# Monetary Policy with Interest on Reserves

John H. Cochrane*

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**Abstract**

I analyze monetary policy with interest on reserves and a large balance sheet. I show that conventional theories do not determine inflation in this regime, so I base the analysis on the fiscal theory of the price level. I find that monetary policy can peg the nominal rate, and determine expected inflation. With sticky prices, monetary policy can also affect real interest rates and output, though higher interest rates raise output and then inflation. The conventional sign requires a coordinated fiscal-monetary policy contraction. I show how conventional new-Keynesian models also imply strong monetary-fiscal policy coordination to obtain the usual signs. I address theoretical controversies. A concluding section places our current regime in a broader historical context, and opines on how optimal fiscal and monetary policy will evolve in the new regime.

*University of Chicago Booth School of Business, Hoover Institution, NBER, Cato Institute. I thank Bob Hall, Stavros Panageas, Siddhartha Sanghi, Tom Sargent, Tom Saving, John Taylor, and many seminar and conference participants for helpful comments and conversations. I gratefully acknowledge research support from CRSP.

# 1 Introduction

During the last few years, the Federal Reserve has made two changes that will fundamentally affect monetary policy, or at least the theory of monetary policy, going forward. First, the Fed now pays interest on reserves. Second, the Fed has amassed a large balance sheet, buying about \$3 trillion of assets, and creating about \$3 trillion of bank reserves in return. Before the crisis, banks only held about \$50 billion of reserves. Required reserves – the amounts banks must hold at the Fed corresponding to deposits – are still only about \$80 billion, so almost all of the \$3 trillion are excess reserves, held voluntarily by banks.

When interest rates rise, the Fed has announced that it will maintain the large balance sheet, and pay market interest rates on reserves. Indeed, the Fed will attempt to control short-term interest rates primarily by changing the rate it pays on abundant reserves, rather than by controlling the quantity of reserves via open market operations. This plan is articulated in Chairman Bernanke's (2010) testimony and most recently reinforced in the July 2014 FOMC minutes (Federal Reserve (2014)).

The Fed seems still to be deciding how long this new regime will last. While it clearly will not aggressively sell assets to soak up reserves, the Fed may sell off assets gradually. It may also let the balance sheet decline gradually as bonds mature, rather than reinvest maturing bonds to keep the balance sheet large. The Fed is not committed to paying market interest on reserves. Much discussion continues around using the spread between interest on reserves and other rates, and the size of the balance sheet, as policy tools. Some voices want the Fed to return to "normal" quickly, meaning a very small amount of non-interest paying reserves. This paper analyzes an interest-on-reserves regime, in which reserves always pay market interest and the balance sheet remains large. That analysis may be helpful in the policy debate, by analyzing whether there is much to fear from the interest-on-reserves regime and thus any strong reason to return to the previous configuration (whether one regards that configuration as "normal" or not).

There are many reasons why a large balance sheet, with interest on reserves, is a desirable state of monetary affairs. Friedman (1969) explained that the optimal quantity of money is obtained when there is no interest-rate spread between money and bonds. In this case, the "shoe-leather costs" of money management disappear. More important, in my view, are the benefits for financial stability. \$3 trillion of interest-paying reserves represent \$3 trillion of narrow-banking deposits, and \$3 trillion of the most liquid asset one could want on a bank balance sheet. (Cochrane 2014.)

However, interest on reserves and a large balance sheet, together with the spread of interest-paying electronic money, deeply challenge standard monetary policy analysis. We will continue to be satiated in liquidity. Reserves and short-term treasuries are and will remain essentially perfect substitutes at the margin. Reserve demand becomes indeterminate – banks are indifferent to holding another dollar of reserves and another dollar less short-term treasuries.

Standard answers to fundamental questions such as how the Fed controls real and nominal interest rates, what are the channels by which monetary policy affects the economy and the banking system, and how or whether inflation is determined, all fall apart in this regime.

The standard story says that to tighten, the Fed sells treasuries in exchange for reserves. A lower supply of reserves forces banks to work down a reserve demand curve, bidding up the interest rate. With the reserve requirement binding, banks must reduce lending and deposit creation via the money multiplier. Depending on your tastes, a lending channel and Phillips curve then reduce employment and prices, or less money supply and MV=PY do the same.

In the interest-on-reserves regime, however, there need be no open market operations, and reserve requirements will not come within \$3 trillion of binding. Bank lending and money creation will continue to be completely unaffected by the quantity of reserves.

Not everyone believed this story already – banks had so much funding by non-reservable sources, non-bank credit markets and the shadow-banking system were so large, that the \$50 billion of reserves remaining were essentially meaningless. But now we don't have to argue about that point – the story simply cannot apply any more.

We must face monetary policy with no monetary frictions. Interest-paying reserves are just overnight government debt, held entirely as an asset with no additional liquidity value, which incidentally can be transferred electronically at low cost to make transactions.

Standard theory predicts that inflation is not even determined in an interest on reserves regime. Sargent and Wallace (1985) is a classic example. (See also the excellent and wide-ranging Sargent 2010.) Yet, we have several years of experience in the US, and more in other countries, suggesting that inflation is quite stable with fixed interest-rate targets and the same interest on reserves and Treasuries.

In this context, I revisit classic questions. What can monetary policy do, when we are satiated in liquidity? What can't monetary policy do? How will inflation be determined? How should an effective monetary policy work?

I adopt a model with no monetary frictions at all, and the fiscal theory of the price level. Not only does this theory determine the price level without monetary frictions, it is, I will argue, the only existing theory that can do so.

The name "fiscal theory" seems to imply that monetary policy is ineffective. I find quite the opposite: Monetary policy *can* set the nominal interest rate and *can* determine expected inflation. The price level remains determinate even with completely fixed (no Taylor-rule responses needed) interest-rate targets. With long-term debt, monetary policy can determine the nominal term structure of interest rates. Furthermore, rearrangements of the maturity structure of government debt, reminiscent of QE operations, can create some inflation today in exchange for less inflation in the future.

I then add pricing frictions, so that monetary policy can have real effects. One contribution of the paper is to study a model with fiscal-theoretic price determination and sticky prices. I find that an inflationary reduction in expected future surpluses reduces real interest rates and increases output. These are signs conventionally attributed to monetary policy, and the opposite of what one might expect for fiscal policy. But they make sense: If the price level cannot adjust, the real value of one-period government debt cannot change. Hence, if expected surpluses decline, their discount rate, the real interest rate, must decline, and this decline raises output growth.

More importantly, I find that monetary policy with sticky prices can also affect output and the real rate of interest. I also find that monetary policy which desires to stabilize prices should raise and lower the interest rate target one-for-one with changes in the underlying real rate of interest, a standard optimal-policy result. However, I find Fisherian responses: an interest rate rise increase raises real interest rates, but raises consumption and output, before raising inflation.

Why do we think otherwise? I argue that events in real-world experience combine monetary and fiscal shocks, as both monetary and fiscal authorities respond to shocks. Combining monetary and fiscal policies, in a coordinated monetary-fiscal tightening, produces responses like those we seem to see in historical experience.

Comparing these results to a standard new-Keynesian model, I find the "monetary policy shock" in the latter also implicitly assumes a coordinated contractionary fiscal policy. Without that fiscal policy change, the standard new-Keynesian model also has the Fisherian result that interest rate rises cause more, not less, inflation.

I find that ties between fiscal and monetary policy are and will remain more important than conventionally acknowledged. For example, the presence of a large stock of outstanding Treasury debt, of relatively short maturity, means that interest rate changes will have large impacts on the Federal budget. The mark-to-market losses on the Fed's portfolio, which monetary analysts have worried about, are tiny in comparison. Fiscal considerations will limit monetary policy in ways that the Federal Reserve is barely thinking about at all.

My concluding comments point out the many unanswered questions that remain for our new regime, and this style of analysis. What does good or optimal monetary policy look like? How can we better structure the coordinated monetary and fiscal policy regime, in particular to better commit and communicate the fiscal underpinnings of a stable price level?

# 2 Inflation and interest rate targets in a frictionless model

I start with the simplest possible model, to answer the most fundamental questions: Can the Fed control nominal interest rates? How will inflation be determined in the interest on reserves regime?

## 2.1 Valuation formula

I base this analysis on the valuation formula for government debt, which states that the real value of nominal debt equals the present value of the primary surpluses that will pay off that debt,

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{1}$$

Here, $B_{t-1}$ is the nominal value of government debt outstanding at the beginning of time t, $P_t$ is the price level, $\beta = 1/(1+r)$ is a constant real interest rate, and $s_t$ are real primary surpluses.

I provide a general-equilibrium derivation of (1) below, and I address below many common misunderstandings and objections to the use of equation (1). But it's better to get on with the analysis first, and defend the first equation later.

Equation (1) establishes that the price level is determinate, even with no monetary frictions at all, so long as the government follows a policy that suitably controls nominal debt $\{B_t\}$ and primary surpluses $\{s_t\}$. We will quickly see that this is the case.

## 2.2 Monetary policy and inflation in the simple model

I further specialize to the case that there is only one-period debt: $B_{t-1}$ is sold at time $t-1$ and comes due at time $t$. The U.S. maturity structure, including reserves, is in fact pretty short, with most debt rolling over in less than two years. So, we can apply these simple equations as a first approximation if we think of the "period" as at least two years. I return to longer-term debt below.

To examine the roles of "monetary" and "fiscal" policy, I take expected and unexpected components of (1),

$$\frac{B_{t-1}}{P_{t-1}}\left(E_t - E_{t-1}\right)\left(\frac{P_{t-1}}{P_t}\right) = \left(E_t - E_{t-1}\right)\sum_{j=0}^{\infty}\beta^j s_{t+j}. \tag{2}$$

$$\frac{B_{t-1}}{P_{t-1}}E_{t-1}\left(\frac{P_{t-1}}{P_t}\right) = E_{t-1}\sum_{j=0}^{\infty}\beta^j s_{t+j}. \tag{3}$$

Equation (2) shows us that

- *Unexpected inflation is determined entirely by expectations of future surpluses.*

The face value of one-period debt $B_{t-1}$ is known ahead of time, so the price level must adjust if there is an unexpected shock to the present value of surpluses. By contrast, Equation (3) shows that

- *The government can entirely determine expected inflation by nominal bond sales $B_{t-1}$, even with no change at all in surpluses*

I define "monetary policy" as manipulating government debt $\{B_t\}$ without any change in taxes or spending – surpluses $\{s_t\}$. I define "fiscal policy" as taxing and spending, i.e. determination of the surpluses $\{s_t\}$. A discussion follows. With this terminology,

- *Monetary policy can control expected inflation in this completely frictionless model.*

To clarify the effect, write (3) as

$$\frac{B_{t-1}Q_{t-1}}{P_{t-1}} = E_{t-1}\sum_{j=0}^{\infty}\beta^{j+1}s_t, \tag{4}$$

where $Q_{t-1}$ is the one-period bond price,

$$Q_{t-1} = \frac{1}{1+i_{t-1}} = \beta E_{t-1}\left(\frac{P_{t-1}}{P_t}\right). \tag{5}$$

Now, think about the government's decision to sell additional nominal debt $B_{t-1}$ at the end of time $t-1$. The real value that the government raised by debt sales $B_{t-1}Q_{t-1}/P_{t-1}$ is fixed by the time $t-1$ present value of real surpluses as shown by (4). The price level $P_{t-1}$ is already determined by (1) at time $t-1$, independently of $B_{t-1}$. Thus, if the government sells more $B_{t-1}$, it faces a unit-elastic demand curve; the nominal bond price $Q_{t-1}$ falls one for one with the increase in nominal debt, because expected inflation $E_{t-1}\left(P_{t-1}/P_t\right)$ rises one for one. This operation is just like a share split or a currency revaluation. The government has complete power over units in this frictionless model, which means it can control expected inflation without changing anything real.

Furthermore, writing (4) as

$$\frac{B_{t-1}}{P_{t-1}}\frac{1}{1+i_{t-1}} = E_{t-1}\sum_{j=0}^{\infty}\beta^{j+1}s_{t+j}, \tag{6}$$

we can replace a government decision of the quantity of debt to sell $B_{t-1}$ with constant surpluses with to a much more realistic-sounding interest rate target. To set an interest rate target, the government auctions bonds – it says "the nominal interest rate will be 5%. We sell nominal bonds at 1/0.95 dollars per face value. We will sell any amount demanded at that price." Equation (6) then is a simple reading of private demand – if the government targets nominal interest rates at a level $i$, that equation tells us how many bonds $B_{t-1}$ will be demanded. It reassures us that a finite amount will be demanded, and therefore the nominal interest rate target will work.

In sum, then,

- *In this frictionless model, monetary policy can set a nominal interest rate target, without any adjustments to fiscal policy $\{E_t s_{t+j}\}$.*
- *By setting nominal interest rates, monetary policy completely controls expected inflation.*
- *Inflation and the price level are determinate in the interest-on-reserves regime, even with fixed interest rate targets.*

The combination (3) and (2) thus address classic issues in monetary economics. The Fisher relationship $i_{t-1} = r + E_{t-1}\pi_t$ already says that by controlling nominal interest rates, the government can control expected inflation. But it is not clear by that simple statement just *how* the government controls nominal interest rates in a frictionless world – rationing non-interest-paying reserves doesn't work. Woodford (2003) discusses a cashless limit, but still one in which the Fed sets interest rates by rationing the last dollar of reserves against a two dollar reserve requirement. Ireland (2012) (see also Ireland 2008) writes of such models "the central bank's ability to manage short-term interest rates has rested, ultimately, on its ability to control, mainly through open market purchases and sales of government bonds, the quantity of reserves supplied to the banking system."

Now we see that the government *can* set the nominal interest rate in a fiscal-theory world, without controlling reserves at all, and the action it must do to achieve that result: sell nominal bonds at a fixed price, with fixed surpluses.

Fixed nominal interest rate targets have been troubling since Friedman (1968) argued they would lead to unstable inflation, and Sargent and Wallace (1975) showed they would lead to indeterminacy: $i_{t-1} = r + E_{t-1}\pi_t$ can pin down expected inflation $E_t\pi_{t+1}$ but unexpected inflation $\pi_t - E_{t-1}\pi_t$ can be anything. Here we see that even a completely fixed interest rate target need not lead to inflation instability or indeterminacy. A Taylor-type rule reacting to inflation is not needed.

- *Recognizing the fiscal backing of nominal debt in the government valuation equation solves the indeterminacy problem resulting from interest rate targets, and from the absence of monetary frictions in the interest-on-reserves regime.*

Finally, if it is not clear, this *is* the interest on reserves regime. The quantity $B_{t-1}$ includes treasury debt and reserves, which are perfect substitutes here since reserves pay interest. One could write $B_{t-1} + R_{t-1}$ to emphasize that they are equivalent securities which enter symmetrically in everything. Equation (1) by itself then tells us that the price level is determinate under the interest on reserves regime, and (3) and (2) show us how monetary and fiscal policy contribute to that result, generating $\{B_t\}$ and $\{s_t\}$ which result in a determinate price level. Now, Sargent and Wallace (1975) show that interest on reserves loses control of the price level. Here we see that result overturned.

- *The price level and inflation rate are uniquely determined in the interest on reserves regime.*

This result is a great comfort. One might have thought we need to accept less than the optimum quantity of money and financial stability in order to achieve control of inflation, a long-standing tension analyzed by Sargent (2010). We do not.

These classic results are overturned since they explicitly ignored the fiscal backing provided by equation (1). If you read carefully, they say "unless there is fiscal backing..." Section 5 covers the relation of these results to the classic literature in more detail.

### 2.3 Mapping the simple model to our world

Now, let us step back a bit to interpret this model in the context of current institutions. It's pretty straightforward to think of the primary surplus $\{s_t\}$ as "fiscal policy." The non-fiscal operation in the above simple model consists of the government setting an interest rate target by announcing the price of one-period debt, and then selling whatever quantity is demanded at that price, without changing expected future surpluses as in (6). That doesn't sound like what the Fed does. The Fed sets interest on reserves, and the Treasury auctions fixed quantities of debt.

I will argue here, however, that monetary policy by interest on reserves (and, in fact, the previous regime as well) can be read in to this operation. In essence, the Fed and Treasury coordinate so that the Fed announces the interest rate and the Treasury sells the bonds. This separation is desirable, as it allows the Treasury separately to buy and sell debt in a way that *does* promise changes in future surpluses, which is how it funds current deficits without causing inflation.

#### 2.3.1 Can the Fed control interest rates?

First, can the Fed even control interest rates? It's not a stupid question. The usual answer is "yes, it can work down a reserve demand curve." But that answer is over. Fama (2013) even challenges Fed interest-rate control empirically. Here, I have described how the government as a whole can target nominal rates by expand or contracting the amount of all Federal debt in private hands, as described by (6). But how does the Fed accomplish that?

The Fed can certainly increase the rate it pays on reserves. The question is whether and how those rates will spread to other rates. If the Fed simply announced a higher interest rate on reserves *and* an unlimited quantity, "we pay 5% on reserves, come and get it. Give us your short-term treasuries, we will create reserves and pay you 5% interest," then clearly the interest rate on treasuries must rise to 5% by arbitrage. But then the Fed would potentially lose control of the balance sheet. A "tightening" might well imply a large *increase* in the balance sheet, opposite the usual sign, a point made by Ireland (2012). In this analysis, there is nothing particularly bad about a large balance sheet. But our Federal Reserve plans to pay interest on reserves *and* to control the size of the balance sheet.

Arbitrage relationships should allow the Fed to control interest rates by changing the interest rate on reserves and, if necessary, the discount rate, without expanding the balance sheet. If the Fed pays 5% on reserves, banks should compete to attract depositors, and end up raising deposit rates to 5% minus costs. The banks cannot collectively hold more reserves, but each bank can get the reserves of another banks if it can steal away that bank's depositors. Once deposit rates rise to 5%, depositors should then try to sell Treasuries to get bank deposits, until Treasuries rise to that level. Banks may also try to dump treasuries to hold more reserves. Banks will surely not lend at less than 5% if they can get 5% on reserves.

It's not clear how strong these arbitrage relationships are in practice. Just paying your nanny \$50 will not, by arbitrage, raise all nanny wages to \$50 tomorrow morning. Banks are big, but not that big. The Fed's new reverse repo program essentially allows non-bank financial companies to transfer bank deposits directly to reserves. The introduction of this program reflects the Fed's uncertainty how strong the bank competition and arbitrage mechanism is, and it adds another connection between interest on reserves and Treasury markets. (See Singh 2014 for a good overview of financial "plumbing" and limits to arbitrage under IOR.)

Nonetheless, at the level of this frictionless model, arbitrage relationships should allow the Fed to control all interest rates by paying interest on reserves, and lending freely at the discount window, even if the Fed does not allow the balance sheet to expand and contract. Furthermore, if Treasury rates rise as they should when interest on reserves rises, then the Fed can finance the payment of interest on reserves entirely from the larger interest it receives on Treasuries, in a neat trick of picking itself up by its bootstraps. Mark-to-market losses on the Fed's long term portfolio make the accounting harder, but don't change the basic picture.

Now, back to the Treasury. The Treasury decides the current and expected future surplus or deficit $\{s_{t+j}\}$. These choices determine the real value of debt $Q_{t-1}B_{t-1}/P_{t-1}$ that the Treasury can sell. The Treasury next determines the quantity of debt it needs to sell to finance the current deficit. When it does so, it observes market interest rates. So, when interest rates rise and bond prices $Q_{t-1}$ fall, the Treasury raises the face value of the debt $B_{t-1}$ that decides to sell. Treasury auctions are designed purposely *not* to move markets, and come within a few basis points of existing rates. But by deciding on the face value of the debt $B_{t-1}$ that it sells after observing interest rates, the Treasury essentially calculates the face value of the debt $B_{t-1}$ that equation (6) demands given the Fed's determination of the interest rate.

In this way, the Treasury and Fed acting together do, in fact, institute a system in which the government as a whole sets the interest rate $i_{t-1}$ and then sells whatever face value of the debt $B_{t-1}$ that equation (6) demands, even though the Fed does not directly change the overall quantity of debt, and even though the Treasury seems to sell a fixed quantity, not a fixed price.

### 2.3.2 The Treasury and the Fed: a desirable distance

The expected surplus terms in all these equations suggests a good reason for the strong institutional separation between Treasury and Fed. When the Treasury sells more debt to finance a current deficit, war, or other temporary spending $-s_{t-1}$, it wants to raise more real revenue, and it does not want to cause inflation or cause an adverse move in interest rates. The Treasury thus wants to sell more debt $B_{t-1}$ and communicate a simultaneous rise in promised real surpluses $\{E_{t-1}s_{t+j}\}$.

By contrast, the Fed wants to communicate the opposite expectations: To raise interest rates, it wants the government to sell more debt $B_{t-1}$ with *no* implications about future surpluses. If the government's only tool was nominal debt sales $\{B_t\}$ conducted by a single agency, it would be very hard to tell these two actions apart.

Isolating the debt sales $B_{t-1}$ in two distinct branches of the government is a great way to communicate different expectations of future surpluses of otherwise identical debt sales. In one case the "Fed" sets interest rates, and the Treasury passively sells more face value of debt $B_{t-1}$, with no change in market value of debt. It's clear that no increase in surpluses is promised by the increase in face value of the debt. In the other case, the Treasury directly sells more face value of debt $B_{t-1}$, with more real market value. The government's implicit promise to raise future taxes or cut future spending to pay off that debt is clear.

In the same way, corporations market share splits – fully-diluting increases in shares outstanding with no changes in earnings – and public offerings – increases in shares outstanding that are intended to fully correspond to changes in earnings with no dilutions– in ways that convey the right expectations. In the first case, the corporation wants to change prices only, and in the second case it wants to raise money with as little price impact as possible. Governments market currency reforms or unions – fully-diluting changes in nominal debt with no change in future real surpluses, designed to affect nominal prices and raise no revenue – very differently from debt sales – changes in nominal debt with one for one changes in promised real surpluses, designed to raise revenue with no change in nominal prices – in ways designed to convey the desired implicit promises about surpluses. The increase in debt $B_{t-1}$ is the same in all cases. As we think about better institutional design for monetary policy – which we should really call coordinated monetary-fiscal policy – better communicating the intended promises about future surpluses is a central issue.

### 2.4 Long-term debt and quantitative easing

In this model, there is no difference at all between interest-paying reserves and Treasury debt held directly by the public. The symbol $B_{t-1}$ refers to the sum of the two quantities, and I ignore cash. Equivalently, we study the consolidated government budget, encompassing Treasury and Fed. So without further frictions, (1) tells us that

- *In the interest-on-reserves regime, open-market operations exchanging reserves for short-term government debt have no effect at all. The size of the Fed's balance sheet is irrelevant. Arbitrary amounts of interest-paying reserves are not inflationary.*

If the Fed, or the government, buys real assets such as mortgage-backed securities, increasing debt $B_{t-1}$ but at the same time adding real assets on the right hand side that can either be sold or generate a stream of surpluses, that action also has no effect on the price level. The only effect would be if the assets are not worth what they seem, or later default resulting in a shock to surpluses.

- *Open market operations or quantitative easing operations that buy real assets from the private sector have no effect on the price level.*

In traditional monetary economics, it was thought not to matter what the Fed bought, or if it bought anything at all. Only the increase in reserves or money mattered. The drop-in-the-bucket size of traditional open market operations relative to the supply of liquid debt, plus the Modgliani-Miller theorem for government asset purchases (the private sector still holds the same risk, just through state-contingent taxes) made a lot of sense of that view.

With trillions of excess reserves, however, that doctrine is turned on its head. The usual story for QE effects is not that the increased *reserves* have any effect, but that the asset markets are segmented or illiquid, so the government soaking up large quantities affects the *asset* prices. The "illiquidity" or "friction" is in the asset market, not the money market. (How such effects depend on flows rather than stocks of assets, and how such price impact can be long-lasting, as often viewed by the Fed, are deeper puzzles, or perhaps mistakes.)

Balance-sheet irrelevance is an important result. For example it means that

- *Reserves that pay market interest, in arbitrary quantities balanced by arbitrary less quantities of short-term government debt, are not inflationary. The size of the balance sheet is irrelevant.*

When money pays market interest, MV = PY ceases to control PY because velocity V absorbs any change in the supply of M. Yet commentator after commentator in the last five years has noticed the quantity labeled "money" (reserves) shooting up 5,600%, from $50 billion to $2.8 trillion and worried about hyperinflation.

### 2.4.1 A QE that works

However, changes in the maturity structure of nominal government debt relative to changes in the maturity structure of future surpluses can affect the path of nominal inflation, and thus give a rationale for the effectiveness of "quantitative easing" operations. To investigate these possibilities, I extend the analysis to include long-term debt. Cochrane (2001) undertakes a deeper analysis. I present a simple example here.

Suppose the real interest rate is zero, $r = 0$. Suppose at time $t = 0$ the government issues one-and two-year debt, $B_0^{(1)}$, $B_0^{(2)}$. The government will retire this debt with surpluses $s_1$, $s_2$, and thereafter run balanced budgets $s_t = 0$. At time 1, the government sells or repurchases some additional $t = 2$ debt without changing current or promised surpluses. This will be our "quantitative easing." Denote the amount of time $t = 2$ debt outstanding at the end of time one, after the purchase and sale, $B_1^{(2)}$, so the purchase or sale is in the quantity $B_1^{(2)} - B_0^{(2)}$.

I now expand the definition of "monetary policy" to include changes in the *maturity structure* of nominal debt outstanding in public hands, as well as the size, but with no change in surpluses $\{s_t\}$. So, what can this monetary policy do?

We have to find what the price level $P_1$, $P_2$ will be given debt and surpluses. To do so, we work backward. The flow equation money in = money out for time $t = 2$ in this situation states that the money "printed" to redeem maturing debt $B_2^{(1)}$ must all be soaked up by tax payments net of spending $P_2 s_2$,

$$B_1^{(2)} = P_2 s_2. \tag{7}$$

This equilibrium condition tells us what $P_2$ will be. The nominal bond price $Q_1^{(2)}$ at time 1 for bonds that come due at time 2 is then

$$Q_1^{(2)} = P_1 E_1 \left( \frac{1}{P_2} \right) = \frac{P_1}{B_1^{(2)}} E_1 \left( s_2 \right).$$

The flow equation at time 1 states that money printed to redeem debt $B_0^{(1)}$ must be soaked up by surpluses or by sales of time 2 debt,

$$B_0^{(1)} = P_1 s_1 + Q_1^{(2)} \left( B_1^{(2)} - B_0^{(2)} \right).$$

Substituting $Q_1^{(2)}$,

$$\frac{B_0^{(1)}}{P_1} = s_1 + \frac{B_1^{(2)} - B_0^{(2)}}{B_1^{(2)}} E_1(s_2). \tag{8}$$

Together, (8) and (7) tell us what the equilibrium price level will be at time 1 and 2, as a function of debt sold and surpluses.

To see what policy can do, again take expected values,

$$E_0\left(\frac{1}{P_2}\right)=Q_0^{(2)}=E_0\left(\frac{s_2}{B_1^{(2)}}\right)=E_0\left[\frac{s_2}{B_0^{(2)}+\left(B_1^{(2)}-B_0^{(2)}\right)}\right]$$

$$E_0\left(\frac{1}{P_1}\right)=Q_0^{(1)}=\frac{E_0\left(s_1\right)}{B_0^{(1)}}+\frac{1}{B_0^{(1)}}E_0\left[\left(\frac{B_1^{(2)}-B_0^{(2)}}{B_1^{(2)}}\right)s_2\right]$$

Fixing the surpluses $s_1$ and $s_2$, "monetary policy" can achieve whatever values on the left hand side it desires by the choice of debt $\left\{B_t^{(n)}\right\}$.

- *The maturity structure of debt, together with expected future bond purchases and sales, controls the time-path of expected inflation and the nominal term structure of interest rates, fixing surpluses.*

Expected future sales and purchases are not even needed here. If there are no time-1 sales and $B_1^{(2)}=B_0^{(2)}$, then the maturity structure at time 0 simply sets the time path of inflation and the nominal term structure of interest rates,

$$E_0\left(\frac{1}{P_2}\right)=Q_0^{(2)}=\frac{E_0\left(s_2\right)}{B_0^{(2)}}$$

$$E_0\left(\frac{1}{P_1}\right)=Q_0^{(1)}=\frac{E_0\left(s_1\right)}{B_0^{(1)}}$$

As monetary policy can target short rates, the Fed could just as easily directly target the long rates, or even the entire term structure. Why it does not do so is a bit of a puzzle. If the Fed wants the 10 year rate to be 50 bp lower, why does it just not say "we buy and sell Treasuries at a 2.0% ten year yield. Come and get them?" Or, it could fix the interest rate on term deposits at the Fed. If the Fed can control the overnight rate by controlling interest on reserves, then it can control the term structure by fixing the interest rate on term deposits.

Taking unexpected values, we find at $t=2$ that fiscal policy fully determines the unexpected time 2 price level.

$$B_1^{(2)}\left(E_2-E_1\right)\left(\frac{1}{P_2}\right)=\left(E_2-E_1\right)s_2.$$

But at time 1, we find

$$B_0^{(1)}\left(E_1-E_0\right)\left(\frac{1}{P_1}\right)=\left(E_1-E_0\right)s_1+\left(E_1-E_0\right)\left\{\frac{B_1^{(2)}-B_0^{(2)}}{B_1^{(2)}}s_2\right\}. \tag{9}$$

Equation (9) offers an exciting new opportunity: By unexpectedly selling more time-2 debt, the government dilutes existing claims to time-2 surpluses. This action raises real revenue and that revenue can be used to increase the payoff to period 1 bondholders, lowering inflation at time 1.

There is a catch however: Selling additional long-term debt raises inflation at time 2,

$$\left(E_1-E_0\right)\left(\frac{B_1^{(2)}}{P_2}\right)=\left(E_1-E_0\right)\left(s_2\right). \tag{10}$$

Fixing $s_1$ and $s_2$, for example, a surprise debt sale increasing $B_1^{(2)}$ at time 1 raises $(E_1 - E_0)\left(\frac{1}{P_1}\right)$ in (9) and lowers $(E_1 - E_0)\left(\frac{1}{P_2}\right)$ in (10).

- *"Monetary policy" – a change in the maturity structure of government debt with no change in fiscal stance – can even affect* ***unexpected*** *inflation in the presence of long-term debt. It does so by rearranging the path of inflation, delaying inflation or bringing inflation forward.*

We may read current "quantitative easing" as this policy with the opposite sign. By unexpectedly (relative to when the debt was sold) *buying* long-term debt, the Fed tries to "stimulate," i.e. to *increase* inflation today, in exchange for less inflation later on.

This result also points to a stabilization role for quantitative easing. Extra debt sales $B_1^{(2)} - B_0^{(2)}$ in (9) can be used to offset surplus shocks $(E_1 - E_0)\, s_1$ or $(E_1 - E_0)\, s_2$, thus insulating the price level $(E_1 - E_0)\,(1/P_1)$ from surplus shocks. Active debt management emerges as a policy to stabilize the price level in the face of shocks to surpluses, as well as other shocks, not just to create inflation or disinflation. The general case of these formulas is quite complex (Cochrane 2001), suggesting a very interesting job of maturity management for governments that want to stabilize inflation and, with pricing frictions, output.

# 3 Real rates and sticky prices

The simple frictionless models got us quite far in describing the potential and limits for monetary policy to affect inflation, but leave out any effects on output and real interest rates. In particular, increased interest rates *increase* expected inflation with no real interest rate or output effects so far. Belief in the opposite sign is strong, so a model with some frictions is worth exploring.

It is not at all obvious that monetary policy in an interest on reserves regime will have the traditional effects. The mechanism for interest rate increase, and the mechanism for its transmission to the price level are utterly different from the standard story. Rationing liquidity, rationing bank lending or deposit creation are simply absent. So, experience in the previous regime may be a poor guide. There is a genuine need for theory, and we should not demand that theory produce the traditional response.

## 3.1 A simple sticky-price model

I maintain a model without *monetary* frictions – reserves still are perfect substitutes for overnight Treasury debt, and people hold no non-interest-bearing money overnight. I add *pricing* frictions. To innovate as little as possible, I add them in the simplest possible way: I force producers to state prices ahead of time, and then sell whatever quantity people want at those prices. The point here is not to create an empirically successful model of pricing dynamics, but to explore the basic signs and mechanisms by which monetary policy might work in the absence of monetary frictions.

I set the model out in detail in the Appendix. It is a simplification of Galí (1999). Households consume a CES composite good $c_t = \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}}\, dj\right]^{\frac{\sigma}{\sigma-1}}$ of many varieties. Each household $i$ uses labor $n_{it}$ to produce one variety $y_{it}$ with production function $y_{it} = An_{it}$. Each household must set its price k periods in advance.

The aggregated version of the formal model is easy to work with. The government debt valuation equation remains, unsurprisingly,

$$u'(c_t)\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j u'(c_{t+j}) s_{t+j}. \tag{11}$$

Now $P_t$ is determined at time $t-k$. Therefore, $u'(c_t)$, consumption, output and bond prices, will react when there is a surplus shock. Since prices are completely free after $k$ periods, however, marginal utility can only be expected to diverge from the frictionless value for k periods. With $\bar{c}$ equal to the frictionless level of consumption and output, we have

$$E_{t-j}\left[u'(c_t)\right] = u'(\bar{c}).\ j \geq k.$$

Asset markets are unaffected, so the nominal interest rate still obeys

$$\frac{1}{1+i_t} = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{P_t}{P_{t+1}}\right].$$

Using these simple rules, one can work out dynamics of the aggregate system in response to shocks, as I did with the flexible-price model.

## 3.2 Surplus shocks

I start with the usual expected and unexpected technique to isolate what fiscal and monetary policy can do.

In general, (11) will lead us to a risk premium for the valuation of government debt, since future surplus shocks will correlate with future consumption shocks $E_t\left[u'(c_{t+j})s_{t+j}\right] \neq E_t\left[u'(c_{t+j})\right] E\left[s_{t+j}\right]$. This effect is very interesting for empirical application – as elsewhere, varying risk premiums help us to understand many puzzling features of the data. But here, let us first understand very basic parts of what monetary and fiscal policy can do.

So, consider a simple example: suppose prices are sticky for one period. Suppose there is a once and for all shock to expected surpluses in the far off future. Suppose $s_t = 0$: this is a period in which debt must be rolled over, though possibly with news about future surpluses, but no new deficits must be financed. The unexpected component of (11) then reads

$$\frac{B_{t-1}}{P_t}\left(E_t - E_{t-1}\right)\left[\frac{u'(c_t)}{u'(\bar{c})}\right] = \left(E_t - E_{t-1}\right)\sum_{j=1}^{\infty} \beta^j s_{t+j}. \tag{12}$$

This fiscal-news shock produces a jump in the price level $P_t$ in the frictionless model, (2). But now, $P_t$ cannot change in response to the shock. All the adjustment to the surplus shock now comes by adjustment to $u'(c_t)$ and hence to the real interest rate. A negative (inflationary) shock to expected future surpluses $\left(E_t - E_{t-1}\right)\sum_{j=1}^{\infty} \beta^j s_{t+j}$ lowers $u'(c_t)$, i.e. raises $c_t$. An inflationary fiscal shock thus produces a temporary output expansion. The inflationary shock also lowers the real interest rate,

$$1 + r_t = \frac{1}{q_t} = \frac{1}{\beta}\frac{u'(c_t)}{u'(\bar{c})},$$

where $q_t$ denotes the real bond price.

These are the "usual" signs – inflation is preceded by higher output and lower real interest rates. In this model, that course of events is out of the Fed's control; it is part of fiscal, not monetary policy. But observers accustomed to thinking the Fed controls real interest rates might well think that the Fed lowered real rates, induced the output expansion, and later the inflation; they might think that the ex-post observed fall in surpluses $\sum_{j=1}^{\infty} \beta^j s_{t+j}$ represented a "Ricardian" or passive-fiscal reaction by the Treasury. Little in the data could falsify this impression.

To get some insight into how the real rate change absorbs the surplus shock in place of inflation, write (12) in the form

$$\frac{B_{t-1}}{P_t} = \frac{1}{(E_t - E_{t-1})(1+r_t)} (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+1+j}. \tag{13}$$

The left-most term is the real value of nominal debt coming due in the morning, which must be rolled over. With a constant real interest rate, the real value of debt sold in the evening declined when expected future surpluses declined, as the right-most term declined. To match that decline, the real value of debt coming due in the morning declined, as $P_t$ on the left hand side of (13) rose. But that mechanism is now absent. The real value of debt coming due in the morning $B_{t-1}/P_t$ cannot decline. How can the real value of debt paid off in the morning stay the same, in the face of a decline in expected surpluses? The answer is that the real interest rate also declines, the bond price rises, so the lower expected surpluses now have the same real value and the bonds are rolled over.

The same mechanism in nominal terms: We can imagine the government printing up money $B_{t-1}$ to pay off debt at the beginning of period $t$, money which must be soaked up with bond sales by the end of time $t$. (In this simple example with $s_t = 0$.) With flexible prices and a constant real interest rate, the real value of surpluses $E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$ was fixed, and at the same nominal price level $P_t$ these would no longer soak up all the dollars. So at that price level, people tried to buy more goods with their excess dollars. In doing so, they pushed up goods prices until the lower real quantity of debt sold in the afternoon soaked up the excess nominal dollars brought in by $B_{t-1}$.

But now prices cannot rise. People still have more newly-printed money in their pockets $B_{t-1}$ than will be soaked up by debt sales. What happens? First, they try to buy more goods and services as before. With prices fixed one period in advance, this extra "aggregate demand" now leads to greater output, not higher prices. But the greater output does not soak up any money in aggregate. More money spent by the buyer is received by the seller, and at the end of the day the excess money $B_{t-1}$ relative to bond sales that will soak it up is still there. So, if money holders cannot bid up the price of *goods,* they bid up the price of *bonds* instead. "Asset price inflation," takes the place of goods inflation. The real interest rate decline / real bond price rise continues until the excess cash is now all soaked up by bond sales at an unchanged price level.

Given that real interest rate rise, the output increase is determined by the intertemporal first order condition $1/(1+r_t) = E_t [\beta u'(c_t) / u'(c_t)]$. In words, with a lower interest rate, people want to buy more today and save less for tomorrow.

- *An inflationary fiscal shock – decline in future surpluses – causes an increase in output, and a decline in the real interest rate.*

Though now (I hope) obvious in terms of the model, these are unconventional predictions. Without the model, we might have thought that a decline in expected future surpluses, a decline in the government's ability to service its debt, would lead to an *increase* in the interest rate, and a *reduction* in the value of government debt. Instead, in equilibrium, real interest rates rise and there is no change in the real value of government debt. The nominal value of government debt cannot change in a (default-free) rollover, and with sticky prices, the real value of government debt cannot change. If expected future surpluses decline, then the rate at which those surpluses are discounted must decline as well.

This observation may help to make sense of many paradoxes in the data, such as during the financial crisis in which economists note bad news about current and future surpluses, but interest rates decline and government bond prices rise anyway in a "flight to" government debt.

### 3.3 Monetary policy with one-period stickiness

Next consider "monetary policy," debt sales with no change in surplus, with sticky prices, deriving from the expected value of (11). Again assume prices stuck for one period, ignore the risk premium, and remember the rules that marginal utility can only be expected to vary for one period, $E_{t-1}\left[u'(c_t)\right] = u'(\bar{c})$. We obtain

$$\frac{B_{t-1}}{P_t} = E_{t-1}\sum_{j=0}^{\infty}\beta^j s_{t+j}. \tag{14}$$

This equation functions much as its flexible price counterpart (3). The only difference is that $1/P_t$ is known at time $t-1$ so lies outside the $E_{t-1}$. This equation tells us that by varying debt $B_{t-1}$, with fixed surpluses, the government can control the *actual* price level at time $t$, $P_t$ with sticky prices, just as it controlled the expected price level at time $t$, $E_{t-1}(1/P_t)$, with flexible prices. Again, this action is like a share split or currency reform. As before, the government can set the price rather than the quantity, and follow a nominal interest rate target. The nominal rate and real rate are related by

$$(1 + r_{t-1}) = (1 + i_{t-1})\frac{P_{t-1}}{P_t}, \tag{15}$$

the difference being that $P_t$ is fixed at $t-1$, not expected.

With the real rate $r_{t-1}$ determined by fiscal shocks at time $t-1$, (and, in a fuller model, real shocks), a fixed nominal rate target $i_{t-1} = \bar{\imath}$ will result in price level volatility: If the real rate $r_{t-1}$ rises and the Fed holds the nominal rate $i_{t-1}$ constant, the price level $P_t$ must decline in (15). Hence,

- *If the Fed wants to reduce price volatility, it should move the nominal rate target one-for-one with the real rate.*

This advice has much of the flavor of "Wicksellian" advice such as Woodford (2005) that the nominal rate target (Taylor rule intercept) should follow rises and falls in the "natural" rate.

### 3.4 Interest rate policy with real effects

However, monetary policy cannot affect output or the real interest rate with one period debt, and prices stuck for one period. Consumption $u'(c_{t-1})$ is already set by $B_{t-2}$ and time $t-1$ expected surpluses. And marginal utility $u'(c_t)$ can only move for one period, $B_{t-1}$ cannot affect $E_{t-1}u'(c_t)$.

When prices are sticky for more than one period, however, monetary policy can affect real quantities. Suppose now that prices must be set $k$ periods in advance. As there are no asset market distortions, the government debt valuation equation remains,

$$u'(c_t)\frac{B_{t-1}}{P_t} = E_t\sum_{j=0}^{\infty}\beta^j u'(c_{t+j})s_{t+j}. \tag{16}$$

Now, $P_t$ must be determined at time $t-k$, and marginal utility and the real interest rate can diverge from the frictionless value for $k$ periods.

The general algebra for this case does not yield much intuition, so I present a simple example. Start at a steady state $P_t = \bar{P}$ with interest rate target

$$\frac{1}{1+i} = \beta = \frac{1}{1+\delta},$$

where the last equality defines $\delta$. Suppose that at time $t=1$ the government unexpectedly changes the interest rate target to a path $\{(1+i)\,\Delta_t\}$. The sequence $\{\Delta_t\}$ captures the dynamic path of interest rates following the policy shock. A permanent increase in the nominal rate is captured by a constant $\Delta_t = \Delta$. The usual AR(1) response, in which the interest rate change gradually fades away, is captured by $\Delta_t = \Delta(1+\rho^t)$ for some $\rho < 1$. Other sequences $\{\Delta_t\}$ let us explore the kinds of dynamic real and nominal responses that the central bank can engineer by changing the dynamic nature of the policy rule, and thereby change expectations of the persistence of policy shocks.

The finite length of price stickiness in this model provides a clean dividing line between the short run and the long run. Prices $P_1, P_2, ...P_k = \bar{P}$ cannot respond to the interest-rate innovation. Prices $P_{k+1}$ and beyond can respond. Conversely, consumption $c_1, c_2, ..c_k$ can respond to the interest-rate innovation, but consumption $c_{k+1} = \bar{c}$ and beyond cannot respond.

To fill in fiscal policy in the simplest way, suppose that surpluses are $s_t = 0,\ t < k$, and will be constant $s_t = \bar{s},\ t \geq k$. The presence of surpluses $s_t$ during the price-sticky period leads to small variations in the value of these surpluses $u'(c_t)s_t$ at these dates as consumption varies, which cloud the basic story. Letting the debt be paid off by far in the future surpluses simplifies later algebra. Having surpluses resume just as prices become unstuck makes no difference. We could let $s_t = 0,\ t < k+K$ and all variables but the quantity of debt would be the same. Denote $\bar{S} = \sum_{j=0}^{\infty}\beta^j\bar{s} = \bar{s}/(1-\beta)$. The steady state implies from (16) that nominal debt $\bar{B}_{t-1} = \beta^{k-t}\bar{P}\bar{S}$ for $t \leq k$ and $\bar{B}_{t-1} = \bar{P}\bar{S}$ for $t \geq k$.

Our job is to solve (16) together with the consumer's optimality condition

$$\frac{1}{1+i_t} = \beta E_t\left[\frac{u'(c_{t+1})}{u'(c_t)}\frac{P_t}{P_{t+1}}\right] \tag{17}$$

for for the sequence $u'(c_1), u'(c_2), ..u'(c_k)$, $B_1, B_2, ...$, and $P_{k+1}, P_{k+2}, ...$given this nominal interest rate path. From the marginal utility path, we can find the path of the real rate of interest. The algebra is straightforward but tedious, so I present it in the Appendix.

Figure 1 shows the effects of a 1 percentage point rise in interest rates at time 1, when prices are sticky for $k = 4$ periods and followed by interest rates reverting with an 0.9 autoregression coefficient.

In the model without pricing frictions, this change would just raise inflation to $P_t/P_{t-1} = \Delta_{t-1}$ immediately, with no change to consumption or the real interest rate. Now the price level cannot

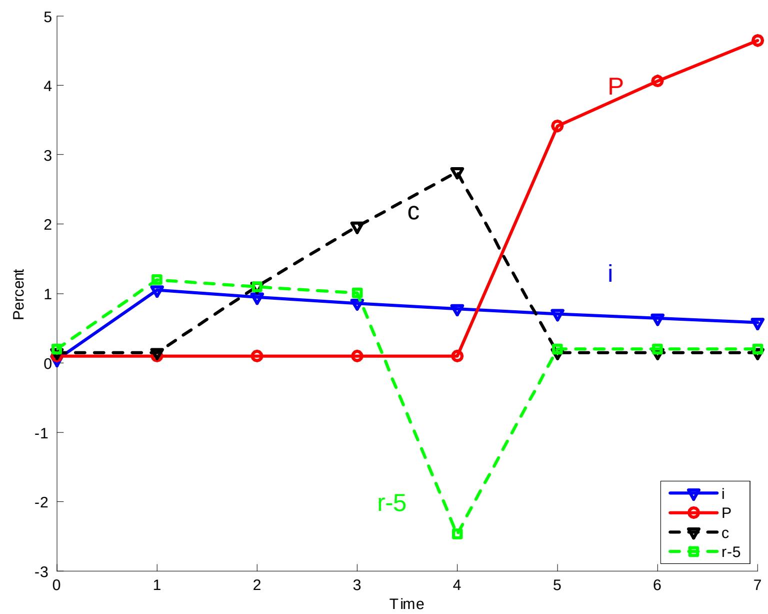


Figure 1: Effects of an interest rate rise in a simple sticky-price model. At time 1, the government unexpectedly raises the nominal interest rate one percentage point. Interest rates then revert with an 0.9 autocorrelation coefficient. Prices are fixed four periods in advance. The steady state interest rate is $\delta = 0.05$.

move for four periods. When the price level is finally free at $t = 5$, it immediately jumps to the frictionless level. All the repressed inflation arrives at once.

During the period of price stickiness, the real interest rate rises by exactly the rise in the nominal rate. This rise in real rate sets off a boomlet in consumption. Consumption growth rises to match the higher real interest rate. However, at the end of the price-stickiness period, consumption jumps back to its frictionless value, there is therefore a period of strong negative real interest rate induced by strong expected inflation and a constant nominal rate.

The jumps at the end of the sticky-price period are of course not realistic. In a more realistic, Calvo-style model, price stickiness would evaporate gradually. So we should expect a consumption boom with little inflation, then consumption to revert to normal slowly as inflation picks up.

In sum, we see that

- *In a model with price frictions, but no monetary frictions, a rise in nominal interest rate target, with no change in fiscal surpluses, can induce real interest rate and output dynamics.*

The scenario plotted in Figure 1 does not conform to the usual story told about interest-rate based monetary policy. This monetary policy is expansionary throughout – first consumption rises, then inflation rises. There is no period in which the rise in real interest rate lowers the level of current consumption, or temporarily lowers inflation. Consumption and inflation look a lot like you might imagine Friedman (1968) to describe a monetary expansion. The only difference, Friedman's monetary expansion would have started with a period of lower interest rates by working down a money demand curve. But there is no money demand curve in this frictionless model.

The prediction that raising nominal interest rates is expansionary is pretty central to the basic structure of this model, combining no monetary frictions, no fiscal response, and price stickiness. First, we have to see that consumption at time 1 $c_1$ cannot fall. At time 1, the basic equation (16) reads

$$u'(c_1)\frac{B_0}{P_1} = E_1 \sum_{j=0}^{\infty} \beta^j u'(c_{1+j}) s_{1+j}. \tag{18}$$

Debt $B_0$ is predetermined. The price level $P_1$ cannot change, by price stickiness. And, with surpluses equal to zero through the time of price stickiness and marginal utility mobility, the right hand side can't change either. Even if we did not make that assumption, discount-rate changes in the value of surpluses for times less than $k$ would be small.

When the nominal interest rate $i_1$ rises, the real interest rate $r_1$ rises, so consumption growth $c_2/c_1$ must rise. If the level $c_1$ could fall, this rise would correspond to a recovery from a recession. If consumption $c_1$ does not move and growth $c_2/c_2$ rises, well, $c_2$ must rise. Both the level and the growth rate of consumption must rise.

In sum, this model alerts us to the neo-Fisherian possibility that perhaps the sign of monetary policy effects is changed in a model without monetary frictions:

- *With no monetary frictions in the interest on reserves regime, raising interest rates is expansionary for both inflation and output.*

The basic logic is pretty simple: raising nominal interest rates either raises inflation or raises real interest rates. If it raises real interest rates, it must raise consumption growth. The prediction is only counterintuitive because for so long we have persuaded ourselves of the opposite, despite the Fisher equation and the consumer's first order condition linking consumption growth to the real rate.

### 3.5 Mixing monetary and fiscal policy

These Fisherian implications for monetary policy seem to violate common views about how monetary policy behaves – that interest rate increases with sticky prices should lower the level of consumption and lower inflation. In standard models, the interest rate rise might eventually raise inflation by the Fisher effect, but impulse- response functions usually imply that the Fed has actually lowered the interest rate target by that time, so the inflation decline is permanent.

But the pure separation between "monetary policy" with no change in surpluses and "fiscal policy" that only changes surpluses, while convenient for conceptual analysis, is a misleading in analyzing actual policy actions or historical events. Historical events and policy interventions always mix monetary and fiscal shocks, and monetary and fiscal policy react to the same underlying shocks.

We can produce something like the standard view by mixing a simultaneous monetary and fiscal policy shock. Figure 2 presents calculations. To make the example clearer, I assume that the Fed raises interest rates by one percentage points for two periods only.

The top left panel of Figure 2 shows the response to this pure monetary policy shock, with no change in fiscal policy, as I presented in Figure 1. The price level is stuck for 4 periods, then jumps up 2% to the "repressed inflation" implied by two periods of 1% higher interest rates. The real rate follows the nominal rate while prices are sticky, then jumps down in the period that prices (expectedly) become unstuck. Consumption growth and level rise when real interest rates rise; the level of consumption reverts back to its previous value when price become unstuck. In short, an interest rate rise is expansionary throughout, first for consumption and output, then for inflation.

The top right panel of Figure 2 presents a pure contractionary fiscal shock. At time 1, people learn that the surpluses past period 4, originally expected to be $\bar{s}$, now will be $\bar{s}' > \bar{s}$, 3% higher. In the model without price stickiness, this change would produce an immediate 3% downward jump in the price level. Now, the price level is stuck for 4 periods, and then jumps down. In the meantime, consumption jumps down instead. This is just a four-period version of the shock discussed in section 3.2.

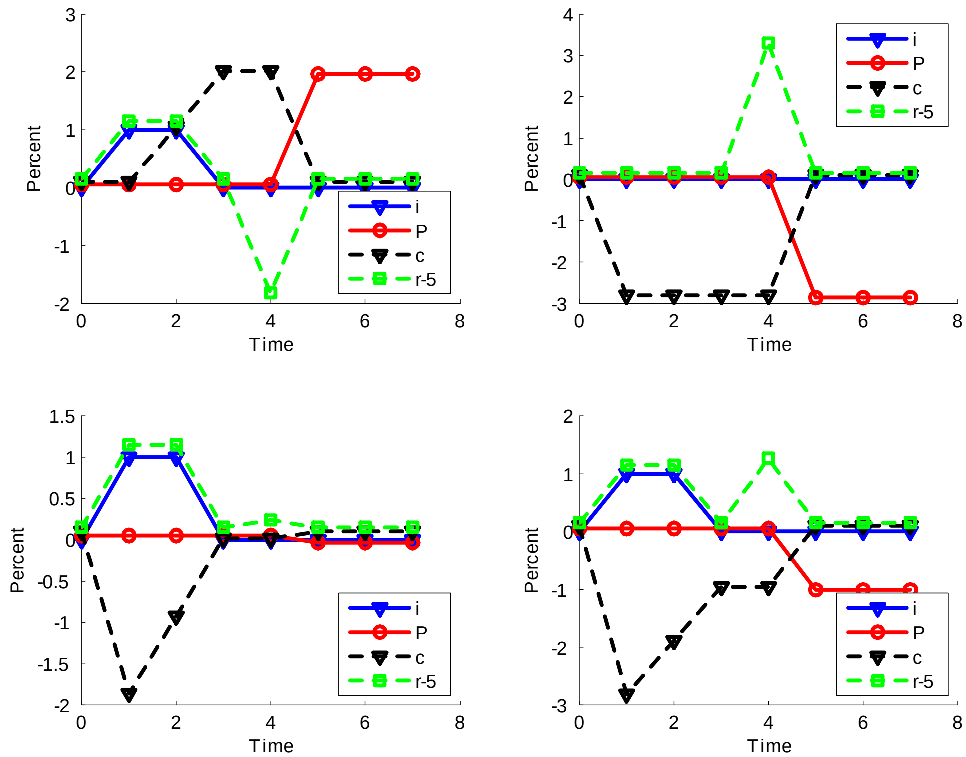


Figure 2: Responses to joint monetary-fiscal tightening. Top left: Monetary policy only. The nominal interest rate rises 1% for two periods, no fiscal policy change. Top right: Fiscal policy only. Long-run surpluses rise 3%, no interest rate change. Bottom left: Monetary policy with a small fiscal change. Interest rate rises 1%, surpluses rise 2%. Bottom right: Monetary policy with a larger fiscal change. Interest rates rise 1%, surpluses rise 3%.

Now, we are looking for a shock that raises nominal, and hence real, interest rates, thus raises consumption growth rates, but lowers the level of consumption. To that end, in the bottom left panel, I mix the 1% interest rate rise from the top left panel with a 2% fiscal contraction. The fiscal policy shock pulls the initial level of consumption down, and the long run price level down.

This *joint* monetary-fiscal policy produces a recession in the level of consumption, by the fiscal contraction. Consumption then regains its original level, due to the growth rate induced by the high interest rate. In this example, there are very small output or price level dynamics left over after period 4.

Clearly, one can produce long run dynamics – positive or negative inflation – anyway one wishes (or, the data indicate) by mixing the size of the fiscal shock with the size, persistence and long-run sign of the interest rate change. The bottom right panel of Figure 2 presents a graph indicative of the kind of VAR evidence found for "monetary policy shocks" that have unstated fiscal accompaniment. Here I add a 3% fiscal tightening along with two periods of 1% interest rate rise. Now, we see a 3% negative shock to output coincident with the higher real interest rates. The higher real rates help consumption to recover faster than it would be with the fiscal shock alone, in the top right panel. But since the 3% fiscal shock is larger than the $2 \times 1\%$ interest rate increases, the "tightening" is followed by a decrease in inflation in period 5, before prices settle to a lower level.

One could also produce a reduction in inflation by the interest-rate response to its own shock. If interest rates fall in the long run, either directly or indirectly by Taylor-rule response to the emerging fiscal deflation, then by Fisherian logic the initial tightening will be followed by eventual disinflation.

In sum, though pure monetary policy without any change in surpluses is Fisherian and expansionary, a joint monetary-fiscal "tightening" consisting of interest rate increases coordinated with a long-run fiscal tightening, produce the kind of responses with which we are familiar. However, such a response hides a far different menu of causal possibilities. If the Fed *wants* to inflate, in this model, and without the usual fiscal coordination, it needs to *raise* interest rates, and leave them there.

(Schmitt-Grohé and Uribe 2013 also reach the conclusion that interest rate increases are inflationary, though in a different new-Keynesian model, writing that their model "shows that raising the nominal interest rate to its intended target for an extended period of time, rather than exacerbating the recession as conventional wisdom would have it, can boost inflationary expectations and thereby foster employment." Ireland 2012 offers a new-Keynesian model with interest on reserves, finding unusual dynamics in response to tightening. )

# 4 Comparison with a new-Keynesian model

A natural reaction at this point is, wait a minute. We have a whole range of models which specify the reaction of the economy to interest-rate policy, with no mention of monetary frictions or fiscal backing: The whole New-Keynesian Taylor-rule DSGE literature, epitomized by Woodford (2003). Why not just reference those models and go on to other questions?

In fact, however, this class of models *does* rely heavily on fiscal backing. When you look at them, these models generate inflation predictions by imagining that monetary policy leads to fiscal policy responses, and their results depend crucially on the nature of the assumed fiscal response.

## 4.1 Fiscal backing in a simple New-Keynesian model

Consider the absolutely simplest new-Keynesian model, as presented in Woodford (2003) (and, in detail, in Cochrane 2011) with no pricing frictions. The model consists of a Fisher equation, a Taylor-type rule by which the Fed sets the nominal rate, and a serially correlated monetary policy shock:

$$i_t = r + E_t \pi_{t+1} \tag{19}$$

$$i_t = r + \phi_\pi \pi_t + x_t$$

$$x_t = \rho x_{t-1} + \varepsilon_t. \tag{20}$$

The equilibrium condition for this model is

$$E_t \pi_{t+1} = \phi_\pi \pi_t + x_t.$$

There are multiple equilibria. Any

$$\pi_{t+1} = \phi_\pi \pi_t + x_t + \delta_{t+1};\; E_t(\delta_{t+1}) = 0 \tag{21}$$

is a valid solution.

The New-Keynesian tradition sets $\phi_\pi > 1$. All but one solution now explodes, $\|E_{t+1}\pi_{t+j}\| \to \infty$. Ruling out nominal explosions, one selects the unique locally-bounded solution

$$\pi_t = -\frac{1}{\phi_\pi - \rho} x_t$$

and interest rates thus follow:

$$i_t = -\frac{\rho}{\phi_\pi - \rho} x_t$$

Equivalently, this equilibrium chooses the shock

$$\delta_t = -\frac{\varepsilon_t}{\phi_\pi - \rho}. \tag{22}$$

Figure 3 presents the response to a one percentage point monetary tightening, $\varepsilon_1 = 1$, in this simple canonical model[1]. (The plots use the borderline case $\phi_\pi = 1$ for all solutions. This saves a lot of plots and discussions exploring both the $\phi_\pi > 1$ and $\phi_\pi < 1$ cases. The response functions depend smoothly on $\phi_\pi$, so are visually indistinguishable for $\phi_\pi$ slightly above or below one. ) The monetary policy shock $x_t$ is positive and slowly declines following the AR(1) pattern. The lower lines marked "New-Keynesian" plot the response of interest rates and inflation to this shock. Inflation jumps down; the tightening lowers inflation as the standard story says. The actual nominal interest rate also falls, which seems like counterintuitive sort of "tightening." But the actual interest rate falls *less* than $\phi_\pi$ times inflation. This represents "tightening" relative to the Taylor rule. The dynamics come entirely from the mean-reversion of the shock. A permanent 1% shock leads to an immediate and permanent decline of interest rates and inflation.

Thus, this completely frictionless model, based only on the Fisher equation with a constant real rate, produces lower inflation from positive shock to the interest-rate rule.

The valuation equation for government debt

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{23}$$

*is* part of this model. It just got brushed in to the footnotes with an assumption that the Treasury will always pass lump sum taxes $\{s_t\}$ to validate whatever solution $\{P_t\}$ emerges. The inflation

[1] The family of response functions are given by

$$\left(\pi_t + \frac{1}{\phi_\pi - \rho} x_t\right) = \phi_\pi^{t-1} \left(\pi_1 + \frac{1}{\phi_\pi - \rho} x_1\right).$$

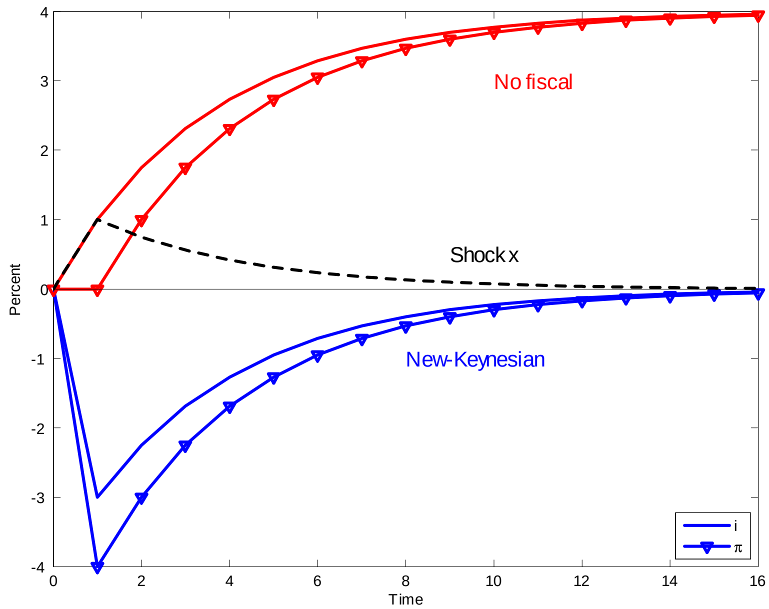


Figure 3: Responses to a monetary tighteing in the standard and fiscally-constrained solutions of a simple new-Keynesian model. $\rho = 0.75, \phi_\pi = 1$.

drop at time $t = 1$ is an *unexpected* drop, as (22) makes clear. As we have seen, with one-period debt the only way to produce an unexpected drop in inflation by (23) is to imagine a change in fiscal policy,

$$\frac{B_{t-1}}{P_{t-1}}(E_t - E_{t-1})\left(\frac{P_{t-1}}{P_t}\right) = (E_t - E_{t-1})\sum_{j=0}^{\infty}\beta^j s_{t+j}. \tag{24}$$

Thus, to produce the unexpected -4% inflation in response to a monetary policy shock shown in Figure 3, this model *must* also specify that fiscal policy produces a 4% increase in the net present value of primary surpluses, to validate a 4% increase in the real value of government debt.

The inflation response is thus really a response to two, simultaneous, shocks: a Taylor-rule shock and an expected surplus shock. In the US context, with \$12 billion dollars of outstanding public debt, that means that the Treasury must be expected to come up with about \$500 billion of extra tax increases or spending cuts, in present value terms, to validate the desired dis-inflationary effects of a 1% interest rate rise, a not inconsiderable amount of fiscal-monetary coordination.

The fiscal coordination is crucial. From the point of view of (23), the *mechanism* by which "monetary policy" produces the downward jump in "aggregate demand" or increased demand for government debt, and thus the *mechanism* by which it produces disinflation, is by inducing this fiscal reaction.

## 4.2 Simple model, no fiscal backing

What if that fiscal backing is not forthcoming? Or, what if people just stop *expecting* it when they see a monetary policy shock, or if the fiscal backing, familiar in the past, would run in to Laffer curve or political limits in a high-debt environment?

Equations (24) and (21) allow a nice view of this conundrum: we can index all the multiple solutions to the new-Keynesian model by their implied fiscal backing. For example, the case of no fiscal response, $(E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j} = 0$, that I studied above, is the case $(E_{t+1} - E_t)\, \pi_{t+1} = \delta_{t+1} = 0$.

Figure 3 also includes this "fiscal-neutral" solution to the model, plotted in red. This solution is simply computed as $\pi_1 = 0$, $\pi_t = \phi_\pi \pi_{t-1} + x_t$, $x_t = \rho^{t-1}$. In this solution, inflation does not jump in the period of the shock – that's how we identified the equilibrium choice. Then interest rates follow obvious dynamics generated from the policy rule and Fisher equation

Now, the fiscal-neutral solution gives positive inflation in response to monetary tightening. Which is, in retrospect, quite a natural prediction. This is a purely frictionless model. Real rates are constant, so there is no mechanism for real rates to lower "demand." In a totally frictionless model, all the Fed can do when it raises the nominal rate is to raise expected inflation. So of course raising the nominal rate raises inflation. The mystery here is, how did the standard new-Keynesian solution produce a downward jump in inflation from a completely frictionless model, with fixed real rate, fixed output, and super-neutrality, yet somehow raising the nominal rate lowers inflation? The answer is now clear: The new-Keynesian solution assumed that the monetary change would *also* be accompanied with an important fiscal tightening, and this fiscal tightening produced the inflation decline.

### 4.3 The three-equation new-Keynesian model

The system (19)-(20) may seem too simple to examine this issue. But the same points hold in more realistic models.

To demonstrate this point, I examine solutions to the standard three-equation new-Keynesian model,

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \sigma \left( i_t - E_t \pi_{t+1} \right) \\
\pi_t &= \beta E_t \pi_{t+1} + \gamma y_t \\
i_t &= \phi_\pi \pi_t + x_{it}
\end{aligned}
$$

$$
x_{it} = \rho_i x_{it-1} + \varepsilon_{it}.
$$

Figure 4 presents responses to a monetary policy shock in this model. (The algebra is in the Appendix.) As one might expect, and similarly to the simple model of Figure 3, the monetary tightening lowers inflation and output. Again, interest rates actually jump down, but less than inflation, so this shock does represent a tightening. Again, the solution depends on a jump downwards in inflation, which requires a fiscal tightening.

Figure 5 presents a "fiscal-neutral" solution of the same model. Here again, I just picked the equilibrium in which $(E_t - E_{t-1})\, \pi_t = 0$.

This change produces radically different inflation and interest-rate responses. Inflation cannot now "jump" down during the period of the shock. The tightening now produces an actual rise in nominal rates. Nominal rates and inflation then chase each other into positive territory, much as they did in Figure 3. Real rates rise, and the real rate and output responses are not that different from the standard new-Keynesian case.

Even this standard New-Keynesian model produces Fisherian results, that a rise in interest rates *increases* inflation when not accompanied by a contractionary fiscal shock.

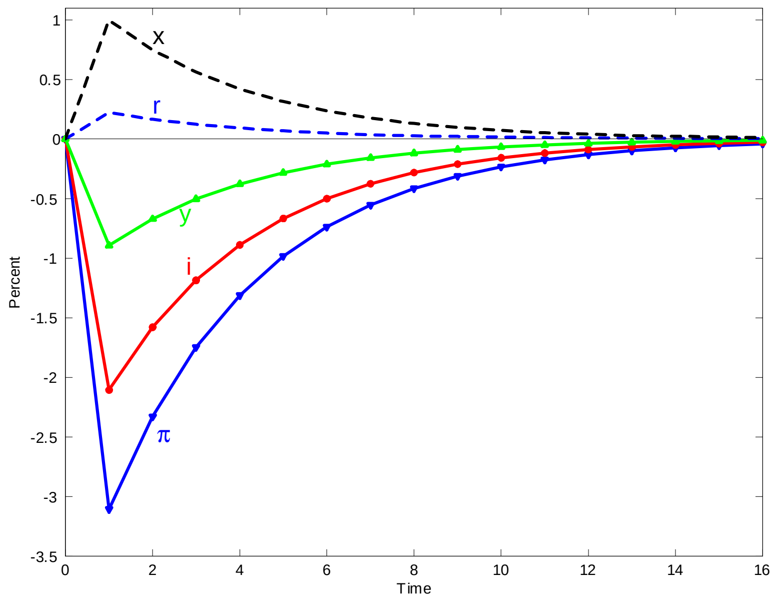


Figure 4: Response of standard new-Keynesian model to a 1% monetary policy shock. $\rho = 0.75$, $\gamma = \sigma = \phi_\pi = 1$.

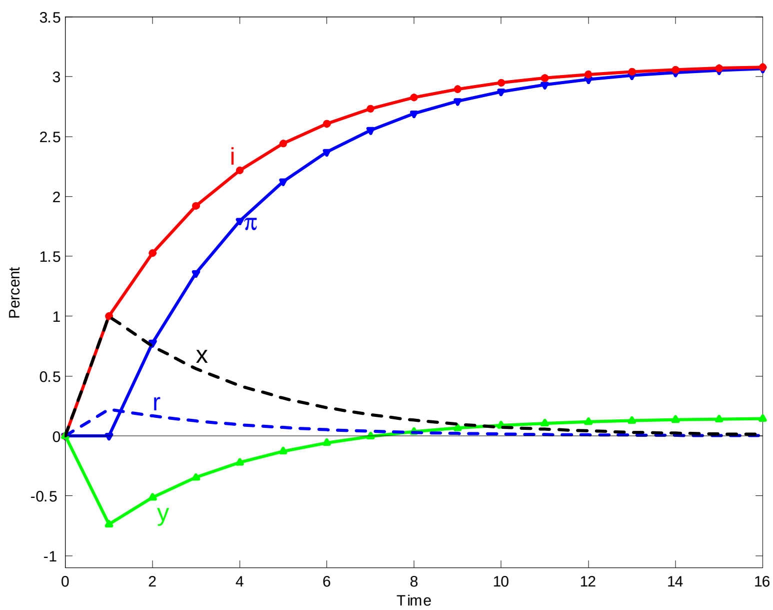


Figure 5: "Fiscal-neutral" response to a 1% monetary policy shock in a new-Keynesian model. I choose the equilibrium with no shock to inflation, $\pi_{t+1} - E_t\pi_{t+1} = 0$.

A disclaimer: properly integrating fiscal backing into models of this sort is more complex than simply adding the frictionless valuation equation, as I have implicitly done here to make a clear illustrative calculation. Since real interest rates change, a change in monetary policy without change in expected surpluses will have a discount-rate effect on the value of government debt. I have implicitly plotted a monetary policy change with just enough change in surpluses that the present value of surpluses is not affected after interest rate changes. More importantly, the details of asset markets, budget constraints, and the nature of price stickiness need to be specified explicitly, as I did in the Appendix for the model with prices set k periods in advance, along with the maturity structure of government debt and state-contingent changes in that maturity structure.

The point here is not to construct a second fully fleshed out model, but to show that the standard new-Keynesian model also stands firmly on fiscal foundations, and that changing those foundations fundamentally changes the model's predictions.

# 5 Theory and literature

There are three basic approaches to monetary policy and price level determination, in the context of modern institutions, fiat money and a central bank: Money supply and demand, MV=PY, interest-rate control in new-Keynesian models, and the fiscal theory of the price level which I use here.

Here, I contrast the three approaches as they apply to the interest on reserves regime. Since the fiscal theory of the price level is controversial, I allay some common theoretical and empirical objections to its use.

## 5.1 Monetary theory

Write money demand $MV(r - r^{TB}) = PY$, where $r$ is the return on money and $r^{TB}$ is the return on Treasury bills. When we observe the rate on money equal to the rate on bills, $r = r^{TB}$, then the economy is satiated in money. Money demand then becomes a correspondence; in $M$ vs. $r$ space, a vertical line. People will hold any amount of $M$, above the satiation point, at zero interest spread. Money (reserves) and treasuries are perfect substitutes, and exchanging the two has no effect on prices, output, interest rates, or anything else. Conversely, for a fixed $M$, any price level can be observed; the price level becomes indeterminate by this equation.

In some models of money, we are never completely satiated. But in those models the interest on reserves can never completely equal the interest rate on Treasuries. In fact, we have observed substantial periods in which the interest on reserves has exceeded the interest on treasuries, which has even been slightly negative. Apparently treasuries, which anyone can hold, can be more liquid or "money-like" than reserves, which only banks can hold.

Sargent and Wallace (1985) is a classic paper making this point, warning that, "Indeterminacy of equilibrium is a possibility because the proposal eliminates the interest differential between...reserves, and other assets. ... it tends to produce an indeterminate demand for reserves and hence for the monetary base." They add, "This source of indeterminacy is widely recognized."

In the relevant case that interest paid on reserves comes from earnings on the Fed's portfolio of Treasuries, and nominal interest rates are positive, Sargent and Wallace show that there is no equilibrium. They survey (section 5) alternative models including cash in advance and money in the utility function, and again find that interest on reserves leads to price-level indeterminacy. Ennis (2014) is a recent detailed general-equilibrium model also showing indeterminacy with interest on reserves without fiscal price level determination. Ennis argues that capital constraints can substitute for reserve control.

Sargent and Wallace, like all well-posed monetary models, contains a version of the government debt valuation equation (1). They focus on financing the interest on reserves as a result. They show that there is a continuum of $r$ (interest rate) and $v$ (tax rate) pairs that generate an equilibrium. Their indeterminacy result (Proposition 1) states that for any $r$, there exists a $v$ that makes it an equilibrium. But one can reverse the implication: Fixing $v$, there is only one $r$, and thus one equilibrium. In the notation of this paper, if $B_{t-1}/P_t = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$, then for any $P_t$ there exists a $\{s_t\}$ such that that $P_t$ is an equilibrium. But fixing $\{s_t\}$, only one $P_t$ is an equilibrium. To get an absence of equilibrium, Sargent and Wallace implicitly assumed the "passive fiscal" special case, in Leeper's (1991) terminology, that wipes out the government debt valuation equation. That case is entirely appropriate to their point – monetary policy alone, with passive fiscal policy, leaves an indeterminate equilibrium with interest on reserves.

But the contrary point remains as well. With an active fiscal policy, we have a unique, determinate equilibrium, with interest on reserves and satiation in money. In fact, to have a determinate equilibrium in this environment, we *must* have an active fiscal policy. One can regard that statement either as positive or normative. Positive: we entered satiation quite a while ago, yet inflation is if anything puzzlingly stable. So we must be in an active fiscal regime, and our job is to figure out how it works, and think about how to make it work better. Normative: We are clearly going to be in a monetary satiation regime, so we had better figure out quickly how our fiscal-active regime is going to work.

### 5.1.1 What about cash?

Cash does not pay interest. Fama (1983) finds price level determinacy with interest on reserves, by anchoring price level determination in a demand for non-interest-bearing currency and control of that currency.

Cash still exists in rather surprising quantity – about a trillion dollars, or more than \$3,000 per capita, 77% of it in hundred-dollar bills[2]. But you and I, corporate businesses, and financial markets use trivial amounts of cash. The legal, and especially corporate and financial economies, have moved to electronic, interest-bearing money. Almost all of us pay by credit cards or debit cards, linked to accounts that will, when interest rates rise, pay interest, and are mostly settled by netting between our banks – an essentially electronic accounting system. Cash really is only used in any substantial quantity for illegal transactions, undocumented people, and store of value in foreign mattresses.

For this reason, as a modeling approximation, it seems wiser to think of cash holdings as disconnected from nominal (legal) GDP, than to found control of nominal GDP on control of cash balances not used for most of GDP. Empirically, cash holdings just trundle with little apparent connection to the economy and, especially, the financial system. Unredeemed coupons, unused subway cards, sock-drawer change, that stack of receipts you've been putting off submitting for reimbursement, and, more seriously, invoices and some trade credit are also non-interest paying claims. But they're not tightly connected to output or price level determination. Controlling the inventory of unredeemed coupons would not control the price level.

Furthermore, the Fed does not control the quantity of cash, as Fama prescribes. For MV = PY to control PY, the Fed must control the M, as well as V being defined and stable. The Fed allows banks freely to exchange cash for reserves.

For these reasons, it makes more sense, I think, to abstract from cash – along with unredeemed

[2] http://federalreserve.gov/paymentsystems/coin_data.htm

coupons and the rest of my humorous list of non-interest-bearing claims – and think of a monetary system based entirely on interest-paying reserves, and consisting entirely of interest-paying electronic money. Reserves, not cash, are really our fundamental numeraire and means of final payment/. We certainly don't want to embark on the alternative abstraction – that the functioning of monetary policy and the control of inflation centrally revolves around the demand for cash, almost all of which is held for illegal purposes.

More generally, some monetary frictions do remain. There are tiny spreads between treasuries and reserves. There are on-the-run and other small liquidity spreads in treasuries. But I think it would be a mistake to base our basic analysis of big questions of monetary policy – can monetary policy affect GDP and the price level, and if so how – on these ephemeral frictions, using models that, if those frictions were to disappear, would not be able to describe monetary policy and price level determination at all. Instead, it seems more sensible to base our analysis on a theory that is valid in a world with no monetary frictions at all, and then add frictions as necessary to understand second-order effects.

### 5.2 Interest rate targets

This discussion about money may seem quaint, because our Federal Reserve explicitly targets interest rates rather than monetary aggregates, and obviously will continue to do so. So, the central class of theory needed is a theory that describes how Fed manipulation of interest rate targets, not $M$, controls the price level.

Even without interest on reserves – when the Fed controls interest rates through open market operations – a theory of how pure interest rate targeting controls inflation with passive fiscal policy took a long time to construct. Friedman (1968) warned verbally that an interest rate target would lead to unstable inflation. Sargent and Wallace (1975) showed that inflation is indeterminate with an interest rate target. The Fisher relation $i_t = r + E_t\pi_{t+1}$ means that controlling the interest rate can determine expected inflation, but unexpected inflation $\pi_{t+1} - E_t\pi_{t+1}$ can be anything. Sargent and Wallace show that this basic logic survives in a carefully specified general equilibrium model – considering all the equations of the model except a valuation equation (1), $\pi_{t+1} - E_t\pi_{t+1}$ is still not tied down.

As I pointed out above, adding back an active fiscal policy via (1), we resolve Sargent-Wallace indeterminacy, as well as Friedman's instability. Even fixed interest rate targets control both expected and unexpected inflation. The question here is, can one proceed to describe inflation determination by interest rate control without active fiscal policy instead, and how does the approach I followed compare with that more conventional approach?

McCallum (1981) and Hall (1984, 2002) suggested that an interest rate target that varies more than one for one with inflation $i_t = r + \phi\pi_t,\ \phi > 1$, is sufficient overcome Friedman and Sargent-Wallace's difficulties. Taylor (1999) formalizes this logic in a backward-looking old-Keynesian model, showing that an active interest rate policy $i_t = r + \phi\pi_t,\ \phi > 1$ makes inflation and the economy stable, addressing Friedman's (1968) concern, while a passive policy $\phi < 1$ has the opposite effect. I do not follow that path here, as even Taylor admits the model, with backward-looking expectations and a static IS curve, is "ad-hoc."

New-Keynesian models, summarized in Woodford (2003) and described above, are now the standard way to model an economy under interest rate targets. In this model, the Fed deliberately introduces *in*stability to the economy so that all but one path explodes. Choosing the one nonexplosive path, we obtain determinacy: A pure interest rate target can, apparently, determine the inflation rate. This kind of model, unlike standard Keynesian models, has exquisite and explicit micro-foundations. It also has a version without pricing frictions as well as no apparent monetary frictions, outlined above, so it is a much more promising candidate for this kind of exercise.

"Indeterminacy" and "instability" are distinct issues. A model $\pi_{t+1} = 1.5\pi_t + \varepsilon_{t+1}$, where $\varepsilon_{t+1}$ is an economic shock, is determinate but unstable. A model $\pi_{t+1} = 0.5\pi_t + \delta_{t+1}$, where $\delta_{t+1}$ is an expectational error, so all we know is $E_t\delta_{t+1} = 0$, are stable but indeterminate. Old-Keynesian models in the Friedman-Taylor tradition have backward-looking dynamics, or adaptive expectations, for example a Fisher equation $i_t = r_t + \pi_t$, not $i_t = r_t + E_t\pi_{t+1}$. As a result, they suffer from instability but not indeterminacy. The Taylor rule stabilizes their dynamics. New-Keynesian models in the Woodford tradition have forward-looking agents, for example a Fisher equation $i_t = r_t + E_t\pi_{t+1}$. They suffer from indeterminacy, and the Taylor rule *de*-stabilizes their dynamics to try to restore determinacy.

Cochrane (2011, 2014) argues that Sargent-Wallace indeterminacies remain in new-Keynesian models even with $\phi > 1$. The rule that only locally-bounded equilibria are valid is not usually part of economics. More importantly, the assumption that people expect our Fed to deliberately *de*-stabilize the economy seems strained, is not verifiable or learnable in data, and is loudly not how our Fed describes its role. The assumption requires the Fed to precommit to take actions that ex-post are ruinous for its own objectives.

Now, Woodford (2003) and the surrounding new-Keynesian literature explicitly recognizes that the government valuation equation (1) is part of the model. However, they assume that the Treasury adjusts surpluses $\{s_t\}$ to validate any price level, so that equation has no force in inflation determination, "passive" fiscal policy in Leeper's (1991) taxonomy.

One can reinterpret any new-Keynesian model solution with fiscal backing in place of explosive off-equilibrium threats by the Fed. To do so, restore "active" fiscal policy so that (1) uniquely determines the price level. Then, coordinate fiscal and monetary policies, so that fiscal policy *choose* to follow the Fed's price level target. The difference: this fiscal policy would not validate other paths, such as an off-equilibrium deflation requiring large taxes to pay off bondholders. Then we observe the same equilibrium output and inflation as the new-Keynesian model predicts.

To be specific, the New-Keynesian Taylor rule is $i_t = i_t^* + \phi(\pi_t - \pi_t^*)$ where $i_t^*$ represents the interest rate target, including monetary policy shocks such as the $x_{it}$ of my above new-Keynesian models, $\pi_t^*$ represents the inflation target, and $i_t$ and $\pi_t$ represent how the Fed would respond to off-equilibrium inflation. (For the relation between this and other statements of the Taylor rule, see King 2000, Cochrane 2014.)

As a minimal modification, we might think of the $\phi(\pi_t - \pi_t^*)$ reaction as a Sargent-Wallace (1981) style game of chicken between Federal Reserve and Treasury. Rather than view hyperinflation as a threat which might "coordinate expectations" of the private sector on the unique nonexplosive equilibrium $\pi_t^*$, as Woodford (2003) suggests, we can regard it as a threat against the Treasury, to induce the Treasury to follow an appropriate fiscal policy.

Better, I think, since it avoids all such subgame-imperfect threats, is simply to study coordinated active-fiscal and monetary policy without modeling the coordination game. Replace this the new-Keynesian rule with $i_t = i_t^*$, and let an active fiscal policy choose inflation $\pi_t^*$. If fiscal policy agrees to follow the Fed's inflation target, and can do so, then we observe *exactly* the same equilibrium outcomes $\{i_t^*, \pi_t^*\}$ as the standard new-Keynesian model predicts.

One might object that Taylor-rule regressions such as Clarida, Galí and Gertler (2000) establish the $\phi(\pi_t - \pi_t^*)$ part of the Taylor rule, but this is not the case. We only see $i_t^*$ and $\pi_t^*$, never off-equilibrium threats, so Taylor-rule regressions only document the correlations between $i_t^*$ and $\pi_t^*$, not the $\phi(\pi_t - \pi_t^*)$ reaction.

While we can observe the same equilibrium as predicted by new-Keynesian models, however, we do not have to do so. For example, fiscal policy might not agree to tighten when the Fed changes the interest rate target, and we might see the response of Figure 5 not Figure 4. Or, fiscal policy, facing a Laffer limit, might not be able to back up monetary policy. More deeply, recognizing that fiscal and monetary policy are separate but coordinated changes deeply our understanding of how each operates. As we have seen, monetary policy without the usual fiscal coordination is expansionary throughout, raising interest rates raises inflation.

### 5.3 Big picture

Fundamentally, there are three possibilities for price-level determination with fiat money. First, money might be valued because it is uniquely useful in transactions and limited in supply; the quantity theory; MV=PY. But under the interest on reserves regime, the whole point is that money is not scarce. We will be satiated in liquidity and hold far more than needed for transactions. And already, interest rate targets do not limit money supply.

Second, money might be valued because it is backed by convertibility to a real good or asset. The government debt valuation equation reveals that apparently fiat money is in fact backed by the present value of surpluses which will retire government debt. Backing theories survive intact as monetary frictions disappear.

The new-Keynesian Taylor-rule model represents an attempt to construct a third kind of theory, in which fiat money is valued and inflation determined based on active interest-rate setting alone, with neither scarcity in exchange or backing. My survey concludes that this interpretation of the equations is unsuccessful. It too is really a theory of fiscal backing, with a particular monetary-fiscal coordination mechanism by which Fed actions lead the Treasury to adjust surpluses as the Fed wishes.

Ad-hoc backward-looking and mostly static Keynesian ISLM equations with a Taylor rule also give stable and determinate inflation responses to interest rate targets, while ignoring the government debt valuation equation. This kind of analysis remains popular in policy circles, and underlies most of the verbal explanations the Fed gives of its actions and their effects on the economy. However, it doesn't anymore qualify as an "economic" theory. For thinking about how an economy will work, out of sample, with the disappearance of crucial frictions, with profoundly new institutions, and how it might work better with different institutions, it is better to start with something a bit more structural.

I conclude that an analysis of inflation based on the government debt valuation equation (1) is the only currently available framework for understanding inflation in the interest-on-reserves regime, i.e. at the limit that monetary frictions vanish.

### 5.4 Fiscal-theory controversies

The fiscal theory of the price level represented by (1), or a slightly more general version which applies when consumption is not constant over time

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j},$$

has a long tradition. Like much of economics, it starts with Adam Smith, who wrote that "A prince, who should enact that a certain proportion of his taxes be paid in a paper money of a certain kind, might thereby give a certain value to this paper money." ( *Wealth of Nations*, Vol. I, Book II, Chapter II). It has long been recognized that fiscal and monetary policy must be coordinated. One of Milton Friedman's (1949) earliest writings was a "*Fiscal* and Monetary Framework for Economic Stability," (my emphasis). Pesek and Saving (1963) recognized that "once we introduce bonds and the taxes necessary to pay interest on these bonds, the theory of the rate of interest becomes 'monetary' and 'fiscal' as well."

The modern theory has its roots in Sargent and Wallace (1981) and then pure statements and elaboration in Leeper (1991), Woodford 1995, Cochrane (1998, 2001, 2005, 2011b) and Sims (2001, 2005). Sims (2013) is an excellent treatment covering issues similar to those covered here. Cochrane (2005) answers many common objections in detail.

Use of the valuation formula (1) to think about inflation is clouded in myriad unnecessary controversies. I address some here. (See also Cochrane 2005, 2011b on these points.)

It is helpful to derive (1) in a fully-specified model, which I do in the Appendix. The representative consumer maximizes $E\sum_t \beta^t u(c_t)$ and has a constant endowment $y$. This specification produces a constant real interest rate $1 + r = 1/\beta$. The government sells one-period nominal debt with face value $B_{t-1}$ at the end of time $t-1$. It redeems debt with money at the beginning of time $t$, then soaks up that money at the end of time $t$ with lump-sum real surpluses $s_t$ and bond sales with value $Q_t B_t$, where $Q_t$ is the one-period bond price. Interest is paid overnight, and people do not want to hold non-interest paying money overnight, so money printed in the morning must be soaked up in the afternoon,

$$B_{t-1} = P_t s_t + \beta E_t \left( \frac{P_t}{P_{t+1}} \right) B_t,$$

or, in real terms.

$$\frac{B_{t-1}}{P_t} = s_t + \beta E_t \left( \frac{B_t}{P_{t+1}} \right). \tag{25}$$

Iterating forward and applying the consumer's transversality condition, we obtain the basic equilibrium condition (1). (At a zero interest rate, people may be willing to hold money overnight, but money and bonds are now the same assets so the same equations hold.)

Equation (1) is not a "budget constraint." It is a valuation equation, an equilibrium condition. Its ingredients include the household budget constraint and first-order conditions. It works the same was as the valuation equation by which stock prices adjust the present value of expected dividends. There is no "budget constraint" that *forces* the government to respond to a deflation in $P_t$ by raising surpluses, any more than a stock price "bubble" forces a company to raise earnings to justify the stock price. And just as well, because there is a Laffer curve limiting surpluses, but there is no limit to deflation, so there must be some price at which (1) is violated while budget constraints can never be violated.

Equation (1) has a natural "aggregate demand" interpretation. (Woodford 1995). If the real value of nominal debt is less than the present value of surpluses, then people try to spend their debt and money on goods and services. But collectively, they can't, so this "excess aggregate demand" just pushes up prices until the real value of debt is again equal to the present value of surpluses. Aggregate demand is nothing more or less than demand for government debt. By the private-sector budget constraint the only way to spend more on everything else is to spend less on government debt. This equation also expresses a "wealth effect" of government debt.

Though the literature spends a lot of time thinking about "regimes" and testing for them, there is really not much point to that exercise. As a simple example, suppose we modify the model to add a demand

$$M_t V = P_t y \tag{26}$$

for money held overnight that does not pay interest. Equation (1) now includes a seignorage term,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t \left[ s_{t+j} + \frac{M_{t+j} - M_{t+j-1}}{P_{t+j}} \right], \tag{27}$$

or equivalently

$$\frac{B_{t-1} + M_{t-1}}{P_t} = \sum E_t \left[ m_{t,t+j} \left( s_{t+j} + \frac{i_{t+j}}{1 + i_{t+j}} \frac{M_{t+j}}{P_{t+j}} \right) \right].$$

Both equations (26) and (27) must hold in equilibrium.

Now, following Leeper (1991) we often talk of a money-dominant "regime" as one in which the Fed sets $M_t$, $P_t$ follows from (26), and then the Treasury sets $\{s_t\}$ in (27) to validate the Fed-chosen $P_t$, and a fiscal-dominant "regime" as the opposite case. One might say that in the absence of monetary frictions, the government must "switch" to a "fiscal-dominant regime."

But both equations (26) and (27) hold in both regimes, so there is no testable content to the regime specification from observations of $\{M_t, B_t, P_t, s_t\}$, (Cochrane 1998). This observation should already alert us to the sterility of the "regime" investigation.

One can read the equations of a supposedly fiscal-passive regime as instead verifying the power and necessity of the fiscal backing. Monetary policy only affects inflation *because, and only because* it induces changed expectations of fiscal surpluses. The change in "aggregate demand" that ultimately affects the price level comes only from the induced change in fiscal surpluses. Is it the foot on the gas pedal, or the engine which ultimately causes the car to go? If a man (Fed) induces a horse (Treasury) to pull a cart by putting a carrot under the horse's nose, does that mean the man pulls the cart?

The same points hold if the Fed follows an interest rate target. Again, a valuation equation like (1) holds, and the Treasury is assumed to adjust $\{s_t\}$ to validate the model's price-level predictions. If the Treasury will not or cannot follow through, the hypothesized price level won't happen. The interest rate only affects the price level because of the induced fiscal response. There is no testable content to whether the Treasury or Fed drives the "regime."

Money and fiscal policy must always be coordinated. Monetary contractions without fiscal support and coordination fail. Fiscal contractions with loose money stop inflations (Sargent and Wallace 1981). If the Fed were to try a 50% deflation now, this would mean doubling the real value of publicly-held debt from \$12 trillion to \$24 trillion, and the value of the government's credit guarantees by additional trillions. The "passivity" of fiscal policy would be sorely tested.

As these examples emphasize, for (1) to hold and play a central role in price determination, one does not have to, and one should not, think of surpluses $\{s_t\}$ as being "exogenous," or set without regard to other variables, including prices. Equation (1) tells us what the equilibrium price level must be, conditioned on the equilibrium $\{B_t\}$ and $\{s_t\}$. That is all. By analogy, we have gotten used to using the standard asset pricing equation $p_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}x_{t+1}\right]$ without needing to assume that consumption $\{c_t\}$ or payoff $\{x_t\}$ are exogenous, fixed, endowments, and so forth, understanding that all elements of the equation are endogenous and simultaneously determined.

Some economists regard fiscal price determination as a matter for extremes; currency crashes and hyperinflations maybe, but not normal times. But even in "normal times" monetary and fiscal policy must be coordinated; monetary policy only works if the fiscal backing – the response of surpluses $\{s_t\}$ – is there, even when that response is "small." And cyclical variations in aggregate demand, the right hand side of (1), are not usually thought of as being that "small."

Perhaps the "exogenous" confusion is behind this point. Equation (1) holds even when the surpluses $s$ are within the government's control, and the government could choose to raise surpluses if it wished, not just when the top of the Laffer curve or other disaster means the government loses control of surpluses.

It's tempting and useful for comparative-statics exercises to think about fixing $\{s_t\}$ or $\{B_t\}$ holding the others constant, as I have done above. However, real monetary and fiscal policy is always coordinated, and most events contain large movements in both quantities at the same time. Good monetary policy institutions carefully think through fiscal-monetary coordination. For example, wars and recessions feature big increases in debt $B_t$ with big negative current surpluses $s_t$. But these events come with big increases in expected future surpluses $E_t s_{t+j}$, because governments want to raise real revenue, not cause inflation. So $\{s_t\}$ follows a response that is negative now, and positive later, to such a shock, nothing like an AR(1), and $\{s_t\}$ and $\{B_t\}$ move together in response to such typical economic shocks.

### 5.5 Interest on reserves

Most of the recent literature focuses on the desirability of the interest-on-reserves regime. Stein (2012), Kashyap and Stein (2012), Keister, Martin, and McAndrews (2008) and Goodfriend (2002, 2011) praise the regime, because it allows the Fed to purchase and sell assets, without changing interest rates, and vice versa. As Kashyap and Stein put it, the Fed can separate interest rate changes used to "manage the inflation-output tradeoff" from balance sheet policy by which the Fed will "regulate the externalities created by socially excessive short-term debt issuance on the part of financial intermediaries." Likewise, Goodfriend praises the fact that the interest on reserves regime "frees monetary policy to fund credit policy independently of interest rate policy."

In a series of thoughtful speeches, Charles Plosser (2009, 2010, 2012, 2013) disagrees strongly that opening these doors is a good idea. For example, Plosser (2010, p.8) writes: "the composition of the portfolio has changed for the explicit purpose of supporting a particular sector of the economy – housing – which breaks entirely new ground. The public and market participants may believe that the Fed can and will use its purchases to pursue other sorts of credit policies than has been its practice in the past."

All of these views add something I have left out of the model, *financial* frictions by which Federal Reserve purchases affect asset prices or flows, at least temporarily. Curdia and Woodford (2010, 2011) and Gertler and Karadi (2011) write explicit new-Keynesian DSGE models with financial frictions, in which optimal policy involves changes in the size and composition of the balance sheet. Here, briefly and in Cochrane (2014), in depth, I praise the financial stability benefits of interest on reserves, but with an entirely different mechanism. I focused on the financial stability benefits of the Fed's *liabilities*, abundant interest-paying reserves which create narrow-banking deposits, rather than the potential benefits of asset-market manipulation in the Fed's *asset* choices. These authors, and Plosser especially, also bring up important political-economy considerations, which I ignore. The Fed's powers are limited as the price of its independence. Greater power, especially in politically-sensitive areas, may cost independence.

Federal Reserve policy in the future goes so far past "monetary," that the label will no longer be appropriate. As in these author's focus, active management of the financial system and financial flows is likely to occupy much of the Fed's attention, and inexorably to be mixed with inflation and macroeconomic goals. If, as I have argued, reserve requirements no longer have any effect on bank lending and deposit creation, why not use capital requirements to reimpose such control? Why not use the Fed's abundant regulatory powers to tell banks how much to lend and who to lend to? The "macroprudential" policy idea really amounts to a set of temptations, or intriguing possibilities, depending on your view, for the Fed to control financial and thereby economic activity. And these ideas will be increasingly tempting as pure "monetary" policy, setting short-term interest rates, loses power.

But all that discussion is beyond the scope of this paper. None of these authors are concerned with the basic questions of interest rate and price level control I have focused on here. Stein (2012) even uses the fiscal theory for price-level determination.

# 6 Concluding comments

The interest-on-reserves regime with a big balance sheet is an attractive extension of a decades-long period of financial and monetary innovation. It gives us interest-paying money, the end of monetary frictions, and the foundation of a more stable financial system in which government short-term debt drives out private short term debt, much as government notes drove out banknotes in the 19th century. But this apparently small extension of our institutions challenges the core of traditional monetary theory. One might argue that this monetary theory had ceased to matter already. But interest on reserves and a large balance sheet force us to confront that fact.

Some of the questions and doctrines I have addressed: The government need not lose control of inflation in this regime. We *can* have price level control with no control of "money," no rationing of liquidity, no limit on central bank balance sheets, no limit of private intermediation, and under interest rate targets, even targets that violate the Taylor principle. We can enjoy full interest on "monetary" assets. We can be satiated in liquidity. The Federal Reserve has the power to target nominal interest rates in this regime, though whether it can simultaneously control the size of its balance sheet is more open to question. Fortunately the size of its balance sheet is also irrelevant to monetary affairs, so long as we stay comfortably above the bound of satiation in reserves. Interest-paying reserves are not inflationary. The money multiplier, the link between open market operations and lending, and velocity both become meaningless.

I have explored these issues with extremely simple models, in order to make the mechanisms transparent. Once one trusts the basic mechanisms, one can go on to explore more realistic models, both for matching data and for evaluating policy.

The first task is to match empirical impulse-response functions. Though I have shown how purely "monetary" policy without fiscal coordination can produce changes in real interest rates and output, I found that an interest rate rise produced by monetary policy alone is expansionary, first for output and then for inflation. To produce the classical sign, that output falls in response to an interest rate rise, and that inflation then declines, I had to pair the interest rate rise with a fiscal contraction. Examining a standard simple new-Keynesian model, I verified the same result: monetary policy only produces a contraction in that model, if we assume a simultaneous contractionary fiscal policy shock. My sticky-price model *is* a standard simple new-Keynesian model, just with a different equilibrium concept, so this is not a surprising result.

Now, perhaps more complex models will reverse this result. Perhaps they will not. Perhaps we will find that temporary output and inflation reduction is only a feature of monetary frictions, and once those frictions disappear so does the conventional sign. Perhaps the model is right and an interest rate rise without fiscal coordination is expansionary. In the world, monetary and fiscal policy are always coordinated – changes in expectations of $\sum_t m_t s_t$ accompany all monetary moves, and fiscal policy responds to the same underlying economic shocks as does monetary policy,, so all of the "monetary policy shocks" we have studied in the data combine fiscal and monetary policy shocks.

I have not touched the question of optimal monetary and fiscal policy. Something simple such as a Taylor rule guide for good policy has not yet emerged from this analysis. To study optimal policy, however, we need a model with shocks that policy is trying to offset. We also need realistic pricing frictions, and ideally we need to consider distorting taxes and a maturity structure of debt.

First of all, to understand the data as well as to understand optimal policy, we have to understand why and how interest rate increases have been correlated with fiscal contractions.

The bottom left panel of Figure 2 suggests that a coordinated monetary-fiscal tightening is a good way to control the real economy without affecting the price level at all. That is, perhaps exactly what our authorities are trying to do. Perhaps this combination of monetary and fiscal policies is something like an optimal response to a real shock. Presence of shocks like this also helps to explain why it is so hard to find inflationary effects of monetary policy shocks in VARs.

But if all one wants to do is to reduce inflation, then the kind of coordination shown in the bottom left panel makes less sense. The contrast between the upper right and lower right-hand panels of Figure 2 suggest that expansionary monetary policy can offset the effects of a fiscal contraction, but expansionary monetary policy also offset fiscal policy's disinflationary effects as well. If all you want to do is reduce inflation, this does not seem like an ideal policy.

That line of thought suggests inevitably that we will end up with one kind of coordinated monetary - fiscal policy for offsetting recessions and fighting real shocks, and a different kind of coordinated monetary-fiscal policy for fighting inflation or deflation, rather than a one-size-fits-all monetary-fiscal coordination.

In the model considered here, a better course for reducing inflation would be to announce a *reduction* in nominal interest rates, to occur after prices are able to move. Perhaps that prediction is not so unreasonable. As we look over the period since 1980, inflation has been on a slow downward trend, along with nominal interest rates. Perhaps that's pretty much what happened, and central banks' discovery of the power of forward guidance, transparency, inflation-targeting rules, and so forth can be captured in this prediction of the model.

The weak spot of applying the fiscal theory to understand events is the nebulousness of the expected present value of future surpluses, just as the weak spot of finance is the nebulousness of the expected present value of future dividends. For both theory and empirical work, time-varying discount rates and risk premiums loom large in the present value of future surpluses.

But the fiscal theory did not spring in a vacuum from the day Ben Bernanke received authorization to pay interest on reserves. Rather, it describes a long period of historical and institutional evolution. And one can see how new, better monetary policy institutions might follow smoothly.

The gold standard seems like a pure monetary policy, but it is not. Since no government ever backed 100% of its nominal debt with gold, the gold standard was a way to communicate and commit the government to raise the appropriate surpluses to pay off its nominal debt. If people wanted to redeem notes for gold, the government would raise the gold with current or, via borrowing, future taxation. The gold standard is impractical, of course, since we want to stabilize the CPI not the price of gold. And its history is full of crashes, when the essentially fiscal "commitments" fell flat. Foreign exchange pegs are similar fiscal commitments.

The disinflation of the 1980s in the US can be seen as a classic coordinated fiscal-monetary tightening. The higher real interest rates raised interest payments on the debt by two percentage points of GDP for a decade, and the disinflation was a bonanza for holders of long-term debt. These fiscal resources came from somewhere. In the US, monetary contraction was quickly followed with fiscal and regulatory reforms. The following period of strong growth produced fiscal surpluses through the late 1990s. These surpluses paid for the 1980s bondholder bonanza. Without that fiscal backing, the disinflation would have failed as so many others.

The success of inflation targeting, in particular in Sweden and New Zealand, (Grimes 2013, Svensson 2010) can be read in the same light. In each case, the inflation target for monetary policy accompanied a far-reaching fiscal reform. And we can read the inflation target equally as a commitment by the Treasury, to fund debt at the targeted level of inflation, as it is a commitment by the central bank to target that level of inflation via interest rate policy. The quick declines in inflation following the announcement of inflation targets, without requiring a painful period of high interest rates, is suggestive of this story. (It's worth recalling that the US disinflation of the 1980s, though it did feature high real rates, was much faster than contemporaneous Phillips curves predicted.)

With this historical context and in this interpretation of events, we are at a ripe moment to widen the set of tools for monetary and fiscal policy, and to reconsider the regime and coordination mechanism, taking the next step past inflation targeting.

Now that we have real or indexed debt, for the central bank to target the *spread* between real and nominal rates rather than the level of the nominal rate is an attractive possibility. The spread more directly controls expected inflation in an environment where real rates vary. Targeting the spread, by standing willing to buy and sell nominal vs. indexed debt, amounts to a gold-standard-like rule, which promises to exchange something real for something nominal at fixed value. But it targets the expected CPI rather than the price of gold. In the fiscal theory, writing the value of real debt as $b_{t-1}$, the basic equation becomes

$$b_{t-1} + \frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{28}$$

and, moving back a period,

$$b_{t-1} + \frac{B_{t-1}}{P_{t-1}} \frac{Q_{t-1}}{\beta} = b_{t-1} + \frac{B_{t-1}}{P_{t-1}} E_{t-1} \left( \frac{P_{t-1}}{P_t} \right) = E_{t-1} \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{29}$$

If the Fed targets the spread $Q_{t-1}/\beta$, that policy obviously targets expected inflation. If people think inflation will be higher than the target, they will sell a lot of nominal debt in exchange for real debt. But (28) shows, if the quantity of nominal debt $B_{t-1}$ relative to $b_t$ declines, then expected inflation must also decline. So it has a natural stabilizing mechanism, just as allowing the quantities of paper notes and gold outstanding does.

More generally, we can add control of the real vs. nominal composition of government debt to our list of "QE" tools, though like the gold standard or foreign exchange rate peg it is obviously one with fiscal implications. My section 2.4 already pointed to state-contingent adjustments of the maturity structure of government debt as a potentially important tool for joint monetary-fiscal policy to respond to shocks. So, as this discussion indicates, I suspect optimal monetary-fiscal policy will broaden the recommended set of tools substantially, beyond short-term nominal interest rate targets.

The gold standard and exchange rate pegs are plagued by crises and defaults when the underlying fiscal commitments can't be met. Pure nominal debt means that shocks to surpluses are met by inflation, which transfers wealth from all savers to borrowers and which causes many distortions in sticky-price economies. Government debt with explicitly variable coupons would allow fiscal adjustments without explicit default, crisis, or inflationary consequences. Then, adjusting the coupon rate in response to shocks becomes a vital policy tool.

I have highlighted that inflation targets can be interpreted as fiscal commitments. A Taylor rule for *fiscal* policy would formalize this commitment. For example, purely real (indexed) debt seems to leave out price level determination in (28). But if we have purely real debt and a rule that surpluses must adjust to the price level,

$$b_{t-1} = E_t \sum_{j=0}^{\infty} \beta^j \left( s^0_{t+j} + \alpha P_{t+j} \right),$$

where $s_{0t}$ is a potentially stochastic temporary deficit/surplus, and $\alpha P_{t+j}$ represents a rule by which long-run tax rates or the cyclically adjusted budget must respond to the price level, we again have a determinate price level.

None of these ideas are really new and radical. They represent a continuation of the long trend of monetary-fiscal policy coordination, and building of better institutions to manage that coordination.

We started with what seemed like minor and rather technical issues, whether the Fed pays interest on bank reserves, and whether in order to raise interest rates, the Fed needs to sell off its balance sheet, or whether the Fed can just raise interest on reserves and keep the huge balance sheet. We have ended up, really, at a once per generation redefinition of role and nature of monetary policies, and of the institutions that generate price stability and financial stability, the proper role of a central bank, the question of what monetary policy can do, what it can't do, what it should do, and what it shouldn't do.

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# 8 Appendix

## 8.1 Frictionless Model

This section sets out a very simple, but complete, model of frictionless price determination, to verify that analysis using (1) is not incomplete. This is a simplified version of Cochrane (2005).

The representative household maximizes

$$E\sum_{t=0}^{\infty}\beta^t u(c_t).$$

The household receives a constant endowment $y_t = y$.

It is easier to imagine a sequence of events in each period or day, though that sequencing is not important to the model. Each evening, the government sells a face value $B_{t-1}$ of nominal debt due the next period. Each morning $t$ then, the government prints up $B_{t-1}$ new dollars to pay off the outstanding debt. Households receive the dollars, sell their endowments $y$ for dollars and buy goods $c_t$ for dollars. At the end of the day, they must pay lump sum taxes less transfers $P_t s_t$ in dollars.

I fix the real value of net taxation. This is realistic: With standard income taxes, the nominal amount of taxes are a rate times nominal income, $P_t s_t = \tau P_t y_t$, and if the price level doubles so does the nominal amount of taxes. However, I wish to leave tax distortions out of the model.

The government also sells new debt $B_t$ at a nominal bond price $Q_t$, thereby soaking up cash. The government sets $\{B_t, s_t\}$. The household chooses $\{c_t\}$ and along the way demand for bonds and money, and the price level and asset prices clear markets.

The household period budget constraint is

$$B_{t-1} + P_t y = P_t c_t + P_t s_t + Q_t B_t + M_t$$

where $M_t$ is money held overnight. I assume that the nominal interest rate is positive, so the household chooses zero overnight money holdings, $M_t = 0$ and thus

$$B_{t-1} + P_t y = P_t c_t + P_t s_t + Q_t B_t.$$

The household's first order condition with respect to $c_t$ and $c_{t+1}$ yield

$$Q_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}\frac{P_t}{P_{t+1}}\right] = \beta E_t\left[\frac{P_t}{P_{t+1}}\right]$$

where in the second equality I have used the equilibrium condition $c_t = y$. Dividing by $P_t$ and substituting, the money in = money out condition reads

$$\frac{B_{t-1}}{P_t} + y = c_t + s_t + \beta E_t\frac{B_t}{P_{t+1}}$$

Iterating forward,

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty}\beta^j\left(c_t - y + s_t\right) + \lim_{j\to\infty}\beta^j E_t\frac{B_{t+j}}{P_{t+j+1}}$$

I impose the usual condition that the last term is zero. This transversality condition is a condition for *household* optimality, and a constraint on household borrowing from the government. If it is positive, then the household can increase consumption over income by simply not buying so much government debt. If it were negative, then the household could roll over debt forever. If we just assume that the government borrows but never lends and the price level is positive, that condition is assured.

Then, the equilibrium condition $c_t = y$ at every date implies

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_t.$$

I took some time to derive this equation in order to emphasize that it is *not* an "intertemporal government budget constraint." It combines the household budget constraint, the household's desire not to hold money, the household optimality condition, and equilibrium in the goods market. If the household wished to die holding money, the government could print money and leave money outstanding. If the household wished to hold ever increasing amounts of government debt, the government would never have to pay its debts, and the transversality condition would not hold. If the household were a growing set of overlapping generations, that condition would not, in fact hold. The fact that OLG models relieve the government of its "budget constraint" shows it isn't a "constraint" in the first place.

While I described a "day," and nominal debt exchanged for money and back again, that feature is clearly inessential. People can transact directly with maturing government debt, and pay taxes or buy new debt by delivering maturing government debt $B_{t-1}$. While this looks like a cash-in-advance economy it is not by one crucial difference: the securities market is always open, and people can hold zero cash overnight.

## 8.2 Sticky-Price Model

In this section I build an explicit model in which prices are set one period in advance. The sticky-price setup is a simplification of Galí (1999). The contribution is to combine that price-stickiness framework in a model of fiscal price determination.

Each household derives utility from the consumption of a range of goods $j$. Its objective is

$$\max_{\{c_{jt}, n_t\}} E \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - n_t \right]; \; c_t = \left[ \int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}.$$

The households' period budget constraint is

$$B_{t-1} + \pi_t = \int_{j=0}^{1} p_{jt} c_{jt} dj + S_t + Q_t B_t$$

and a transversality condition I will describe below. The household enters the period with $B_{t-1}$ face value of government debt, receives profits from selling goods,described below, purchases a range of goods from other households, pays nominal taxes less transfers $S_t$, and buys new bonds $B_t$ at price $Q_t$.

### 8.2.1 Demand for varieties

We can solve the household problem in two steps: First, find the allocation across goods $c_{jt}$ conditional on the overall level of purchases $c_t$, and then find the optimal allocation across time $c_t$ and labor supply decision $n_t$. We can find the first step by the associated cost minimization problem,

$$\min_{\{c_{jt}\}} \int_{j=0}^{1} p_{jt}c_{jt}dj \text{ s.t. } c_t = \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}$$

The first order conditions for buying good $j$ are

$$p_{jt} = \lambda \left[\int_{j=0}^{1} c_{jt}^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{1}{\sigma-1}} c_{jt}^{-\frac{1}{\sigma}}$$

$$p_{jt} = \lambda \left(\frac{c_{jt}}{c_t}\right)^{-\frac{1}{\sigma}} \tag{30}$$

where $\lambda$ is the Lagrange multiplier. Raising both sides to the $1-\sigma$ power and integrating to evaluate the multiplier, we have

$$\int p_{jt}^{1-\sigma}dj = \lambda^{1-\sigma}\left(\frac{1}{c_t}\right)^{\frac{\sigma-1}{\sigma}} \int c_{jt}^{\frac{\sigma-1}{\sigma}} dj$$

$$\left[\int p_{jt}^{1-\sigma}dj\right]^{\frac{1}{1-\sigma}} = \lambda^{-\sigma}\left(\frac{1}{c_t}\right)\left[\int c_{jt}^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{\sigma-1}}$$

$$\left\{\left[\int p_{jt}^{1-\sigma}dj\right]^{\frac{1}{1-\sigma}}\right\}^{-\frac{1}{\sigma}} = \lambda.$$

Defining the price index

$$p_t \equiv \left[\int p_{jt}^{1-\sigma}dj\right]^{\frac{1}{1-\sigma}},$$

and substituting $\lambda$ in to (30), we obtain the conditional (on $c_t$) demand curve for each good,

$$\frac{c_{jt}}{c_t} = \left(\frac{p_{jt}}{p}\right)^{-\sigma}. \tag{31}$$

Total expenditure is

$$\int p_{jt}c_{jt}dj = \frac{c_t}{p^{-\sigma}} \int p_{jt}^{1-\sigma}dj = \frac{c_t}{p^{-\sigma}} p_t^{1-\sigma} = p_t c_t.$$

This lovely result allows us to express the consumer's problem in terms of aggregates. Now, the consumer's problem simplifies to

$$\max_{\{c_t,n_t\}} E\sum_{t=0}^{\infty}\beta^t\left[u(c_t)-n_t\right]$$

with budget constraint

$$B_{t-1}+\pi_t = p_t c_t + S_t + Q_t B_t$$

### 8.2.2 Production

Each household also owns a firm, which produces only one variety of good using the household's labor, with production function

$$y_{it} = An_t$$

and facing the demand curve given by (31) from all the other households. The household earns $\pi_t = p_{it}y_{it}$. The household's problem is then

$$\max_{\{c_t, n_t, p_{it}\}} E\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - n_t\right] \text{ s.t.}$$

$$\begin{aligned} B_{t-1} + p_{it}y_{it} &= p_t c_t + S_t + Q_t B_t \\ y_{it} &= An_t \\ \frac{y_{it}}{y_t} &= \left(\frac{p_{it}}{p_t}\right)^{-\sigma} \end{aligned}$$

I use $y_t$ in the last equation to emphasize that each household takes the aggregate consumption = output and all the other household's pricing decisions as fixed when making its own output and consumption decisions.

Given the constraints, we can let the household choose price, quantity or labor supply. This being a "sticky price" model, I express the decision in terms of price

$$\max_{\{c_t, n_t, p_{it}\}} E\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A}\left(\frac{p_{it}}{p_t}\right)^{-\sigma}\right] \text{ s.t.}$$

$$B_{t-1} + p_{it}y_t\left(\frac{p_{it}}{p_t}\right)^{-\sigma} = p_t c_t + S_t + Q_t B_t \tag{32}$$

### 8.2.3 Flexible prices

In the flexible-price case, the household can set its price at time $t$. The first order condition for $p_{it}$ is then

$$\frac{y_t}{A}\sigma\left(\frac{p_{it}}{p}\right)^{-\sigma}\frac{1}{p_{it}} = -\lambda_t(1-\sigma)y_t\left(\frac{p_{it}}{p}\right)^{-\sigma} \tag{33}$$

where $\lambda_t$ is the Lagrange multiplier on the nominal period $t$ budget constraint (32), the value of a dollar at time t. Simplifying,

$$p_{it} = \frac{1}{A\lambda_t}\frac{\sigma}{\sigma - 1}$$

This optimal price is the same for all households, so all prices are identical, and

$$p_t = \frac{1}{A\lambda_t}\frac{\sigma}{\sigma - 1} \tag{34}$$

With all prices equal, we have $y_{it} = y_t$ and $n_t = y_t/A$.

Substituting this result in the remaining household problem, we obtain

$$\max_{\{c_t, B_t\}} E \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - \frac{y_t}{A} \right]$$

$$B_{t-1} + p_t y_t = p_t c_t + S_t + Q_t B_t.$$

Now we can solve the household problem and equilibrium condition. The first order condition with respect to $c_t$ gives

$$u'(c_t) = p_t \lambda_t = \frac{1}{A} \frac{\sigma}{\sigma - 1}. \tag{35}$$

The latter equality comes from the pricing decision (34). This is a frictionless economy, so as in our endowment economy consumption is constant with no real shocks, no matter what happens to nominal quantities.

The first order condition with respect to $B_t$ gives

$$Q_t \lambda_t = E_t \lambda_{t+1}$$

so the bond price satisfies

$$\begin{aligned} Q_t &= E_t \left[ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right] \\ &= \beta E_t \left[ \frac{p_t}{p_{t+1}} \right] \end{aligned}$$

($p_t$ is known at time $t$, but this is prettier.)

Substituting, the flow budget constraint becomes

$$B_{t-1} + p_t y_t = p_t c_t + S_t + \beta E_t \left[ \frac{p_t}{p_{t+1}} \right] B_t.$$

### 8.2.4 Taxes and Present values.

The government charges net lump-sum *real* taxes in the amount $s_t$ so $S_t = p_t s_t$. This is not an unnatural assumption. For example, if the government charged a rate $\tau$ on nominal income $S_t = \tau p_t y_t$, then the real tax revenue would be fixed $s_t = \tau y_t$. I specify lump sum taxes to avoid dealing with distortions.

Dividing by $p_t$

$$\frac{B_{t-1}}{p_t} + y_t = c_t + s_t + \beta E_t \left[ \frac{B_t}{p_{t+1}} \right]$$

and iterating forward,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j \left( c_{t+j} - y_{t+j} + s_{t+j} \right) + \beta E_t \left[ \frac{B_{t+k}}{p_{t+k+1}} \right]$$

I impose that the limit of the term on the right hand side is zero. In the positive direction, this is a condition for *consumer* optimality. If not, the consumer could increase consumption and hence utility. In the negative direction, this is a standard no-Ponzi condition preventing the consumer from borrowing larger and larger amounts. $B > 0$ and $p > 0$ – the government does not lend, and prices must be positive – serve the same purpose.

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j \left(c_{t+j} - y_{t+j} + s_{t+j}\right)$$

Finally, we impose the equilibrium condition,

$$c_t = y_t.$$

This condition determines the overall price level,

$$\frac{B_{t-1}}{p_t} = E_t \sum_{j=1}^{k} \beta^j s_{t+j}$$

just as in the endowment-economy model.

### 8.2.5 Prices set one period in advance

To create a sticky-price version of this model, I require that each household set its price $p_{it}$ one period in advance. The household is committed to supply whatever demand there is at the posted price. The demand curve faced by each household producer is still

$$\frac{y_{it}}{y_t} = \left(\frac{p_{it}}{p_t}\right)^{-\sigma}.$$

so, with all prices still equal, individual demand will equal aggregate demand. But now aggregate demand $y_t$ can vary over time.

The first order condition of the problem (32), which I repeat here,

$$\max_{\{c_t, n_t, p_{it}\}} E \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \frac{y_t}{A}\left(\frac{p_{it}}{p_t}\right)^{-\sigma}\right] \text{ s.t.}$$

$$B_{t-1} + p_{it} y_t \left(\frac{p_{it}}{p_t}\right)^{-\sigma} = p_t c_t + S_t + Q_t B_t$$

with respect to $p_{it}$ in this case becomes, in place of (33),

$$E_{t-1} \frac{y_t}{A} \sigma \left(\frac{p_{it}}{p}\right)^{-\sigma} \frac{1}{p_{it}} = -E_{t-1}\left[\lambda_t (1-\sigma) y_t \left(\frac{p_{it}}{p}\right)^{-\sigma}\right]$$

which we simplify to

$$p_{it} = \frac{1}{A E_{t-1}\left(\lambda_t\right)} \frac{\sigma}{\sigma - 1}.$$

again all prices are identical, and

$$p_t = \frac{1}{A E_{t-1}\left(\lambda_t\right)} \frac{\sigma}{\sigma - 1} \tag{36}$$

Output now is $y_{it} = y_t$ and thus labor supply $n_t = y_t/A$. The household problem simplifies then to

$$\max_{\{c_t, B_t\}} E \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - \frac{y_t}{A} \right]$$

$$B_{t-1} + p_t y_t = p_t c_t + S_t + Q_t B_t.$$

The first order condition with respect to $c_t$ still gives

$$u'(c_t) = p_t \lambda_t.$$

However, the new pricing rule (36) now means (35) becomes

$$E_{t-1} \left[ u'(c_t) \right] = p_t E_{t-1} \left( \lambda_t \right) = \frac{1}{A} \frac{\sigma}{\sigma - 1}. \tag{37}$$

This is really the crucial difference. *Expected* marginal utility is constant. But nominal shocks will have real effects. A too low price will induce too much output, and too much consumption.

The first order condition with respect to $B_t$ gives

$$Q_t \lambda_t = E_t \lambda_{t+1}$$

as before, and thus

$$Q_t = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right].$$

However, from (37), we can no longer conclude that $c_t$ and the real interest rate are constant.

The flow budget constraint becomes

$$B_{t-1} + p_t y_t = p_t c_t + p_t s_t + E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{p_t}{p_{t+1}} \right] B_t$$

$$\frac{B_{t-1}}{p_t} + y_t = c_t + s_t + \beta \frac{E_t u'(c_{t+1})}{u'(c_t)} \frac{B_t}{p_{t+1}}$$

where I have used the fact that $p_{t+1}$ is known at time $t$. Using (37),

$$u'(c_t) \frac{B_{t-1}}{p_t} = u'(c_t) \left[ c_t - y_t + s_t \right] + \beta E_t \left[ u'(c_{t+1}) \right] \frac{B_t}{p_{t+1}}$$

$$u'(c_t) \frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} \beta^j u'(c_{t+j}) \left[ c_{t+j} - y_{t+j} + s_{t+j} \right]$$

equilibrium $c_t = y_t$ requires that $p_t$ obey

$$u'(c_t) \frac{B_{t-1}}{p_t} = E_t \sum_{j=0}^{\infty} \beta^j \left[ u'(c_{t+j}) s_{t+j} \right]$$

Now, in this simple model with one-period price stickiness, we have from (37) that $E_t u'(c_{t+j}) = \frac{1}{A} \frac{\sigma}{\sigma - 1}$ for $j \geq 1$. If the covariance between marginal utility and surpluses is zero, then

$$u'(c_t) \frac{B_{t-1}}{p_t} = u'(c_t) s_t + \frac{1}{A} \frac{\sigma}{\sigma - 1} E_t \sum_{j=1}^{\infty} \beta^j s_{t+j}$$

$$\frac{u'(c_t)}{u'(\bar{c})}\frac{B_{t-1}}{p_t} = \frac{u'(c_t)}{u'(\bar{c})}s_t + E_t\sum_{j=1}^{\infty}\beta^j s_{t+j}$$

in this case, marginal utility $u'(c_t)$ must do all the adjusting when there is a surplus shock, as the price level cannot move.

### 8.2.6 Algebra for the k-period sticky price example.

As a reminder, surpluses are $s_t = 0,\ t < k$. Before time 1, surpluses were expected to be $s_t = \bar{s}$, $t \geq k$. At time 1, expected surpluses change to $s_t = \bar{s}'$, $t \geq k$. Denote $\bar{S} = \sum_{j=0}^{\infty}\beta^j\bar{s} = \bar{s}/(1-\beta)$ and $\bar{S}' = \bar{s}'/(1-\beta)$. The steady state implies from (16) that nominal debt $\bar{B}_{t-1} = \beta^{k-t}\bar{P}\bar{S}$ for $t \leq k$.

To derive the path shown in Table 1 and Figure 1, express equations (16) and (17) at each date, substituting in $\bar{c}, \bar{P}, \bar{B}$, where appropriate, and remember the rules, $B_0 = \bar{B}_0$, $P_1, P_2, ..P_k = \bar{P}$, and $u'(c_k) = u'(c_{k+1}) = ... = u'(\bar{c})$ do not change.

For $t = 1$,

$$u'(c_1)\frac{B_0}{P_1} = E_1\sum_{j=0}^{\infty}\beta^j u'(c_{1+j})s_{1+j}$$
$$\frac{1}{1+i_1} = \beta\frac{u'(c_2)}{u'(c_1)}\frac{P_1}{P_2}.$$

With $B_{t-1} = \beta^{k-t}\bar{P}\bar{S}$ for $t \leq k$ and substituting in $\bar{c}, \bar{P}, \bar{B}$, we have

$$u'(c_1)\frac{\beta^{k-1}\bar{P}\bar{S}}{\bar{P}} = \beta^{k-1}u'(\bar{c})\bar{S}' \tag{38}$$
$$\beta\frac{1}{\Delta_1} = \beta\frac{u'(c_2)}{u'(c_1)}\frac{\bar{P}}{\bar{P}}. \tag{39}$$

Equation (38) tells us right away that consumption at time 1 is determined only by the fiscal shock,

$$u'(c_1) = u'(\bar{c})\frac{\bar{S}'}{\bar{S}}.$$

Equation (39) then gives us time 2 consumption,

$$u'(c_2) = \frac{1}{\Delta_1}u'(c_1) = \frac{1}{\Delta_1}u'(\bar{c})\frac{\bar{S}'}{\bar{S}}.$$

For $t = 2$,

$$u'(c_2)\frac{B_1}{\bar{P}} = \beta^{k-2}u'(\bar{c})\bar{S}' \tag{40}$$
$$\beta\frac{1}{\Delta_2} = \beta\frac{u'(c_3)}{u'(c_2)}\frac{\bar{P}}{\bar{P}}. \tag{41}$$

Equation (41) tells us time 3 consumption

$$u'(c_3) = \frac{1}{\Delta_1\Delta_2}u'(c_1) = \frac{1}{\Delta_1\Delta_2}u'(\bar{c})\frac{\bar{S}'}{\bar{S}}$$

and (40) tells us the less interesting time 1 debt required by the interest rate peg (recall the initial values $\bar{B}_{t-1} = \beta^{k-t}\bar{P}\bar{S}$)

$$\frac{1}{\Delta_1}u'(\bar{c})\frac{\bar{S}'}{\bar{S}}B_1 = \beta^{k-2}u'(\bar{c})\bar{P}\bar{S}\frac{\bar{S}'}{\bar{S}}$$
$$B_1 = \Delta_1\bar{B}_1$$

For $t = 3$,

$$u'(c_3)\frac{B_2}{\bar{P}} = \beta^{k-3}u'(\bar{c})\bar{S}'$$
$$\beta\frac{1}{\Delta_3} = \beta\frac{u'(c_4)}{u'(c_3)}\frac{\bar{P}}{\bar{P}}.$$

hence, similarly,

$$u'(c_4) = \frac{1}{\Delta_1\Delta_2\Delta_3}u'(\bar{c})\frac{\bar{S}'}{\bar{S}}$$

and

$$\frac{1}{\Delta_1\Delta_2}u'(\bar{c})\frac{\bar{S}'}{\bar{S}}B_2 = \beta^{k-3}u'(\bar{c})\bar{P}\bar{S}\frac{\bar{S}'}{\bar{S}}$$
$$B_2 = \Delta_1\Delta_2\bar{B}_2$$

For $t = k$, we know that $u'(c_{k+1}) = u'(\bar{c})$, so instead

$$u'(c_k)\frac{B_{k-1}}{\bar{P}} = \beta u'(\bar{c})\bar{S}' \tag{42}$$
$$\beta\frac{1}{\Delta_k} = \beta\frac{u'(\bar{c})}{u'(c_k)}\frac{\bar{P}}{P_{k+1}}. \tag{43}$$

Now, $u'(c_k) = (\Delta_1..\Delta_{k-1})^{-1}u'(\bar{c})\bar{S}'/\bar{S}$ is already determined, and $u'(c_{k+1}) = u'(\bar{c})$ as well, but not the price $P_{k+1}$ is free. So, rather than determine $u'(c_{k+1})$, equation (43) implies

$$P_{k+1} = (\Delta_1\Delta_2..\Delta_k)\,\bar{P}\frac{\bar{S}}{\bar{S}'}.$$

Equation (42) continues to flesh out the debt required to support the interest rate target,

$$B_{k-1} = \Delta_1\Delta_2...\Delta_{k-1}\bar{B}_{k-1}$$

For $t = k + 1$,

$$\frac{B_k}{P_{k+1}} = \bar{S}' \tag{44}$$
$$\beta\frac{1}{\Delta_{k+1}} = \beta\frac{P_{k+1}}{P_{k+2}}, \tag{45}$$

and similarly for $t = k + 2, t = k + 3, ...$

Table 1 gives the evolution of each variable in this scenario. I present the algebra below.

| t: | Before | Shock | After shock | | |
|---|---|---|---|---|---|
| t: | 0 | 1 | 2 | 3 | ... |
| $1+i_t$ : | $1+\delta$ | $(1+\delta)\,\Delta_1$ | $(1+\delta)\,\Delta_2$ | $(1+\delta)\,\Delta_3$ | ... |
| $u'(c_t)/u'(\bar{c})$ : | 1 | $\bar{S}'/\bar{S}$ | $1/\Delta_1\bar{S}'/\bar{S}$ | $1/\left(\Delta_1\Delta_2\right)\bar{S}'/\bar{S}$ | ... |
| $1+r_t$ : | $1+\delta$ | $(1+\delta)\,\Delta_1$ | $(1+\delta)\,\Delta_2$ | $(1+\delta)\,\Delta_3$ | ... |
| $P_t$ : | $\bar{P}$ | $\bar{P}$ | $\bar{P}$ | $\bar{P}$ | ... |
| $B_t$ : | $\bar{B}_t$ | $\Delta_1\bar{B}_t$ | $\Delta_1\Delta_2\bar{B}_t$ | $\Delta_1\Delta_2\Delta_3\bar{B}_t$ | ... |

Panel A: $t=0$ and initial response

| t: | | After shock | Last stuck P | P unstuck | etc. |
|---|---|---|---|---|---|
| t: | ... | k-1 | k | k+1 | ... |
| $1+i_t$ : | ... | $(1+\delta)\,\Delta_{k-1}$ | $(1+\delta)\,\Delta_k$ | $(1+\delta)\,\Delta_{k+1}$ | ... |
| $u'(c_t)/u'(\bar{c})$ : | ... | $1/(\Delta_1\Delta_2...\Delta_{k-2})\bar{S}'/\bar{S}$ | $1/(\Delta_1\Delta_2...\Delta_{k-1})\bar{S}'/\bar{S}$ | 1 | ... |
| $1+r_t$ : | ... | $(1+\delta)\,\Delta_{k-1}$ | $(1+\delta)\,/(\Delta_1\Delta_2...\Delta_{k-1})\left(\bar{S}'/\bar{S}\right)$ | $(1+\delta)$ | ... |
| $P_t$ : | ... | $\bar{P}$ | $\bar{P}$ | $\Delta_1\Delta_2...\Delta_k\bar{P}(\bar{S}/\bar{S}')$ | ... |
| $B_t$ : | ... | $\Delta_1\Delta_2...\Delta_{k-1}\bar{B}_t$ | $\Delta_1\Delta_2...\Delta_k\bar{B}_t$ | $\Delta_1\Delta_2...\Delta_k\Delta_{k+1}\bar{B}_t$ | ... |

Panel B: Response as sticky prices end

Table 1. Responses to a surprise increase in nominal interest rates from $1+\delta$ to $\{(1+\delta)\,\Delta_t\}$, together with a fiscal shock from $\bar{s}_t$ to $\bar{s}'_t$.

### 8.3 Three-equation new-Keynesian model

This section sets out the algebra for the three-equation new-Keynesian model. Cochrane (2011a, online appendix) has a more extensive but more cumbersome treatment.

The model is

$$
\begin{aligned}
y_t &= E_t y_{t+1} - \sigma\left(i_t - E_t\pi_{t+1}\right) + x_{dt} \\
\pi_t &= \beta E_t\pi_{t+1} + \gamma y_t + x_{\pi t} \\
i_t &= \phi_\pi \pi_t + x_{it}
\end{aligned}
$$

In vector form,

$$
\begin{aligned}
\left[\begin{array}{c} y_{t+1} \\ \pi_{t+1} \\ x_{dt+1} \\ x_{\pi t+1} \\ x_{it+1} \end{array}\right] &= \left[\begin{array}{ccccc} \frac{1}{\beta}\left(\beta+\sigma\gamma\right) & -\frac{\sigma}{\beta}\left(1-\beta\phi_\pi\right) & -1 & \frac{\sigma}{\beta} & \sigma \\ -\frac{\gamma}{\beta} & \frac{1}{\beta} & 0 & -\frac{1}{\beta} & 0 \\ 0 & 0 & \rho_d & 0 & 0 \\ 0 & 0 & 0 & \rho_\pi & 0 \\ 0 & 0 & 0 & 0 & \rho_i \end{array}\right] \left[\begin{array}{c} y_t \\ \pi_t \\ x_{dt} \\ x_{\pi t} \\ x_{it} \end{array}\right] + \left[\begin{array}{c} \delta_{yt+1} \\ \delta_{\pi t+1} \\ \varepsilon_{dt+1} \\ \varepsilon_{\pi t+1} \\ \varepsilon_{it+1} \end{array}\right] \\
X_{t+1} &= AX_t + \varepsilon_{t+1}
\end{aligned}
$$

The central issue in this class of models is that the model only determines $E_t y_{t+1}$ and $E_t\pi_{t+1}$. Older Keynesian models had lagged values on the right hand side, and thus no indeterminacy issues.

The solution can be found by eigenvalue decomposing the transition matrix,

$$X_{t+1} = Q\Lambda Q^{-1}X_t + \varepsilon_{t+1}$$

$$Q^{-1}X_{t+1} = \Lambda Q^{-1}X_t + Q^{-1}\varepsilon_{t+1}$$

$$Z_{t+1} = \Lambda Z_t + V_{t+1}$$
$$z_{jt+1} = \lambda_j z_{jt} + v_{jt+1}$$

New Keynesian models overcome indeterminacy with the rule that we pick nonexplosive solutions $\lim_{k\to\infty} E_t Z_{t+k} \to 0$. This rule means that for $\lambda_j > 1$, we must have $z_{it} = 0$. Then, the dynamics of the $X$ variables can be written in terms of the first $K < N$ nonzero $z$ as

$$\begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{Kt} \\ \vdots \\ x_{Nt} \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & \cdots & q_K \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \\ \vdots \\ z_{Kt} \end{bmatrix}$$

$$\begin{bmatrix} z_{1t+1} \\ z_{2t+1} \\ \vdots \\ z_{Kt+1} \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_K \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \\ \vdots \\ z_{Kt} \end{bmatrix} + \begin{bmatrix} v_{1t+1} \\ v_{2t+1} \\ \vdots \\ v_{Kt+1} \end{bmatrix}$$

where $q_j$ denote the columns of $Q$.

$Z$ is a linear combination of $X$, so $z_{jt} = 0$ is a relationship linking endogenous variables $\pi_t, y_t$ to shocks $x_t$. Equivalently, $z_{jt+1} = 0$ means $v_{jt+1} = 0$. $v_{jt+1}$ is a linear combination of $\delta$ and $\varepsilon$ shocks, so this requirement picks the shocks $\delta$ that index alternative equilibria.

The eigenvalues of the transition matrix are

$$\lambda = \lambda_-, \lambda_+, \rho_d, \rho_\pi, \rho_i$$
$$\lambda_\pm = \frac{1}{2\beta}\left(1+\beta+\sigma\gamma \pm \sqrt{(1+\beta+\sigma\gamma)^2 - 4\beta\left(1+\sigma\gamma\phi_\pi\right)}\right)$$

The eigenvectors of the first two (model) eigenvalues are

$$\begin{bmatrix} 1-\beta-\sigma\gamma+\sqrt{(1+\beta+\sigma\gamma)^2 - 4\beta\left(1+\sigma\gamma\phi_\pi\right)} \\ 2\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \lambda_-,$$

$$\begin{bmatrix} 1-\beta-\sigma\gamma-\sqrt{(1+\beta+\sigma\gamma)^2 - 4\beta\left(1+\sigma\gamma\phi_\pi\right)} \\ 2\gamma \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \lambda_+$$

The eigenvectors of the shock eigenvalues are

$$\begin{bmatrix} 1-\rho_d\beta \\ \gamma \\ (1-\rho_d)(1-\beta\rho_d) + \sigma\gamma\left(\phi_\pi - \rho_d\right) \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \rho_d$$

$$\left[\begin{array}{c} \sigma\left(\rho_\pi-\phi_\pi\right) \\ 1-\rho_\pi \\ 0 \\ \left(1-\rho_\pi\right)\left(1-\beta\rho_\pi\right)+\sigma\gamma\left(\phi_\pi-\rho_\pi\right) \\ 0 \end{array}\right] \leftrightarrow \rho_\pi$$

$$\left[\begin{array}{c} -\sigma\left(1-\rho_i\beta\right) \\ -\sigma\gamma \\ 0 \\ 0 \\ \left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right) \end{array}\right] \leftrightarrow \rho_i$$

In the standard new-Keynesian equilibrium selection, we assume $\phi > 1$.Then both $\lambda_+ > 1$ and $\lambda_- > 1$, two $z$ are equal to zero so we determine both $y$ and $\pi$. The model dynamics can then be written

$$\left[\begin{array}{c} y_t \\ \pi_t \end{array}\right] = \left[\begin{array}{ccc} 1-\rho_d\beta & \sigma\left(\rho_\pi-\phi_{\pi,0}\right) & -\sigma\left(1-\rho_i\beta\right) \\ \gamma & 1-\rho_\pi & -\sigma\gamma \end{array}\right] \left[\begin{array}{c} z_{dt} \\ z_{\pi t} \\ z_{it} \end{array}\right]$$

$$\left[\begin{array}{c} z_{dt} \\ z_{\pi t} \\ z_{it} \end{array}\right] = \left[\begin{array}{ccc} \rho_d & 0 & 0 \\ 0 & \rho_\pi & 0 \\ 0 & 0 & \rho_i \end{array}\right] \left[\begin{array}{c} z_{dt-1} \\ z_{\pi t-1} \\ z_{it-1} \end{array}\right] + \left[\begin{array}{c} v_{dt} \\ v_{\pi t} \\ v_{it} \end{array}\right]$$

where the $z$ and the $x$ are related by

$$\begin{aligned} x_{dt} &= \left[\left(1-\rho_d\right)\left(1-\rho_d\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_d\right)\right] z_{dt} \\ x_{\pi t} &= \left[\left(1-\rho_\pi\right)\left(1-\rho_\pi\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_\pi\right)\right] z_{\pi t} \\ x_{it} &= \left[\left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right)\right] z_{it}. \end{aligned}$$

Similarly the shocks $v$ are related to fundamental shocks $x$ by

$$\begin{aligned} \varepsilon_{dt} &= \left[\left(1-\rho_d\right)\left(1-\rho_d\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_d\right)\right] v_{dt} \\ \varepsilon_{\pi t} &= \left[\left(1-\rho_\pi\right)\left(1-\rho_\pi\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_\pi\right)\right] v_{\pi t} \\ \varepsilon_{it} &= \left[\left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right)\right] v_{it}. \end{aligned}$$

It's interesting to carry along the $i$ response. From

$$i_t = \phi_\pi\pi_t + x_{it},$$

we can simply append the $i$ to the response variables as

$$\left[\begin{array}{c} y_t \\ \pi_t \\ i_t \end{array}\right] = \left[\begin{array}{ccc} 1-\rho_d\beta & \sigma\left(\rho_\pi-\phi_{\pi,0}\right) & -\sigma\left(1-\rho_i\beta\right) \\ \gamma & 1-\rho_\pi & -\sigma\gamma \\ \gamma\phi_\pi & \left(1-\rho_\pi\right)\phi_\pi & -\sigma\gamma\rho_i+\left(1-\rho_i\right)\left(1-\rho_i\beta\right) \end{array}\right] \left[\begin{array}{c} z_{dt} \\ z_{\pi t} \\ z_{it} \end{array}\right]$$

For the response to a monetary policy shock, we only need the last column.

In the end, then, I plot the response to a monetary policy shock by simulating forward

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} -\sigma\left(1-\rho_i\beta\right) \\ -\sigma\gamma \\ -\sigma\gamma\rho_i + \left(1-\rho_i\right)\left(1-\rho_i\beta\right) \end{bmatrix} \begin{bmatrix} z_{it} \end{bmatrix}$$

$$z_{it} = \rho_i z_{it-1} + v_{it}$$

$$v_{it} = \frac{\varepsilon_{it}}{\left(1-\rho_i\right)\left(1-\rho_i\beta\right) + \sigma\gamma\left(\phi_\pi - \rho_i\right)}$$

$$z_{it} = \frac{x_{it}}{\left(1-\rho_i\right)\left(1-\rho_i\beta\right) + \sigma\gamma\left(\phi_\pi - \rho_i\right)}$$

### 8.3.1 Fiscal solution

In the fiscal solution, we pick the inflation shock directly, from the change in present value of future surpluses. Ideally, the present value should contain interest rates and risk premiums as well as surpluses. Monetary policy may affect the present value of surpluses by changing discount rates, even if it cannot change surpluses. We should also have a serious analysis of monetary and fiscal policy coordination. For my illustrative calculation, I will simply choose to pair monetary policy with no change in present value of surpluses, as I have done in the other illustrative calculations, resulting in $\pi_{t+1} - E_t\pi_{t+1} = 0$. This choice generally does not mean no change in surpluses, but a change in surpluses that matches the change in discount rate effects on their present values.

Since we pick one innovation $\delta_{\pi_{t+1}} = 0$, we only need one eigenvalue greater than one. Hence, following the usual rules, we need a "passive" monetary policy $\phi < 1$. This choice implies $\lambda_+ > 1$ but $\lambda_- < 1$. Hence, the model dynamics keep an additional eigenvector,

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} 1-\rho_d\beta & \sigma\left(\rho_\pi - \phi_\pi\right) & -\sigma\left(1-\rho_i\beta\right) & k \\ \gamma & 1-\rho_\pi & -\sigma\gamma & 2\gamma \\ \phi_\pi\gamma & \phi_\pi\left(1-\rho_\pi\right) & \left(1-\rho_i\right)\left(1-\rho_i\beta\right) - \sigma\gamma\rho_i & 2\gamma\phi_\pi \end{bmatrix} \begin{bmatrix} z_{dt} \\ z_{\pi t} \\ z_{it} \\ z_{\lambda t} \end{bmatrix}$$

$$k = 1 - \beta - \sigma\gamma + \sqrt{\left(1+\beta+\sigma\gamma\right)^2 - 4\beta\left(1+\sigma\gamma\phi_\pi\right)}$$

and we add

$$z_{\lambda t+1} = \lambda_- z_{\lambda t} + \delta_{\lambda t+1}$$

Now we can compute responses to fiscal shocks, identified by the innovation in $\pi_{t+1}$, and to other shocks orthogonalized, i.e. holding fiscal shocks and thus the innovation in inflation constant.

To impose no shock to inflation, we must have

$$\begin{bmatrix} \gamma & 1-\rho_\pi & -\sigma\gamma & 2\gamma \end{bmatrix} \begin{bmatrix} v_{dt} \\ v_{\pi t} \\ v_{it} \\ v_{\lambda t} \end{bmatrix} = 0$$

In my calculations, when there is only a monetary policy shock $v_i$, this means

$$\begin{bmatrix} -\sigma\gamma & 2\gamma \end{bmatrix} \begin{bmatrix} v_{it} \\ v_{\lambda t} \end{bmatrix} = 0,$$

i.e. we pair the $v_i$ shock with a contemporaneous shock

$$v_{\lambda t} = \frac{\sigma}{2} v_{it}.$$

In sum, then, to find the response to a monetary policy shock, we simulate

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} -\sigma\left(1-\rho_i\beta\right) & 1-\beta-\sigma\gamma+\sqrt{\left(1+\beta+\sigma\gamma\right)^2-4\beta\left(1+\sigma\gamma\phi_\pi\right)} \\ -\sigma\gamma & 2\gamma \\ \left(1-\rho_i\right)\left(1-\rho_i\beta\right)-\sigma\gamma\rho_i & 2\gamma\phi_\pi \end{bmatrix} \begin{bmatrix} z_{it} \\ z_{\lambda t} \end{bmatrix}$$

$$\begin{aligned} z_{it+1} &= \rho_i z_{it} \\ z_{\lambda t+1} &= \lambda_- z_{\lambda t} \\ z_{i1} &= v_{i1} = 1/\left[\left(1-\rho_i\right)\left(1-\rho_i\beta\right)+\sigma\gamma\left(\phi_\pi-\rho_i\right)\right] \\ v_{\lambda 1} &= z_{\lambda 1} = \sigma/2\ v_{i1} \end{aligned}$$