risk reduced ($300 or $200 v. $400 or $100)

- Here: financial derivatives give value (how much? present value?)
Model for Single Period

• Suppose:
  – Price: $p(\omega)$
  – Cost: $c$
  – Max sales: $l + kp(\omega)$ ($k > 0$ or $< 0$)
  – Decision: $x$ (amount to hedge, i.e., sell forward)

• Objective

$$\max (E(p)-c)x + E[(p-c)^+(l+kp-x)^+$$
$$+ (c-p)(l+kp-x)^-]$$
Single Period Results

• When does hedging add value?
  – For $k < k^*$, hedge.
  – For $k \geq k^*$, do not hedge.

• When prices are supply-driven, hedging can be beneficial in securing higher prices when demand is high.

• When prices are demand-driven, hedging can negate the value of potential cost advantage over the market.
Overall Observations

• Farmer:
  – Financial and operational together

• Miner:
  – Financial alone (but only for risk reduction)

• One-period model
  – Hedging when correlation of price and quantity is below a threshold

• Next: dynamic model
Optimal Investment and Operations with Currency Exchange Risk

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Foreign Currency Exchange Issues

- Foreign exchange risk costly (e.g., Laker/LVMH)
- Operational hedges may be valuable
- Supplier risk also important
- Questions:
  - What is the value of foreign capacity?
  - What are alternatives for optimal capacity configurations?
  - What is an optimal operating policy?
Capacity Alternatives

- **Domestic Investment**: No flexibility
- **Domestic Parent Plant, Foreign Subsidiary**: Semi flexible
- **Two Plant Model**: Fully flexible
Notation

- $i =$ index of the originating market, 1 domestic, 2 foreign;
- $j =$ index of the destination market, 1 domestic, 2 foreign;
- $r =$ domestic risk free rate;
- $r_f =$ foreign risk free rate;
- $s_i =$ sales price (in respective currency) of the product in country $i$;
- $c_i =$ production cost of the product in country $i$;
- $d_i =$ demand for the product in country $i$;
- $k_i =$ production capacity (in units) in country $i$, $\vec{k} = (k_1, k_2)$ is the investment vector;
- $Y_t =$ Foreign exchange rate given by (dom. currency)/(for. currency);
- $X_{ij}(t), i, j = 1, 2 =$ - the amount of product produced in market $i$ and sold at market $j$ at time $t$.
- $Z_t^i =$ 1 if capacity in market $i$ is available, = 0 if not available.
Assumptions

- Continuous monitoring and instantaneous shifts (if not, put in switching costs)
- No inventory holding (contracted or spot market)
- Focus on stochastic exchange rate - other data known
- Fixed same-country margin (contract prices and costs, but can include transportation costs)
- Positive margins if in same country
- No shutdown or abandonment
Valuation

- No flexibility
  - Stream of cash flows in two currencies

- Semi-flexibility
  - Continuous option to use lowest-cost source
  - Integral of option value over time gives value of flexibility

- Full flexibility
  - Option value over solution of transportation problem at each point in time
  - Integration over time of parametric LP solution
Value Calculations

No Flexibility:

\[ V_{\text{noflex}} = E\left[ \int_0^\infty e^{-rt}((s_1 - c_1)d_1 + (s_2 Y_t - c_1)d_2)Z_t^1 dt \right] \]

Semi-flexibility \((k_1 \geq d_1 + d_2)\):

Instantaneous cash flow:

\[ p(t) = (s_1 - c_2)d_1 Z_t^1 + \max(d_2(s_2 Y_t - c_1)Z_t^1, (d_2 - k_2)(s_2 Y_t - c_1)Z_t^1 + k_2 Y_t(s_2 - c_2)Z_t^2) \]

\[ V_{\text{semi-flex}}(\vec{k}) = E\left[ \int_0^\infty e^{-rt}p(t) dt \right] \]
\[ = E\left[ \int_0^\infty e^{-rt}((s_1 - c_1)d_1 + d_2(s_2 Y_t - c_1))Z_t^1 \right. \]
\[ + k_2 c_2 \max(0, \left( \frac{c_1}{c_2} - Y_t \right)Z_t^2)) dt\]
Full-Flexible Capacity Valuation

Max \((s_1 - c_1)X_{11} + (s_2Y_t - c_1)X_{12} + (s_1 - c_2Y_t)X_{21} + (s_2 - c_2)Y_tX_{22}\) (1)

s.t. \(X_{11} + X_{12} \leq k_1\); \(X_{11} + X_{21} = d_1\); \(X_{12} + X_{22} = d_2\); \(X_{ij} \geq 0\), for all \(i\) and \(j\). (6)

Solution:

- Ordering of margins determines possible set of optimal bases
- Maximum-margin allocation is optimal
- Each ordering corresponds to 3 potential optimal bases
- At most 5 optimal bases for given \(c_i, s_i\)
- Can compute with discretization
Volatility Effect

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\text{Net Worth} \\
\bar{V}_{\text{semi-flex}} \cdot C(d_1 + d_2, d_2)
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Volatility Effect Observations

Either \( \exists \sigma^* \) such that foreign capacity investment is optimal for \( \sigma > \sigma^* \)

or

\( \nexists \sigma \) such that foreign capacity is optimal

With transaction (and other) costs, the critical value \( \sigma^* \) declines

Allows for evaluation with bounds on costs and selling prices

Assumes stable (zero drift) exchange rate, costs, and selling prices
Optimal Capacity Investments

- **Observations on value of capacity**
  - Value of capacity is linear in capacity levels $k = (k_1, k_2)$ for semi-flexible case
  - In fully flexible case, $B^{-1}(h(k, d))$, where $B$ is an optimal basis at time $t$
  - Overall, integral over $t$ for value is piecewise-linear in $k$ with breakpoints at $d_1, d_2, d_1 + d_2$

- **Assumptions on cost of capacity**
  - Concave in $k$, e.g., fixed plus proportional

- **Result:**
  - Optimal capacity investments at extreme points of the demand possibilities with fixed demand
  - Alternatives for random demand (varying breakpoints)
Optimal Plant Configurations (No Switch Cost/Fixed Demand)

Domestic parent:
\[ \vec{k} \in \{(0,0), (d_1 + d_2, 0), (d_1, d_2), (d_1 + d_2, d_2)\} \]

Two plant cases:
\( \vec{k}^* \) is one of:
- (0, 0)
- \((d_1 + d_2, 0)\)
- \((0, d_1 + d_2)\)
- \((d_1, d_2)\)
- \((d_2, d_1)\)
- \((d_2, d_1 + d_2)\)
- \((d_1 + d_2, d_1)\)
- \((d_1 + d_2, d_1 + d_2)\).
Operational Model \((k_2 = d_2)\)

Assumptions:

- Switching times \(\tau_i\)
- Switching amounts \(\xi_i\)
- Impulse control \(u = (\tau, \xi)\)
- \(X(t)\) amount at 2
- Dynamics: \(dX(t) = 0, \tau_i \leq t < \tau_{i+1}\)
- Forced switches at times \(\eta^1_i, \eta^2_i\)

Objective: maximize

\[
J^u(s, x, y) = E^{s, x, y}\left[ \int_0^\infty e^{-r(s+t)}\left[\left(-c_1 + c_2Y_t\right)Z^2_tX_tdt + \sum_{i=1}^{\sup\{i: \tau_i < \infty\}} e^{-r(s+\tau_i)}(\alpha + \beta|\xi_i|) \right.ight. \\
\left.\left. + \sum_{i=1}^{\sup\{i: \eta^1_i < \infty\}} e^{-r(s+\tau_i)}(\alpha + \beta d_2) + \sum_{i=1}^{\sup\{i: \eta^2_i < \infty\}} e^{-r(s+\tau_i)}(\alpha + \beta \xi(i)) \right]\right]
\]
Optimal Policies

Characterization:

- The continuous-time problem has a unique viscosity solution
- The Markov chain discretization converges to the unique viscosity solution
- The discretization provides characterization of optimal policies

Policies characterized by continuation region $D = (X^*, Y^*)$

- Within $D$, no un-forced switches
- When $Y_t$ reaches boundary of $Y^*$ given $X_t$, switch to $X_t$ in $X^*$
- Switches force $X_t = 0$ or $d_2$
## Example Results

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Aytekin/Birge - Supplier and Exchange Risk
Conclusions and Next Steps

Results:
- Valuation of flexible resources with unreliable suppliers
- Characterization of optimal capacity configurations
- Maximum value on foreign capacity over all exchange rate volatilities
- Bang-bang policies for operational model with affine switching costs

Next steps:
- Sensitivity of results to failure rates
- Capacity results with switching costs
- Extensions to additional markets