Operational Decisions, Capital Structure, and Managerial Compensation

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20 October 2006

*Work with Xiaodong Xu, Deutsche Bank, NY; Supported by National Science Foundation under Grant DMI-0100462.
Outline

• Basic Issues
• Single-stage Framework
• Single-stage Results
• Capital Structure Observations
• Principal-Agent Model and Incentive Compensation
• Conclusions
Basic Issues

- Financing affects production decisions (and v.v.)
- Traditional analyses ignore interactions
- No market valuation for private firms
- Bankruptcy provision necessary for evaluation
- Capital structure relationship explained in different ways
- Incentive compensation can affect operational and financial decisions
Financing Effects - Single Stage

News vendor with debt \((D)\) and equity \((E)\)

- Convert to risk-neutral equivalent demand distribution \((F)\)
- Suppose debt priced at market rate
- Find optimal mix of debt and equity

Basic single-stage problem:

\[
\begin{align*}
\text{maximize} & \quad p \left( \int_0^x s \, dF(s) + x \int_x^\infty dF(s) \right) - cx(1 + r_f) \\
\text{subject to} & \quad cx \leq E + (1 + r_f)^{-1} \left( \int_0^{s^b} ps \, dF(s) + D(1 + r) \int_{s^b}^\infty dF(s) \right), \\
& \quad x \geq 0.
\end{align*}
\]
Risk-Neutral Equivalent Forms

Simple Setup:
- Assume future cash flow proportional to future demand $d$ at $t = 1$
- $N$ future states
- Suppose a maximum $U$ (and minimum 0)
- Current value in $[c_L, c_U]$
- Market prices $s$ (future $S_i, i = 1, \ldots, N$)
- Riskfree rate $r_f$
No Arbitrage Model

Implications:

- Above $c_L$, Buyer cannot purchase share $0 \leq x \leq 1$ of $d$:
  - Using $y$ of market instruments at $s$, $B$ of riskfree
  - End with non-negative value in all states and positive in some (No free lunch)

Buyer Model Formulation:

$$0 = \max_{x,y,B} \left\{ \sum_i p_i (d_i x - S_i^T y - e^{rf} B) \right\},$$
subject to
- $-cx + s^T y + B = 0$,
- $-d_i x + S_i^T y + e^{rf} B \leq 0, \forall i, 0 \leq x \leq 1$. 
Dual Problem

Formulation:

\[ 0 = \min_{\lambda, \pi \geq 0, \rho} \rho \]
subject to

\[ \sum_i d_i p_i + \lambda c + \sum_i \pi_i d_i - \rho \leq 0, \]
\[- \sum_i S_i p_i - \lambda s - \sum_i \pi_i S_i = 0, \]
\[- e^{-rf} - \lambda - \sum_i \pi_i e^{rf} = 0. \]

Solution: \( \lambda^L, \pi^L, \rho^L \) s.t.

\[ c \geq \sum_i d_i (p_i + \pi^L_i) / (-\lambda^L), \]
\[ s = \sum_i (p_i + \pi^L_i) S_i / (-\lambda^L), \]
\[- \lambda^L = e^{rf} (1 + \sum_i \pi^L_i). \]

Resulting prices: \( q^L_i = \frac{p_i + \pi^L_i}{\sum_i (p_i + \pi^L_i)} \)

- \( q^L \geq 0, \sum_i q^L_i = 1, c_L = e^{-rf} \sum_i d_i q^L_i, s = e^{-rf} \sum_i S_i q^L_i \)
- I.e., All prices consistent with \( q^L \) (risk-neutral or equivalent martingale measure (EMM))
Seller’s Problem and Overall Result

Seller Model Formulation – for \( c \leq c_U \):

\[
0 = \min_{x,y,B} \left\{ \sum_i p_i (-d_i x + S_i^T y + e^{rf} B) \right\},
\]

subject to

\[
c x - s^T y - B = 0, \quad d_i x - S_i^T y - e^{rf} B \leq 0, \forall i, 0 \leq x \leq 1.
\]

Dual Solution: \( \lambda^U, \pi^U, \rho^U \) s.t.

\[
c_U = \sum_i d_i (p_i + \pi^U_i) / (-\lambda^U), \quad s = \sum_i (p_i + \pi^U_i) S_i / (-\lambda^U L),
\]

\[-\lambda^U = e^{rf} (1 + \sum_i \pi^L_i).
\]

Resulting prices: \( q^U_i = \frac{p_i + \pi^U_i}{\sum_i (p_i + \pi^U_i)}, \quad c_U = e^{-rf} \sum_i d_i q^U_i, \quad s = e^{-rf} \sum_i S_i q^U_i \)

Conclusion:

- Convex combination of \( q^L \) and \( q^U \) consistent with market prices and any value \( c_L \leq c \leq c_U \)
- Can find correlation to market for a CAPM view of value
- Alternative values can produce range of consistent decisions
Debt Pricing

Debt payoff:

\[ Y_D(x, D) = \begin{cases} 
D(1 + r(D)) & \text{if } s \geq s^b, \\
\alpha ps & \text{if } s^b > s,
\end{cases} \]

Expected payment \( E(Y_D) = D(1 + r_f) \)

\[ D(1 + r_f) = D(1 + r) \int_{s^b}^{\infty} f(s)ds + \alpha \int_{0}^{s^b} psf(s)ds. \]

Implications: Debt is priced fairly, \( r \) includes payment for bankruptcy cost (not risk aversion)
Single Stage Model

No bankruptcy cost or tax shield

• $D$ and $E$ do not affect optimal solution

• Modigliani-Miller (MM) result that capital structure is irrelevant

• **Problem:** Financial distress, interest deductibility, varying tax rates for income, dividends, cap. gains

  $\Rightarrow$ Market imperfection

Bankruptcy and tax assumptions

• Proportional bankruptcy cost $\alpha$

• Corporate tax rate $\tau$

• No loss carry-overs or personal tax effects
• Capital structure and production interdependent
• Production more critical than capital structure
• Low-margin companies especially exposed to mis-specifying leverage
**Debt Ratio as Function of Profitability**

**Traditional views:**
- Tradeoff theory: More profitable firms have higher debt ratios
- Pecking-order theory (Myers): Less profitable firms have higher debt ratios
- Empirical evidence (Fama and French): Debt ratio declines with profitability

**Results of this model:**
- Capital structure is convex function of operating margin
- Book and market leverage may increase at both high and low operating margins
Model Results: Leverage as Function of Margin

![Graphs showing leverage as a function of production cost for book leverage ratio and market leverage ratio.](image)
Empirical Results as Function of Operating Margin

Data sources:
- Value-line public firms
- Operating margin, book, and market leverage

Results:
- Strong evidence ($p = 0.0008$) for declining leverage at low margins
- Weak evidence ($p = 0.09$) for increasing leverage at high margins
Multiple Stage Framework

Key Observations

• Equity and debt investments over time
• Special relevance for valuation of private equity
• Need to include bankruptcy possibility
• Need to include growth and contingencies

Traditional valuation models

• Discount dividend models \(V^0_E\) equity value at 0, \(\tilde{d}_t\) dividend, \(\tilde{T}V_T\) terminal value, \(\rho_e\) discount factor

\[
V^0_E = \sum_{t=1}^{T} \rho^{-t}_e E_0(\tilde{d}_t) + \rho^{-T}_e E_0(\tilde{T}V_T)
\]

• Multiple models \(V_F\) firm value, \(S_F\) firm parameter (e.g., sales), \(Comp\) - competitor values

\[
V_F = V_{Comp} * \text{relative size} = V_{Comp} * (S_F/S_{Comp}) = (V_{Comp}/S_{Comp}) * S_F
\]
Max \[ \sum_{t=0}^{H} \sum_{s \in S_t} e^{-r_t} p^s_t (e^s_t - e^s_{+}) + e^{-r_T} \sum_{s \in S_T} p^s_T \theta(x^s_T, u^s_T) \]

s.t. \[ x^{s-,t-1} + y^{s-,t-1} \geq x^{s,t} + z^{s,t} \quad \forall \ t \ s \]
\[ u^{s-,t-1} + e^{s-,t-1} - e^s_{-} - r^{s-,t-1} d^{s-,t-1} - K I^{s-,t-1} - c y^{s-,t-1} \]
\[ + p z^{s,t} - \tau i^{s,t} \geq u_+^{s,t} - u_-^{s,t} \quad \forall \ t \ s \]
\[ i^{s,t} - [(p - c) z^{s,t} - r^{s-,t-1} d^{s-,t-1} - K I^{s-,t-1}] \geq 0 \quad \forall \ t \ s \]
\[ u_+^{s,t} + e_+^{s,t} - e_-^{s,t} + d^{s,t} - c y^{s,t} - K I^{s,t} \geq 0 \quad \forall \ t \ s \]
\[ M I^{s,t} - [d^{s,t} + e_-^{s,t} + e_+^{s,t} + x^{s,t} + y^{s,t} + z^{s,t} + i^{s,t} + u_+^{s,t}] \geq 0 \quad \forall \ t \ s \]
\[ M (1 - I^{s,t}) - u_-^{s,t} \geq 0 \quad \forall \ t \ s \]
\[ I^{s-,t-1} - I^{s,t} \geq 0 \quad \forall \ t \ s \]
\[ z^{s,t} \leq q^{s,t} \quad \forall \ t \ s \]
\[ d^{s-,t-1} (1 + r^{s-,t-1}) \int_{q^b}^{\infty} dF_t(q) + \alpha p \int_{0}^{q^b} q dF_t(q) = d^{s-,t-1} (1 + r_f) \quad \forall \ t \ s \]
\[ y^{s,t} \geq 0 \quad x^{s,t} \geq 0 \quad U_E \geq e_+^{s,t} \geq 0 \quad e_-^{s,t} \geq 0, \quad I^{s,t} \in \{0, 1\}, \]
\[ u_+^{s,t} \geq 0 \quad u_-^{s,t} \geq 0 \quad i^{s,t} \geq 0 \quad z^{s,t} \geq 0 \quad U_d \geq d^{s,t} \geq 0 \quad \forall \ t \ s \]
Variables $I^{s,t}$

- Zero value corresponds to bankruptcy at $t$ under $s$
- All subsequent production must end
- Exponential size growth in stages and scenarios

Optimal solution properties - structural result

- Branches with continued operations at optimality form *rational tree*
- For Markovian problems where future only depends on current demand state:

  Bankruptcy in demand state $s$ at $t \Rightarrow$ bankruptcy in all $s' \leq s$ at $t$
Implications of rational trees

- Only consider variables for rational trees
- At highest demand point $s$ with bankruptcy, sales exactly cover debt face value
- Can solve for nonlinear interest rate function $r_t(D)$ for each rational tree at time $t$

\[
(1 + r_f)D = (1 + r_t(D)) D \int_{q^b(r_t(D))}^{\infty} dF_t(q) + \alpha \int_{0}^{q^b(r_t(D))} pqdF_t(q)
\]

Methodology
- Pick rational tree
- Solve for given interest rates $r_t(D)$ at time $t$
- Solve linear program
- Update tree (and/or prune)
Three Stage Example

Key Observations

- Varying interest rates for time and scenario
- Allows bankruptcy in one of middle branches (but not all)
- Value increase over two-stage and traditional methods

Demand: $D = 110.9$, $r_0 = 19\%$  
Earnings: $E = 59.6$

Terminal Value

- 61.1, 5%  
  - 116.7  
  - 185 (Demand)

- 144.9, 18%  
  - 68.64

Bankrupt

- 46

0 Bankrupt

- 133

- 247

- 64

- 246.3

- 246.3
Multistage Results

Equity value for stochastic program, discount dividend, and multiple methods as function of parameters

- Decreasing in production cost, volatility
- Increasing in bankruptcy recovery, terminal value multiple
- Wider gaps for high margins, large volatility, recovery, terminal value

Panels A, B, C, and D show the relationship between equity value and various parameters.
Multistage Compared to Single Stage

Observations:

- Equity value increases with multiple stages
- Leverage decreases with multiple stages
- Bigger gaps for higher margins (lower costs)
Effect of Managerial Compensation

Framework:
• Manager receives either ownership or bonus (call option on value)
• Relative weights given by $\lambda$ (= 0 for all ownership)
• Assume manager acts to maximize compensation
• Observe distortion in decisions

Expected bonus compensation $U(x, D)$:

\[
\text{maximize } U(x, D) = (1 - \tau)(px - cx - rD) \int_x^{\infty} f(s) \, ds \\
+ \int_{s^*}^{x} (1 - \tau)(ps - cx - rD)f(s) \, ds \\
\text{subject to } D(1 + r_f) = D(1 + r)[1 - F(s^b)] + \alpha \int_0^{s^b} ps f(s) \, ds,
\]
\[0 \leq D \leq cx\]

Overall objective: $(1 - \lambda)V(x, D) + \lambda U(x, D)$
General Results

Effect on decisions \((x^m, D^m)\) for manager-optimal \(0 < \lambda < 1\):

- **Aggressive production:** \(x^m > x^*\)
- **Conservative debt:** \(D^m \leq D^*\)

Numerical costs in terms of \((V^* - V^m)/V^*\):

![Graph showing normalized agency cost (%)]
**Production and Leverage Differences**

**Effect of $\lambda$ relative to optimal decisions:**

- Greater effect on production for low-margin
- No debt after $\lambda$ exceeds tax rate $\tau$
Observations on Bonus Compensation

Effect on value:
- Greatest for low-margin firms
- Little impact on high-margin firms
- Potential overall benefit from options for high-growth (high-margin)

Effect on production and leverage:
- Most pronounced on low-margin for production
- Debt conservatism impacts low-margin most
Conclusions

Results:

- Capital restriction may cause sub-optimal production decisions
- Mistaken production choice generally worse than mistaken financing
- Low-margin producers face greater risk in not coordinating finance and operations
- Capital structure may be U-shaped function of operating margin
- Bonus compensation creates aggressive production and conservative debt - worst for low-margin

Caveats:

- Assumed risk-neutral demand transformation and full disclosure on debt offer
- Only single-period debt (no explicit issuing cost); No explicit investment timing
- Multi-stage structural result for one product only
- No competitor and supply chain interactions (in process)