Equity Valuation, Production, and Financial Planning:

A Stochastic Programming Approach

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Abstract

Most of the operations management literature assumes that the firm can always finance production decisions at an optimal level or borrow at a constant interest rate; however, operational decisions are constrained by limited capital and often critically depend on external financing. This paper proposes an integrated corporate planning model, which extends the forecasting-based discount dividend pricing method into an optimization-based valuation framework to make production and financial decisions simultaneously for a firm facing market uncertainty. We also develop an efficient algorithm to solve the integer stochastic programming model with nonlinear constraints. Compared with the traditional valuation and planning models, our method yields higher

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equity valuations, indicating that valuation without considering contingent decisions is inherently inaccurate.

Key Words: Production Planning; Financial Planning; Stochastic Programming; Debt Pricing; Capital structure

1 Introduction

The operations management literature tends to focus on the areas of capacity expansion, inventory control, and supply chain management without considering the effects of financial constraints or capital structure on a firm’s operating decisions. In contrast, while financial economists have long considered the capital structure of a company, they usually assume that investment or production decisions are exogenously determined. The separation between operational and financial decisions in both literatures simplifies complex problems and can be justified by the seminal work of Modigliani and Miller (1958), which proves that a firm’s investment and financial decisions can be made separately in a perfect capital market; however, due to the existence of taxes, agency costs, and asymmetric information, the capital market is not perfect in reality (see Harris and Raviv (1991) for a general review on the theory of capital structure). Furthermore, many firms’ growth potential is constrained by limited internal capital and critically depends on bank loans, equity issues, or venture capital investments. In this practical context, a firm’s operational decisions may in fact be closely related to its financial choices.

In this paper, we propose an integrated corporate planning model for making production and financial decisions simultaneously to maximize the value of the firm within a dynamic uncertain environment. While an extensive optimization literature in the areas of production and financial
management has developed over the last several decades, a general framework linking production planning, capital structure decisions, market demand prediction, and the interactions among those decisions is not yet available. Indeed, the financial management literature has in general developed in isolation from research on production planning, while the operations management literature has not typically considered the effects of financial constraints or capital structure on a firm’s operating decisions.

Planning systems within a company are usually organized under the term corporate planning, which can be divided roughly into marketing planning, financial planning, and production or operations planning. Production planning addresses decisions on the acquisition, utilization, and allocation of resources to meet customer requirements in the most efficient way. Typical decisions include purchasing parts and supply from vendors, setting inventory levels, deciding production lot sizes, and scheduling personnel. Optimization models can provide decision support in this context. General references that review such models in production planning include Graves, et al. (1993) and Silver, et al. (1998). In our case, to model and analyze the interactions between production and financing decisions, we focus on high level decisions of production and inventory quantities.

Applications of optimization in corporate financial planning include: currency hedging for multinational corporations; asset allocation for pension plans and insurance companies; risk management for large public corporations, etc. Many articles in this literature have illustrated that stochastic programming models are flexible tools for describing such financial optimization problems under uncertainty. Kusy and Ziemba (1986), for example, describe stochastic linear programs for bank asset/liability management; Mulvey and Vladimirou (1992) propose a multi-period stochastic network model for the purpose of asset allocation; Carriño, et al., (1994) formulate the asset/liability management problem of a Japanese insurance company as a multi-period stochastic linear program.
Although these planning systems have achieved some success, most existing models are either re-
stricted to deterministic environments or focus on a single function of the company while ignoring
the interactions among different planning units.

Recently, several researchers in the operations management community have begun to address
the interface between production and financial decisions. Among them, Lederer and Singhal (1994)
consider joint financing and technology choices when making manufacturing investments and show
that considerable value can be added to investments through financing decisions. In another model,
Birge (2000) adapts contingent claim pricing methods to incorporate risk into capacity planning.
Babich and Sobel (2004) consider capacity expansion and financial decisions to maximize the ex-
pected present value of a firm’s IPO income. Buzacott and Zhang (2004) incorporate financial
capacity into production decisions using asset-based constraints on the available working capital
in a leader-follower game. These papers, however, either concentrate on a single period analysis or
ignore the effects of bankruptcy on a firm’s financing and production decisions.

In a previous paper (Xu and Birge (2004)), we explored financing and operational decision
interactions for a single period. We showed that integrating production and financial decisions can
indeed be significant in this model when taxes and bankruptcy costs are included. We also showed
that low-margin producers can especially increase firm value through financing and production
decision integration. This setting, however, did not consider the possibility of additional borrowing,
equity investments, or the value of inventory beyond the single period. Our current model aims to
determine how these additional factors affect management decisions in a multi-period setting.

In addition to coordinating production and financial decisions in an integrated planning frame-
work, our paper has a non-traditional, from the operations perspective, criterion for measuring the
performance of the firm: maximizing the equity value instead of maximizing profit or minimizing
cost as is used in most traditional operation management literature. We consider firms whose management teams operate on behalf of the shareholders, and take actions to maximize the expected cash flow to equity holders. This operating criterion has been widely accepted as an objective of the firm in the finance and economics literature and is, in fact, the fiduciary responsibility of a corporate board.

We propose a new framework for equity valuation that synthesizes two pricing approaches in the accounting literature: the direct multi-period and the relative single-period valuation techniques (see Penman (1998), Lee (1999), Liu, et al. (2002), and Gilson (2000)). The general characteristic of the multi-period method is that the valuations are based on projected dividends or earnings, and involve a present value computation of future forecasted cash flows. The dividend discount model (DDM), the discounted cash flow model (DCF), and the residual income model (RIV) are among the most commonly used direct valuation approaches. The second method takes a relative valuation approach, which obtains the firm value by applying a multiple from a comparable firm to a target firm’s value driver, such as earnings, sales, cash flows, or book values. We integrate these two techniques into a unified framework in which a multi-period valuation model is used to calculate the value of the cash flow during a finite planning horizon, while the relative comparative method is used to estimate the terminal value of the firm beyond the planning horizon.

Our paper also extends the traditional passive-forecasting based valuation technique, particularly the discount dividend model, into an active-optimization based valuation framework by applying stochastic programming methodology. The traditional valuation methods in the accounting or financial literature usually assume the firm faces forecasted cash flows or the value of the equity follows a given stochastic process, leaving the company without managerial flexibility. In reality, however, firm valuation is inherently an interdisciplinary concept, involving skills that span
accounting, finance, economics, marketing, and corporate strategy. The company should have the right to alter production scale, adjust capital structure, and even to abandon operations in disadvantageous situations; hence, effective decision making can create value. Valuation without contingent decision making is then inherently inaccurate.

The remainder of this paper is organized as follows. In the next section we give a brief review of the accounting firm valuation models and show how we can introduce these methods into operations management and how we can improve the accuracy of these pricing methods. In Section 3, we first describe the integrated corporate planning (ICP) problem in detail. A multistage stochastic programming model is developed to analyze the interactions between production and financial decisions. We also identify some of the challenges involved in developing a solution strategy. In Section 4, we analyze properties of the ICP model and propose an efficient algorithm. In Section 5, a numerical example and computational results are discussed. Finally, we conclude our work in Section 6.

2 Basic Model of Firm Valuation

In this section we first give a brief introduction to the commonly used multi-stage valuation models and the single-period comparative multiples. We then integrate these two techniques into a unified framework in which a multi-period valuation model is used to calculate the value of the cash flows during a finite planning horizon, while the relative comparative method is used to estimate the terminal value of the firm beyond the forecast horizon. We also point out the problems associated with the traditional dividend discount model.

Multi-period valuation models can be expressed in terms of projected future payoffs. The most
The most popular of these valuation models is the DDM model. In this model, the equity value of the firm equals the present value of future expected dividends, i.e., \( V_0^E = \sum_{t=1}^{\infty} \rho_e^{-t} E_0(\tilde{d}_t) \), where \( V_0^E \) is the price of equity at time 0, \( d_t \) is the net dividend paid at the end of each future period, \( t \), \( E_0 \) indicates an expectation conditional upon information at time 0, and \( \rho_e \) is the cost of equity. This formula requires forecasting dividends to infinity but in practice finite forecast horizons are used. This clearly presents implementation problems. For a finite horizon \([0, T]\) one requires a terminal value determination, \( TV_T \), to correct the error introduced by truncating the forecasting horizon, such that the calculated price is equal to

\[
V_0^E = \sum_{t=1}^{T} \left[ \frac{\rho_e^{-t} E_0(\tilde{d}_t) + \rho_e^{-T} E_0(TV_T)}{\rho_e - g} \right],
\]

where \( E_0(TV_T) \) denotes the expected terminal value of the dividends after period \( t + T \). The appropriate terminal value essentially requires a forecast of the price at time \( T \).

The Gordon growth model attempts to satisfy the terminal value calculation for the DDM by capitalizing terminal dividends at an assumed growth rate \( g \); the terminal value is then given by \( E_0(TV) = E_0(\tilde{d}_{T+1})/(\rho_e - g) \). Although this is a simple and convenient way of valuing equity, its use is limited to firms with a stable growth rate. The value of the firm, in this case, is extremely sensitive to the inputs of \( g \).
In our model, we apply the multiple method to calculate the terminal value, i.e., the value for the cash flows subsequent to the horizon year. In the comparative multiple valuation method, the price of the target firm is given by applying a market multiple averaged from a group of comparable firms to the target firm’s corresponding value driver. In doing so, the analyst treats the value driver of the target firm as a summary statistic for the value of the firm. This approach is justified by the logic that two companies in the same sector with the same earnings should have the same value. For example, if net earnings are the best way to measure relative size and a similar firm can be found,

\[ V_F = P_C \times \text{relative size} = P_C \times \left( \frac{E_F}{E_C} \right) = \left( \frac{P_C}{E_C} \right) \times E_F, \]

where \( V_F \) is the value of firm, \( P_C \) is the market price of comparable firm, \( E_F \) is net earnings of the firm to be valued; and \( E_C \) is net earnings of a comparable firm. Among the value drivers, the price-to-earnings, market-to-book, price-to-operating-earnings, enterprise-value-to-sales, and enterprise-value-to-operating earnings ratios are most commonly used.

Although the DDM is the most popular equity valuation model, it is ill-suited for a company facing a market demand distribution with large variance. Since the first order mode, which is commonly used in DDM to forecast future cash flows, is not enough to characterize uncertainty, this approach cannot provide enough information to hedge market risk. Another shortcoming of DDM is that it does not properly account for managerial options. An underlying assumption of this model is that the firm is passive to the uncertain evolution of information. For example, a manager cannot take actions to avoid bad scenarios, or expand production capacity in good realizations.

Without managerial flexibility, the DDM model only passively considers the firm’s future cash flows. In reality, the company can alter production scale, adjust capital structure, and even cease operation. In the following section, we introduce an active valuation framework as an integrated
corporate planning model to allow such managerial flexibility. We then apply stochastic programming methodology to the modified DDM model to improve pricing accuracy and take advantage of the value of information.

3 Problem Description and Formulation

The purpose of this section is to develop an integrated corporate planning model for a company facing financial constraints. We consider a discrete-time, finite-horizon, partial equilibrium model, in which the firm takes output price, risk-free interest rate, and the market demand evolution process as given. The objective is to maximize the expected discounted value of net cash flow to shareholders subject to period-by-period constraints that model resource evolution.

The company is assumed to produce a single product for simplification. The unit production cost is $c$ and the selling price is $p$, where both $c$ and $p$ are constant through time. The stochastic demand, realized at the end of period $t$, has a risk-neutral equivalent cumulative distribution $F_t(q)$. At the beginning of every operating period $t \in [0, H - 1]$, the company observes an inventory level $x$ and cash level $u$. If it is optimal to continue operations, a production decision $y$ is made. If optimal to default on debt, the firm is liquidated; all growth options of the firm and its cash flow producing ability are lost.

Due to internal financial constraints, the company may not be able to reach an optimal investment level. We assume the firm has four potential sources of funds: internal savings, current cash flow, single period debt, and external equity. Whenever the firm’s desired investment exceeds internally generated cash flow, the company can obtain external funds, up to debt capacity, at a premium.
At every decision epoch, the firm’s manager first selects an investment or default policy to maximize the value of shareholders’ claims. As in Leland (1994), and Duffie, et al. (2001), we assume default is triggered by the decision of shareholders to cease raising additional equity to meet the debt payment. If the manager decides to default on the debt obligation, either it is not optimal for the equity holders to continue operation or the company’s financing ability cannot meet the interest payment plus the par value of debt at the end of an operating period; the company is then forced into bankruptcy with inventory sold at a proportion of market price.

If the firm decides to finance part of its investment by selling corporate debt at the beginning of period \( t \), the end-of-period payoff to the debt-holders, \( Y_D \), is uncertain because it depends on the market demand and the firm’s operational decisions. If equity value falls below the face value of debt, debt holders take ownership of the firm and pay bankruptcy costs. Similar to Leland (1994), our paper takes a proportional bankruptcy form with bankruptcy cost represented by \((1 - \alpha)pq \forall q < q^b\), where \( q^b = D(1 + r)/p \) is the bankruptcy point in terms of sales and \( 0 \leq \alpha \leq 1 \) represents the asset recovery rate after bankruptcy. If bankruptcy occurs, a fraction \( 1 - \alpha \) of the operating income represents the loss due to bankruptcy costs; therefore, the end-of-period debt-holders’ payoff is

\[
Y_D(D) = \begin{cases} 
D(1 + r(D)) & \text{if } q \geq q^b, \\
\alpha pq & \text{if } q < q^b,
\end{cases}
\]

where \( r(D) \) is the nominal interest rate charged by debt-holders for lending \( D \).

Because of market uncertainty and bankruptcy costs, the debt-holders’ actual income may be less than the firm’s promised payment. Following Dotan and Ravid (1985), we assume the debt can be priced with a risk-neutral equivalent distribution, so that we can analyze optimal decisions as if the firm was in a risk-neutral world; hence, the interest rate paid to debt holders must guarantee that
the expected payments under the risk-neutral equivalent distribution equals the return obtained at
the risk-free rate, i.e., \( E(Y_D) = D(1 + r_f) \). The explicit form is

\[
(1 + r_f)D = (1 + r)D \int_{-\infty}^{\infty} dF_t(q) + \alpha \int_{-\infty}^{\infty} p q dF_t(q). \tag{2}
\]

The firm’s operating profit during a certain period equals \((p - c)z - rd - K\), i.e., sales profit
less interest expense and fixed operating cost, where \( r \) is the interest rate charged by debt holders
for debt level \( d \). Notice that the realization of the sales, \( z \), should be less than the initial inventory
\( x \) plus the output level \( y \) of the current period, i.e., \( y = \min\{x + y, q\} \), where \( q \) is the realization
of the random demand. For simplicity, we assume there is no backordering. We also assume the
firm is taxed at a flat rate of \( \tau \in (0, 1) \) on taxable corporate income \( i = \max[0, (p - c)z - rd - K] \).
The debt payments are assumed fully deductible with no tax loss offset or carry forward provisions.
The firm then decides how much residual income should be paid out as dividend. If the default
decision is taken, the company is liquidated immediately; otherwise, the operation continues until
the end of period \( H \).

3.1 Chronology

To briefly summarize the model, the firm’s decisions at stage \( t \in [0, H - 1] \) are as follows:

1. Observe current period initial inventory level \( x^{s,t} \), and cash position \( u^{s,t} = u_{+}^{s,t} - u_{-}^{s,t} \);

2. Find an optimal production decision \( y^{s,t} \) and optimal financing decisions \( \{e^{s,t}, d^{s,t}\} \) to maxi-
mize the value of the equity \( V^{s,t}(x^{s,t}, u^{s,t}) \);

3. If \( V^{s,t}(x^{s,t}, u^{s,t}) \geq 0 \), continue operating the company, and

   (a) Borrow an amount \( d^{s,t} \), issue new equity or pay dividends at level \( e^{s,t} = e_{+}^{s,t} - e_{-}^{s,t} \);
(b) Pay the fixed operating cost $K$, and variable cost $cy^{s,t}$ to produce $y^{s,t}$ units of goods;

(c) Observe the market demand $q^{s,t}$ and satisfy demand at level $z^{s,t} = \min [x^{s,t} + y^{s,t}, q^{s,t}]$
to obtain revenue $p_{z^{s,t}} - cy^{s,t}$;

(d) Pay $(1 + r^{s,t})d^{s,t}$ to the debt holders for principal and interest;

(e) Realize operating profit $i^{s,t} = (p - c)z^{s,t} - r^{s,t}d^{s,t} - K$, and pay out $\tau \max [i^{s,t}, 0]$ as corporate tax;

else if $V^{s,t}(x^{s,t}, u^{s,t}) < 0$, stop operations, and go to bankruptcy.

If the company does not go to bankruptcy by the end of the last period, the comparative multiple method introduced in Section 2 is applied to calculate the terminal value $V^{s,H}(x^{s,H}, u^{s,H})$.

3.2 Integrated Corporate Planning Formulation

We depict future uncertainty in the form of an event tree. Every node in the scenario tree at time $t \in [0, H]$ represents a realization of the uncertainty with a positive probability. There are no intersections between different branches because the bankruptcy processes of the company are path-dependent. We call a path in the event tree between time 0 and time $t$ a scenario at time $t$, referred to by $\{s, t\}$. We denote $S_t$ as the set of all possible scenarios at time $t$ and the probability of the $s^{th}$ outcome in period $t$ as $p^s_{r,t}$. Let $N^{s,t}$ be the decision node associated with scenario $\{s, t\}$. Each scenario $\{s, t\}$ has an ancestor scenario $\{s^-, t - 1\}$ and one or more descendants $\{s^+, t + 1\} \forall s^+ \in D^{s,t}$. With the notation and procedures introduced above, the ICP problem can be formulated as follows:
Max \( \sum_{t=0}^{H} \sum_{s \in S_t} e^{-rt} p^s_r (e_s - e^s_\theta) + e^{-rT} \sum_{s \in S_T} p^s_r \theta(x^{s,T}, u^{s,T}) \) \( \quad \) (3)

s.t. \( x^{s,t-1} + y^{s,t-1} \geq x^{s,t} + z^{s,t} \) \( \quad \) \( \forall t \in [1,H], \ s \in S_t, \) \( \) (4)

\( u^{s,t-1} + e^{s,t-1} - e^{s,t-1} - r^{s,t-1} d^{s,t-1} - K I^{s,t-1} - c y^{s,t-1} \)
\( + p z^{s,t} - \tau i^{s,t} \geq u^{s,t} - u^{s,t}_- \)
\( \quad \) \( \forall t \in [1,H], \ s \in S_t, \) \( \) (5)

\( i^{s,t} - [(p-c) z^{s,t} - r^{s,t-1} d^{s,t-1} - K I^{s,t-1}] \geq 0 \)
\( \quad \) \( \forall t \in [1,H], \ s \in S_t, \) \( \) (6)

\( u^{s,t}_+ + e^{s,t}_+ - e^{s,t}_- + d^{s,t} - cy^{s,t} - K I^{s,t} \geq 0 \)
\( \quad \) \( \forall t \in [0,H-1], \ s \in S_t, \) \( \) (7)

\( M I^{s,t} - \left[d^{s,t} + e^{s,t}_- + e^{s,t}_+ + x^{s,t} + y^{s,t} + z^{s,t} + i^{s,t} + u^{s,t}_+\right] \geq 0 \)
\( \quad \) \( \forall t \in [0,H], \ s \in S_t, \) \( \) (8)

\( M(1 - I^{s,t}) - u^{s,t}_- \geq 0 \)
\( \quad \) \( \forall t \in [0,H], \ s \in S_t, \) \( \) (9)

\( I^{s,t-1} - I^{s,t} \geq 0 \)
\( \quad \) \( \forall t \in [1,H], \ s \in S_t, \) \( \) (10)

\( z^{s,t} \leq q^{s,t} \)
\( \quad \) \( \forall t \in [1,H], \ s \in S_t, \) \( \) (11)

\( d^{s,t-1}(1 + r^{s,t-1}) \int_{q^b}^{\infty} dF_i(q) + \alpha p \int_{0}^{q^b} qdF_i(q) = d^{s,t-1}(1 + r_f) \)
\( \quad \) \( \forall t \in [1,H], \ s \in S_t, \) \( \) (12)

\( y^{s,t} \geq 0 \quad x^{s,t} \geq 0 \quad U_E \geq e^{s,t}_+ \geq 0 \quad e^{s,t}_- \geq 0 \quad I^{s,t} \in \{0,1\}, \)

\( u^{s,t}_+ \geq 0 \quad u^{s,t}_- \geq 0 \quad i^{s,t} \geq 0 \quad z^{s,t} \geq 0 \quad U_d \geq d^{s,t} \geq 0 \)
\( \quad \) \( \forall t \in [1,H], \ s \in S_t, \) \( \)

The first term in the objective function represents the present value of the cash flow from time 0 to \( H, \) which is equal to the dividend payout net of equity issuing proceeds. The last item, \( \theta(x^{s,T}, u^{s,T}) = u^{s,T} + \beta x^{s,T} + \rho z^{s,T}, \) is the terminal value of the firm if the default decision has not been taken by the end of the planning horizon, \( \beta \) is the salvage factor of the inventory, and \( \rho \) is the equity-sales multiple. Taking sales as a value driver is simply used here for illustration. Any value driver may be used.

The inventory transfer constraints (4) link inventory between successive periods in each scenario,
i.e., the inventory level of a node at stage $t$ is determined by its parent node’s inventory $x^{s^{-},t-1}$ plus the surplus of production $y^{s^{-},t-1}$ over sales $z^{s,t}$ at the beginning of stage $t$. The greater or equal relations are employed because we assume the salvage value of the product is zero once the company goes to bankruptcy; hence, equality will not hold at bankruptcy nodes.

The cash transfer constraints are given by (5), which describes the relationship among net operation income, capital investment, and cash increments, i.e., the net operating income in each period is the sum of capital investment and the dividend payment $e^{s^{-},t-1} - e^{s^{-},t-1}$, where the net operating income of a scenario at stage $t$ is equal to the sales revenue $pz^{s,t}$ minus production cost $cy^{s,t}$, interest cost $r^{s^{-},t-1}d^{s^{-},t-1}$ and tax cost $\tau^{s,t}$. The capital investment consists of two parts: the working capital investment $c(y^{s^{-},t-1} - z^{s,t})$ and the cash increment $u^{s,t} - u^{s^{-},t-1}$. Because both $u^{s,t}$ and $u^{s^{-},t-1}$ are nonnegative variables, $u^{s,t} > 0$ indicates that the value-to-go of the equity is negative; in this case, it is not optimal for the equity holders to invest further in the firm; the best policy is to abandon the firm and let it fall into bankruptcy.

The tax constraints (6) give the amount of taxable income. If sales income exceeds the sum of the production cost, fixed operating cost, and interest cost, the company incurs a profit and is subject to a tax obligation. Notice that we assume debt payments are fully tax deductible, so that debt provides a tax shield which should be deducted from the operating income. If the company operates at a break-even or loss position, the taxable income is assumed to be zero.

The first set of resource constraints form the production constraints. The capital required to support the production operation comes from three categories: the internal cash $u^{s,t}$, the net equity issuing income $e^{s,t} - e^{s^{-},t}$, assumed only available to the incumbent shareholder, and the debt financing proceeds $d^{s,t}$. Constraints (7) ensure that the total amount of cash income from these financial instruments can cover the production and fixed operating costs.
Bankruptcy constraints (8) and (9) imply that, after the company goes into bankruptcy, the contribution of the future cash flow to the wealth of the equity holders is zero. If the company does not go to bankruptcy at a certain scenario, Constraint (8) will have no effect on the decisions of the company since we assume $M \uparrow +\infty$; otherwise, if $u_{s,t}^{\ast} < 0$, Constraints (8) and (9) will stop the operation of the firm. Constraints (10) then indicate that once the firm goes into bankruptcy, it cannot resume operations again.

Constraints (11) specify that realized sales should not be greater than the market demand, $q_{s,t}^{\ast}$. The nonlinear interest equilibrium constraints (12) indicate that the interest rates charged by the debt holders are functions of the market demand distribution and the firm’s operational decisions. Since the amounts of debt crucially depend on the interest costs, the firm’s financing decisions are then related to its production decisions. The company’s output levels are also contingent on its financing ability; the firm’s production and financial decisions are then made simultaneously. All the variables in the above model are subject to nonnegativity constraints. We also assume there is an upper bound on the dividend payout and the debt issue.

4 Model Properties and Algorithmic Strategy

In this section, we detail an algorithm for solving the integrated corporate planning model (3)-(12). We first analyze properties of the ICP model and show that, for each rational operating strategy, there exists a unique equilibrium interest rate for every decision node. We then propose a two-stage algorithm to find an optimal solution by taking advantage of the rational-operations property. This approach significantly decreases the computational complexity to find an optimal solution.
4.1 Properties of the Integrated Corporate Planning Model

Compared with traditional stochastic programming problems, a key difficulty in solving Model (3)-(12) is that the input interest rate parameter vector, \( r \equiv \{ r_{s,t} | N_{s,t} \in N^t, \ t \in [0, T-1] \} \), depends on the optimal solutions of the model, as indicated by Equation (12). An intuitive strategy is to remove the nonlinear constraints (12) from the ICP model, iteratively updating the interest rate parameters by substituting the solutions of the previous iteration into Equation (12), and stopping when an equilibrium is achieved. Our following analysis, however, indicates that, for each rational tree (defined later in this section), there exists a unique equilibrium \( r \), maximizing the objective value of model (3)-(12); furthermore, closed-form solutions for \( r \) can be given. Hence, it is not necessary to use the iterative strategy.

Lemma 4.1 gives properties of the equity value \( V_{s,t} \) as inventory level, cash position, and demand realizations change.

**Lemma 4.1** Let \( V_{s,t} \) be the equity value of the company under scenario realization \( s \in S^t \) at stage \( t \in [0, H] \); then, \( V_{s,t} \) is nondecreasing in (i) inventory level \( x_{s,t} \), (ii) cash position \( u_{s,t} \); and (iii) demand realization \( q_{s,t} \).

**Proof.** From the ICP model, it is clear that increasing \( x_{s,t} \), \( q_{s,t} \), or \( u_{s,t} = u_{s,t}^{+} - u_{s,t}^{-} \) expands the feasible region of the corresponding subproblem starting from node \( N_{s,t} \); therefore, the associated objective value is nondecreasing in \( x_{s,t} \), \( u_{s,t}^{+} \) and \( q_{s,t} \). \( \blacksquare \)

Denote \( D_{s,t} \) as the set of descendants of scenario \( s^{-} \) at time \( t - 1 \). Also, let \( N_{s,t} \), \( N'_{s,t} \) be two sibling scenarios belonging to \( D_{s,t} \). Lemma 4.2 states that if \( N_{s,t} \) is the node with higher demand realization, then it is not optimal to operate the firm under the lower demand while shutting it down in the state with higher demand.
Lemma 4.2 \( \forall N^{s,t}, N^{s',t} \in D^{s,t-1}, t \in [2, \ldots, H], \) if \( q^{s,t} \geq q^{s',t} \), then \( I^{s,t} \geq I^{s',t} \).

Proof. Let \( X = X^{s,t}, s \in S^t, t \in [0, \ldots, H] \) be the optimal solution of the ICP model, and \( X^{s,t}, X^{s',t} \) be the subproblem solution associated with scenario \( s \) and \( s' \) respectively. Denote \( V^{s,t} \) as the equity value at node \( N^{s,t} \). Suppose \( I^{s,t} < I^{s',t} \) is an optimal strategy; we must have \( V^{s',t} \geq 0 \) since the company will stop operating only if its equity value falls below zero. From (iii) of Lemma 4.1, we know \( V^{s,t} \geq V^{s',t} \) since the realized demand at node \( N^{s,t} \) is higher than that of node \( N^{s',t} \), and all the other decisions and status variables up to time \( t - 1 \) are exactly the same for both nodes; therefore, if \( V^{s',t} \geq 0 \), keeping the company operating at node \( N^{s,t} \) increases the value of the firm’s equity, which contradicts the assumption that \( I^{s,t} < I^{s',t} \) is the optimal strategy. Hence, \( I^{s,t} \geq I^{s',t} \) if \( q^{s,t} \geq q^{s',t} \).

To facilitate our analysis and exploit the special structure of the ICP model, we use the following definitions.

Definition 4.3 Let \( N^{s,t+1}, N^{s',t+1} \in D^{s,t}, q^{s',t+1} \geq q^{s'',t+1}, t \in [0, H - 1], \)

(i) A scenario tree, \( T \), is called a stopping tree if it has an associated stopping policy \( I \in \mathcal{I} \equiv \{ I^{s,t} \in \{0, 1\} \mid N^{s,t} \in \mathcal{N}^t, t \in [0, H]\} \).

(ii) A stopping tree, \( T_R \), is rational if both \( I^{t,s} \geq I^{t+1,s'} \) and \( I^{t+1,s'} \geq I^{t+1,s''} \) hold.

(iii) A tree, \( T_E \), is in equilibrium if it is rational and satisfies the debt pricing constraints,

\[
(1 + r_f) d^{s,t} = (1 + r^{s,t}) d^{s,t} \sum_{s' \in D^{s,t}} P^{s',t+1} I^{s',t+1} + \alpha \sum_{s' \in D^{s,t}} q^{s',t+1} P^{s',t+1} [1 - I^{s',t+1}] \quad (13)
\]

Definition 4.3 indicates that, if a stopping tree is not a rational tree, then the equity value given by that operating strategy is dominated by another strategy. The equilibrium definition further specifies that, at every decision node, the optimal decisions satisfy the debt pricing equation (13).
For every stage $t \in [0, H - 1]$, the rational tree realization $T_R$ pre-specifies the stopping region. If demand realization $q^t$ is less than the lowest operating demand $q^t_b$ specified by $T_R$, then the initial cash position of that scenario is negative, i.e., $u'(q^t) < 0, \forall q^t < q^t_b$.

To show that there exists an equilibrium interest rate vector $r$ for each rational tree $T_R$, we start with analysis of the properties of the optimal policy of the relaxed ICP model (3)–(11) without the debt pricing constraints (12).

**Lemma 4.4** Let $V^{s,t}(u, x)$ be the value-to-go function at stage $t \in [0, \ldots, H - 1]$ of a rational realization $T_R$, with initial cash position $u$ and inventory level $x$; then,

(i) given inventory level $x$ and production decision $y$, the sales volume is $z = \min [q, x + y]$,

(ii) if $s^b = \max \{s \mid N^{s,t} \in D^{s^-, t-1}, I^{s,t} = 1\}$, then $u^{s^b,t} = 0$,

(iii) given values of the production and dividend decisions, the debt level is

$$d = \max[0, cy + e + K - u].$$

**Proof.** (i) To show that $z = \min[q, x + y]$ is optimal, we first show that $V^{s,t}$ is nondecreasing in $z$. From the cash transition constraints $u' = u - e + pz - cy - rd - K - \tau i$, it is clear that $u'$ and $e$ are non-decreasing in $z$. Note also that $V^t(u + c\Delta, x - \Delta) \geq V^t(u, x)$, and that $V^{s,t}$ is nondecreasing in $z$. Since $z \leq q$ and $z \leq x + y$ are the only constraints on $z$, the optimal sales decision is $z = \min[q, x + y]$.

(ii) Suppose $u' \neq u^{s^b,t} = 0$ is another optimal policy. Since the company can raise money from the financial market at the beginning of each stage as long as the firm does not default on its debt payment, the value of $V_t(u, x)$ can be increased by the following strategy: (1) if $d \geq u'$, decreasing debt usage by $u'$ will increase $V_t(u, x)$ by $ru'$; (2) if $d = 0$, increasing the dividend payout by $u'$ will boost the value of $V_t(u, x)$ by $(1 - \rho)u'$; (3) if $0 < d < u'$, the company can combine strategies
(1) and (2), the increased value will be \( rd + (1 - \rho)(u' - d) \). Since all the above strategies achieve higher values of \( V_t(u, x) \) and satisfy the rational policy requirement given by \( T_R \), \( u' > 0 \) could not be an optimal policy; hence, \( u^{*t} = 0 \).

(iii) Let \( d' \neq d \) be another optimal debt policy. If \( d' < d \), either the company takes negative debt, \( d' < 0 \), or the debt level is not sufficient to support the production and dividend decisions. Hence, \( d' < d \) is infeasible. If \( d' > d \), from (ii) the company incurs an additional interest cost, \( r(d' - d) \), which decreases the value of \( V_t(u, x) \); hence, \( d' = d \).

Lemma 4.4 indicates that a property of optimal decisions of the ICP model is that the initial cash position corresponding to the scenario with the lowest demand realization is zero. Recognizing this fact, Lemma 4.5 shows that, for the optimal decisions of the relaxed ICP problem (3)–(11), the debt interest rate is negatively correlated with the debt amount for each decision node.

**Lemma 4.5** Let \( r^{s,t} \) be the interest parameter of node \( N^{s,t} \) for a rational realization \( T_R \) and let \( d^{s,t} \) be the corresponding optimal debt decision, then \( d^{s,t} \) is a monotone decreasing function of \( r^{s,t} \).

**Proof.** We first show that \( z^{s,t} = q^{s,t} \). Notice that \( q < q^{s,t} \) is the bankruptcy region pre-specified by the rational policy \( T_R \) corresponding to \( V_t(u, x) \). From part (i) of Lemma 4.4, we must have \( z = q \leq (x + y) \); otherwise, the market demand state corresponding to \( q' = x + y \leq q^{s,t} \) does not include bankruptcy, which contradicts the assumption that \( q < q^{s,t} \) is the bankruptcy region; hence, \( z^{s,t} = q^{s,t} \).

From Lemma 4.4, we know (i) \( cy = u + d - e - K \), (ii) \( u^{s,t} = 0 \). Substituting \( i = \max[0, (p - c)q^b - rd - K] \) and \( y = (u + d - e - k)/c \) into \( u^{s,t} = 0 \), we have

\[
d = \begin{cases} 
\frac{(p - \tau(p - c))q^b + \tau K}{1 + (1 - \tau)r} & \text{if } i > 0 \text{ (or } r < r^b \text{)} \\
\frac{pq^b}{1 + r} & \text{o.w.} 
\end{cases}
\]  

(14)
where \( r^b = (p - c)q^b - K \). We now show that \( r < r^b \) is equivalent to \( i > 0 \). If \( i > 0 \), substituting equation (14) into \( rd < (p - c)q^b - K \) and reorganizing items, we have \( d < cq^b + K \), i.e., \( r < r^b \). On the other hand, \( r \geq r^b \) leads to negative taxable income, i.e., \( i = 0 \). Substituting \( y = (u + d - e - k)/c \) into \( u_{st} = 0 \) yields (1 + \( r^s,t \)) \( d \geq (pq^b) \), which is Equation (14) when \( i = 0 \). From Equation (14), \( d_{s,t} \) is a monotone decreasing function of \( r_{s,t} \).

To show that there exists a unique equilibrium interest vector \( r \), we only need to show that the debt-interest relationship specified by Equation 13 is positively correlated. 

**Lemma 4.6** Let \((r_{s,t}, d_{s,t}) \), \( \forall N_{s,t} \in N^t \), \( t \in [0, H - 1] \) be a debt-interest pair associated with an equilibrium tree realization, then \( d_{s,t} \) is a monotone increasing convex function of \( r_{s,t} \).

**Proof.** Let \( N'_{s',t+1} \in D_{s,t} \) be the descendant nodes of \( N_{s,t} \); also let \( I_{s',t+1} \in \{0, 1\} \) be the associated rational operating decision of node \( N'_{s',t+1} \). We know at equilibrium that \( r_{s,t} \) and \( d_{s,t} \) satisfy

\[
(1 + r_f) d_{s,t} = (1 + r_{s,t}) d_{s,t} \sum_{N'_{s',t+1} \in D_{s,t}} P_{r_{s',t+1} I_{s',t+1}} + a \sum_{N'_{s',t+1} \in D_{s,t}} q_{s',t+1} P_{r_{s',t+1} I_{s',t+1}} [1 - I_{t+1, s'}], \tag{15}
\]

where \( q_{s',t+1} \) and \( P_{r_{s',t+1}} \) are the demand realization and probability density associated with scenario \( s' \). For a given rational operating strategy \( T \), \( I_{s',t+1} \) is constant. For simplicity, let \( a = \sum_{N'_{s',t+1} \in D_{s,t}} P_{r_{s',t+1} I_{s',t+1}} \) and \( b \) be the second term of the right hand side of equation (15). Notice that \( 1 + r_f - (1 + r_{s,t})a > 0 \), so that \( \frac{dd_{s,t}}{dr_{s,t}} = \frac{ab}{[1 + r_f - (1 + r_{s,t})a]^2} \geq 0 \) and \( \frac{d^2d_{s,t}}{dr_{s,t}^2} = \frac{-2a^2b}{[1 + r_f - (1 + r_{s,t})a]^3} \leq 0 \); therefore, \( d_{s,t} \) is convex in \( r_{s,t} \). \( \blacksquare \)

**Proposition 4.7** There exists a unique equilibrium for each rational operating strategy \( T_R \in T_R \).

**Proof.** From Lemma 4.5 and Lemma 4.6, there exists a unique equilibrium for every \( N_{s,t} \in N^t \), \( \forall t \in [0, \ldots, H] \) such that Equations (4.3) are satisfied. \( \blacksquare \)
Lemma 4.5 and 4.6 not only show the unique existence of the equilibrium interest rates \( r_E \), but also give a method to calculate these parameters, which solve Equations (13) and (14). Notice that the equity value of the rational stopping tree dominates that of the non-rational stopping tree; we only need to consider rational trees instead of all stopping trees to find the optimal decisions, which significantly decreases the computational burden. This is formally stated in the following lemma.

**Proposition 4.8** Let \( T^* \equiv \arg \max_{T: T \in \mathcal{T}_S} V_E(u, x) \) be a stopping tree which maximizes the value of the equity, then \( T^* \in \mathcal{T}_E \), where \( \mathcal{T}_E := \{ T_E \} \) is the set of equilibrium scenario trees.

**Proof.** To show \( T^* \in \mathcal{T}_E \), we only need to show that \( V^* \) is not achievable if \( T \notin \mathcal{T}_E \). From Definition 4.3 there are three cases: Equation (13) does not hold, Lemma 4.2 is not satisfied, or both. In the first case, the ICP problem is infeasible. In the second case, we know the optimal value cannot be achieved from Lemma 4.2; therefore, \( T^* \in \mathcal{T}_E \).

### 4.2 Algorithm and Example

We now detail a 2-stage algorithm for the ICP model, which identifies the set of rational scenario trees, \( \{ \tilde{T}_R \} \), in the first stage, and then runs over \( \{ \tilde{T}_R \} \) to find optimal decisions during the second stage.

**Step 1:** Identify a rational scenario realization set \( \mathcal{T}_R \) such that

\[
I_{t,s} \geq I_{t+1,s'} \quad \text{and} \quad I_{t+1,s'} \geq I_{t+1,s''}, \quad t \in [0, \ldots, H - 1].
\]

Let \( J = ||\mathcal{T}_R|| \), denote the \( j^{th} \) element of \( \mathcal{T}_R \) by \( \tilde{T}_j \). Set \( j = 0 \).

**Step 2:** Let \( j = j + 1 \). If \( j > J \), go to step (3); Set the interest rate parameter set \( r_j \) from the solutions of Equation (14) and (15). Solve the ICP problem with rational stopping realization \( \tilde{T}_j \); denote \( V(\tilde{T}_j, r_j) \) and \( X(\tilde{T}_j, r_j) \) as the corresponding objective value and decision respectively.
Step 3: Identify the index of the optimal rational scenario tree as \( j^* = \arg\max_{j \in [1, \ldots, J]} V(T^j_R, r_j) \); optimal decisions corresponding to the optimal value \( V(T^j_R, r_j) \) are \( X(T^j_R, r_j) \).

The following proposition demonstrates the efficiency of our integer decomposition strategy by comparison with the computational complexity of the worst case for the branch-and-bound method.

**Proposition 4.9** Let \( m \) and \( n \) be the number of stopping trees and rational scenario trees of the ICP model respectively, then \( n < k^{\frac{H(H+1)}{2}} \ll 2^{\frac{H+1}{k-1}} = m \) if \( H \) and \( k \) are large, where \( k \) is the number of children of each node, and \([0, H]\) is the length of the planning horizon.

**Proof.** Let \( \{T_S\} \) and \( \{T_R\} \) be the sets of stopping trees and rational trees respectively. From Definition 4.3 we know the size of the rational tree is \( n \equiv ||\{T_R\}|| < \prod_{t=0}^{H-1} k^{t+1} = k^{\frac{H(H+1)}{2}} \); therefore, \( n < k^{\frac{H(H+1)}{2}} \ll 2^{\frac{H+1}{k-1}} = m \), where \( m \equiv ||\{T_S\}|| = 2^{\frac{H+1}{k-1}} \) is the total number of stopping trees. \( \blacksquare \)

We illustrate the above valuation/planning procedure by considering an ICP model for a hypothetical firm. In our example, the firm’s planning horizon is three periods. The equity-sales multiple is used to calculate its future cash flows at Time 3. The industrial average cost of capital and the risk-free interest rate are assumed to be 0.1 and 0.05 respectively. We assume the market demand evolution follows a geometric diffusion process with a growth rate of 0.1 and volatility of 0.5 per year. The current market demand level is set to 100 units per year. The unit commodity production cost is $0.40 and the selling price is $1. Both initial cash positions and inventory levels are set to zero. The salvage value of inventory at the end of the planning horizon is assumed to be 50% of its production cost. We also let the terminal value multiples be one, i.e., the value of future cash flows to equity holders equals the firm’s sales revenue during the second stage. The base case bankruptcy recovery rate is 60% and the fixed operating cost is $40 per stage.
For simplicity, we assume there are three scenarios for each decision node; hence, there are 13 binary integer variables in this example. The total number of stopping tree is $2^{13}$, while the total number of rational trees is only 85, indicating significant decreases in computational complexity even in such a small-scale situation. For each rational stopping tree, the interest rate parameter set solves Equations (14) and (15). We use the ILOG CPLEX solver (Version 8.0, 2002) to find the optimal decisions for each rational tree and choose the tree with highest equity value as the optimal operating strategy.

Figure 1 illustrates the results of the optimal decisions. The optimal operating strategy for the equity holder at Stage 2 is to abandon the firm if the demand realization is poor, corresponding to $N^{2,3}$; otherwise, it is optimal to continue operation. At the sixth scenario of Stage 3, the value of the equity is not sufficient to cover the debt payment; therefore, the debt holders force the firm to
bankruptcy if the uncertain realization is \( N^{3,6} \); otherwise, the firm can always pay back the debt plus interest in full. Notice that in the ICP model, rather than subjectively fixing the borrowing rate, we use the decision-adjusted interest rates for different market realizations. The interest rates associated with the three decision nodes \( N^{1,1} \), \( N^{2,1} \), and \( N^{2,3} \) are 0.19, 0.05, and 0.18, respectively.

Given the optimal operating strategy and the debt interest costs, we solve the ICP model and find the optimal production and financial decisions. Notice that the optimal interest rate charged at node \( N^{2,1} \) is the risk-free rate. The low interest rate should give the equity holders incentive to take an all-debt financing strategy; however, the firm actually finances the production mainly by a new equity issue, 102.87, which is almost twice the amount of debt. The firm takes this action because the non-default operating strategy (i.e., risk-free interest cost), actually restricts the firm from aggressive debt policy, which might eventually lead to default in the case of low demand realizations; also notice that, in this example, the optimal market leverage ratio is not a fixed target but rather a dynamic one that changes over time as market demand situations change.

5 Numerical Results

In this section, we present numerical results to compare with alternative models. To evaluate the performance of the ICP model described in Section 3, we compare against a fixed interest (FI) model, which removes the debt pricing constraints in the ICP model and assumes that the company can always borrow at a fixed rate. We also consider a mean value (MV) model in which all random variables are replaced by their means. A brief discussion on the sensitivity analysis of the models is given by changing the demand and financial market environmental factors. To evaluate the effects of the planning horizon on the firm’s valuation and capital structure choices,
we compare performance both in single and multiple-period settings. We conclude that a longer planning horizon increases firm valuation and leads to lower leverage ratios.

As discussed in the previous section, the ICP model outperforms the static FI model. The positive value of the stochastic solution over the MV model also suggests we should adopt a stochastic programming technique in the discount dividend model. Our results indicate that the financial and market demand factors have significant effects on production decisions and equity valuation. The joint financing and market demand effects suggest the necessity of an integrated corporate planning model.

The base case numerical example is identical to the previous example in Section 4 except that we let the volatility of the underlying demand process be 0.5 per year. We also change the fixed operating cost to $10 per operating period. The base case terminal value multiples are also set to zero; the company effectively operates as a two-stage project.

We first explore how operating environmental factor variations affect a firm’s decisions and valuation. Figure 2 displays the performance of the ICP, FI, and MV models as functions of production cost $c$, demand volatility $\sigma$, bankruptcy recovery rate $\alpha$, and terminal valuation multiple $\rho$ (equity-sales ratio). Notice that the equity price given by the ICP model is always greater than or equal to the results of FI model because the ICP model identifies the best production and financial strategy while FI model specifies a fixed debt interest cost that the production decisions must satisfy. The constant interest rate assumption restricts production flexibility and decreases the feasible region for decisions.

Substituting the decisions of the traditional DDM into the FI model gives the MV solution. Figure 2 shows that the values given by the MV model are always dominated by the other two models, which indicates the DDM model can be improved significantly by incorporating more
Figure 2: Equity value as a function of production cost, demand volatility, bankruptcy recovery rate, and terminal value multiple for three different planning models.

Panel A of Figure 2 also shows that the equity value is negatively correlated with production cost $c$; the differences among the three valuation models also decrease as production cost increases. A rise in production cost increases the marginal production cost, which leads to a lower production level. The gap between the ICP and FI models becomes smaller as costs increase because the lower output levels due to higher costs are associated with lower debt interest rates.

Panel B illustrates that equity value is a decreasing function of market volatility. The firm’s scenario realizations. Specifically, the difference between the objective value of the FI model and the MV model is called the \textit{value of the stochastic solution}, which is always nonnegative (see Birge (1982)). Another explanation for the improvement is that DDM only uses first-order information to calculate the value of the equity. Such a simplification of future uncertainty leads to inferior decisions and lower equity valuation.
future cash flows become riskier as volatility increases, which decreases profit margin and leads to higher debt cost; hence, equity value has a downward trend. Another observation from Panel B is that the higher the volatility, the better the performance of the ICP model compared with the FI and MV models. The ICP model allows the firm to stop operating under poor conditions, while in the FI model, the firm must meet the solvency requirement under all scenarios, significantly limiting the high-end possibilities for the firm.

For different bankruptcy recovery rates and terminal valuation multiples, Figure 2 shows that, as expected, equity value is positively correlated with these two factors. An increase in the bankruptcy recovery rate clearly decreases debt costs and increases profit margin, leading to higher equity value.

Panel D of Figure 2 illustrates that the firm’s value and decisions are sensitive to the growth factor, indicating that the demand trend plays an important role in decision making. This result is intuitive since the value of a firm is not only determined by the income during its planning horizon but also by the future cash flow beyond that period. If the firm has a strong growth trend, it might not be optimal to shut down under poor demand realizations during early stages, since the value of the future cash flows could exceed the losses incurred during the initial planning horizon. When the growth multiple reaches a certain level, the optimal policy for the company is to continue operations under all scenarios. In this case, the ICP model becomes identical to the FI model. Hence, the equity values given by the ICP and FI model converge as the multiple value increases.

Figure 3 indicates that production cost, demand volatility, bankruptcy recovery rate, and the terminal valuation multiple play important roles in capital structure decisions. Panel A of Figure 3 shows that the financial leverage ratio is positively correlated with production cost. This observation suggests that a low-margin company should take aggressive financial decisions, while a high-margin firm should follow a conservative debt policy. Because lower margin means higher production
Figure 3: The market leverage ratio as a function of production cost and terminal value multiple for varying market volatility and bankruptcy recovery rates.

cost, the firm’s production output level decreases with decreasing margin. For a company facing uncertain demand, a lower production level decreases the risk of future cash flow; therefore, the debt holder charges a small risk premium to compensate for bankruptcy risk. To take advantage of a low cost of debt, the firm prefers to use more debt in its capital structure. Another explanation is that high-margin firms usually expect large future investments. To balance current and expected financing costs, high-margin firms tend to conservative financing policies to raise low-cost debt in the future.

Another observation from Panel A of Figure 3 is that the financial leverage ratio is negatively related to demand volatility, which is consistent with the trade-off model, in which firms with more volatile earnings and net cash flows have less leverage and lower dividend returns. More volatile earnings imply lower expected tax rates and high expected bankruptcy costs, which push firms toward less leverage and lower dividend payouts. We also observe that the higher the demand volatility, the more significant the effect of production cost on capital structure. As production cost increases, the value of equity decreases while the debt usage increases, leading to high leverage ratios. On the other hand, a rise in market volatility drives up interest cost and the percentage of
debt financing declines. Although an increase in market volatility has a negative effect on the equity valuation, the drop in debt usage is even sharper; hence, the effect of production cost increases on capital structure becomes more significant for firms facing higher demand uncertainty.

The effects of bankruptcy recovery rate and terminal valuation multiples on capital structure are illustrated by Panel B of Figure 3. The optimal debt leverage ratio is a decreasing function of both factors. Since a lower bankruptcy recovery rate reduces the debt-holders’ cash flow in the case of default on the firm’s debt payment, this yields higher debt cost; hence, the firm is reluctant to raise the debt level when the interest commitment increases the likelihood of bankruptcy. On the other hand, a higher recovery rate lowers the borrowing cost, while raising production and increasing debt. Our findings support the trade-off theory, which implies that firms use debt conservatively when the expected financial distress costs are high.

In our optimization-based valuation framework, we apply the multiple method to calculate the terminal value, i.e., the value for the cash flows subsequent to the horizon year. The value driver is a summary statistic for the value of the future cash flows; therefore, the higher the value of the multiple, the stronger the potential growth trend of the company. Panel B of Figure 3 indicates that leverage ratio is negatively correlated with value of the multiple. These observations agree with the practice that firms with strong growth ability prefer lower leverage ratios. An intuitive explanation is that firms with high value multiples are expected to have large growth rates and profit margins, implying larger equity value and lower leverage.

In an earlier paper, Xu and Birge (2004), we considered joint production and financial decisions in a single period model that did not allow future investment. To explore the differences between the single and multiple period planning models, Figure 4 shows the equity and market leverage ratio as a function of production cost for the single-stage and two-stage cases. The two-stage planning
model yields higher equity valuation than the single-stage case. The main reason is that a longer planning horizon allows firms to observe realizations of future uncertainty before taking contingent actions. The manager can base decisions on new information, which provides an option for the firm to hedge market risk by adjusting the investment level or ceasing operations in undesirable situations. In the multistage setting, the firm can also take advantage of debt or equity financing by, for example, waiting for future favorable conditions to expand financing.

Another observation from Figure 4 is that the debt-to-market-leverage ratio is higher in the single-stage model than in the multistage model. At first, this difference might appear counter-intuitive since the multistage model actually allows greater overall debt capacity with larger investment and production alternatives than the single-stage model. Those expanded opportunities, however, produce higher equity valuation as explained earlier. This increase overcomes the debt increase and leads to lower leverage in the multistage case.
6 Conclusions

In this paper, we develop an integrated corporate planning (ICP) model to make production and financial decisions simultaneously for a company facing demand uncertainty. Financial and production decisions are linked in this model because the firm’s operational decisions depend critically on its financing ability to support optimal production and the operational decisions affect the firms’ financing costs and choices. We model the corporate planning problem with a multistage stochastic program. At each decision node the managers make operational and financing decisions: stop or continue operations; determine amounts of loans, dividend payout or new equity issues; and set levels for product output. A difficult part of this problem is that the debt interest rate is a nonlinear function of the operating decisions, while we also need the debt interest rate as an input parameter to find optimal operating decisions.

To find an optimal solution of the ICP model with nonlinear financial constraints and binary integer variables, we first identify the rational integer realization sets, which significantly decreases computational complexity from a standard implementation. We then show that, for each set of rational realizations, there is a unique equilibrium interest rate satisfying the nonlinear financial constraint for each decisions node. From solving the model under these conditions, our sensitivity analysis indicates that the decisions of the ICP model outperform the traditional production planning model and the discount dividend model.

Our main conclusions are: (a) production and financial decisions should be made simultaneously in an integrated interactive framework; (b) the traditional DDM method only passively calculates the present value of forecasted cash flow without capturing the potential value of managerial flexibility; (c) the ICP framework enables a firm to coordinate production and financial decisions
simultaneously and extends the passive pricing method into an active valuation framework; (d) the ICP model can consider debt costs as endogenous decision variables instead of exogenous parameters; (e) compared with a single-period static model, the multistage setting yields higher equity valuation and lower leverage ratios.

Appendix (Notation)

\[ V^{s,t}(x^{s,t}, u^{s,t}) : \text{equity value function under scenario } s \text{ at period } t \]

Parameters:

\[ M: \text{constant positive parameter, } M \uparrow \infty \]

\[ q^{s,t}: \text{market demand under scenario } s \text{ at period } t \]

\[ r^{s,t}: \text{single period debt interest cost under scenario } s \text{ at period } t \]

Status variables:

\[ x^{s,t}: \text{inventory level at the beginning of period } t \text{ under scenario } s \]

\[ u^{s,t}: \text{cash position at the beginning of period } t \text{ under scenario } s, \quad u^{s,t} = u^+_s - u^-_s \]

Decision variables:

\[ y^{s,t}: \text{production decision at the beginning of period } t \text{ under scenario } s \]

\[ d^{s,t}: \text{debt issued by the company at the beginning of period } t \text{ under scenario } s. \]

\[ e^{-}_s: \text{dividend paid at period } t \text{ under scenario } s \]

\[ e^{+}_s: \text{stock issued at period } t \text{ under scenario } s \]

\[ z^{s,t}: \text{realized sales of product at period } t \text{ under scenario } s, \quad z^{s,t} = \min(x^{s,t}, q^{s,t}) \]

\[ i^{s,t}: \text{taxable operating income at period } t \text{ under scenario } s, \quad i^{s,t} = \max[ (p - c)z^{s,t} - r^{s,t-1}d^{s,t-1} - K^{s,t-1}, 0 ] \]
\[ I^{s,t}: \text{operating indicator variable,} \ I^{s,t} = \begin{cases} 1 \text{ if } u^{s,t} \geq 0, \\ 0 \text{ if } u^{s,t} < 0. \end{cases} \]

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