Optimal Dynamic Portfolio Construction with Transaction Costs

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General Theme

• Dynamic portfolio optimization is difficult due to:
  • Consistency
  • Many parameters to estimate
  • Non-stationarity
  • Non-normality
  • Transaction costs across time

• Continuous time models provide guidance in how to control directly

• By equating transaction costs from continuous time with discrete time analogues, close-to-optimal portfolios are possible
Basic Model

• Basic setup:
Continuous-Time Results


Results: No trading in a region $H$; boundary at some distance from optimal no-transaction-cost point (for CRRA utility: $x^* = (1/\gamma) \Sigma^{-1}(\mu-r)$, Merton line)

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General Result

\[ x_1(t) \]

Merton line

No-trade region

Time

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Equivalence in Discrete Time

General observation: The continuous time solution is (approximately) equal to a discrete-time problem with a fixed boundary.

\[ x_1(t) \]

Merton line

No-trade region

Boundary here: same as for one period to \( T^* \).
Discrete Time (Single Period) Problem
Effective Result in Terms of Average Number of Re-balances

Observation: $T^*$ is approximately the average time between re-balances or $1/T^*$ is approximately the expected number of re-balances in a single period.

- Can normalize to a single period and use $\pi/T^*$ for transaction cost.

- (Note: can learn $T^*$ along with $\mu$, $\Sigma$)
Empirical Setup

Testing results:
- Simulate portfolios over time
- Vary $T^*$
- Observe final objectives and number of re-balances as a function of $T^*$

$\mu = 0.08, \Sigma = 0.04*I, \tau = .01.$

100 trials for each run; 100 periods.
Comparisons on final Sharpe ratio
Adjustment for Different Volatilities

• The value of $T^*$ will vary depending on $\mu$ and $\Sigma$ (which may vary over time).

• An approximation to deal with variation is to assign $T^*$ as a function (e.g., a multiple $\lambda$) of the portfolio volatility $(x^T \Sigma x)^{1/2}$.

• Now, the learning can occur on $\mu$, $\Sigma$, $\lambda$. 

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Extensions

• Integrate with estimation of other parameters (and possibly with non-log-normal distributions)
• Use GARCH-type models of the volatility to capture periods of faster turnover
Conclusions

• Transaction costs create difficulties for portfolio optimization
• Finding the no-trade region is difficult in higher dimensions
• Finding an effectively equivalent single-period formulation with appropriate modification of the transaction cost can approximate the continuous-time solution
Thank you!