Efficient Solutions of Multi-stage Stochastic Programs for Stochastic Unit Commitment and Transmission Repair

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Themes

• Multi-stage stochastic programs can model many dynamic situations including hidden Markov chains
• Multi-stage stochastic programs are notoriously difficult to solve without some additional problem structure
• Can achieve tractable formulations with non-anticipativity constraints in buckets of observable characteristics
Outline

• Basic formulations
• Alternative formulations to capture hidden elements
• Traditional computational difficulties
• Uses with Monte Carlo procedures
• Results
• Extensions
General Form in Discrete Time

- Find \( x=(x_1,x_2,...,x_T) \) and \( p \) (allows for "robust formulation") to

\[
\text{minimize} \quad E_p \left[ \sum_{t=1}^{T} f_t(x_t,x_{t+1},p) \right]
\]

\[ \text{s.t.} \quad x_t \in X_t, \quad x_t \text{ nonanticipative}, \quad p \in P \text{ (distribution class)} \]

\[ P[ h_t (x_t,x_{t+1},p_t) <= 0 ] >= a \text{ (chance constraint)} \]

General Approaches:
Simplify distribution (e.g., sample) and form a mathematical program:
- Solve step-by-step (dynamic program)
- Solve as single large-scale optimization problem

Use iterative procedure of sampling and optimization steps
Electric Power Examples

• Stochastic unit commitment
  – $x_t$ corresponds to generation levels as well as other variables for unit operations (e.g., up, down, time-up, time-down)
  – High variability in demand and supply
  – May also have dependence on hidden elements (e.g., unobserved system characteristics)

• Transmission system repair/restoration
  – Many unobservables (e.g., line conditions) that are only observed as function of decisions

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Incorporating Hidden Markov States

• General situation
  – Dynamics of random parameters depend on current system states
  – States such as local weather and line conditions may not be fully observed
  – Decisions cannot depend on unobserved characteristics

Representation:

Standard non-anticipativity for filtration $\mathcal{F}_t$: $x_t = E_{\mathcal{F}_t}[x_t]$
HMM non-anticipativity for observable filtration $\mathcal{G}_t$: $x_t = E_{\mathcal{G}_t}[x_t]$

Result: Replace $\mathcal{F}_t$ with $\mathcal{G}_t$ as explicit non-anticipativity constraints.
Uncertainty Issues: U.S. Wind Power

Source: AWEA, 2013 MISO 2012
Wind Variability

MISO Hourly Wind Power - January 2012
New U.S. Solar Electric Installations

Source: SEIA 2012
Why Stochastic Programming?

- Weather-driven renewables are hard to forecast and increase the uncertainty in the electric power grid

- Stochastic programming could serve as a tool to address the increased uncertainty in power system and electricity market operations

- Stochastic programming is a powerful tool in dealing with uncertainty, but it has advantages and disadvantages
  
  **Pluses**
  - is based on axioms of foundational decision theory
  - considers uncertainty holistically rather than focusing on worst case scenarios
  - can effectively hedge against randomness

  **Minuses**
  - requires probabilistic inputs which may be hard to obtain or estimate
  - can be computationally hard to solve stochastic programming models
Scenario Generation

• Focus on wind power uncertainty with scenarios
  – Stochastic unit commitment model requires scenario representation of wind power forecast
  – Scenario generation from Markov chain model (where some values may be unobserved)
  – May attempt to reduce scenarios but…

• Random scenario selection performs better than both scenario reduction algorithms
  – Scenario reduction reduces scenario variance and level of hedging in UC strategy

• Increasing the number of scenarios improves performance
  – Computational burden also increases, 15-20 times longer run-time with 100 scenarios
Stochastic Unit Commitment

Minimize \{\text{fuel cost + start-up cost + load shedding penalty}\}

<table>
<thead>
<tr>
<th>Decision Variables</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>First stage:</strong></td>
<td>• Load balance</td>
</tr>
<tr>
<td>Unit on/off</td>
<td>• Min up-time/down-time</td>
</tr>
<tr>
<td><strong>Second stage:</strong></td>
<td>• Ramp up/down</td>
</tr>
<tr>
<td>Thermal dispatch</td>
<td>• Transmission limits</td>
</tr>
<tr>
<td>Wind dispatch</td>
<td>• Generation capacity limits</td>
</tr>
<tr>
<td>Transmission flow</td>
<td>• Spinning reserves</td>
</tr>
</tbody>
</table>
Simplified Finite Sample Model

• Assume \( p \) is fixed and random variables represented by sample \( \xi_i^t \) for \( t=1,2,...,T, \ i=1,...,N_t \) with probabilities \( p^i_t, a(i) \) an ancestor of \( i \), then model becomes (no chance constraints):

\[
\begin{align*}
\text{minimize} & \quad \sum_{t=1}^{T} \sum_{i=1}^{N_t} p^i_t \ f_t(x^{a(i)}_t, x^{i}_{t+1}, \xi_i^t) \\
\text{s.t.} & \quad x^{i}_{t} \in X^{i}_{t}
\end{align*}
\]

Observations?

• Problems for different \( i \) are similar – solving one may help to solve others
• Problems may decompose across \( i \) and across \( t \) yielding
  • smaller problems (that may scale linearly in size)
  • opportunities for parallel computation.
Stochastic Unit Commitment

$$\min_{u, x, f, w} \sum_{s \in S} \sum_{t=1}^T \sum_{i \in I} \left[ g(x^s_{it}) \cdot u_{it} + h(u_{it}, u_{i,t-1}) \right]$$

s.t. $u, x, f, w \in C_s, s \in S$

$$u^s_{it} = u_{it} \forall i, \forall s \in S, t \in T$$

$u$: Unit on/off

$x$: Generation output

$f$: Flow

$w$: Wind dispatch

$p_s$: Probability of scenario $s$

$P_s$: Scenario set

$S$: Set of thermal generators

$I$: Number of periods

$T$: Technological constraints
6-Bus system* with
• 2 thermal generators
• 3 loads

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>U</th>
<th>b (MBtu/MW)</th>
<th>c (MBtu/MW²)</th>
<th>Pmax (MW)</th>
<th>Pmin (MW)</th>
<th>Ini. State (h)</th>
<th>Min Off (h)</th>
<th>Min On (h)</th>
<th>Ramp (MW/h)</th>
<th>Start Up (MBtu)</th>
<th>Fuel Price (S/MBtu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>176.95</td>
<td>13.51</td>
<td>0.0004</td>
<td>220</td>
<td>100</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>55</td>
<td>1</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>129.98</td>
<td>32.63</td>
<td>0.001</td>
<td>100</td>
<td>10</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>50</td>
<td>200</td>
</tr>
</tbody>
</table>
Wind Power Day-Ahead Forecast Scenarios

- 10 wind scenarios
- Derived from EWITS data with KDF, MC sampling, and scenario reduction
- Wind unit capacity is set so that it can satisfy 30% of the daily load
Basic UC Model

Unit 1 is always on.

Unit 2 is on when the wind generation is low.

Wind is dispatched down (curtailed) early morning and late night.
## The Expected Value of Perfect Information (EVPI)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Probability</th>
<th>Total cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>0.10</td>
<td>61,306</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>0.06</td>
<td>64,503</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>0.09</td>
<td>59,321</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>0.07</td>
<td>61,067</td>
</tr>
<tr>
<td>Scenario 5</td>
<td>0.11</td>
<td>61,996</td>
</tr>
<tr>
<td>Scenario 6</td>
<td>0.19</td>
<td>58,074</td>
</tr>
<tr>
<td>Scenario 7</td>
<td>0.13</td>
<td>61,944</td>
</tr>
<tr>
<td>Scenario 8</td>
<td>0.10</td>
<td>59,577</td>
</tr>
<tr>
<td>Scenario 9</td>
<td>0.08</td>
<td>58,850</td>
</tr>
<tr>
<td>Scenario 10</td>
<td>0.07</td>
<td>53,268</td>
</tr>
<tr>
<td><strong>Perfect information solution</strong></td>
<td></td>
<td>59,913</td>
</tr>
<tr>
<td><strong>Stochastic solution</strong></td>
<td></td>
<td>60,427</td>
</tr>
</tbody>
</table>

*The expected value of perfect information (EVPI)*

515
### The Value of a Stochastic Solution (VSS)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Load Curtailment</th>
<th>Total cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>0.00</td>
<td>61,306</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>3.90</td>
<td>77,523</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>0.00</td>
<td>59,321</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>1.72</td>
<td>66,755</td>
</tr>
<tr>
<td>Scenario 5</td>
<td>0.46</td>
<td>62,950</td>
</tr>
<tr>
<td>Scenario 6</td>
<td>0.00</td>
<td>58,074</td>
</tr>
<tr>
<td>Scenario 7</td>
<td>0.00</td>
<td>61,944</td>
</tr>
<tr>
<td>Scenario 8</td>
<td>0.00</td>
<td>59,577</td>
</tr>
<tr>
<td>Scenario 9</td>
<td>0.00</td>
<td>58,850</td>
</tr>
<tr>
<td>Scenario 10</td>
<td>0.00</td>
<td>53,526</td>
</tr>
<tr>
<td><strong>Expected value solution</strong></td>
<td></td>
<td><strong>61,247</strong></td>
</tr>
<tr>
<td><strong>Stochastic solution</strong></td>
<td></td>
<td><strong>60,427</strong></td>
</tr>
</tbody>
</table>

*The value of stochastic solution (VSS) 880*
Alternative Approach with Bundling of Scenarios

• Stochastic programming models tend to give better results with more scenarios, capturing the full range of uncertainty.

• Unit commitment is a multi-stage decision problem in electricity market operations (day-ahead, reliability, real-time).

• To solve the problem with a large number of scenarios and to capture the multi-stage decision process we consider bundling. We observe that:
  – the scenarios can be bundled according to their deviation from the average forecast.
  – the bundles might be different across the time horizon.

• The idea is:
  – to enforce the non-anticipativity constraints for the bundles only
Bunching Form

\[
\min_{u, x, f, w} \sum_{s \in S} p_s \sum_{t=1}^{T} \sum_{i \in I} \left[ g(x_{it}^s) \cdot u_{it}^s + h(u_{it}^s, u_{i,t-1}^s) \right]
\]

s.t. \( u, x, f, w \in C_s, s \in S \)

\[
u_{il}^s = u_{il}^b \quad \forall i, \forall b \in B, \forall s \in S_b, t \in T_{\text{block}}, T_{\text{block}} \subset \{1, \ldots, T\}
\]

\( B \): Set of bundles

\( S_b \): Set of scenarios in bundle \( b \)

\( T_{\text{block}} \): Time periods in a time block
Bundling Approach

• Tradeoff
  – More variables versus ability to capture uncertainty

• Advantages of bundling
  – Captures multi-stage decision process
    • no need to enforce formal tree structure
  – Reduces the need for scenario reduction
    • can take into account extreme scenarios
  – May reduce computational burden
    • relaxation of traditional 2-stage formulation

• Three approaches
  – Non-anticipativity constraints across scenarios
  – Non-anticipativity constraints across bundles
  – Non-anticipativity constraints across bundles at the end of the blocks
Bundles for 100 Scenarios (Day-Ahead Forecast)
– According to the deviations from the average forecast
• < 25% quantile -> Bundle 1
• < 50% quantile -> Bundle 2
• < 75% quantile -> Bundle 3
• < 100% quantile -> Bundle 4
### Bundle UC Model: Objective

#### 6 Bus Function and Run-time

<table>
<thead>
<tr>
<th>Extensive Form</th>
<th>“Across scenarios”</th>
<th>“Across bundles”</th>
<th>“Across bundles at the end of time blocks”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective</td>
<td>62,401</td>
<td>62,162</td>
<td>61,860</td>
</tr>
<tr>
<td>Execution time (sec)</td>
<td>18.15</td>
<td>23.37</td>
<td>23.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Progressive Hedging*</th>
<th>“Across scenarios”</th>
<th>“Across bundles”</th>
<th>“Across bundles at the end of time blocks”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objective</td>
<td>62,401</td>
<td>62,162</td>
<td>61,846</td>
</tr>
<tr>
<td>Execution time (sec)</td>
<td>635.29</td>
<td>400.56</td>
<td>399.19</td>
</tr>
<tr>
<td>Number of PH iterations</td>
<td>50</td>
<td>26</td>
<td>29</td>
</tr>
</tbody>
</table>

The bundling approach gives
- Lower expected operating cost
- Improved run-time and fewer iterations (under PH)

Test System - 1996

Test System Task Force * of the Methods Subcommittee

It should be noted that in developing and adopting the various parameters for RTS-96, there was no intention to develop a test system which was representative of any specific or typical power system. Forcing such a requirement on RTS-96 would result in a system with less universal characteristics and therefore would be less useful as a reference for testing the impact of different evaluation techniques on diverse applications and technologies. One of the important requirements of a good test system is that it should represent, as much as possible, all the different technologies and configurations that could be encountered on any system. RTS-96 therefore has to be a hybrid and atypical system.

SYSTEM TOPOLOGY

The topology for RTS-79 is shown in Figure 1 and is labeled "Area A." Since the demand for methodologies that can analyze multi-area power systems has been increasing lately due to increases in interregional transactions and advances in available computing power, the task force decided to develop a multi-area reliability test system by linking various single RTS-79 areas. Figure 2 shows a two-area system developed by merging two single areas - "Area A" and "Area B" through three interconnections. As shown the two areas are interconnected by the following new interconnections:

- 51 mile 230 kV line connecting bus # 123 and bus # 217
- 52 mile 230 kV line connecting bus # 113 and bus # 215

-96 24-Bus

- 24-Bus
- 32 generators – thermal, hydro
- 34 lines
- 17 loads
Solution Approaches – Computational efficiency

– 24 Bus, 100MW and 250MW wind farms located in Nodes 7, 8.
– 10 Scenarios

<table>
<thead>
<tr>
<th>Scenario Type</th>
<th>Solution Time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Across scenarios</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>1665</td>
</tr>
<tr>
<td>Across bundles</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>871</td>
</tr>
<tr>
<td>Bundles at end of time blocks</td>
<td>2418</td>
</tr>
<tr>
<td></td>
<td>962</td>
</tr>
</tbody>
</table>
Price Effects

- Stochastic model produces smoother price responses
Conclusions and Future Work

- Stochastic programming is a powerful tool in solving problems with uncertainty
  - Has the potential to address uncertainty from renewables in operational decisions
- Computational effort is a challenge
  - We propose addressing this by bundling forecast scenarios and reducing the number of non-anticipativity constraints within a progressive hedging framework
  - The formulation also captures some of the multi-stage nature of the unit commitment problem
- Future work includes
  - Developing methods for more effective bundling of scenarios
  - Solving larger problems with more scenarios and stochastic variables
  - Investigate potential for improved pricing and financial incentives under stochastic scheduling
Thank you!

• Questions?