# The New-Keynesian Liquidity Trap

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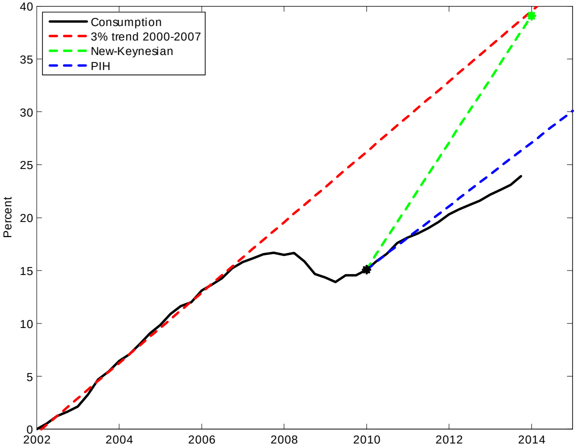
# New Keynesian models: Diagnosis

- Consensus NK diagnosis: "Natural" rate $r << -2\% \Rightarrow i = 0$, $\pi = 2\%$, $i - \pi$ too high.
- Fix $E_t c_{t+\infty}$. "Too high" $i - \pi \Rightarrow c$ grows too fast, level too low.
- Why do we not see more $\pi$? $\Rightarrow$ model!



# New Keynesian models: Policy

- Policy: Many laws of economics change sign at the zero bound.

  1. Commitments to future policy raise GDP with no action today.
  2. *Expected* inflation raises output.
  3. Wasted government purchases, even if financed by taxes, can have very large multipliers.
  4. Technical regress, lower productivity, broken windows raise output.

- These work by by raising $\pi$, lowering $i - \pi$, lowering growth $E_t \Delta c_{t+j}$ and thus raising the level of consumption and output.

$$c_t = E_t \lim_{T \to \infty} c_{t+T} - \sigma^{-1} \int_{s=0}^{\infty} \left( i_{t+s} - r_{t+s} - \pi_{t+s} \right) ds$$

  NK model is *not* static, income–driven Keynesian. MPC = 0. IS = Intertemporal substitution. $r$, not $Y$, equilibrates.

- Puzzles:

  1. Promises *further* in the future have *larger* effect today.
  2. Diagnosis and policy get *stronger* as frictions *diminish*, with $\infty$ limit.
  3. $\rightarrow$Though stickiness is the central friction causing our troubles, don't fix it! Making prices sticker is good.

# New-Keynesian model (Werning 2012)

$$\frac{dx_t}{dt} = \sigma^{-1}(i_t - r_t - \pi_t) \quad (1)$$

$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa x_t. \quad (2)$$

$$r_t = \text{“natural” rate}$$

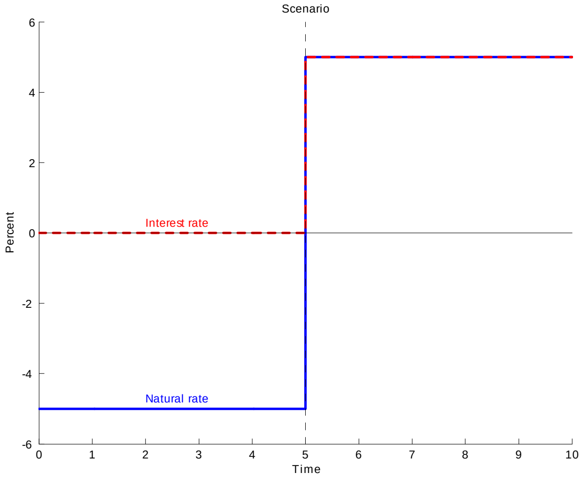
From discrete time,

$$\begin{aligned} x_t &= E_t x_{t+1} + \sigma^{-1}\left[i_t - r_t - E_t \pi_{t+1}\right] \\ \pi_t &= e^{-\rho} E_t \pi_{t+1} + \kappa x_t \end{aligned}$$

or

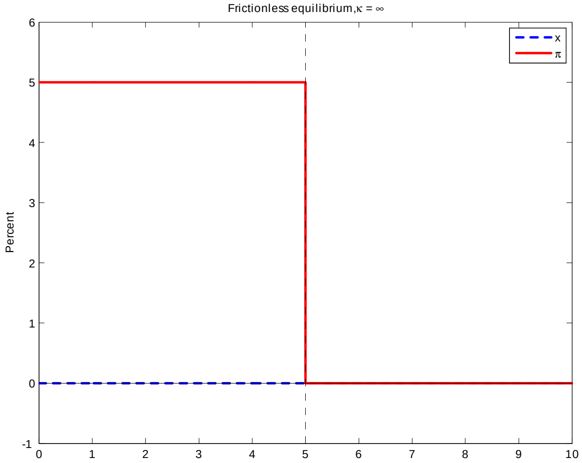
$$\pi_t = \kappa \int_{s=0}^{\infty} e^{-\rho s} x_{t+s} ds$$

# Scenario – negative natural rate / liquidity trap




- Task: find $\{\pi_t, x_t\}$. (*Taylor rule analysis follows.*)

# The frictionless equilibrium

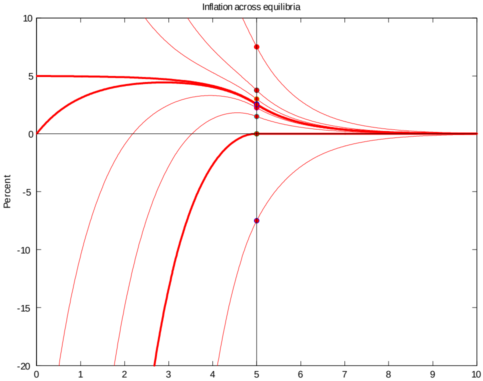



- $d\pi_t / dt = \rho\pi_t - \kappa x_t$. $\kappa \Rightarrow \infty$; $x_t = 0 \; \forall \pi_t$
- $dx_t / dt = \sigma^{-1} \left( i_t - r_t - \pi_t \right) \Rightarrow \pi_t = i_t - r_t$
- Higher $\pi$ exactly matches $r < 0$. Zero bound has no output effect.

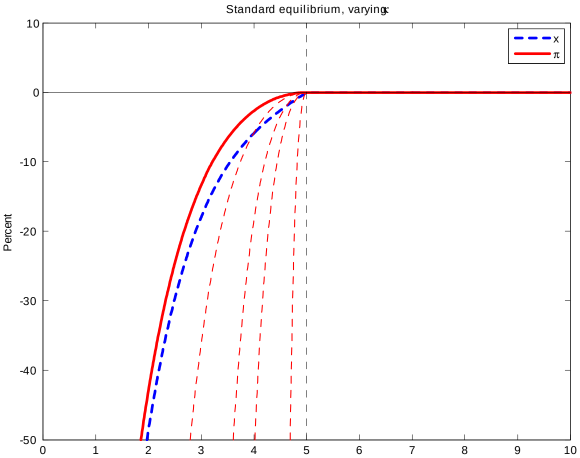
# Solution with price stickiness

$$\frac{d}{dt}\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -\kappa & \rho \end{bmatrix}\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} i r_t \\ 0 \end{bmatrix}$$

- Two eigenvalues, $\lambda^m < 0$, $\lambda^p > 0$; two free constants.
- $t > T$ : Set $e^{\lambda^p t}$ term to zero. $t \leq T$ : Match solutions at $t = T$.
- One-dimensional family of solutions. Index by $\pi_T$ (really $E_t \pi_T$).

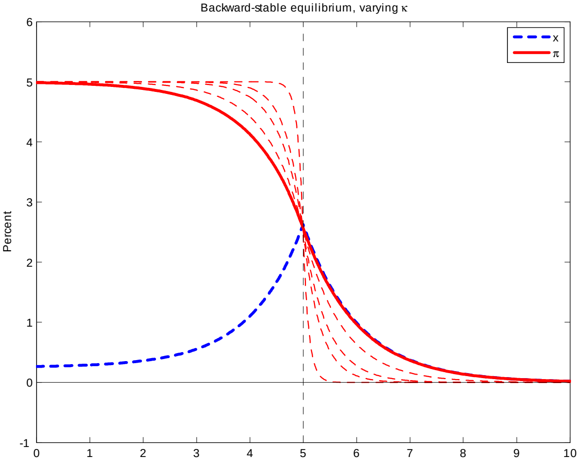


# The standard equilibrium choice $\pi_T = 0$



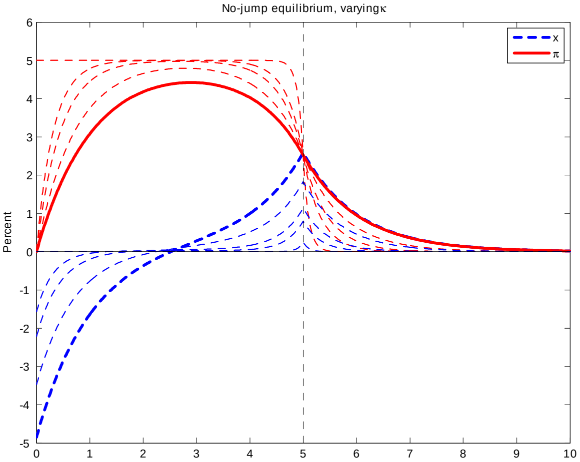

- Depression (with growth). Deflation (rapidly changing).
- Backward-explosive $\lambda \Rightarrow$ key to big predictions.
- Less friction$\rightarrow$worse! Limit$\neq$limit point! Less frictions $\rightarrow$ faster.

# The backwards-stable equilibrium




- Choice: $\pi_T$ s.t. *backward* approach to steady state.
- No (demand-side) recession. No big growth/deflation & dynamics.
- Nice frictionless limit!
- *Exactly the same interest-rate path.*

# The no-inflation-jump equilibrium



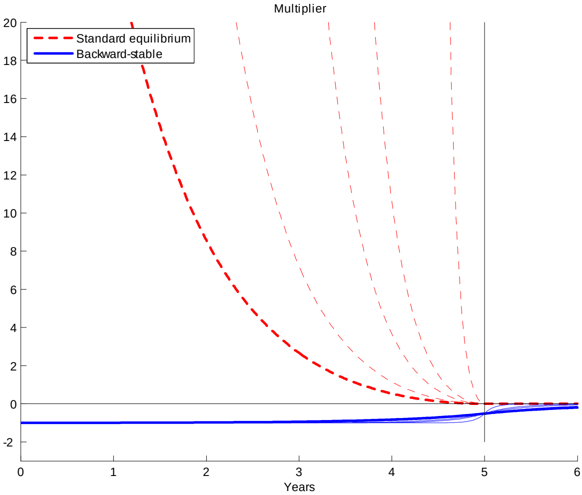

- Choose $\pi_T$ so that $\pi_0 = 0$.
- Also no deflation, small output effects.
- *All* equilibria with limited $\pi_0$, $x_0$ jump have normal frictionless limits, small effects for small price stickiness.

# Magical multipliers and paradoxical policies

$$\frac{dx_t}{dt} = \sigma^{-1}(i_t - r_t - \pi_t)$$
$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa(x_t + \mathbf{g}_t).$$

- $g$ = useless spending, technical regress, high wage mandates, capital destruction, etc.
- Phillips curve disturbance: All else constant, more $g$ means more $\pi$. More $\pi$ means less $i - \pi$.
- $\pi$, intertemporal substitution channel, *not* consumption function!
- $\{\pi_t\}$ constant means $\partial x / \partial g = -1$. Effect needs $\pi$, $d\pi_t/dt$.
- Consider $g_t = g > 0$ for $t < T$, then $g_t = 0$ for $t > T$. Calculate $\partial x_t / \partial g$, private-output multiplier....

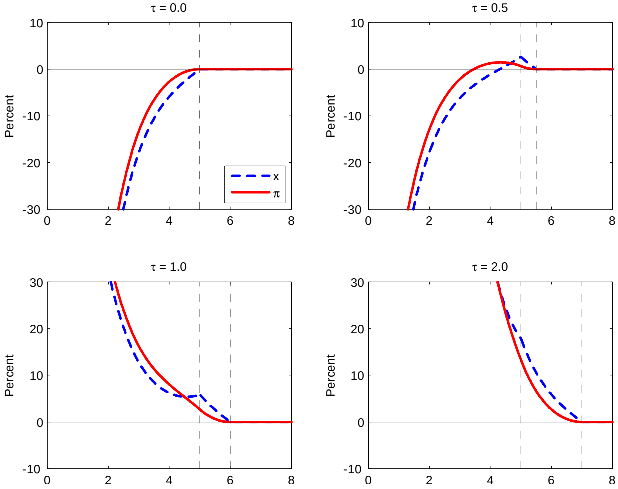
# Magical multipliers




- $\pi_T = 0$ equilibrium: $\partial x_t / \partial g_t$ is big! Bigger as frictions *decrease*!
- Other equilibria: $\partial x_t / \partial g_t = -1$.
- Equilibria without big $\pi$, $d\pi / dt$ do not have big multipliers.

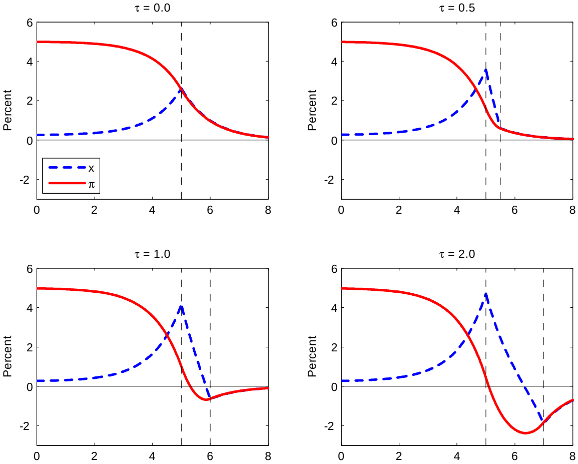
# Low rate promise – standard equilibrium

- Interest rate stays at zero for time $\tau$ after the trap ends




- Far-away promises have huge effects.

# Low rate promise – backwards-stable equilibrium




- Faraway promises have little current effects→frictionless.

# So far

- Reminder: the (expected, equilibrium) interest rate path is the same for all solutions.
- The diagnosis of large recession (with large $E_t \Delta c_{t+1}$) and deflation (with large $d\pi_t / dt$) relies crucially on which equilbrium we choose.
- Unusual (magical) policies rely crucially on which equilibrium we choose.
- Equilibria that are bounded backwards, bounded impulse-response $\pi_0$, and have frictionless limits do not produce large recessions or unusual policies.
- So why pick $\pi_T = 0$? What about Taylor rules?...

# Werning's equilibrium selection

Why $\pi_T = 0$?
Answer: expectations of an equilibrium-selection policy, *apart from* interest rate policy

For $t \geq T$ the natural rate is positive, $r(t) = \bar{r} > 0$, so that, as mentioned above, the ideal outcome $(\pi(t), x(t)) = (0, 0)$ is attainable. I assume that the central bank can guarantee this outcome so that $(\pi(t), x(t)) = (0, 0)$ for $t \geq T$.[7] Taking this as given, at all earlier dates $t < T$ the central bank will find it optimal to set the nominal interest rate to zero. The resulting no-commitment outcome is then uniquely determined by the ODEs (1a)–(1b) with $i(t) = 0$ for $t \leq T$ and the boundary condition $(\pi(T), x(T)) = (0, 0)$.

This situation is depicted in Figure 1 which shows the dynamical system (1a)–(1b) with $i(t) = 0$ and depicts a path leading to $(0, 0)$ precisely at $t = T$. Output and inflation are both

[6] In this section, I proceed informally. With continuous time, a formal study of the no-commitment case requires a dynamic game with commitment over vanishingly small intervals.

[7] Although this seems like a natural assumption, it presumes that the central bank somehow overcomes the indeterminacy of equilibria that plagues these models. A few ideas have been advanced to accomplish this, such as adhering to a Taylor rule with appropriate coefficients, or the fiscal theory of the price level. However, both assume commitment on and off the equilibrium path. Although this issue is interesting, it seem completely separate from the zero lower bound. Thus, supposing that $(\pi(t), x(t)) = (0, 0)$ can be guaranteed for $t \geq T$ allows us to focus on the interaction between no commitment and a liquidity trap scenario.

# Taylor rule

- Why $\pi_T = 0$?
- NK: *Two* Fed policies, "interest rate policy" $i_t^*$ and "equilibrium selection policy" to pick one $\pi_t^*$ from $\{\pi_t\}$ consistent with $i_t^*$.
- Taylor form selection. Fed specifies $i_t^*$ and *also* specifies $\pi_t^*$ from $\{\pi_t\}$ consistent with $i_t^*$, then adds

$$i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*).$$

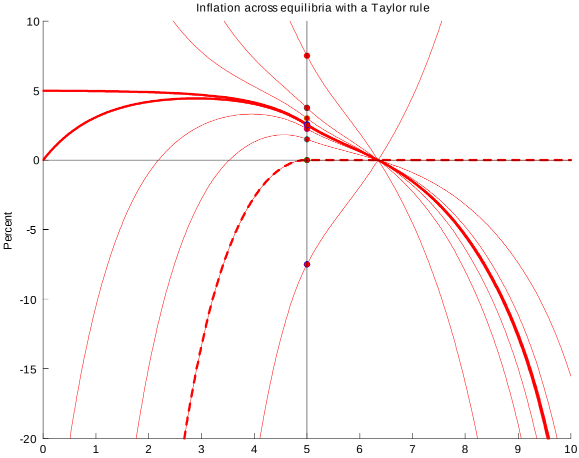
  $\|\phi_\pi\| > 1+$ "no explosive solutions" $=$ select $\pi_t^*$.
- The same as "Wicksellian," (optimal) "stochastic intercept," "intercept reacts to shocks," "temporary deviations" policy

$$i_t = (i_t^* - \phi_\pi \pi_t^*) + \phi_\pi \pi_t = \bar{\imath}_t + \phi_\pi \pi_t$$

- $\rightarrow$Expectations about equilibrium-selection policy/stochastic intercept/Wicksellian response, *not* about expected interest rates (we see and expect $i_t^*$ only), drive the whole result.
- Continuous time: needs partial adjustment rule

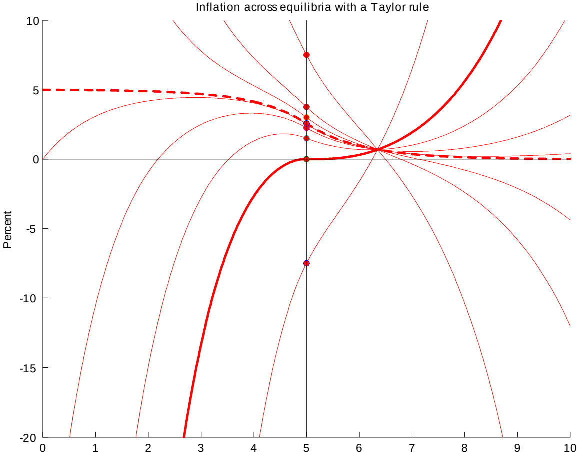
$$\frac{di_t}{dt} = \left\{ \begin{array}{cc} \theta\left[\phi\left(\pi_t - \pi_t^*\right) - \left(i_t - i_t^*\right)\right] & \text{if } i_t > 0 \\ \max\left\{\theta\left[\phi\left(\pi_t - \pi_t^*\right) - \left(i_t - i_t^*\right)\right], 0\right\} & \text{if } i_t = 0 \end{array} \right\};$$

# Taylor rule and the standard solution




- Not: "if $\pi_T > 0$, Fed tightens too much, lowers inflation too fast"
- Yes: "if $\pi_T > 0$ people expect the Fed to explode the economy." *Or those equilibria are not ruled out.*

# A Taylor rule could select the glidepath too




- Equilibrium selection policy to choose $\pi_t^*$ (glidepath) is just as possible.

# Precommitment and Dr. Stragelove

- Why do people believe the Fed would choose $\pi^*_T = 0$, not a benign glidepath?
  1. Werning: $i^*_T = r$, $\pi^*_T = 0$ from lack of precommitment.
  2. Me: But $i_t - i^*_t = \phi_\pi(\pi_t - \pi^*_t)$ takes a *huge* (Dr. Strangelove, subgame-imperfect) precommitment. Fed cannot precommit to $\pi^*_t > 0$ but can precommit to blow up the economy for $\pi_t \neq \pi^*_t$?
  3. Conclusion: lack of precommitment makes no sense once we recognize equilibrium selection policy.
- Old points:
  1. Why rule out non-local equilibria (JPE 2011)?
  2. Does the Fed do this / do people the Fed does this?
     - 2.1 We never observe $\pi \neq \pi^*$ so cannot learn $\phi_\pi$.
     - 2.2 The Fed loudly says it stabilizes, creating $\|\lambda\| < 0$, not destabilizes creating $\|\lambda\| > 0$.
     - 2.3 The Fed uses the word "glidepath" when fighting inflation.
     - 2.4 Is there really any such thing as "equilibrium selection policy?" (No "equilibrium selection" opeds!)

# Which equilibrium II/Save the model

- Bottom line:
  1. NK diagnosis and policy are very sensitive to equilibrium selection.
  2. There are many plausible equilibria that do not deliver big recession/deflation or policy magic.
- Which equilibrium?
  1. Fiscal: Jump to negative inflation means a huge transfer to bondholders. (No-jump equilibrium has zero fiscal implications.) Inflation target is a *fiscal* promise, constraint on *Treasury*.
  2. Philosophical? No backward explosions, smooth frictionless limit?
  3. Data: which equilibrium selection rule accounts for stagnation? Was there policy magic in 1930, 1950, 1975?
  4. Data: Measure equilibrium selection from time - 0 response $\pi_0$?
- Goal: *Save* the NK model (forward looking, microfounded) by solving multiple equilibrium problem, choosing a sensible one (frictionless limits).
  1. Builds nominal distortions on top of real models.
  2. Gets rid of the policy magic. Alas, if you like magic.

# Solution with price stickiness

$$\frac{d}{dt}\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -\kappa & \rho \end{bmatrix}\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} ir_t \\ 0 \end{bmatrix}$$

- $t > T$ $(ir_t = 0)$

$$\begin{bmatrix} \kappa x_t \\ \pi_t \end{bmatrix} = \infty \begin{bmatrix} \lambda^p & \lambda^m \\ 1 & 1 \end{bmatrix}\begin{bmatrix} e^{\lambda^m(t-T)} z_T \\ e^{\lambda^p(t-T)} w_T \end{bmatrix}; \qquad \begin{matrix} \lambda^m < 0 \\ \lambda^p > 0 \end{matrix}$$

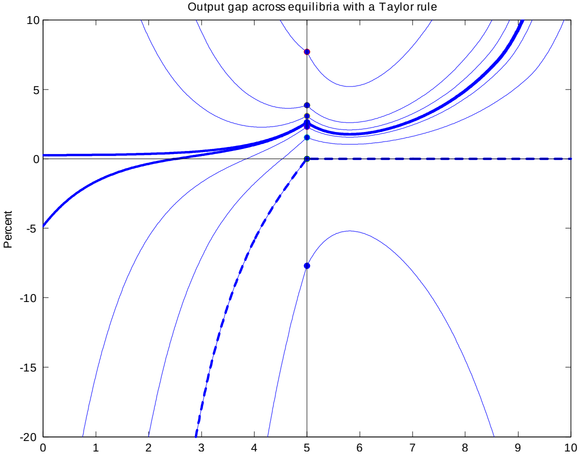
  $w_T = 0$ (no quarrel today). Leaves $z_T = \pi_T$, multiple stable equilibria $e^{\lambda^m(t-T)}\pi_T$, index by $\pi_T$

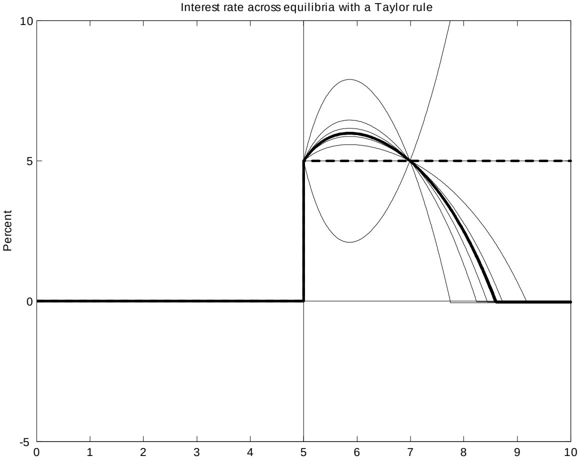
- $t < T$: choose $z_T, w_T$ to paste at $\pi_T$.

$$\begin{bmatrix} \kappa x_t \\ \pi_t \end{bmatrix} = ir \begin{bmatrix} \rho \\ 1 \end{bmatrix} - ir \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}\begin{bmatrix} e^{\lambda^m(t-T)} \\ e^{\lambda^p(t-T)} \end{bmatrix} + \pi_T \begin{bmatrix} \lambda^p \\ 1 \end{bmatrix} e^{\lambda^m(t-T)}.$$

- Both $e^{\lambda^m(t-T)}$ and $e^{\lambda^p(t-T)}$ terms are potentially active. $e^{\lambda^m(t-T)}$ explode going *back* in time. $\{\pi_T\}$ adds $e^{\lambda^m(t-T)}$ terms.

# Taylor rule and the standard solution





Interest rate across equilibria with a Taylor rule
Percent
10
5
0
-5
0
1
2
3
4
5
6
7
8
9
10

# A Taylor rule could select the glidepath too