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Money as stock[☆]

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Abstract

The fiscal theory determines the price level from the value of nominal government debt as a claim to government primary surpluses, just as private stock is valued as a claim to corporate profits. Valuation equations are not constraints, so this theory does not mistreat the government's intertemporal budget constraint. I anchor the analysis in a simple cash in advance model. When money demand falls to zero, I show that the price level can still be determined by the government debt valuation equation.

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1. Introduction

1.1. *The fiscal theory and the stock analogy*

Suppose that Microsoft stock becomes numeraire, unit of account, and medium of exchange. When you buy coffee, you deliver a fraction of a Microsoft share, or a

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banknote, check or electronic transfer that promises such payment. Bonds promise future delivery of a share of Microsoft stock. Clearly, such a monetary system can establish a well-determined price level. We would start to understand that price level via the usual frictionless stock valuation equation,

$$\frac{\text{Number of shares}}{\text{Price level}} = \text{Expected present value of future dividends or earnings.} \quad (1)$$

(The price level, shares per good, is the inverse of the share price, so it goes in the denominator.)

The fiscal theory of the price level recognizes that nominal debt, including the monetary base, is a residual claim to government primary surpluses, just as Microsoft stock is a residual claim to Microsoft's earnings. If surpluses are not sufficient, the government must default on or inflate away the debt. Therefore, we can determine the price level via the valuation equation for government debt,

$$\frac{\text{Nominal government debt}}{\text{Price level}} = \text{Expected present value of primary surpluses.} \quad (2)$$

An equivalent view is that money is valued because the government accepts it for tax payments—the “public” part of “This note is legal tender for all debts, public and private.” If the government requires money for tax payments at the end of the day, money will be valued in trade during the day. Starr (1974) presents the first formal analysis of a tax theory of value that I know of, though as usual one can find centuries-old verbal expressions of the basic ideas. Starr (2000) quotes Adam Smith: “A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money.” (*Wealth of Nations*, Vol. I, Book II, Chapter II).

Most economic analysis of the price level relies instead on the quantity theory—a special demand for transactions-facilitating assets, combined with an artificially limited supply of such assets. As the stock analogy and the tax story make clear, the fiscal theory can determine the price level with no money demand or other frictions, no restrictions on open market operations (the composition of nominal government debt across maturities or transactions-facilitating status), no restriction on private note issue, no restrictions on transactions technology or financial innovation, and no explicit redemption promises (gold or commodity standards, foreign exchange pegs or currency boards).

These facts make the fiscal theory attractive, as we live in fiat-money economies with rapidly diminishing frictions and rampant financial innovation that undermine the foundations of the quantity theory (most recently, see Friedman, 1999), passive monetary policies (interest rate rather than money stock targets), and yet roughly stable prices. The fiscal roots of recent currency collapses are all too evident. Is there any open market operation, without fiscal reform, that could have saved the Ruble, the Bhat, or the Argentine Peso?

More importantly, a frictionless fiscal model can provide a useful benchmark for more complex and realistic analyses with frictions. Throughout economics, frictionless competitive models are the benchmark, the foundation upon which we add interesting frictions. Yet monetary economics has so far crucially relied on a big friction at the short end of the yield curve in order even to start talking about a price level.

1.2. Theoretical controversies, and this paper

Despite (or perhaps because of) this promise, the fiscal theory remains a focus of theoretical controversy. Most prominently, the fiscal theory is said to assume that the government can violate its budget constraint at off-equilibrium prices. This is either an unusual special first-mover or large-agent advantage for the government, a novel game-theoretic concept of economic equilibrium (Bassetto, 2002), or, less charitably, a fundamental violation of the rules of Walrasian equilibrium.

Kocherlakota and Phelan (1999, conclusion) write

The key force behind the fiscal theory is that a government is fundamentally different from households. Households need to satisfy their budget constraint for all prices, regardless of whether or not those prices are equilibria. A government does not.

Buiter (2002) writes

... the ‘fiscal theory of the price level’ is fatally flawed. An economic misspecification is the source of the problem... it denies that government’s intertemporal budget constraint must hold as an identity, that is, for all admissible values of the variables entering the budget constraint. Instead, it requires it to be satisfied only *in equilibrium* (p. 459).

The fiscal theory of the price level rests on a fundamental confusion between equilibrium conditions and budget constraints. It therefore does not constitute a valid starting point for further research in monetary economics (p. 478).

Ljungqvist and Sargent (2000, p. 507) state that the fiscal theory assumes that the government violates a budget constraint at off-equilibrium prices, and that the government is special in its ability to do so. Bohn (1999) expresses the same view. Marimon (2001) characterizes the fiscal theory as “a theory that does not respect Walras’ law,” which is the same thing.

Even Woodford (2001) expresses a mild version of this view. Woodford (p. 691) answers “Mustn’t fiscal policy satisfy an intertemporal budget constraint?” by arguing that such a constraint exists, it is violated by the government at off-equilibrium prices, but that the special nature of the government allows this. Woodford starts by pointing out that private agents who can violate a constraint will post infinite demands, but governments, desirous of producing an equilibrium, might not do so. Then (bottom of p. 692) he points out that in overlapping generations models, the government “budget constraint” does not even hold *in equilibrium*, so

we should be used to specifying models with violated budget constraints. Finally, he makes the government-is-special argument,

[T]he government should not optimize subject to given market prices and a given budget constraint, as private agents are assumed to in the theory of competitive equilibrium. For the government is a large agent, whose actions can certainly change equilibrium prices, and an optimizing government surely should take account of this in choosing its actions.

To be fair, I must add [Cochrane \(1999\)](#) to the list. That paper called the valuation equation an “intertemporal budget constraint,” as did all but the final draft of [Cochrane \(2001\)](#). It only seems obvious in retrospect.

The main contribution of this paper is to address these and related theoretical criticisms of the fiscal theory. The theory can work in a perfectly standard and well-specified Walrasian economic model, one in which the government has no special status, and one in which all budget constraints are satisfied at both equilibrium and disequilibrium price levels. It may or may not apply to a given time and place but it is at least a theoretically coherent possibility.

This is the key insight: Eq. (2) is, like the stock equation (1), a *valuation equation*, a *market-clearing condition*, it is not a *constraint*. If a bubble pushes stock prices up, no budget constraint forces Microsoft to raise subsequent earnings. Microsoft can satisfy the obligations that define equity in a Walrasian model—to pay the promised state-contingent dividend stream—completely ignoring the evolution of its stock price. Analogously, if an off-equilibrium deflation doubles the value of nominal debt, no budget constraint forces the government to raise subsequent taxes to pay off bondholders. The government can satisfy the obligations that define nominal debt in a Walrasian model—to redeem maturing debt for cash, to auction new debt for cash, and to accept cash for tax payments—for arbitrary price paths, as we will see.

More directly, Microsoft can double the number of shares without changing its earnings stream. This is a stock split. Everyone understands that the price per share will halve in response to the split. This fact shows that the valuation equation is not a constraint: you cannot double your demand for Porsches, counting on the price to halve. Analogously, the government can double nominal debt (including the monetary base) without changing the corresponding real surplus stream. This is a currency reform. Everyone understands that the price level doubles. If Eq. (2) were a budget constraint, an equation determining allowable sequences of nominal debt and real surpluses *given* prices, a currency reform would be impossible. I think this example makes the point most clearly: if a government can conduct a currency reform, then the valuation equation cannot be a budget constraint.

[Sims \(1999\)](#) and [Marimon \(2001\)](#) also point to the analogy between government debt and private equity. Marimon notices (p. 3) that the value of private equity can grow without bound at off-equilibrium prices, though he views this as an observation that firms *are* allowed to violate budget constraints rather than an indication that the valuation equation is not a ‘constraint’ in the first place.

1.3. A model

To make these points formally, I write a standard cash in advance model with one small modification: I reopen the securities market at the end of the day. This modification allows consumers to hold no money overnight. Money demand is precisely zero. Nonetheless, I show that this economy can have a finite price level, in which (2) is the central equilibrium condition that determines the price level. Spelling out the model provides reassurance that the fiscal theory really does “obey the rules of Walrasian equilibrium.” It addresses criticisms such as [Buiter \(1999, p. 8\)](#) that fiscal theorists would see the error of our ways if we were to write down completely specified models rather than concentrate on two equilibrium conditions.

The model is a second contribution of the paper. Most fiscal theory research including [Leeper \(1991\)](#), [Sims \(1994\)](#), [Woodford \(1995, 1998, 2001\)](#), and [Dupor \(2000\)](#) (see also the references in [Buiter, 2002](#)) studies economies in which there *is* a money demand and monetary friction. It rehabilitates money *supply* policies that are often thought to lead to an indeterminate price level, such as real-bills doctrines, interest rate pegs or providing “enough money to accommodate the needs of trade,” or it prunes multiple equilibria in models with hyperinflationary dynamics.

This model goes one step further (or back, depending on how you look at it, since the contribution is to remove ingredients rather than to add them): the price level can be determined with no money *demand* either. [Woodford \(1998\)](#) studies interest rate rules in a cashless limit. This paper studies the limit point.

With the model in hand, I return to a careful analysis of the “violating the budget constraint” question. I also address a few of the myriad criticisms and confusions surrounding the fiscal theory, including these: (1) Is this all just [Sargent and Wallace \(1981\)](#) all over again? (2) Does not Maastricht and the US experience of the 1990s tell us that governments *will* follow Ricardian policies, even if they do not *have* to?

2. Model

2.1. Preview

The model is a simplified version of the familiar cash-in-advance framework in [Sargent’s \(1987\)](#) textbook. I use this framework because it is a reasonable abstraction of the current U.S. payments system, and because it maintains a close connection to a familiar setup.

Let B_{t-1} = one period nominal debt, issued at $t - 1$, coming due at t , p_t = price level, \tilde{s}_t = real primary government surplus including any seignorage, $m_{t,t+j} = \beta^j u'(e_{t+j})/u'(e_t)$ = marginal rate of substitution or stochastic discount factor, e_t = endowment or consumption, M_t = money supply, v = velocity. The government determines the sequences of debt, money and surplus, $\{B_t, M_t, \tilde{s}_t\}$. In the cash in advance model, the price level is determined by two equilibrium conditions, a money

demand equation and the government debt valuation equation

$$M_t v = p_t e_t, \quad (3)$$

$$\frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t(m_{t,t+j} \tilde{s}_{t+j}). \quad (4)$$

We see a problem immediately: (3) and (4) are two equations in one unknown, p_t . Equilibria only exist for a restricted set of $\{B_t, M_t, \tilde{s}_t\}$ processes for which both (3) and (4) can hold. Therefore, fiscal and monetary policies must be *coordinated*, as Sargent (1987, p. 168) emphasizes.

It is useful to think of stylized *regimes* that achieve this coordination. In a monetary regime, the Fed determines $\{M_t\}$, and hence determines the price level $\{p_t\}$ with (3). The Treasury then adjusts surpluses $\{\tilde{s}_t\}$ so that Eq. (4) holds at this price level. If the Fed engineers a deflation, the Treasury must raise taxes in order to pay off the Fed's gift to bondholders.¹ I will refer to this case as the *monetary regime* of the *monetary model*.

In a fiscal regime, the Treasury fixes $\{\tilde{s}_t\}$ and $\{B_t\}$. The government valuation equation (4) determines the price level. The Fed must then "passively" set $M_t = p_t e_t / v$, "accommodating the needs of trade." I will refer to this case as the *fiscal regime* of the *monetary model*.

I go beyond passive money *supply* to show that there is a determinate price level even if there is no money *demand*. I reopen the securities market at the end of the day, so that consumers can avoid holding money overnight, though they still need it during the day to shop. This change eliminates Eq. (3). Given $\{B_t, s_t\}$, and a fiscal regime, the government valuation equation (4) can alone determine a positive the price level. I refer to this case as the *frictionless model*.

Once we see that the frictionless model can determine the price level, it is clear how a completely cashless economy can work as well. After all, redeeming bonds for cash and returning the cash to pay taxes and buy bonds is a wash transaction. The economy can work just as well if transactions are mediated by claims to maturing government debt, and if old debt is exchanged directly for new debt and to pay taxes. I refer to this case as the *cashless model*.

Though these regime stories are useful, there is nothing game-theoretic in the definition and characterization of equilibria. All that matters in the end is whether the government has produced a sequence $\{B_t, M_t, s_t\}$ that results in a unique, positive, price level sequence $\{p_t\}$ that simultaneously solves (3), if it is present, and (4). Needless to say, the actual process by which the government arrives at a coordinated policy is much messier than the simple monetary and fiscal regimes imagine.

¹Explicit cash-in-advance models typically state that seignorage revenues are rebated to consumers by lump-sum transfers. The government makes seignorage revenue $(M_{t+1} - M_t)/p_t$, but gives it to consumers ("injects it"), which is an equal and opposite lump-sum transfer $s_t = -(M_{t+1} - M_t)/p_t$. In this way, the surplus including seignorage \tilde{s}_t is zero at all dates. (4) reads $0/p_t = 0$ and thus holds for any money-determined price level. See footnote 4 of Lucas and Stokey (1987), and a nice treatment in Lucas (1984, pp. 36–37).

2.2. Model specification and equilibrium

2.2.1. Choices

The government chooses a state-contingent sequence for one-period nominal debt, money and primary surpluses, $\{B_t^s, M_t^s, s_t\}$. B_t^s, M_t^s, s_t are each random variables, and the notation $\{x_t\}$ denotes a sequence of random variables $x_1, x_2, \dots, x_t, \dots$. Section 3.2 discusses constraints on $\{B_t^s, M_t^s, s_t\}$. I start the economy at $t = 0$, so B_{-1}, M_{-1} are fixed. At a minor cost in complexity, we can start the economy with $B_{-1} = M_{-1} = 0$ and some other trading mechanism (foreign money, gold), and describe a first period in which money and debt are issued.

The notation $\{B_t^s, M_t^s, s_t\}$ does not imply statistical exogeneity; the government may choose to pick one or more elements of $\{B_t^s, M_t^s, s_t\}$ by a feedback rule from other variables. In particular, real-world governments typically target interest rates, exchange rates, or inflation, and typically do not make explicit plans that distinguish nominal debt and real surpluses. However, as it has been theoretically convenient for a century to write models in terms of nominal money stock and real income, and then feed descriptively realistic policy rules through that framework at a later stage, the analogous specification remains the most convenient description with which to analyze the theoretical issues here.

Identical households maximize a standard utility function,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t).$$

I assume $u'(c) > 0$ on $0 \leq c < \infty$ to ensure that budget constraints are not slack, and $u''(c) < 0$ so that optimal choices are described by first order conditions.

2.2.2. Timing and trade

The representative household enters period t with money balances M_{t-1} and one period nominal discount bonds with face value B_{t-1} . Any news is revealed. The household then goes to the asset market. The household redeems maturing bonds, pays net lump-sum taxes $p_t s_t$, buys new bonds B_t and leaves with money M_t^d . Each household receives a nonstorable endowment e_t in the goods market. The household cannot consume its own endowment, and must therefore buy the endowments of other households. To do so, the household splits up into a worker and a shopper. The shopper takes the money M_t^d and buys goods c_t subject to a cash in advance constraint,

$$p_t c_t \leq M_t^d v. \quad (5)$$

For the moment, $v = 1$, but it is useful to introduce the parameter v and consider what happens as it changes later. The worker sells the endowment e_t in return for money, and gets cash $p_t e_t$ in return.

In the monetary model, the shopper and worker go home and eat c_t . They must hold overnight any money $M_t^d - p_t c_t$ left over from the shopper, and the money $p_t e_t$

earned by the worker,

$$M_t = M_t^d + p_t(e_t - c_t). \tag{6}$$

The frictionless model makes one small change: the securities market reopens at the end of the day. The household can return to the securities market, and trade any unwanted cash for more bonds (the household’s bank offers overnight repurchase agreements and plentiful free ATMs). Thus, the household does not face the constraint (6); it can use cash during the day without holding it overnight. The absence of the constraint (6) is the only difference in the economic setup of the two models. There is no interest on intraday bond holdings or cash loans. This is, roughly, the current institutional arrangement. No intraday interest also results if we think of the “day” as an arbitrarily short trading interval, say a minute of each hour, on the way to a continuous time model.

2.2.3. Household budget constraints

The household can trade arbitrary contingent claims in the asset market. I represent contingent claims prices by a real stochastic discount factor $m_{t,t+j}$. For example, the price of a 1 period nominal discount bond at time t is

$$Q_t = p_t E_t \left(m_{t,t+1} \frac{1}{p_{t+1}} \right). \tag{7}$$

Discount factors chain together, $m_{0,t+j} = m_{0,t} m_{t,t+j}$ so once time-zero prices $\{m_{0,t}\}$ have been determined, the prices in markets that reopen at time t follow. Since all households are identical, claims not provided by the government are in zero net supply and their presence or absence has no effect on the equilibrium prices or allocations. Therefore, I do not list such claims in the household budget constraints or the definition of equilibrium.

Households are forbidden to issue money, to keep them from arbitraging zero interest money against interest bearing bonds,

$$M_t \geq 0. \tag{8}$$

The household’s period to period budget constraint then states that the nominal value of money and bonds at the beginning of period, plus any profits in the goods market, must equal the nominal value of bonds purchased, money held overnight, and net tax payments,

$$B_{t-1} + M_{t-1} + p_t(e_t - c_t) = Q_t B_t + M_t + p_t s_t. \tag{9}$$

As usual in infinite period models with dynamic trading, the household’s debt demands $\{B_t\}$ must obey the transversality condition

$$\lim_{T \rightarrow \infty} E_t \left(m_{t,T} \frac{B_{T-1}}{p_T} \right) = 0. \tag{10}$$

This condition rules out arbitrage between dynamic trades in spot markets and trades in long-dated securities—you cannot get a free lunch by shorting an

overpriced security, and buying the dividends separately. Equivalently, it means that (9)–(10) are equivalent to the present value budget constraint,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j}(s_{t+j} + c_{t+j} - e_{t+j}). \tag{11}$$

2.2.4. Definition of equilibrium

An equilibrium is a set of initial stocks B_0, M_0 , and sequences for quantities $\{c_t, M_t^d, M_t, B_t, s_t\}$ and prices $\{m_{0,t}, p_t\}$ such that

1. (Household optimization) Given prices $\{p_t, m_{0,t}\}$, initial stocks B_{-1}, M_{-1} , and the tax and endowment streams $\{s_t, e_t\}$, the choices $\{B_t, M_t^d, M_t, c_t\}$ maximize expected utility subject to the budget constraints (9)–(10), the cash in advance constraint (5), and the no-printing-money constraint (8). In the cash-in-advance model, the household must also meet the constraint (6) that money coming from the goods market must be held overnight.
2. (Market clearing) $c_t = e_t, M_t = M_t^s, B_t = B_t^s$ at each date and state of nature.

2.2.5. Characterization of equilibrium: monetary model

The consumer’s first-order conditions, budget constraints, and market-clearing imply the following characterizations, familiar from Lucas (1984) and Sargent (1987):

1. The marginal rate of substitution is equal to the stochastic discount factor,

$$\beta^j \frac{u'(e_{t+j})}{u'(e_t)} = m_{t,t+j}. \tag{12}$$

Hence, nominal bond prices are given by

$$Q_t = \beta E_t \left[\frac{u'(e_{t+1})}{u'(e_t)} \frac{p_t}{p_{t+1}} \right]. \tag{13}$$

2. Any equilibrium with positive nominal interest rates ($Q_t < 1$), must have a binding cash constraint,

$$M_t v = p_t c_t = p_t e_t. \tag{14}$$

3. The government debt valuation equation holds,

$$\frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t \left[m_{t,t+j} \left(s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_{t+j}} \right) \right] \tag{15}$$

or, equivalently,

$$\frac{B_{t-1} + M_{t-1}}{p_t} = \sum E_t \left[m_{t,t+j} \left(s_{t+j} + \frac{r_{t+j}^f}{1 + r_{t+j}^f} \frac{M_{t+j}}{p_{t+j}} \right) \right], \tag{16}$$

where r_t^f denotes the one period nominal interest rate,

$$1 + r_t^f \equiv 1/Q_t.$$

Expression (15) counts seignorage revenue as it is made; expression (16) counts the interest differential between money and debt as the flow of seignorage.

Fact 1 follows from the household’s first order conditions for buying one less consumption good, investing in a contingent claim, and then consuming more at $t + j$. Following [Sargent \(1987\)](#), there is no asset-pricing distortion with this timing convention. In order to raise consumption c_t the household must also get more money M_t^d , but cash overnight M_t will be unaffected because $p_t c_t$ changes by the same amount as M_t^d changes (see Eq. (6)). With positive nominal interest rates, money is strictly dominated by bonds, so the household will hold as little money as possible overnight. In the CIA model, that quantity is $M_t = p_t e_t / v$; goods market equilibrium gives $e = c_t$, and hence Fact 2. To derive Fact 3, use the bond price definition (7), iterate forward the consumer’s period to period budget constraint (9), impose the condition (10), and impose market clearing ($e_t = c_t, M_t = M_t^s$).

The pair (14) and (15) together determine the price level in terms of variables chosen by the government. Existence of equilibrium and the case of zero nominal interest rate are well treated by [Lucas \(1984\)](#) and [Sargent \(1987\)](#), so I will not delay getting to the point of this paper with a review.

2.2.6. Characterization of equilibrium: frictionless model

1. The marginal rate of substitution (12) is still equal to the stochastic discount factor or contingent claims prices,

$$\beta^j \frac{u'(e_{t+j})}{u'(e_t)} = m_{t,t+j}. \tag{17}$$

2. Any equilibrium with positive nominal interest rates ($Q_t < 1$), must have no money

$$M_t = 0. \tag{18}$$

No equilibrium may have negative nominal interest rates, $Q_t > 1$.

3. The government debt valuation equation holds, now

$$\frac{B_{t-1} + M_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t(m_{t,t+j} s_{t+j}). \tag{19}$$

The consumer’s flow budget constraint (9) is not changed, so first-order condition behind fact 1 is the same. Removing constraint (6) that cash from sales must be held overnight, the minimum cash that the household can hold overnight is zero, so (18) replaces the quantity equation (14). Eq. (18) is still a money demand equation, but it now holds for any price level and so does not help in price level determination. A negative nominal interest rate is an arbitrage opportunity, and leads to infinite money and negative infinite bond demand, and so cannot be an equilibrium. Eq. (19) specializes (16). In periods with positive nominal rates $r_{t+j}^f > 0$, we have $M_{t+j} = 0$, so the seignorage term drops because M is missing. In periods with zero nominal rates, $r_{t+j} = 0$, seignorage drops because there is no interest differential between money and bonds.

2.2.7. Existence of equilibrium

There are specifications of the utility function, endowment processes, and government choices $\{B_t^s, M_t^s, s_t\}$ that result in equilibria of the frictionless model with determinate, finite price levels. I can prove this statement most transparently by giving a simple example. Suppose $u(c) = c^{1-\gamma}$, $e_t = e$, $B_t^s = B$, $M_t^s = 0$, $s_t = s$, all positive and constant over time. Obviously, we must have $c_t = e$. From (17), the discount factor is constant,

$$m_{t,t+1} = \beta.$$

From (19), the price level must be constant and positive,

$$p_t = p = (1 - \beta) \frac{B}{s}.$$

Nominal interest rates are positive, $Q_t = \beta < 1$ so money demand equals money supply $M = 0$. $\lim_{T \rightarrow \infty} \beta^T B/p = 0$ so the transversality condition (10) is satisfied. The consumer’s first-order conditions and transversality conditions are necessary and sufficient for an optimum. Thus, we have found sequences $\{c_t, M_t^d, M_t, B_t, s_t, Q_t, p_t\}$ and M_0, B_0 that satisfy the definition of an equilibrium. Furthermore, given all the other variables, $\{p_t\}$ is unique.

Not all specifications of the utility function, endowment process and government choices $\{B_t^s, M_t^s, s_t\}$ result in equilibria, as pathological utility functions and “uncoordinated” or otherwise nonsensical policy do not lead to equilibria in the monetary model. Here, I discuss the issues, but I do not attempt a characterization of the weakest possible restrictions on utility functions and exogenous processes that result in an equilibria.

As in all dynamic models, the endowment process and utility function must be such that equilibrium marginal rates of substitution $m_{t,t+j} = \beta^j u'(e_{t+j})/u'(e_t)$ are defined. For example, we can’t have occasionally negative endowments in a model with power utility.

Eq. (19) and market clearing ensure a unique, positive, equilibrium price level sequence $\{p_t\}$, if the government always chooses a positive amount of nominal debt at each date, $\infty > B_t^s + M_t^s > 0$ and a surplus whose present value is positive $\infty > \sum_{j=0}^{\infty} E_t(m_{t,t+j} s_{t+j}) > 0$. It is not necessary that all these sequences are positive. There can be equilibria with negative debt, surpluses or money supplies, but one must rule out $0/0 = 0$ problems in (19).

One-period bond prices are determined from $Q_t = E_t(m_{t,t+1} p_t / p_{t+1})$. For there to be an equilibrium, the government must choose a price level sequence, via its choices of $\{B_t^s, M_t^s, s_t\}$, so that the expectation exists, and so that the nominal interest rate is nonnegative, $Q_t \geq 1$. If it chooses the price level sequence so that the nominal interest rate is negative, households will try to hold infinite cash and infinite negative amounts of debt.

2.2.8. Ricardian and non-Ricardian regimes

Finally, and most famously, suppose that the government chooses to alter surpluses systematically so that (19) holds for any price level. For example, in a

one-period version of the model or if no new debt is sold, the valuation equation (19) is

$$\frac{B_{t-1}}{p_t} = s_t.$$

If the government *chooses* to increase the real surplus one-for-one with the price level, holding $S_t = p_t s_t$ constant, then there is either no equilibrium (if $S_t \neq B_{t-1}$) or the equilibrium is indeterminate (if $S_t = B_{t-1}$). This is a “Ricardian” policy regime (Woodford, 1995).

Although the government collects nominal taxes, our tax regime is not naturally Ricardian. The government collects a fixed tax *rate* times nominal income, so nominal tax payments and nominal expenditures both rise as the price level rises. Inflation-induced distortions in the tax code are far from the one-for-one offset needed to induce a Ricardian regime.

Thus, the government must also choose a “non-Ricardian” policy in order for there to be an equilibrium price level in a fiscal regime. The government must not choose a $\{s_t\}$ process that responds to prices in such a way that (15) or (19) hold for any price level.

The non-Ricardian regime requirement is exactly analogous to the requirement of a monetary regime that the Fed not “accommodate the needs of trade,” raising M one for one with changes in py/v . *One* of money supply = money demand and the government valuation equation can be passive. If *both* are passive, the price level is no longer determined.

2.3. Comments on the frictionless model

2.3.1. Realism and aggregate demand

Would you notice anything different if you woke up in a fiscal regime? Probably not. If the price level is too low, people have more money in their pockets than they need to make tax payments $p_t s_t$. They try to buy goods, driving up the price level. Thus, inflation is still “too much money chasing too many goods.”

Alternatively, consider the consumer’s present value budget constraint (11),

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} [s_{t+j} + (c_{t+j} - e_{t+j})].$$

If the price level is too low, then the value of debt on the left-hand side exceeds the present value of tax payments $E_t \sum_{j=0}^{\infty} m_{t,t+j} s_{t+j}$ on the right-hand side. The consumer thinks he has “wealth” to buy more consumption than endowment. He will try to do so. We will see “aggregate demand” giving “inflationary pressures,” as the Fed loves to say. (Woodford, 1995, emphasizes this interpretation, reminiscent of Patinkin’s “wealth effect” of government debt.) More generally, the equilibrium conditions (3) and (4) both hold in the monetary and fiscal regimes of the monetary model. *No* time series test can distinguish them.

2.3.2. *Less cash, no cash*

The cash in advance constraint plays no essential role in the equilibrium. For any equilibrium of the frictionless model as stated so far, the same equilibrium (same sequences for M_t, B_t, c_t, p_t , etc.) holds if we halve or eliminate the cash constraint and halve or eliminate intraday cash M^d at the same time. Equivalently, the parameter v appears nowhere in the solution for the price level; we can choose $v = 2$ or ∞ .

This fact has many interesting interpretations. We can let the security market be open constantly, and let all operations (debt purchases or sales, tax payments) happen at any time during the day. We can allow private note issue. Formally, we can allow agents to sell claims to government debt, redeemable in the securities market, and allow those claims to be used to satisfy the cash in advance constraint in transactions. This is an important feature. In the quantity theory tradition, keeping a determinate price level requires rigorous control of private note issue, or other transactions-facilitating assets that compete with government money. Checking accounts are controlled via reserve requirements, banknotes and small-denomination bearer bonds are forbidden. We can also allow improvements in transactions technologies. Any amount of the goods can become “credit goods” or “credit card goods” whose purchase is paid for by agreements to exchange bonds in the asset markets, with no effect on the price level.

We can even consider a completely cashless economy. Dollars can be numeraire though not medium of exchange. The equilibrium price level is obviously unaffected if, instead of redeeming a bond for a dollar, and then immediately using that dollar to pay taxes and buy new bonds, consumers exchange maturing bonds directly for tax payments and new bonds. Bonds can still promise to pay “a dollar.” The *right* to a dollar, i.e. the right to be relieved of a dollar’s worth of tax payments, is valuable, even if that right is never exercised in equilibrium, and no dollars are held. (Cash-settled commodity futures and options work somewhat the same way.) Maturing government bonds, or checks that settle using maturing government bonds, can be the medium of exchange. Completely cashless models (such as Sims, 1997; Cochrane, 1999, 2001) are in fact much simpler to analyze, though the conceptual hurdles are evidently larger. (For example, Buiter, 2002, p. 476 finds that “A theory capable of pricing phlogiston, something that does not exist, except as a name, is an intellectual bridge too far.”)

2.3.3. *Long-term debt*

Long-term bonds complicate the algebra and can fundamentally change the dynamic properties of the equilibrium, but do not alter the existence of equilibrium or the basic mechanism. Cochrane (2001) analyzes this case.

A perpetuity is a simple and useful example of long-term debt. Suppose that the government issues D_0 perpetuities at time 0, and the perpetuities pay in aggregate a potentially state-contingent coupon B_t at each date t . Then, the price level at each date is

$$\frac{B_t}{p_t} = s_t. \quad (20)$$

The value of the perpetuities is

$$\frac{D_0 \times \text{time } t \text{ perpetuity price}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \frac{B_{t+j}}{p_{t+j}} = E_t \sum_{j=0}^{\infty} m_{t,t+j} s_{t+j}. \quad (21)$$

In this case, the price level is related directly to the coupon and surplus at *each date*, Eq. (20), rather than to the total outstanding debt and present value of future surpluses in Eq. (21). Also, issuing more debt devalues existing long-term claims to future surpluses, so changes in the maturity structure of government debt—“open market operations”—can affect the price level path.

The different dynamics come down to the convention that maturing debt—coupon payments or dollars—are the numeraire rather than the long-term bonds themselves. We could use 30 year treasury bonds as numeraire. We happen not to do so. The volatility of long-term bond prices makes this a wise, and perhaps not coincidental, choice.

3. Objections

3.1. Where is the government's intertemporal budget constraint?

There is no intertemporal government budget constraint in this model—the government's choice of $\{B_t, M_t, s_t\}$ is not made given prices and “subject to” some equation like

$$\frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t(m_{t,t+j} s_{t+j}). \quad (22)$$

How have we achieved an equilibrium without this traditional ingredient?

The analogous stock valuation equation is not a constraint. There are three steps in defining a competitive Walrasian equilibrium: first, one defines what the securities *are*—what state-contingent stream of goods is promised for each share or unit of a security. Second, one finds demand and supply curves for those securities, as well as demand and supply curves for goods. Third, one finds prices that clear markets. The decision of how many shares to issue is a definition of securities. Definitions of securities occur without constraint, before the “auctioneer” announces any prices. We do not try to construct a “supply curve” in which a firm observes the price per share before it decides whether to issue 100 or 1000 shares. If we did so, the firm would issue a lot of shares!

The government's decision of how much money and how many bonds to issue are analogously a definition of units, a definition of securities, a definition of how many “shares” to a given surplus stream there will be. It *must* occur before the auctioneer announces any prices. Money has always been modeled this way. If the “auctioneer” announces the price level before the government announces the money supply, hungry governments will issue a lot of money. Nominal debt—a claim to future money—should be treated the same way.

To analyze this issue, I first show how (22) acts as an market-clearing condition rather than a budget constraint in a variety of situations. I then examine the mechanics underlying (22) in detail—exactly what the government does at off equilibrium prices.

3.1.1. Reaction to off-equilibrium prices

Demand curves must respect budget constraints even at off-equilibrium prices. If (22) were a budget constraint, then the government would have to respond to an off-equilibrium deflation in p_t by raising subsequent surpluses. It would *have* to follow a Ricardian regime.

However, if a bubble raises the price of Microsoft's stock, no budget constraint forces Microsoft to raise subsequent earnings. Once equity is issued, all the firm has to do is to pay the promised state-contingent sequence $\{s_t\}$. It may ignore completely the market price of its stock. Like any other market-clearing condition, the firms' valuation formula is violated at off equilibrium prices.

The analogous conclusion holds for nominal government debt. The government must honor the terms of the security: it must exchange one dollar for each maturing bond, it must accept one dollar in exchange for one dollar's worth of tax liability, and it must auction new debt for dollars and accept dollars in exchange for new debt. It must accept dollars as "legal tender" "for all debts, public" [and private] as it says on the face of a dollar bill. It must also stick to the promised sequence $\{s_t\}$. It can honor these commitments at arbitrary off-equilibrium prices. I trace the mechanics in detail below.

3.1.2. Currency reforms and splits

If Eq. (22) were a constraint on $\{B_t, s_t\}$ given $\{p_t\}$, then the government could not raise debt B_{t-1} without raising expected future surpluses $\{s_{t+j}\}$. However, a corporation may double its shares without changing future earnings, and this action simply halves the equilibrium price. This is a stock split. The corporation knows this in contemplating a stock split; it does not think it has to take the price per share as given and double earnings in order to double its outstanding shares. If the share price does not fall following a split, no budget constraint of the firm is violated.

The government may double its nominal debt without changing future real surpluses. This is a currency reform. This action doubles the price level, and the government knows this in contemplating the reform. If (22) is a budget constraint, currency reforms are impossible. Currency reforms are possible. Hence (22) is not a budget constraint.

Governments *choose* to pair most nominal debt issues with implicit or explicit promises to raise subsequent surpluses. Governments typically issue nominal debt to finance wars or countercyclical spending; they want to raise revenue, and they do not want to raise the price level. If they issue debt without changes in expected future surpluses, they raise the price level and raise no revenue, as in a currency reform. Thus, most of our experience and data consists of debt changes paired with changes in expected future surpluses. (Cochrane, 2001, presents a quantitative example.) But currency reforms and revaluations are *possible*, even if infrequent.

Corporations also sometimes increase shares while changing future dividends, in a new issue (seasoned equity offering). Corporations arrange, market, and account for new stock issues very differently from splits, to signal that total dividends will increase following a new issue but not following a split. Governments similarly arrange and market currency reforms very differently from debt issues, to convey a sharply different set of expectations: that no change in real surpluses will follow a currency reform, and that a proportional change in surplus will follow a debt issue.

3.1.3. Real debt

The government faces budget constraints in other transactions, as do private agents. If the government wants to buy or sell goods and services, foreign debt, indexed debt, real debt, or other already-defined bundles of contingent claims; if the price level is defined in terms of gold or foreign currency, or if it wants to engage in any other transaction, it must obey the relevant constraints, just like private agents.

The distinction between nominal debt and real debt—indexed debt, or debt denominated in a commodity or in foreign currency—is subtle, important, and accounts for most of the confusion. A real bond is a state-uncontingent promise to pay one unit of the numeraire good. In this case, the government valuation equation (22) does constrain the government. Any increase in debt B_t must come with increases in subsequent surpluses $\{s_{t+j}\}$. The government cannot “split” its (say) gold-denominated debt, counting on the price of gold to halve. The government must respond to price changes. For example, if the government issues foreign debt and the exchange rate depreciates, it must raise taxes to pay a larger real amount. A government that issues only real debt is exactly like a firm that issues debt and no equity. If the price level halves, the owners of the debt-only firm must double real interest payments. Nominal government debt walks like debt and quacks like debt, but it is really equity.

Of course, the government or a debt-only firm might also default. In a Walrasian equilibrium, contracts are perfectly and costlessly enforceable, so true default is impossible. Instead, we model defaultable debt as a different and state-contingent security. It is straightforward to extend the analysis to include this kind of explicit default, and it does not change the issues that are the focus of this paper.

3.1.4. Special agents

The stock analogy makes it clear that the fiscal theory assumes nothing special about the government. The government affects a security’s price because it defines that security. It does not have to be a “large agent,” one that “moves first,” one with market power, or one that has some special ability to threaten things at off-equilibrium prices. The smallest firm on the NYSE cuts its stock price in half when it does a 2-1 split.

Why can you or I not demand that the price level adjust to make our (soon to be much expanded) budget constraints hold? Why cannot Argentina insist that the US price level adjust to make Argentina’s budget constraint hold? Buiter (2002, p. 477) writes “one could apply the logic of the FTPL to the household sector and view the household’s intertemporal budget constraint as a condition that need only be

satisfied in equilibrium... This gives us the ‘household intertemporal budget constraint theory of the price level’...” Well, obviously not, but why not?

The answer is: we do not print dollars. Consider a terminal period. If the surplus is low, the government prints dollars to redeem the debt, more dollars than are needed to pay taxes. Agents trying to get rid of dollars will push up the price level. Again, we can still understand inflation as “money chasing goods.” (Though easiest to see in the frictionless world with money, the argument doesn’t hinge on money. If real net taxes decline in a cashless world, consumers have too much nominal debt sitting in their pockets, which they try to exchange for goods, driving up prices.) On the other hand, if my, your, or Argentina’s, dollar debts exceed our capacity to pay them, we can’t print our way out of trouble. To us, dollar debt is real debt—debt. To the government, dollar debt is nominal debt—equity.

3.2. Plans at off-equilibrium prices

It’s obvious that the valuation equation such as (22) is not a constraint to a private equity issuer, since he can pay the state-contingent dividend $\{s_t\}$ completely ignoring the evolution of prices $\{p_t\}$. One period government debt is rolled over every period, however. Since the government must operate in the market each period to roll over debt, it is less obvious that it can ignore the sequence of prices. Therefore, I detail here just how the government valuation equation can be violated at off-equilibrium prices. In the interest of space, I do not treat every possible price sequence, but instead I focus on a few of the more interesting possibilities, and I focus on the central issue, whether (22) should be thought of as a budget constraint.

Of course, private equity can be rolled over too, and the increasing popularity of repurchases in place of dividends makes this a practical as well as theoretical possibility. Each period, the corporation can use the cash formerly allocated to dividend payments, together with the proceeds of a new share issue, to repurchase all the outstanding shares. Rolled-over equity, of course, has exactly the same valuation formula as conventional equity. (This kind of corporate equity exactly mimics one period nominal government debt. Similarly, government nominal debt would exactly mimic conventional private equity if the government issued perpetuities, and if we used the perpetuities rather than their coupons as numeraire.)

The analysis is easiest in the terminal period of a cashless model. Nominal debt B_{t-1} is outstanding. Consumers use maturing nominal debt to pay taxes s_t . The government burns the matured bonds, and then the world ends. The equilibrium price level is determined by

$$B_{t-1} = p_t s_t.$$

Suppose that the auctioneer tries too low a price level p_t . Tax payments $p_t s_t$ would be insufficient to soak up the outstanding debt B_{t-1} . Some debt would be left in consumer’s hands at the end of the world. But this event does not violate the government budget constraint. If people developed a taste for past-due debt—if it acquired numismatic, nutritional, or decorative value—then people would want to keep some of it, and the government’s budget constraint must allow them to do so.

Therefore, the condition that all debt is redeemed derives from *preferences* and *market clearing*, not the government budget constraint. The government need not raise the surplus s_t . (Obviously, adding money and leaving useless money in consumers' hands works the same way.)

Suppose instead that the auctioneer calls out too high a price level p_t . At these prices, $p_t s_t > B_{t-1}$, so the government would receive more than all the outstanding bonds in payment of taxes. The government has already announced that it will burn bonds after they are redeemed, and if extras magically come in, the government can burn them too. The government need not lower the surplus s_t . (The two experiments are asymmetric as people cannot hold negative past-due nominal debt.)

Of course, receiving more than all outstanding bonds is impossible in equilibrium. The government's budget constraint can, and should, ignore this fact. If I have \$4, and the auctioneer announces a price of \$2 for the Hope Diamond, my budget constraint allows me to announce a demand for two Hope Diamonds, completely ignoring the fact that only one exists. We impose one Hope Diamond when we find *prices* such that supply = demand, a market clearing condition. Budget constraints do not respect market clearing conditions. No trade occurs at off-equilibrium prices, so my demand for two Hope Diamonds would never be tested. Economies are not expected to function sensibly at off-equilibrium prices.

There is an aggregate resource constraint here as well: all outstanding debt B_{t-1} is either received by the government, or left in private hands. Denoting the latter quantity $B_{t-1}^{(i)}$, the aggregate resource constraint is

$$B_{t-1} = p_t s_t + Q_t B_t + B_{t-1}^{(i)}.$$

With $B_{t-1}^{(i)} \geq 0$, the government's plan happens to be consistent with the resource constraint for prices below equilibrium. However, as in the last paragraph, even this aggregate resource constraint does not affect the government's plans for prices above equilibrium.

In sum, the fact that nominal debt B_{t-1} is outstanding places *no constraint at all* on the surplus s_t . If the auctioneer announces the market-clearing price, the outstanding debt will be redeemed. If not, the government has perfectly feasible "demands" at any off equilibrium price.

A typical period of the intertemporal cashless model works similarly. The government accepts maturing debt at the new debt auction as well as for tax payments, and it burns any matured debt it receives. The equilibrium price level satisfies

$$B_{t-1} = Q_t B_t + p_t s_t, \tag{23}$$

where $Q_t = E_t(m_{t+1} p_t / p_{t+1})$ is the bond price. If the price level p_t or bond price Q_t are too low, the government can again let matured debt sit in private hands. If the price level p_t or bond price Q_t are too high, the government can again plan to burn the extra debt that comes in, as it burns the equilibrium matured debt. Neither event forces a change in s_t .

The government *auctions* new debt, and accepts whatever price results, as equity issuers auction bundles of contingent claims in Walrasian equilibrium. By this means, it can commit to its plan B_t for any price Q_t or p_t .

An interesting class of off-equilibrium price paths violate the transversality condition

$$\lim_{j \rightarrow \infty} E_t \left(m_{t,t+j} \frac{B_{t+j-1}}{p_{t+j}} \right) = 0 \tag{24}$$

rather than the flow equation (23). (The present value equation (22) is equivalent to the flow equation (23) plus the transversality condition (24).) For example, consider a perfect-foresight cashless model with a constant real interest rate $1 + r$, constant surplus s , so the equilibrium price level from (22) is

$$\frac{B_{t-1}}{p_t} = \frac{1+r}{r} s.$$

Consider an off-equilibrium price path in which the flow equation (23) (relating p_t to p_{t+1}) holds correctly. Rearranging (23), p_{t+1} and the real value of the debt must then evolve as

$$\frac{B_t}{p_{t+1}} - \frac{1+r}{r} s = (1+r) \left(\frac{B_{t-1}}{p_t} - \frac{1+r}{r} s \right). \tag{25}$$

If the overall level of the price path is too low, the real value of debt grows explosively, violating the transversality condition.

The government can let real debt explode. Each period, the government accepts debt for tax payments and sales of new debt; these are the only commitments defining nominal debt. You may say, “consumers eventually will not lend enough to the government to roll over an exploding debt”—just as you might say “consumers do not want to hold past due debt in the last period.” You would be right—but these are market-clearing conditions, not a budget constraints, since they involve consumers. If the consumers *were* willing to lend ever increasing amounts, the government budget constraint must allow them to do so.

Models with money are a little trickier—either the frictionless model with intraday money but a reopened security market, or the monetary model. Here, transactions are mediated with cash—bonds are redeemed for cash, and taxes are paid in cash. The equilibrium price level satisfies the flow equation

$$B_{t-1} = Q_t B_t + p_t s_t + (M_t - M_{t-1}). \tag{26}$$

Again, this equation is not even the aggregate resource constraint. Consumers may fail to redeem all their outstanding debt for money, leaving past due debt in consumers’ hands. And again, the aggregate resource constraint does not bind the government’s demands. If p_t or Q_t rises so much that more cash than $B_{t-1} + M_{t-1}$ would come in, this equation will not hold. Again, the government can tell the auctioneer it will burn the extra cash at these prices.

However, suppose that consumers do redeem all their outstanding debt for money, and consider only off-equilibrium price declines, or rises less than $p_t s_t = B_{t-1} + M_{t-1}$ at which outstanding money and debt are all returned. In this case, money takes the role of past-due debt in the resource constraint, so (26) *does* hold mechanically at any price level.

This change does not force a Ricardian regime, or alter fiscal price determination. Iterating (26) forward and using the arbitrage condition $Q_t = E_t(m_{t+1}(p_t/(p_{t+1})))$, we obtain

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} m_{t,t+j} \left(s_{t+j} + \frac{(M_{t+j} - M_{t+j-1})}{p_{t+j}} \right) + \lim_{j \rightarrow \infty} E_t \left(m_{t,t+j} \frac{B_{t+j-1}}{p_{t+j}} \right).$$

As above, no constraint on the government forces the last term to be zero. The government can keep on redeeming debt for freshly printed cash, accepting cash for tax payments, and auctioning new debt as the real value of that debt explodes.

This change does seem to limit the government's ability to separately announce all three of $\{B_t, M_t, s_t\}$. In the cashless model, the government auctioned a fixed stock B_t , and so could control B_t for any price level. Now, given B_t and s_t , it seems that M_t is out of the government's control. Should we rewrite the statement of the model as "The government chooses two of $\{B_t, M_t, s_t\}$ and the third follows from Eq. (26)?" Not necessarily, because slight variations in the rights governing money and debt at off-equilibrium prices would allow the government to control all three of $\{B_t, M_t, s_t\}$. For example, the government can declare that money, like debt, is only good for one period, and auction the new money stock as well as nominal debt. Now the government can commit to all three of the sequences $\{B_t, M_t, s_t\}$. Old money as well as past due debt will accumulate in consumer's hands at too-low prices. The government can also refuse to redeem debt for cash once it has reached its money stock target.

Much of this section seems to apply to real debt as well. If consumers decide to use real debt as wallpaper, the government does not have to pay it back. If consumers will lend ever increasing amounts of real debt to the government, the value of government debt can explode. Have we concluded that there is no budget constraint at all? No. The issue is not arbitrary physical possibilities. The issue is what plans or demands are implied by the commitments that define real or nominal debt, as a function of prices. The commitments that define nominal debt are to redeem it for money, and to accept that money for tax payments and new debt. We have verified that the government can keep these commitments at off-equilibrium prices. The commitment behind real debt is to pay one unit of numeraire good, no matter what the price level. A decision ex-post not to repay real debt is physically possible, leaving useless debt in consumers' hands, but it would violate the commitments that define real debt. In that sense it violates the "budget constraint." Leaving nominal debt in consumer's hands if the auctioneer announces too low a price is, by contrast, exactly *keeping* the commitments that define nominal debt, and in that sense it does not violate the "budget constraint."

3.3. More objections

3.3.1. Will the government let debt explode?

Granted that the Government *can* ignore off-equilibrium prices, the more interesting question is, *will* it do so? Of course, any government at some point runs out of taxing power and so *must* become Non-Ricardian. But perhaps sensible governments that are not in fiscal distress will invariably *choose* Ricardian regimes. McCallum (2001) argues that declining real debt will lead the government to spend more or tax less, and that rising values of debt will lead it to raise surpluses. Christiano and Fitzgerald (2000, p. 4) notice that “often governments do seem ready to adjust fiscal policy when the debt gets large,” and cite the US government in the 1990s and the Maastricht treaty.

Alas, this intuition and historical experience are not relevant to the question. The rise in the value of U.S. debt from the mid-1970s to 1990 was caused by large deficits, large nominal debt sales at roughly stable prices, not by an unanticipated (and especially “off-equilibrium”) deflation which raised the value of a stable stock of outstanding nominal debt. The Maastricht treaty envisions the same sort of events. When the bonds were sold, they raised revenue, and thus investors must have believed they came with a commitment to raise future surpluses. Without such a commitment, the bonds would have raised prices and no revenue, as in a currency reform or split. Raising the surpluses after the fact is only making good on commitments; fulfilling a time-consistent dynamic equilibrium.

The issue is different: suppose an unexpected deflation hits. Will the Treasury really raise taxes to pay off windfall gains to wealthy bondholders? Or will it simply roll over the debt, letting it grow in value, waiting for good times to come again, and trying various policies to “reflate” the economy, which will devalue the debt rather than pay it off? The experience of the U.S. government in the 1930s and the Japanese government since the early 1990s strongly suggests the latter response.

On the other hand, the Fed very clearly does follow an accommodative monetary policy. Fixed nominal money growth targets that do not respond to inflation is not even a vaguely plausible description of current policy. History and intuition do not obviously suggest a Ricardian regime.

And even this evidence is at best suggestive. The issue is how governments will respond to *off-equilibrium* prices. Walrasian equilibrium describes only the equilibrium; we never see “off equilibrium” behavior for a period, let alone for a long stretch of time for which the government’s resolve can be tested.

More formally, Canzoneri et al. (2001) and Bohn (1998) run regressions of surpluses on debt to GDP ratios. They find that higher debt leads to higher surpluses, and conclude that the U.S. follows a Ricardian regime. But this is exactly the pattern we expect of a government in a Non-Ricardian regime that values price stability. In a recession, the government issues more debt. To raise revenue from this sale, the government must promise to raise taxes when the recession ends. (Taxes, not necessarily tax *rates*. An income tax can make this automatic.) Higher debt in the recession thus precedes higher surpluses in the boom. Analogously, Cohen et al. (2003) find that higher corporate equity values forecast higher subsequent dividends.

This does not mean that the equity valuation formula has become a constraint, or that sensible companies adjust earnings and dividends in response to off-equilibrium revaluations of their stock prices. It does not mean that splits are impossible.

More fundamentally, the equilibrium conditions of the fiscal and monetary regimes of the monetary model are identical; the models are observationally equivalent. No time series test can distinguish them. The only hope I can see to tell them apart is by thinking about why the government chooses observed policies. A currency collapse in a monetary regime is hard to understand—the government needs only the spine to do the required open market operations. A currency collapse makes much more sense in a fiscal regime, where it represents a choice to “default” on bond and money holders rather than to raise distorting or politically difficult taxes.

3.3.2. *If the government can violate its budget constraint, why not set taxes to zero?*

If the government can violate a budget constraint, this obviously opens an optimal-taxation can of worms. If there is no budget constraint, why not set taxes to zero? [Christiano and Fitzgerald \(2000, p. 4\)](#) write

...consider legislators living in a non-Ricardian regime. Understanding that tax cuts or increases in government spending do not necessarily have to be paid for with higher taxes later, they might be tempted to embrace policies that imply too much spending and too much debt.

[Buiter \(1999, abstract\)](#) writes

The fiscal theory of the price level implies that a government could exogenously fix its real spending, revenue and seignorage plans, and that the general price level would adjust the real value of its contractual nominal debt obligations so as to ensure government solvency. When reality dawns, the result could be a painful fiscal tightening, government default, or unplanned recourse to the inflation tax.

Since the valuation equation is not the government’s budget constraint, this issue is resolved. Issuing more debt is no better for the government than diluting (splitting) shares is for Microsoft. Cutting surpluses ex-post devalues outstanding government debt, equivalent to a default. We can and should worry about time-consistency, contract enforcement, and default, but these issues are not special to the fiscal theory.

3.3.3. *What about the historical stability of money demand?*

A generation of economists since [Friedman and Schwartz \(1971\)](#) has pointed to the stability of the money–income relation as support for a monetary explanation of inflation. Most recently, [\(2001, p. 2\)](#) write “the U.S. Inflation of the 1970s and 1980s can be *fully accounted for* by the corresponding increase in M2 (or M1) growth rates, and the return to relatively low inflation can be *explained by* the correspondingly low average rate of money supply growth in that decade.” (My emphasis.) (See also [Lucas, 1996.](#))

The money demand equation is still present in the fiscal regime. However, it determines the quantity of money rather than the price level, as the Fed is passively providing whatever quantity of money is demanded. The observation that the quantity of money tracks nominal income is irrelevant to the regime question. The issue is the direction of causality. Rich men drive fancy cars, but will driving a fancy car make you rich? Furthermore, most of M2 is inside, interest paying, liquid assets, which will survive unchanged in a cashless economy. Monetary economists in a future economy with no cash or reserves may still have pretty graphs of GDP and M2 on their office walls.

3.3.4. Is this all in Sargent and Wallace?

Sargent and Wallace (1981) consider a model with money, in which the equilibrium price level is determined by (14) and (15), reproduced here

$$M_t v = p_t e, \quad (27)$$

$$\frac{B_{t-1}}{p_t} = \sum_{j=0}^{\infty} E_t \left[m_{t,t+j} \left(s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{p_t} \right) \right]. \quad (28)$$

Sargent and Wallace consider a regime in which $\{s_{t+j}\}$ is exogenous (or governments are already maximizing surpluses), but the central bank can still control money. They specify *indexed* debt, so the value on the left-hand side of (28) is unaffected by the price level. (This specification is explicit below Sargent and Wallace's equation (4).) Now, if $\{s_{t+j}\}$ declines, the central bank must generate some seignorage revenue. Since the *present value* of seignorage revenue must rise, the central bank still has an interesting choice; less "inflation now" will bring more "inflation later." Alas, seignorage is a trivial component of government revenue for many advanced economies, leading many to discount this parable as an interesting connection between fiscal difficulties and inflation.

With Sargent and Wallace's main point in front of us, we can see that it is justly famous as a pioneering study of fiscal-monetary links. However, it is no insult to point out that 20 subsequent years of fiscal theorizing have produced some novelty. First, the B on the left-hand side may be nominal debt. Thus, inflation can reduce the value of the debt *directly*, rather than just through a seignorage channel. In fact, with nominal debt, a fall in $\{s_{t+j}\}$ can generate inflation (a rise in the price level) in the frictionless version of the model, with no money demand or seignorage whatsoever—deleting (27) and setting $M = 0$ in (28)—a possibility not present in Sargent and Wallace's analysis. In fact, by removing the link to seignorage revenue, these modifications reinforce Sargent's (1986) basic point of an underlying fiscal cause of hyperinflations and their ends. More deeply, Sargent and Wallace's indexed debt is equivalent to nominal debt and a Ricardian regime; the more recent fiscal theory contemplates non-Ricardian regimes as well. This is the central assumption that lets us study a cashless model.

3.3.5. What if the government makes policy choices that do not lead to equilibrium?

Arbitrary sequences for debt, money and surpluses $\{B_t, M_t, s_t\}$ do not lead to equilibrium, as detailed in Section 2.2. For example, negative money, nonzero money in a frictionless model, negative debt, an uncoordinated sequence, or a sequence that leads to a negative nominal interest rate cannot lead to an equilibrium. Models with an interest-elastic money demand can impose additional restrictions. For example, McCallum (2001) shows in a model with hyperinflationary dynamics that if debt B_{-1} is too low relative to surpluses, then the fiscal equilibrium will lead to deflation and exploding real debt that violates the consumer's transversality condition. To produce an equilibrium, the government has to issue more nominal debt. (See also Kocherlakota and Phelan, 1999.)

Is this a problem? McCallum views it as a critique, and Buiter (2002, p. 476) cites the fact that “Arbitrary restrictions on the predetermined and exogenous variables in the government solvency constraint are required to support a non-negative equilibrium price level sequence” as evidence that fiscal theory models are misspecified.

But this feature is hardly unique to non-Ricardian regimes. In the standard monetary (Ricardian) economy there is no equilibrium if the government insists on a non-positive amount of money, for choices of $\{M_t\}$ that imply fiscally impossible tax revenue or negative nominal interest rates, or if the central bank follows accommodative policies, setting $M = py/v$. It is not a requirement of a Walrasian equilibrium that one must exist for arbitrary policy specifications $\{M_t, s_t, B_t\}$, nor must an equilibrium specify how the government settles on a policy process that does produce an equilibrium. These are interesting questions, but, like contract enforcement, the size of firms, which securities are marketed, and so on, they lie outside conventional Walrasian equilibrium, and their absence does not establish that a Walrasian model is wrong.

4. Extensions and concluding remarks

4.1. Default, and corporate finance

Governments sometimes default on real or nominal debt. The value of defaultable nominal government debt divided by the price level still equals the present value of future surpluses, and this equation can still determine the price level. Thus, the basic points of this paper are unaffected by adding default. However, explicit default is interesting, and it expands the range of phenomena we can address. Changing probabilities of explicit default, and hence changing nominal values of debt, can soak up surplus shocks as well as changes in the price level. Argentina recently chose some of each. (Uribe, 2001, analyzes this choice.)

Explicit default and inflation are costly. Truly equity-like securities—securities whose *relative* price could soak up surplus shocks—would allow state-contingent government finance without those costs. For example, suppose that the government issues a state-contingent perpetuity, or variable coupon debt. Let d_t denote the

number of dollars actually paid per dollar initially promised, and let B_{t-1} denote the coupon coming due at time t . Then, the price level at each date t is determined by

$$\frac{B_{t-1}d_t}{P_t} = s_t.$$

If the government pays $d_t = s_t/B_{t-1}$ cents on the dollar, the price level is determined by this fiscal regime and is constant, despite surplus shocks.

Why do we not see such securities? The stock analogy proves useful again, as the *corporate* finance of the firm's capital structure is not trivial. The difference between bad luck and bad management in determining s_t is not so easy to see. This is why private equity comes with control rights, and why risky defaultable debt comes with direct monitoring by debt holders.

Why do we see nominal government debt? What are the corresponding control rights? Real vs. nominal debt (default vs. inflation) does not matter to government bond holders; they have the same incentives to monitor the government in either case. However, when all private contracts are tied to the value of government debt, we ensure that a much larger number of voters are angry when the government tries a state-contingent default (unexpected inflation), and they will monitor government finances to make sure it does not happen needlessly. It may not be by chance that successful fiat money regimes are almost exclusively found in healthy democracies, and only those governments can undertake substantial borrowing in their own currencies. The “private” part of “all debts, public and private,” so far unimportant to the analysis, may in fact contain the essential control mechanism that enforces time-consistency. But if we could invent alternative monitoring institutions, we could have a true “government equity” that did not require costly inflation.

Following this corporate finance end of the stock analogy, Sims (2001) advocates that Latin American economies *not* dollarize, precisely to leave intact an equity-like security (nominal debt) for government finance. A government in a dollarized economy, like a debt-only firm, must occasionally default on its real debt. Equity—nominal debt—provides a cushion against default that makes it easier to borrow real (foreign) debt. Sims views the costs of unexpected inflation as preferable to the costs of explicit default, and prefers monitoring by domestic voters to monitoring by large international banks. One may argue with either preference, but not with the logic.

4.2. Optimal taxation

Once we get past the budget constraint business, the fiscal theory becomes a simple addendum to the vast and fast-evolving field of optimal government finance. Inflation is state-contingent default as in Lucas and Stokey (1983). All the standard issues of distorting taxes, time-consistency and commitment, optimal state contingent debt, and so forth, glossed over in this paper in order to focus on budget constraint controversies, remain. Christiano and Fitzgerald (2000) start the important task of integrating fiscal theory with the theory of optimal distorting taxation. For example, a currency crash in fact represents a *choice* by the government to devalue outstanding nominal debt rather than to increase

distortionary taxes, rather than a helpless response to exogenous declines in surpluses. To model the costs of state-contingent devaluation via inflation, [Schmitt-Grohè and Uribe \(2001\)](#) mix a small amount of price stickiness with distortionary taxes. They find that the government optimally smooths inflation a great deal, even with relatively large shocks to surpluses. As above, these extensions promise testable restrictions that may help us to distinguish which governments follow fiscal regimes and which do not.

4.3. *Last words on the stock analogy*

The stock analogy brings a “and yet it moves” perspective to fiscal theory debates. We may argue endlessly the economic theory (at this point, almost theology) of the government valuation equation, formerly known as the government budget constraint. Yet government nominal debt is mathematically identical to stock in our models, and stock prices are determinate. (You may think stock prices “too volatile,” but volatility—a time varying expected return—is not indeterminacy.) Nobody thinks that stock prices are entirely determined by a liquidity demand intersected with an artificially limited supply. If stock price determinacy poses theoretical problems, so much the worse for theory.

Alas, the stock analogy also bodes poorly for decisive empirical work. Stock prices do typically move as we expect them to when there is earnings or discount rate news. However, they also fluctuate a great deal in ways that are hard to explain by independent news about future earnings or discount rates. Similarly, we may expect that many pieces of news will move the price level in the right way, as shown by [Sargent’s \(1986\)](#) analysis of the end of hyperinflations and [Burnside et al. \(2001\)](#) analysis that the East Asian currency crashes were precipitated by bad news about prospective deficits. However, we may also expect that much short-term fluctuation in prices and exchange rates, and questions such as the exact timing of devaluations, will be just as difficult to explain from obvious news about surpluses or discount rates as stock prices are difficult to explain from obvious news about earnings or discount rates.

References

- Alvarez, F., Lucas Jr., R.E., Weber, W., 2001. Interest rates and inflation. *American Economic Review* 91, 219–225.
- Bassetto, M., 2002. A game-theoretic view of the fiscal theory of the price level. *Econometrica* 70, 2167–2195.
- Bohn, H., 1998. The behavior of U.S. public debt and deficits. *The Quarterly Journal of Economics* 113, 949–963.
- Bohn, H., 1999. Comment. In: Olivier, B., Julio, J.R. (Eds.), *NBER Macroeconomics Annual 1998*, pp. 384–389.
- Buiter, W.H., 1999. The fallacy of the fiscal theory of the price level. NBER Working Paper No. 7302.
- Buiter, W.H., 2002. The fiscal theory of the price level: a critique. *Economic Journal* 112, 459–480.

- Burnside, C., Eichenbaum, M., Rebelo, S., 2001. Prospective deficits and the Asian currency crisis. *Journal of Political Economy* 109, 1155–1197.
- Canzoneri, M.B., Cumby, R.E., Diba, B.T., 2001. Is the price level determined by the needs of fiscal solvency? *American Economic Review* 91, 1221–1238.
- Christiano, L.J., Fitzgerald, T., 2000. Understanding the fiscal theory of the price level. Federal Reserve Bank of Cleveland. *Economic Review* 36, 1–37.
- Cochrane, J.H., 1999. A frictionless view of U.S. inflation. In: Bernanke, B.S., Rotemberg, J.J. (Eds.), *NBER Macroeconomics Annual*, pp. 323–384.
- Cochrane, J.H., 2001. Long term debt and optimal policy in the fiscal theory of the price level. *Econometrica* 69, 69–116.
- Cohen, R.B., Polk, C., Vuolteenaho, T., 2003. The value spread. *Journal of Finance* 58, 609–614.
- Dupor, B., 2000. Exchange rates and the fiscal theory of the price level. *Journal of Monetary Economics* 45, 613–630.
- Friedman, B.M., 1999. The future of monetary policy: the central bank as an army with only a signal corps? *International Finance* 2, 321–338.
- Friedman, M., Schwartz, A.J., 1971. *A Monetary History of the United States 1867–1960*. Princeton University Press, Princeton, NJ.
- Kocherlakota, N., Phelan, C., 1999. Explaining the fiscal theory of the price level. Federal Reserve Bank of Minneapolis *Quarterly Review* 23, 14–23.
- Leeper, E., 1991. Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies. *Journal of Monetary Economics* 27, 129–147.
- Ljungqvist, L., Sargent, T.J., 2000. *Recursive Macroeconomic Theory*. MIT Press, Cambridge, London.
- Lucas, Jr., R.E., 1984. Money in a theory of finance. *Carnegie-Rochester Conference Series on Public Policy* 21, 9–45.
- Lucas Jr., R.E., 1996. Nobel lecture: money neutrality. *Journal of Political Economy* 104, 661–680.
- Lucas Jr., R.E., Stokey, N.L., 1983. Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12, 55–93.
- Lucas Jr., R.E., Stokey, N.L., 1987. Money and interest in a cash-in-advance economy. *Econometrica* 55, 491–513.
- Marimon, R., 2001. The fiscal theory of money as an unorthodox financial theory of the firm. In: Leijonhufvud, A. (Ed.), *Monetary Theory as a Basis for Monetary Policy*. Palgrave, Houndmills, UK, New York.
- McCallum, B.T., 2001. Indeterminacy, bubbles, and the fiscal theory of price level determination. *Journal of Monetary Economics* 47, 19–30.
- Sargent, T.J., 1986. *Rational Expectations and Inflation*. Harper & Row, New York.
- Sargent, T.J., 1987. *Dynamic Macroeconomic Theory*. Harvard University Press, Cambridge.
- Sargent, T.J., Wallace, N., 1981. Some Unpleasant Monetarist Arithmetic. Federal Reserve Bank of Minneapolis *Quarterly Review* Fall (Reprinted in Sargent 1986).
- Schmitt-Grohé, S., Uribe, M., 2001. Optimal Fiscal and Monetary Policy under Sticky Prices. Manuscript, University of Pennsylvania.
- Sims, C.A., 1994. A simple model for the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory* 4, 381–399.
- Sims, C.A., 1997. *Fiscal Foundations of Price Stability in Open Economies*. Manuscript, Princeton University.
- Sims, C.A., 1999. Domestic Currency Denominated Government Debt as Equity in the Primary Surplus. Manuscript, Princeton University.
- Sims, C.A., 2001. Fiscal consequences for Mexico of adopting the dollar. *Journal of Money, Credit, and Banking* 33, 597–616.
- Starr, R.M., 1974. The price of money in a pure exchange monetary economy with taxation. *Econometrica* 42, 45–54.
- Starr, R.M., 2000. Why is there money? Endogenous derivation of ‘Money’ as the most liquid asset: a class of examples. Discussion Paper No. 2000-25, University of California, San Diego.
- Uribe, M., 2001. A Fiscal Theory of Sovereign Risk. Manuscript, University of Pennsylvania.

- Woodford, M., 1995. Price level determinacy without control of a monetary aggregate. Carnegie-Rochester Conference Series on Public Policy 43, 1–46.
- Woodford, M., 1998. Doing without money: controlling inflation in a post-monetary world. *Review of Economic Dynamics* 1, 173–219.
- Woodford, M., 2001. Fiscal requirements for price stability. *Journal of Money, Credit and Banking* 33, 669–728. NBER Working Paper No. 8072.