Equity Valuation, Production, and Financial Planning

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Outline

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- Single-stage Results
- Capital Structure Observations
- Multistage Framework and Comparisons
- Principal-Agent Model and Incentive Compensation
- Conclusions
Basic Issues

- Financing affects production decisions (and v.v.)
- Traditional analyses ignore interactions
- No market valuation for private firms
- Bankruptcy provision necessary for evaluation
- Capital structure relationship explained in different ways
- Incentive compensation can affect operational and financial decisions
Financing Effects - Single Stage

News vendor with debt \((D)\) and equity \((E)\)

- Convert to risk-neutral equivalent demand distribution \((F)\)
- Suppose debt priced at market rate
- Find optimal mix of debt and equity

Basic single-stage problem:

\[
\text{maximize}_{x,D,E} \quad p \left( \int_0^x s \, dF(s) + x \int_x^\infty dF(s) \right) - cx(1 + r_f)
\]

subject to \(cx \leq E + (1 + r_f)^{-1} \left( \int_0^{s_b} ps \, dF(s) + D(1 + r) \int_{s_b}^\infty dF(s) \right)\),

\(x \geq 0\).
Risk-Neutral Equivalent Forms

Simple Setup:

- Assume future cash flow proportional to future demand $d$ at $t = 1$
- $N$ future states
- Suppose a maximum $U$ (and minimum 0)
- Current value in $[c_L, c_U]$
- Market prices $s$ (future $S_i, i = 1, \ldots, N$)
- Riskfree rate $r_f$
No Arbitrage Model

Implications:

- Above $c_L$, Buyer cannot purchase share $0 \leq x \leq 1$ of d:
  - Using $y$ of market instruments at $s$, $B$ of riskfree
  - End with non-negative value in all states and positive in some
    (No free lunch)

Buyer Model Formulation: For any $c \geq c_L$,

$$0 = \max_{x,y,B} \left\{ \sum_i p_i (d_i x - S^T_i y - e^{rf} B) \right\},$$
subject to

$$-cx + s^T y + B = 0,$$
$$-d_i x + S^T_i y + e^{rf} B \leq 0, \forall i, 0 \leq x \leq 1.$$
Dual Problem

Formulation:

\[ 0 = \min_{\lambda, \pi \geq 0, \rho} \rho \quad \text{subject to} \quad \sum_i d_i p_i + \lambda c + \sum_i \pi_i d_i - \rho \leq 0, \]
\[ -\sum_i S_i p_i - \lambda s - \sum_i \pi_i S_i = 0, \]
\[ -e^{rf} - \lambda - \sum_i \pi_i e^{rf} = 0. \]

Solution: \( \lambda^L, \pi^L, \rho^L \) s.t.

\[ c \geq \sum_i d_i (p_i + \pi^L_i)/(\lambda^L), \]
\[ s = \sum_i (p_i + \pi^L_i) S_i/(\lambda^L), \]
\[ -\lambda^L = e^{rf}(1 + \sum_i \pi^L_i), \forall c \geq c_L \]

Resulting prices: \( q^L_i = \frac{p_i + \pi^L_i}{\sum_i (p_i + \pi^L_i)} \)

- \( q^L \geq 0, \sum_i q^L_i = 1, c_L = e^{-rf} \sum_i d_i q^L_i, s = e^{-rf} \sum_i S_i q^L_i \)
- I.e., All prices consistent with \( q^L \) (risk-neutral or equivalent martingale measure (EMM))
Seller’s Problem and Overall Result

Seller Model Formulation – for $c \leq c_U$:

$$
0 = \max_{x,y,B} \left\{ \sum_i p_i (-d_i x + S_i^T y + e^r f B) \right\},
$$
subject to $cx - s^T y - B = 0, d_i x - S_i^T y - e^r f B \leq 0, \forall i, 0 \leq x \leq 1.$

Dual Solution: $\lambda^U, \pi^U, \rho^U$ s.t.

$$
c_U = \frac{\sum_i d_i (p_i + \pi^U_i)}{(-\lambda^U)},
$$
$$
-\lambda^U = e^r f (1 + \sum_i \pi^U_i).
$$

Resulting prices: $q_i^U = \frac{p_i + \pi^U_i}{\sum_i (p_i + \pi^U_i)}$, $c_U = e^{-r f} \sum_i d_i q_i^U$, $s = e^{-r f} \sum_i S_i q_i^U$

Conclusion:

- Convex combination of $q^L$ and $q^U$ consistent with market prices and any value $c_L \leq c \leq c_U$
- Can find correlation to market for a CAPM view of value
- Alternative values can produce range of consistent decisions
Debt Pricing

Debt payoff:

\[ Y_D(x, D) = \begin{cases} D(1 + r(D)) & \text{if } s \geq s^b, \\ \alpha ps & \text{if } s^b > s, \end{cases} \]

Expected payment \( E(Y_D) = D(1 + r_f) \)

\[ D(1 + r_f) = D(1 + r) \int_{s^b}^{\infty} f(s)ds + \alpha \int_0^{s^b} psf(s)ds. \]

Implications: Debt is priced fairly, \( r \) includes payment for bankruptcy cost (not risk aversion)
Single Stage Model

No bankruptcy cost or tax shield

- $D$ and $E$ do not affect optimal solution
- Modigliani-Miller (MM) result that capital structure is irrelevant
- **Problem:** Financial distress, interest deductibility, varying tax rates for income, dividends, cap. gains

\[ \Rightarrow \text{Market imperfection} \]

Bankruptcy and tax assumptions

- Proportional bankruptcy cost $\alpha$
- Corporate tax rate $\tau$
- No loss carry-overs or personal tax effects
Single Stage Results

- Capital structure and production interdependent
- Production more critical than capital structure
- Low-margin companies especially exposed to mis-specifying leverage
Debt Ratio as Function of Profitability

Traditional views:

• Tradeoff theory: More profitable firms have higher debt ratios
• Pecking-order theory (Myers): Less profitable firms have higher debt ratios
• Empirical evidence (Fama and French): Debt ratio declines with profitability

Results of this model:

• Capital structure is convex function of operating margin
• Book and market leverage may increase at both high and low operating margins
Model Results: Leverage as Function of Margin
Empirical Results as Function of Operating Margin

Data sources:

- Value-line public firms
- Operating margin, book, and market leverage

Results:

- Strong evidence \((p = 0.0008)\) for declining leverage at low margins
- Weak evidence \((p = 0.09)\) for increasing leverage at high margins
Multiple Stage Framework

Key Observations

- Equity and debt investments over time
- Special relevance for valuation of private equity
- Need to include bankruptcy possibility
- Need to include growth and contingencies

Traditional valuation models

- Discount dividend models ($V^0_E$ equity value at 0, $\tilde{d}_t$ dividend, $\tilde{T}V_T$ terminal value, $\rho_e$ discount factor)

$$V^0_E = \sum_{t=1}^{T} \rho_e^{-t} E_0(\tilde{d}_t) + \rho_e^{-T} E_0(\tilde{T}V_T)$$

- Multiple models ($V_F$ firm value, $S_F$ firm parameter (e.g., sales), $Comp$ - competitor values)

$$V_F = V_{Comp} \times \text{relative size} = V_{Comp} \times (S_F/S_{Comp}) = (V_{Comp}/S_{Comp}) \times S_F$$
Key Observations

- Varying interest rates for time and scenario
- Allows bankruptcy in one of middle branches (but not all)
- Value increase over two-stage and traditional methods

\[ D: 110.9, \ r_0: 19\% \]
\[ E: 59.6 \]

Terminal Value

\[ 61.1, 5\% \]
\[ 116.7 \]
\[ 185 \text{(Demand)} \]
\[ 144.9, 18\% \]
\[ 68.64 \]
\[ 133 \]

Bankrupt

34

0  Bankrupt

247

246.3

246.3

68.4

62.6

62.6

46

Xu/Birge - Valuation and Production
Multistage Results

Equity value for stochastic program, discount dividend, and multiple methods as function of parameters

- Decreasing in production cost, volatility
- Increasing in bankruptcy recovery, terminal value multiple
- Wider gaps for high margins, large volatility, recovery, terminal value

![Graphs showing the relationship between equity value and various parameters](image)
Multistage Compared to Single Stage

Observations:

- Equity value increases with multiple stages
- Leverage decreases with multiple stages
- Bigger gaps for higher margins (lower costs)
Effect of Managerial Compensation

Framework:
- Manager receives either ownership or bonus (call option on value)
- Relative weights given by $\lambda$ (= 0 for all ownership)
- Assume manager acts to maximize compensation
- Observe distortion in decisions

Expected bonus compensation $U(x, D)$:

$$
\text{maximize} \quad U(x, D) = (1 - \tau)(px - cx - rD) \int_{x}^{\infty} f(s) \, ds \\
+ \int_{s^*}^{x} (1 - \tau)(ps - cx - rD)f(s) \, ds
$$

subject to

$$
D(1 + r_f) = D(1 + r)[1 - F(s^b)] + \alpha \int_{0}^{s^b} psf(s) \, ds,
$$

$$
0 \leq D \leq cx
$$

Overall objective: $(1 - \lambda)V(x, D) + \lambda U(x, D)$
**General Results**

Effect on decisions \((x^m, D^m)\) for manager-optimal \(0 < \lambda < 1\):

- **Aggressive production:** \(x^m > x^*\)
- **Conservative debt:** \(D^m \leq D^*\)

Numerical costs in terms of \((V^* - V^m)/V^*\):

![Graph showing normalized agency costs for different values of \(c\)]
Effect of $\lambda$ relative to optimal decisions:

- Greater effect on production for low-margin
- No debt after $\lambda$ exceeds tax rate $\tau$
Observations on Bonus Compensation

Effect on value:
- Greatest for low-margin firms
- Little impact on high-margin firms
- Potential overall benefit from options for high-growth (high-margin)

Effect on production and leverage:
- Most pronounced on low-margin for production
- Debt conservatism impacts low-margin most
Conclusions

Results:

- Capital restriction may cause sub-optimal production decisions
- Mistaken production choice generally worse than mistaken financing
- Low-margin producers face greater risk in not coordinating finance and operations
- Capital structure may be U-shaped function of operating margin
- Bonus compensation creates aggressive production and conservative debt - worst for low-margin

Caveats:

- Assumed risk-neutral demand transformation and full disclosure on debt offer
- Only single-period debt (no explicit issuing cost); No explicit investment timing
- Multi-stage structural result for one product only
- No competitor and supply chain interactions (in process)