Discount Rates

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Discount rates

1. Facts: How risk discount rates vary over time and across assets.
   - “Macro,” “Behavioral,” “Segmented/institutional,” “Liquidity”
3. Applications
   - Portfolio theory, Active/passive management, Accounting, Corporate Finance
4. Apology – see long paper for citation, documentation
### Forecasting with DP

<table>
<thead>
<tr>
<th>Horizon $k$</th>
<th>$b$</th>
<th>$t(b)$</th>
<th>$R^2$</th>
<th>$\sigma \left[ E_t(R^e) \right]$</th>
<th>$\frac{\sigma[E_t(R^e)]}{E(R^e)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>3.8</td>
<td>(2.6)</td>
<td>0.09</td>
<td>5.5</td>
<td>0.76</td>
</tr>
<tr>
<td>5 years</td>
<td>20.6</td>
<td>(3.4)</td>
<td>0.28</td>
<td>29.3</td>
<td>0.62</td>
</tr>
</tbody>
</table>

\[
R^e_{t \rightarrow t+k} = a + b \frac{D_t}{P_t} + \epsilon_{t+k}; \quad \sigma \left[ E_t(R^e) \right] \equiv \sigma \left( \hat{b} \times \frac{D_t}{P_t} \right)
\]
Long-Horizon Regression Coefficients and Price Volatility

- Identity: \( dp_t \equiv \log(D_t/P_t); \ \rho = 0.96 \)

\[
dp_t \approx \sum_{j=1}^{k} \rho^{j-1} r_{t+j} - \sum_{j=1}^{k} \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}
\]

- Long-run regressions, and coefficient identity

\[
\sum_{j=1}^{k} \rho^{j-1} r_{t+j} = a + b_{r}^{(k)} dp_t + \epsilon_{t+k}, \text{etc.}
\]

\[
\Rightarrow 1 \approx b_{r}^{(k)} - b_{\Delta d}^{(k)} + b_{dp}^{(k)}
\]

<table>
<thead>
<tr>
<th></th>
<th>( b_{r}^{(k)} )</th>
<th>( b_{\Delta d}^{(k)} )</th>
<th>( b_{dp}^{(k)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct regression, ( k = 15 )</td>
<td>1.01</td>
<td>-0.11</td>
<td>-0.11</td>
</tr>
<tr>
<td>Implied by VAR, ( k = 15 )</td>
<td>1.05</td>
<td>0.27</td>
<td>0.22</td>
</tr>
<tr>
<td>VAR, ( k = \infty )</td>
<td>1.35</td>
<td>0.35</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- Why do prices \((p/d)\) move? 100\% (135\%)! discount rates, 0\% (-35\%)! dividend growth
A Pervasive Phenomenon, and cycles

▶ A pervasive phenomenon:

1. Stocks. DP → Return, not dividend growth
2. Treasuries. Yield → Return, not rising rates
3. Bonds/CDS. Yield → Return, not default
4. Foreign Exchange. Interest spread → Return, not devaluation
5. Sovereign Debt, Foreign Assets. → Return, not repayment, exports

▶ Common element, business cycle association:
   low prices, high returns in recessions. High prices, low returns in booms

▶ “Bubble?” "Prices too high" ↔ Discount rate “too low”
Houses – Price and Rent

<table>
<thead>
<tr>
<th>Date</th>
<th>CSW price</th>
<th>OFHEO price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>6.8</td>
<td>7.0</td>
</tr>
<tr>
<td>1970</td>
<td>7.0</td>
<td>7.2</td>
</tr>
<tr>
<td>1980</td>
<td>7.2</td>
<td>7.4</td>
</tr>
<tr>
<td>1990</td>
<td>7.4</td>
<td>7.6</td>
</tr>
<tr>
<td>2000</td>
<td>7.6</td>
<td>7.8</td>
</tr>
<tr>
<td>2010</td>
<td>7.8</td>
<td></td>
</tr>
</tbody>
</table>

Houses: | $b$ | $t$ | $R^2$
---|-----|-----|-----|
$r_{t+1}$ | 0.12 | (2.52) | 0.15 |
$\Delta d_{t+1}$ | 0.03 | (2.22) | 0.07 |
d$p_{t+1}$ | 0.90 | (16.2) | 0.90 |

Stocks: | $b$ | $t$ | $R^2$
---|-----|-----|-----|
| 0.13 | (2.61) | 0.10 |
| 0.04 | (0.92) | 0.02 |
| 0.94 | (23.8) | 0.91 |
A Pervasive Phenomenon, and cycles

- A pervasive phenomenon:
  1. Stocks. DP $\rightarrow$ Return, not dividend growth
  2. Treasuries. Yield $\rightarrow$ Return, not rising rates
  3. Bonds/CDS. Yield $\rightarrow$ Return, not default
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- Common element, business cycle association:
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- “Bubble?” "Prices too high" $\iff$ Discount rate “too low”
Multivariate Challenges: More variables

1. Many forecasters. Multiple regression? Common forecasters across assets?

\[ r_{t+1}^{stock} = a_s + b_s \times dp_t + c_s \times ys_t + d'_sz_t + \varepsilon_{t+1}^s? \]

\[ r_{t+1}^{bond} = a_b + c_b \times ys_t + b_b \times dp_t + d'_bz_t + \varepsilon_{t+1}^b? \]

2. Are \( E_t(r_{t+1}^i) = b_i \times x_t \) correlated across assets? Factor structure of time-varying expected returns?

3. Relate mean to covariance

\[ E_t(r_{t+1}^i) = \text{cov}_t(r_{t+1}^i f'_{t+1}) \lambda_t \]

4. Can’t just run big regressions!

5. Back to prices (price/dividend) – long-run forecasts?
Understanding prices. short and long-run forecasts

\[ R_{t+1} = a + b \times dp_t \ [ + c \times cay_t ] + \varepsilon_{t+1}; \]

\[ \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = a + b \times dp_t \ [ + c \times cay_t ] + \varepsilon \]
The cross section

1. Chaos

2. CAPM  \[ E(R^{ei}) = \beta_i E(R^{em}) \]

3. Chaos again  \[ E(R^{ei}) = \alpha_i + \beta_i E(R^{em}) \] (value)

4. Fama and French

\[ E(R^{ei}) = \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb) \]

3. Chaos again

\[ E(R^{ei}) = \alpha_i + \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb) \]

(Market, value, size), momentum, accruals, equity issues, beta-arbitrage, credit risk, bond & equity market timing, carry trade, put writing, “liquidity provision,” …
Value effect and factor

4. Fama and French

\[ E(R_{ei}) = \beta_i E(R_{em}) + h_i E(hml) + s_i E(smb) \]
Value (size, and bond factors)

4. Fama and French

\[ E(R^{ei}) = \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb) \]

a. Theories \((m)\) only need to explain the factor

\[ E(R^{ei}) = \ldots + h_i E(hml) \text{ (Fama French)} \]
\[ E(hml) = \text{cov}(hml, m) \text{ (Theory)} \]

b. Value stocks rise and fall together; mean\(\Leftrightarrow\)covariance. (APT). But theories must now explain covariance!

c. Value betas explain other \(E(R^e)\) sorts, e.g. sales growth.

5. Chaos again.. How to repeat FF?

\[ E(R^{ei}) = [\alpha_i] + \beta_i E(R^{em}) + h_i E(hml) + s_i E(smb) \]

(Market, value, size), momentum, accruals, equity issues, beta-arbitrage, credit risk, bond & equity market timing, carry trade, put writing, “liquidity provision,...
The Multidimensional Challenge

- (Market, value, size), momentum, accruals, equity issues, beta-arbitrage, credit risk, bond & equity market timing, carry trade, put writing, “liquidity provision,”...

1. Which of these are *independently* important for $E(R^e)$? (“multiple regression”)
2. Does $E(R^e)$ spread correspond to new factors?
3. Do we need all the new factors? Or again, fewer factors than $E(R^e)$ characteristics?

- How to approach such a highly multidimensional problem?
1. Portfolio sorts are really cross-sectional regressions

\[ E(R^e_i) = a + b \log\left(\frac{b}{m_i}\right) + \varepsilon_i; \quad i = 1, 2, \ldots N \]
1. Portfolio sorts are really cross-sectional regressions

\[ E(R^{ei}) = a + b' C_i + \varepsilon_i; \ i = 1, 2, \ldots N \]

2. Time series and cross-section are really the same thing

\[ R_{t+1}^{ei} = a + b' C_{it} + \varepsilon_{t+1}^i \]

3. Result: Expected return is a function of characteristics

\[ E(R_{t+1}^{ei} | C_{it}) \]

\[ C_{it} = [\text{size, b/m, momentum, accruals, d/p, credit spread} \ldots] \]

4. Covariance with factors is also a function of characteristics

\[ \text{cov}_t(R_{t+1}^{ei}, f_{t+1}) = g(C_{it}) \]

\[ E(R^e | C) = g(C) \times \lambda? \]
Prices?

1. Why $ER/\beta$, not $p$, $PV$?

2. Long-run / price in the “cross-section”?

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = a + b'C_{it} + \varepsilon^i?$$

3. Prices/long run may simplify.

3.1 Campbell-Shiller:

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - dp_t$$

3.2 One-period:

$$R_{t+1} = \frac{D_{t+1}}{P_t} = \left(\frac{D_{t+1}}{D_t}\right) / \left(\frac{P_t}{D_t}\right)$$

$$r_{t+1} = \Delta d_{t+1} - dp_t$$
Theory classification

1. Frictionless
   a. Macroeconomics – macro data.
      i. Consumption
      ii. Investment
      iii. Background risks outside income
      iv. General equilibrium.
   c. Finance – \( E(R)/\beta \), return-based factors; affine models.

2. Frictions
   a. Liquidity.
      i. Idiosyncratic
      ii. Systemic
      iii. Information trading.
   b. Segmented – Different investors in different markets
   c. Intermediated – Leveraged intermediaries.
Consumption/habits

\[ X_t \approx k \sum_{j=0}^{\infty} \phi^j C_{t-j} \text{ ; risk aversion}_t = \gamma \frac{C_t}{C_{t-1}X_t} \]
1 + α \frac{i_t}{k_t} = \frac{market_t}{book_t} = Q_t
Challenges for theories

- Pervasive, coordinated risk premium in all markets, especially unintermediated
- Mean returns are associated with comovement.
- Strong correlation with macroeconomics
“Arbitrages”

Source: Fontana (2010)
Three-month FX swap-implied US dollar rate from euro

Source: Baba and Parker (2008).

"Arbitrages"
Price and volume in the tech “bubble.”

- Price (discount rate) $\Rightarrow$ Volume? Or some Volume $\Rightarrow$ Price, like money?
- Why so much information trading?
Portfolio theory with many factors

- The average investor must hold the market
- Portfolio theory based on differences

\[ E(R^p) \]
\[ \sigma(R^p) \]
\[ \text{cov}(R^p, f) \]
Bonds – a cautionary tale

Price of a bond that matures in year 10 – simulation
Stocks (your endowment) in the crisis

\[
\text{share} = \frac{1}{\gamma} \frac{E(R^e)}{\sigma^2(R^e)}. \quad 0.6 = \frac{1}{2} \frac{0.04}{0.18^2} \implies \frac{1}{2} \frac{0.04}{0.70^2} = 0.04\???
\]
Prices and payoffs: a mean-variance benchmark

If utility is quadratic, \[ \max \{ c_t \} \ E \sum_{t=0}^{\infty} \delta^t \left( -\frac{1}{2} \right) (c_t - c^*)^2 \] and for any amount of time-varying expected returns,

\[ \tilde{E}(x) = \frac{1}{1-\beta} \sum_{j=0}^{\infty} \beta^j E(x_{t+j}) \]
Alphas, betas, and performance evaluation

\[ R_{t}^{ei} = \alpha_{i} + \beta_{i} \text{rmrf}_{t} + h_{i} \text{hml}_{t} + s_{i} \text{smb}_{t} + u_{i} \text{umd}_{t} + \text{vol.}, \text{ carry, beta-arb, issues} \]
Procedures, corporate, accounting, regulation.

- Capital budgeting, valuation

  \[
  \text{value of investment} = \frac{\text{expected payout}}{R^f + \beta \left[ E(R^m) - R^f \right]},
  \]

- Accounting, regulation, capital structure, if prices can change on discount rate news?
Conclusion

- Discount rates vary over time and across assets a lot more than you thought
- We’ve only started
- How do you ask the right question?
Last word